A Spin on Wave Dark Matter

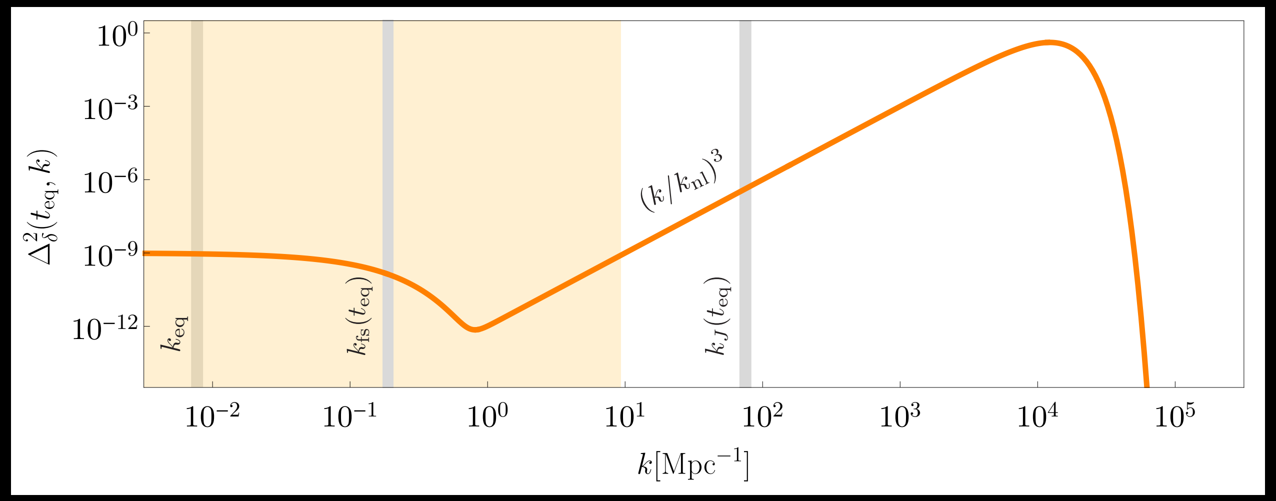
Mustafa A. Amin

RICE

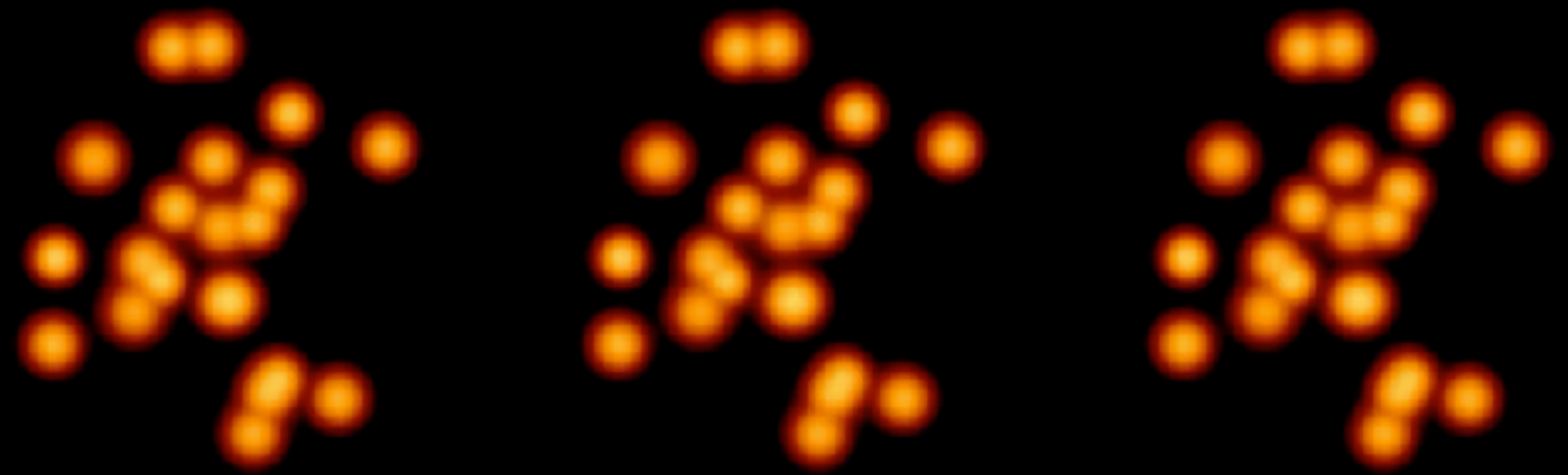
with Jain	2109.04892
Zhang, Jain	2111.08700
Jain, Karur, Mocz	2203.11935
Mirbabayi	2211.09775
Jain	2211.08433
Long, Schiappacasse	2301.11470
Jain, Thomas, Wanischarunarung	2304.01985

talk in 2 parts

1. A lower bound on dark matter mass



2. Spin of wave dark matter from astrophysics?



can we probe the intrinsic spin of wave dark matter?

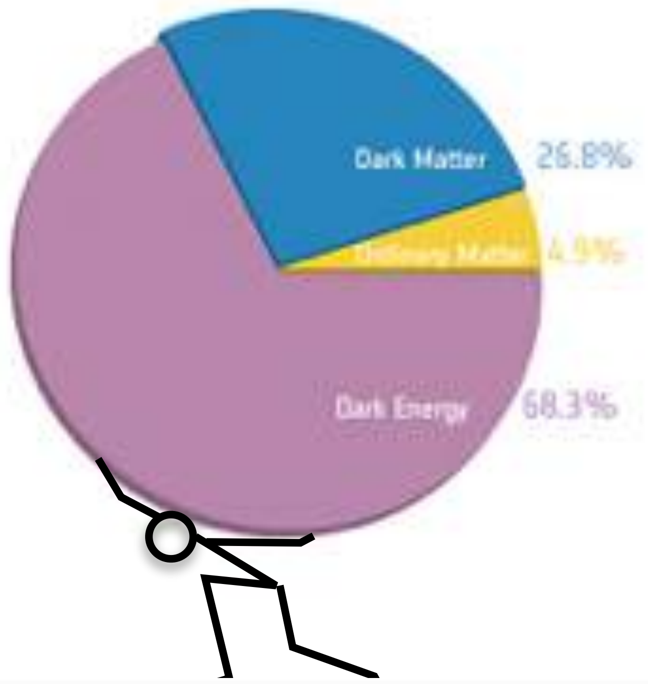
motivation & introduction

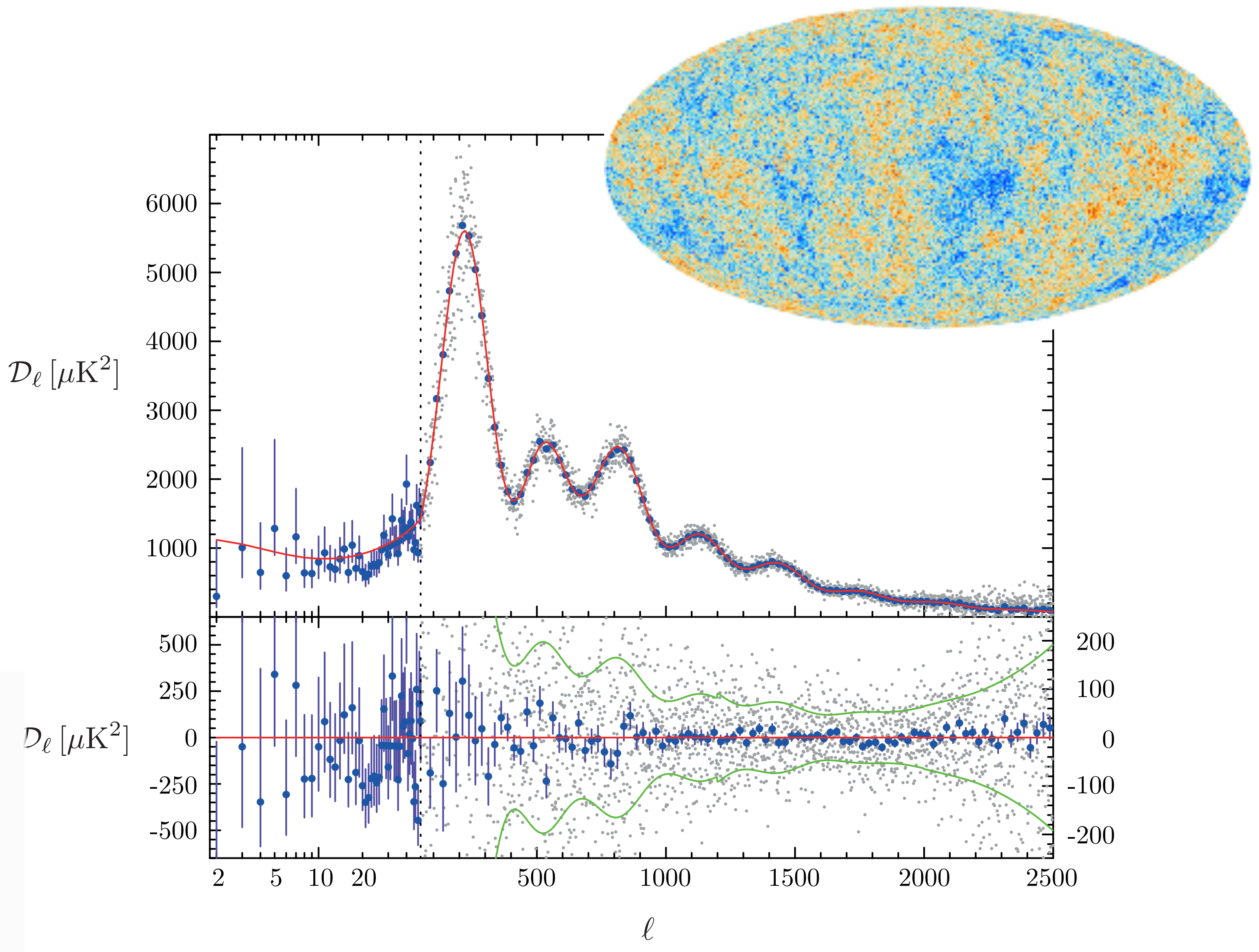
dark matter

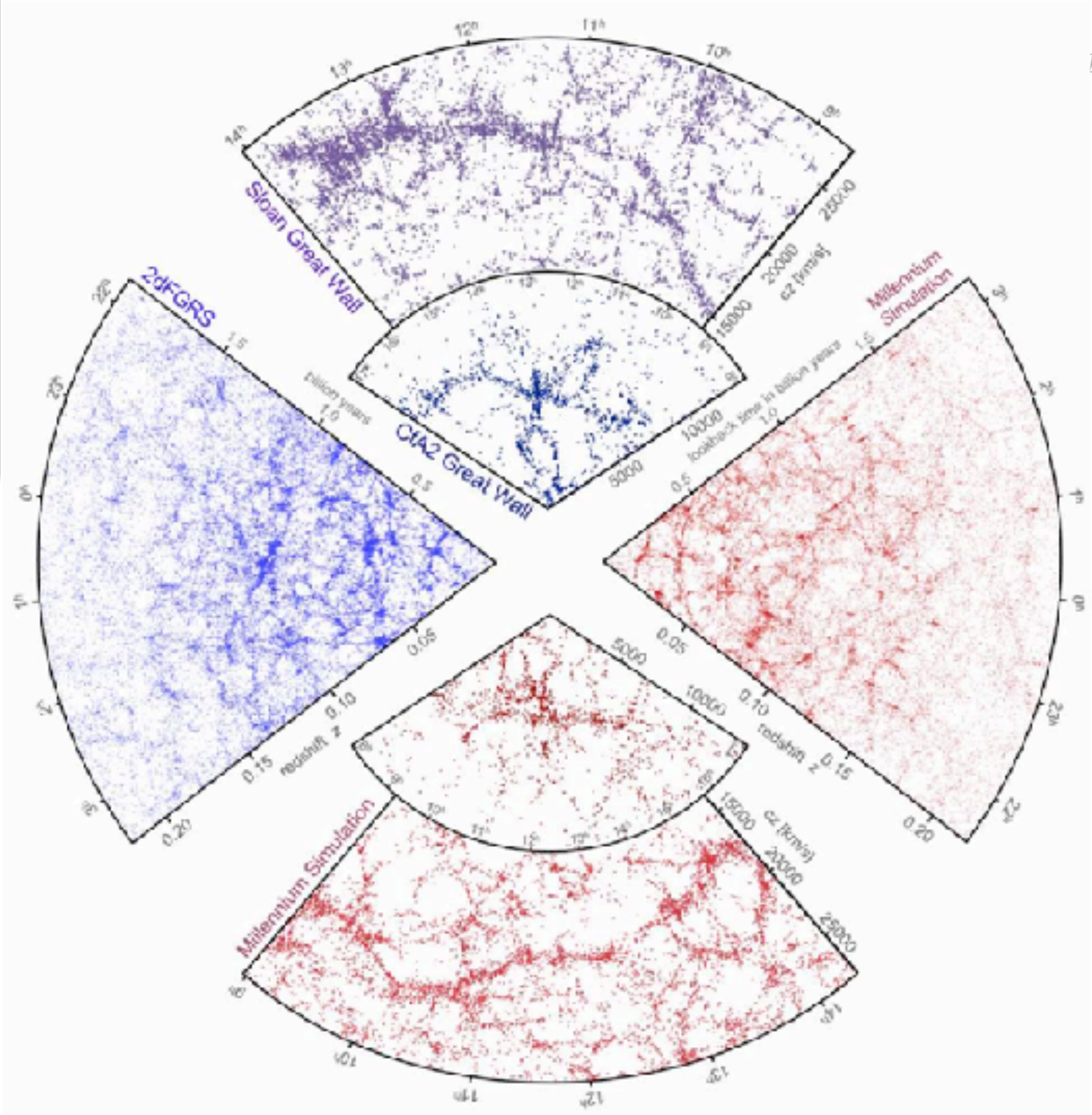
- dark matter exists

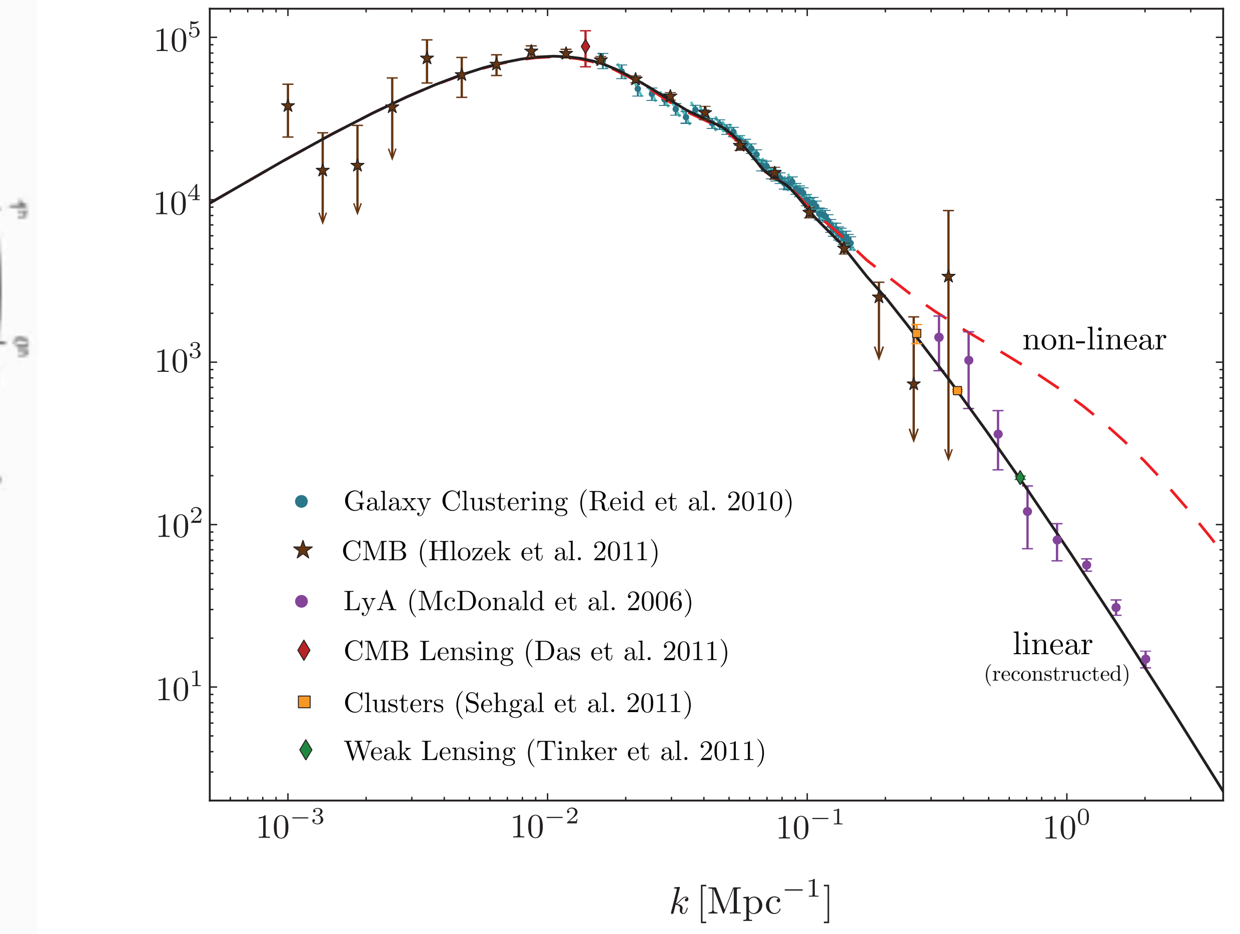
- gravitational interactions

- what is it: spin, mass ?

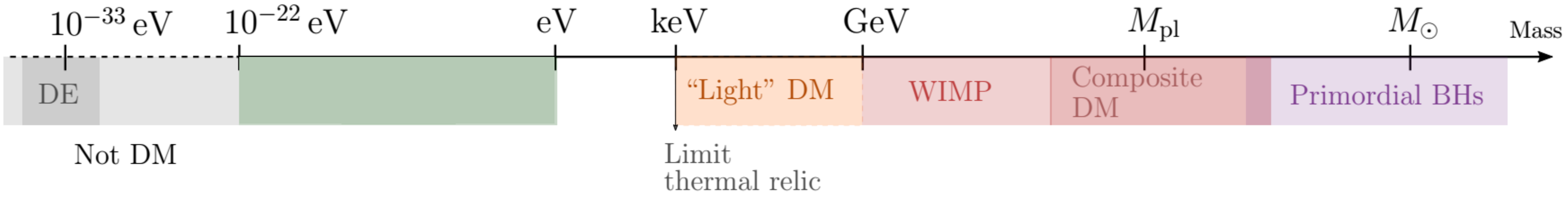






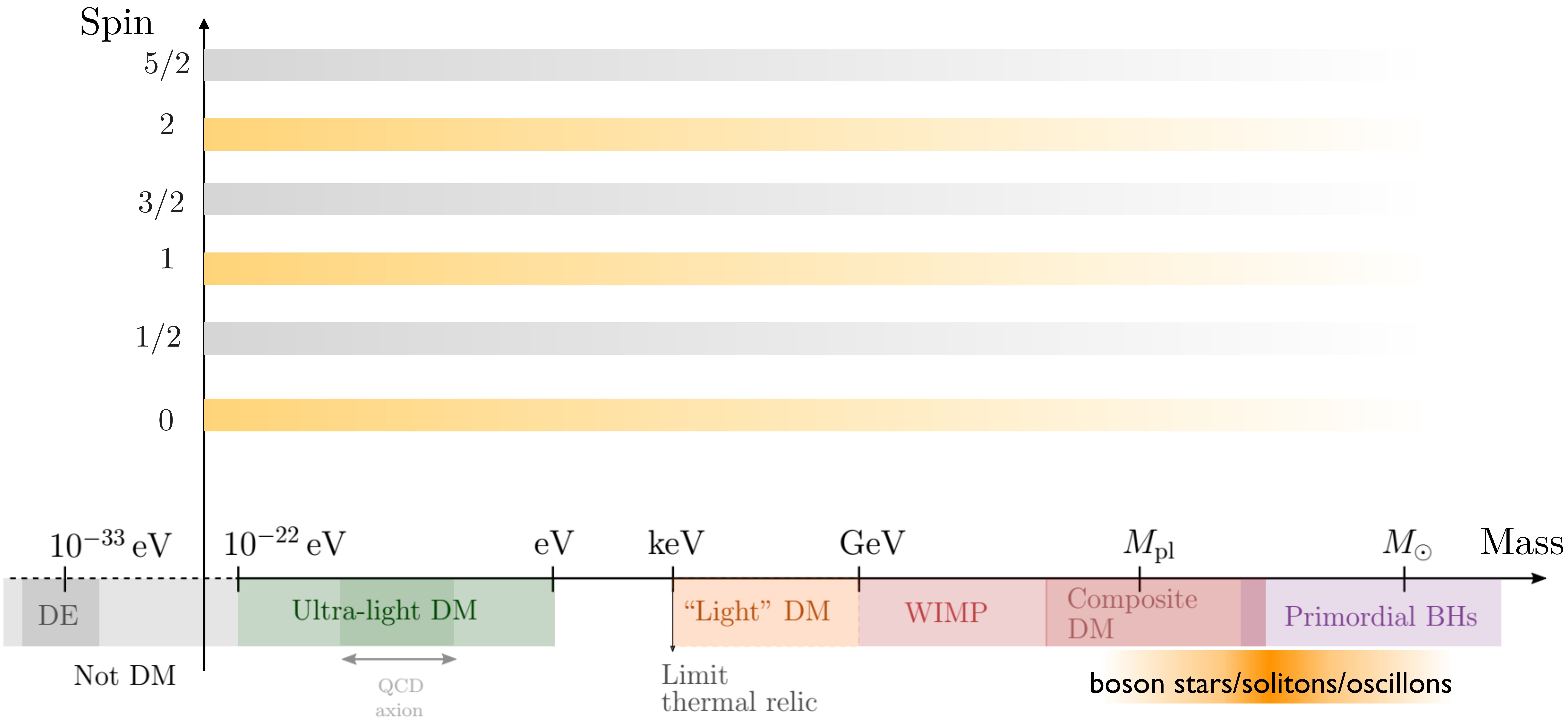


dark matter mass ?

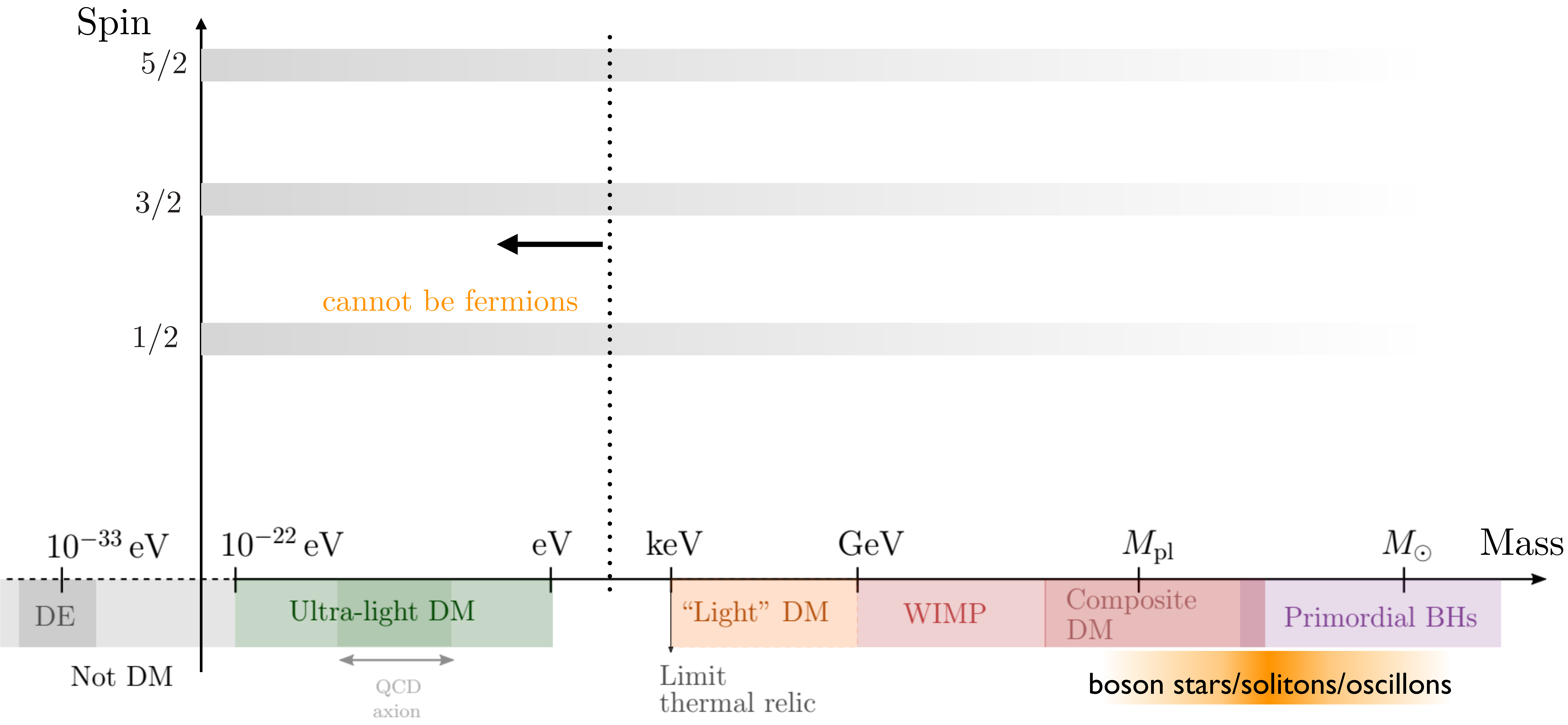


Modified version of image by E. Ferreira

dark matter spin ?



dark matter spin ?

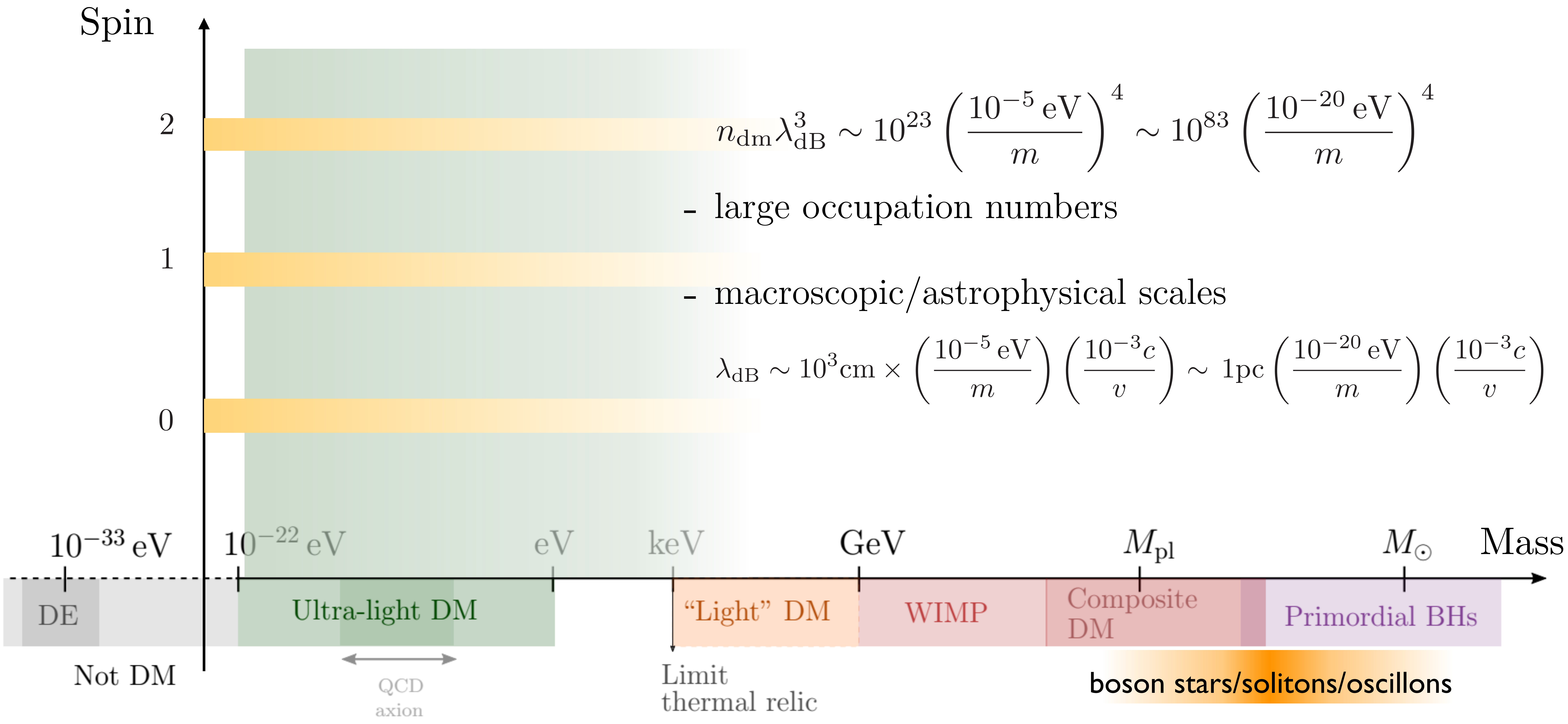


light, bosonic wave dark matter

$$n_{\rm dm}\lambda_{\rm dB}^3 \sim 10^{23}\left(\frac{10^{-5}\,{\rm eV}}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)^4$$

- large occupation numbers
- macroscopic/astrophysical scales

$$\lambda_{\rm dB} \sim 10^3{\rm cm} \times \left(\frac{10^{-5}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right) \sim 1{\rm pc}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$



light bosonic “wave” dark matter

- classical wave description sensible
- linear and nonlinear wave dynamics
- interference, solitons etc.

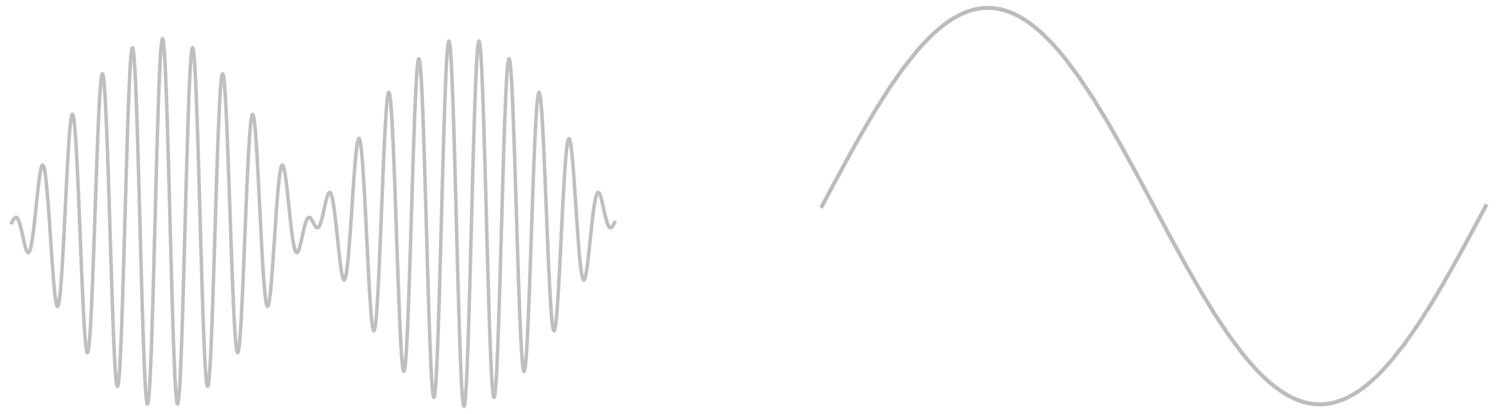
- condensed matter systems (BECs)
- similar ideas already used in early universe cosmology

models

non-relativistic limit = multicomponent Schrödinger-Poisson

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\,\mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\,g^{\mu\nu}W_\mu W_\nu + \frac{c^3}{16\pi G}R + \ldots\right]$$ + non-grav, interactions

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

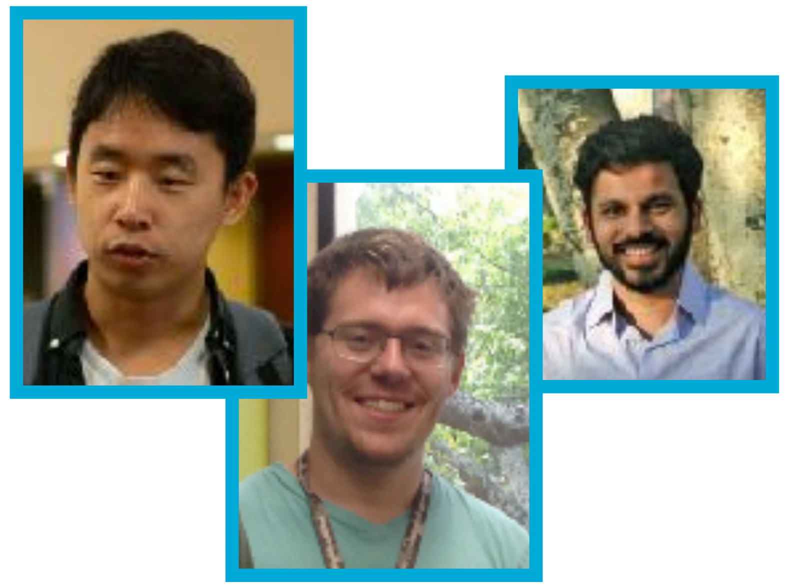


non-relativistic limit

split in "fast" and "slow" parts

$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\mathcal{S}_{nr} = \int \mathrm{d}t\mathrm{d}^3x\left[\frac{i\hbar}{2}\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}} + \text{c.c.} - \frac{\hbar^2}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} + \frac{1}{8\pi G}\Phi\nabla^2\Phi - m\,\Phi\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}\right]$$



Jain & MA (2021)
for spin - 2s+1

Recent work on non-relativistic case :

for scalar, see for example: Eby, Mukaida et. al (2018), Salehian, Zhang et. al (2021), for vector case, see Adshead & Lozanov (2021),
For vectors with non-minimal coupling, see Zhang and Ling (2023). For potential trouble with self-interactions, see Mou and Zhang (2022)

non-relativistic limit = multicomponent Schrödinger-Poisson

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

$$i\hbar \frac{\partial}{\partial t}\mathbf{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\mathbf{\Psi} + m\,\Phi\,\mathbf{\Psi}\,, \qquad \nabla^2\Phi = 4\pi G m\,\mathbf{\Psi}^\dagger\mathbf{\Psi}$$

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1$ scalar case

at this level this is just $2s+1$ equal mass scalar fields

but not when non-gravitational interactions are included!

For including non-grav. Interactions, see: Zhang, Jain and MA(2021), Jain (2022), Jain & MA(2022) and also non-min. coupling Zhang and Ling (2023)

conserved quantities

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$

$$N = \int \mathrm{d}^3x \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}, \quad \text{and} \quad M = mN, \qquad \text{(particle number and rest mass)}$$

$$E = \int \mathrm{d}^3x \Big[\frac{\hbar^2}{2m} \nabla \boldsymbol{\Psi}^\dagger \cdot \nabla \boldsymbol{\Psi} - \frac{Gm^2}{2} \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi} \int \frac{\mathrm{d}^3y}{4\pi |\boldsymbol{x} - \boldsymbol{y}|} \boldsymbol{\Psi}^\dagger(\boldsymbol{y}) \boldsymbol{\Psi}(\boldsymbol{y}) \Big] \,, \qquad \text{(energy)}$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger \,, \qquad \text{(spin angular momentum)}$$

$$\boldsymbol{L} = \hbar \int \mathrm{d}^3x \, \Re \left(i \, \boldsymbol{\Psi}^\dagger \nabla \boldsymbol{\Psi} \times \boldsymbol{x} \right) . \qquad \text{(orbital angular momentum)}$$

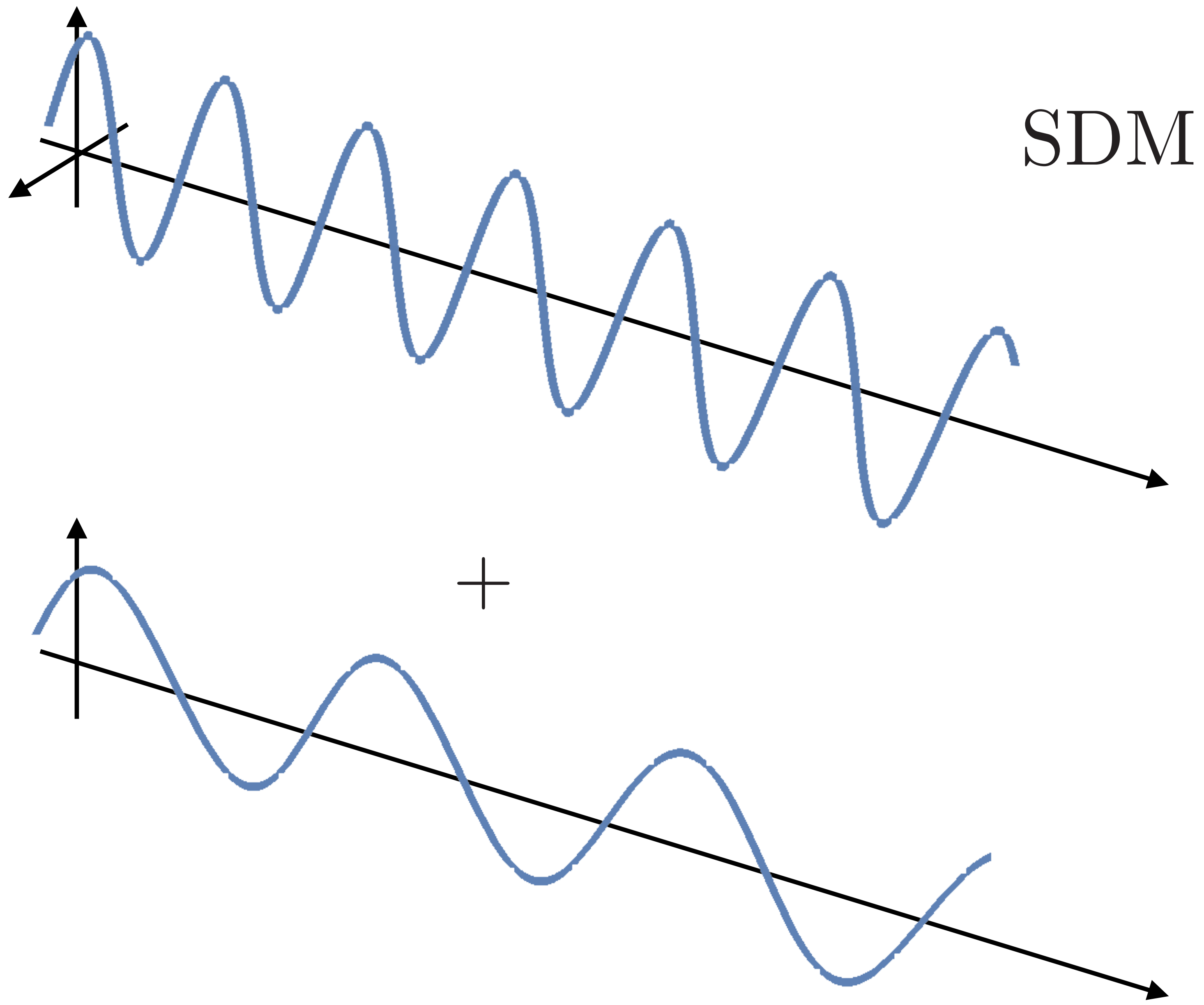
vector vs. scalar DM:
3 phenomenon

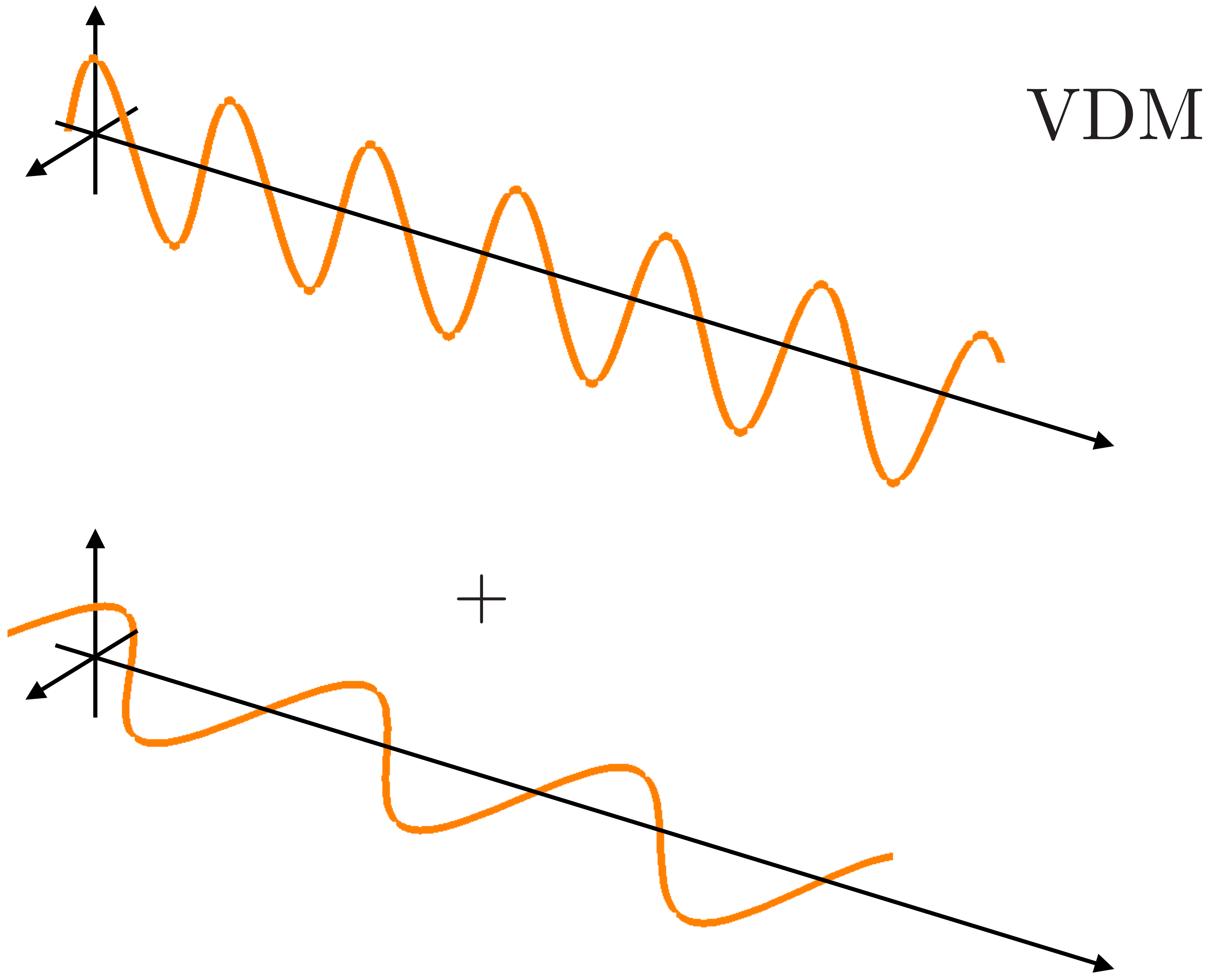
interference

condensation times

polarized solitons

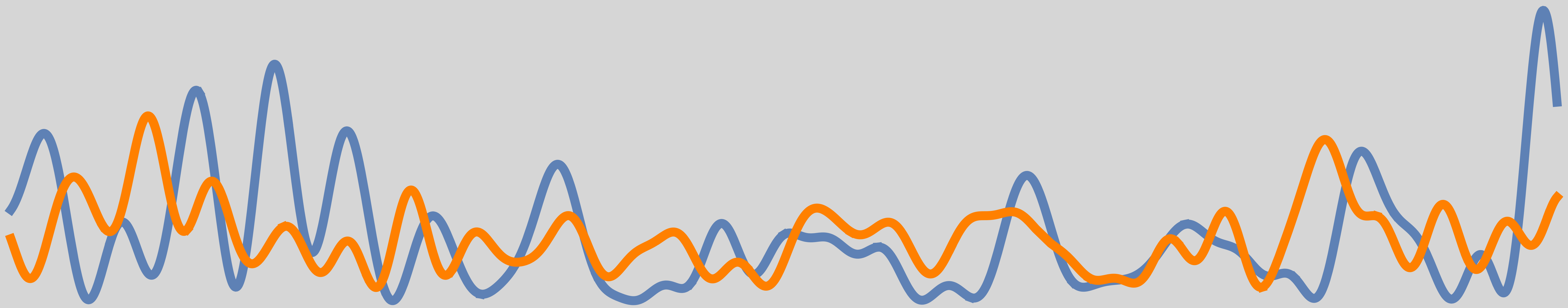
wave interference





$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 \neq |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$

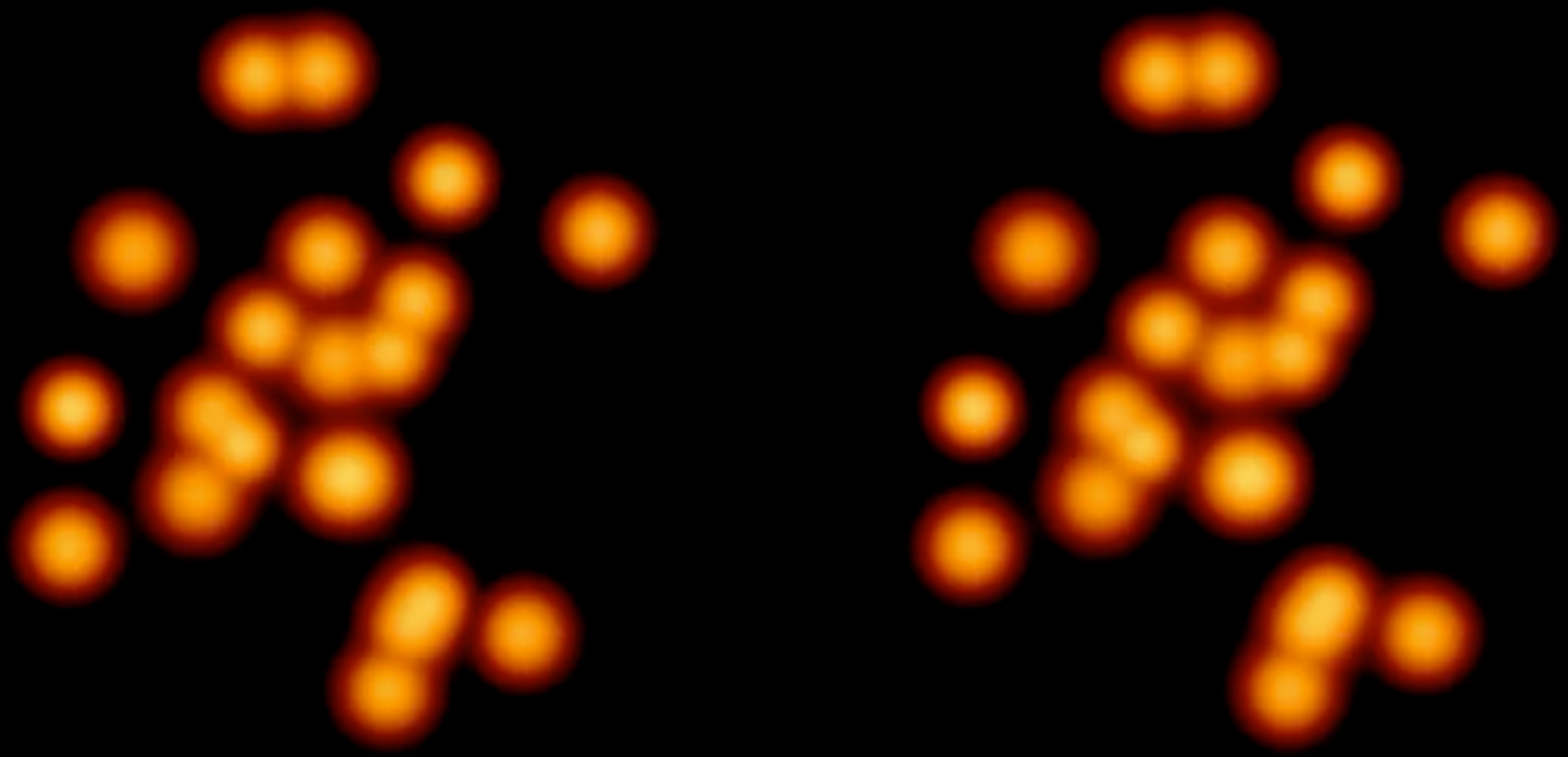
$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 = |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$



reduced interference

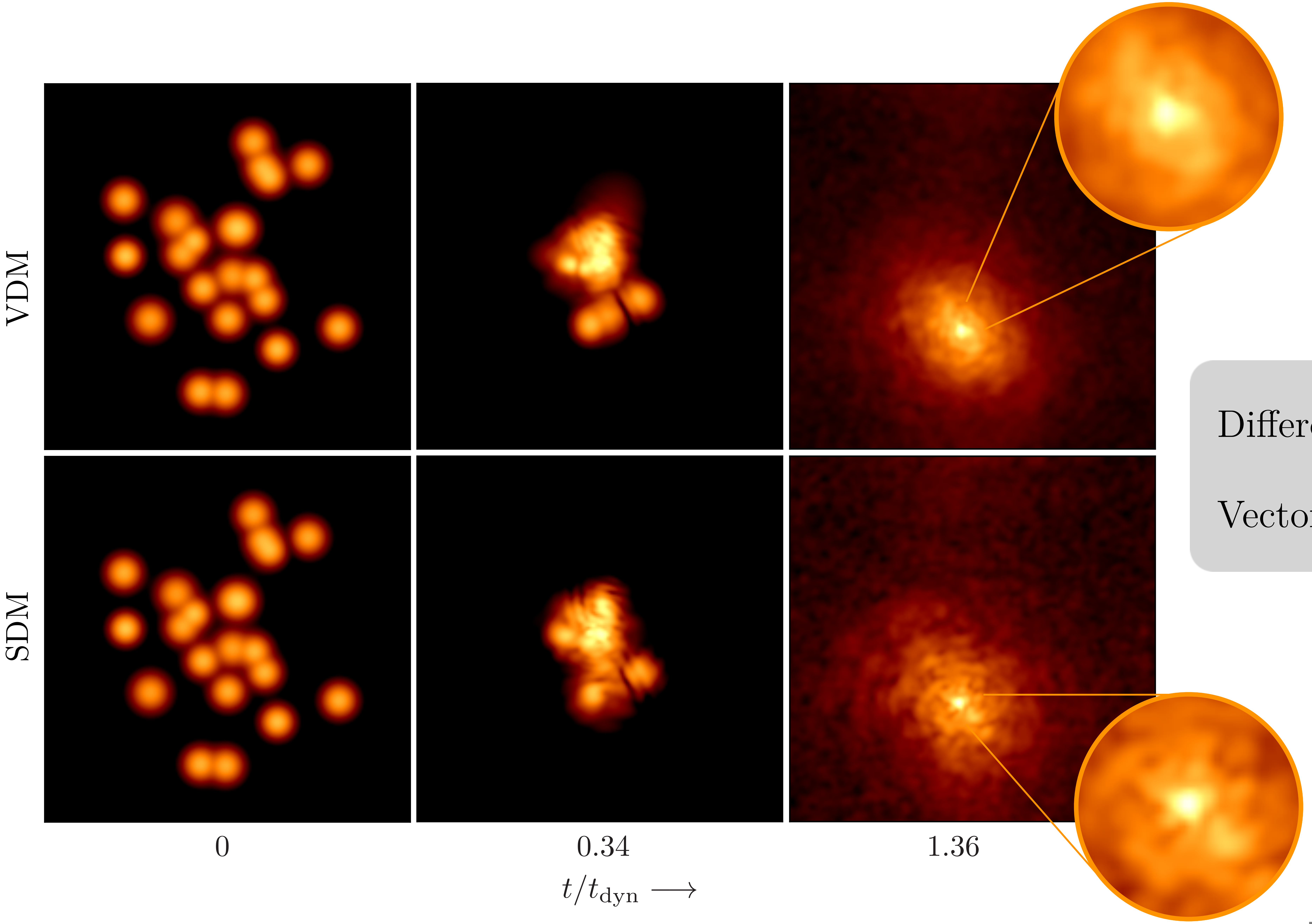
$$\frac{\delta\rho}{\rho} \propto \frac{1}{\sqrt{2s+1}}$$

MA, Jain, Karur & Mocz (2022)



MA, Jain, Karur & Mocz (2022)

MA, Jain, Karur & Mocz (2022)



Difference between

Vector & Scalar Dark Matter

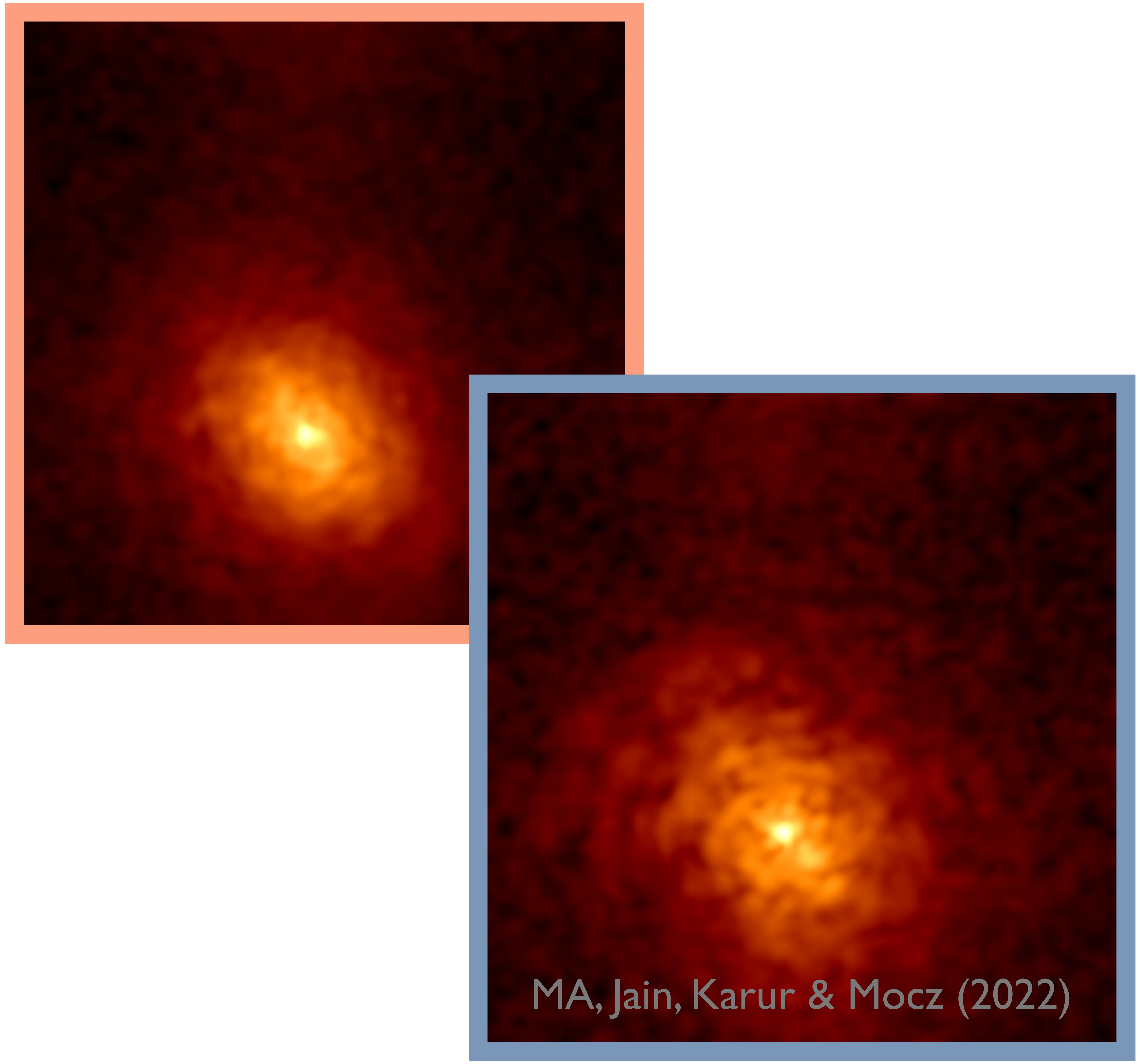
MA, Jain, Karur & Mocz (2022)

gravitational implications (examples)

- dynamical heating of stars

$$m \gtrsim \frac{1}{(2s+1)^{1/3}} \left[3 \times 10^{-19} \mathrm{eV}\right]$$

Dalal & Kratsov (2022)



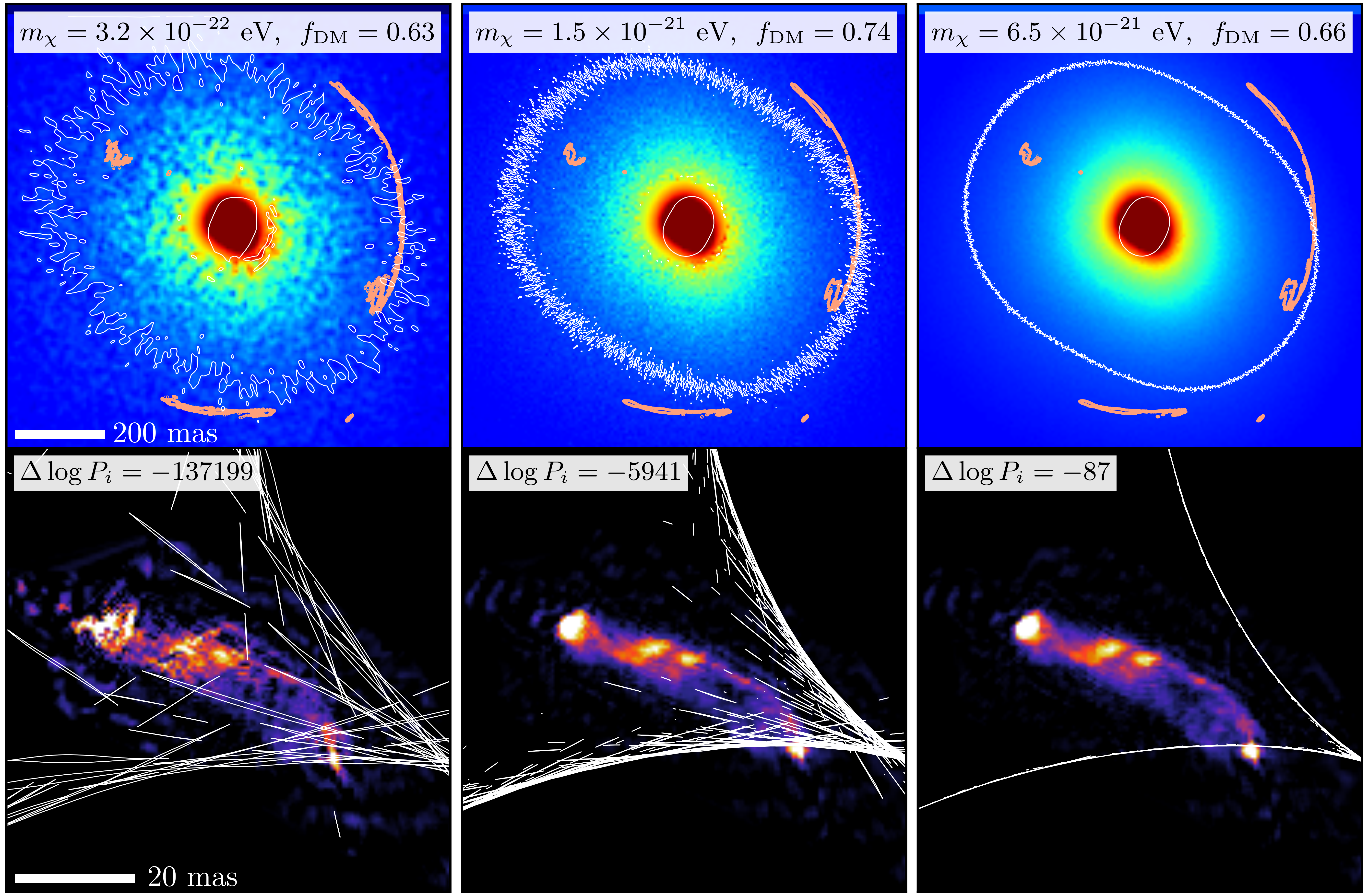
MA, Jain, Karur & Mocz (2022)

gravitational implications (examples)

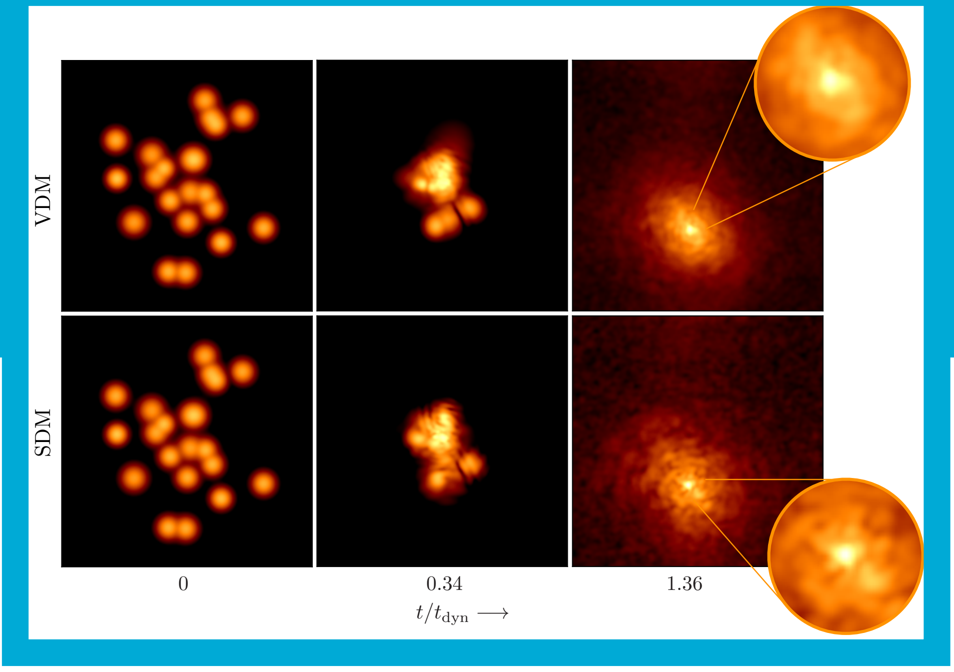
- lensing

$$m \gtrsim \frac{1}{(2s+1)} \left[4.4 \times 10^{-21}\ \mathrm{eV}\right]$$

Powell et. al (2023)

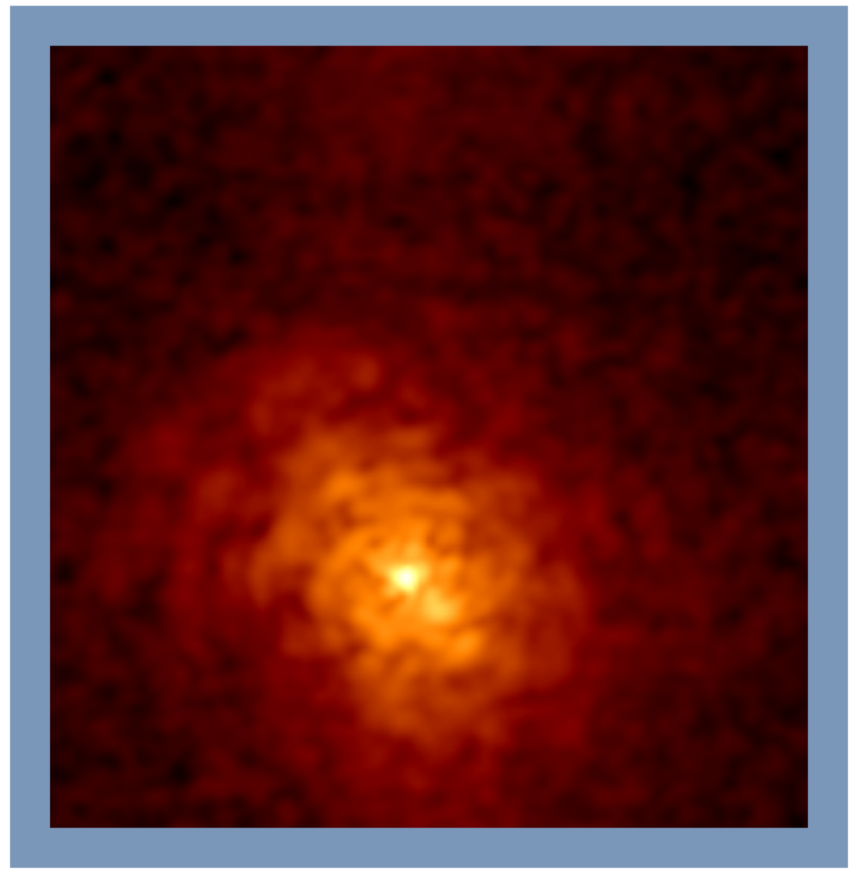


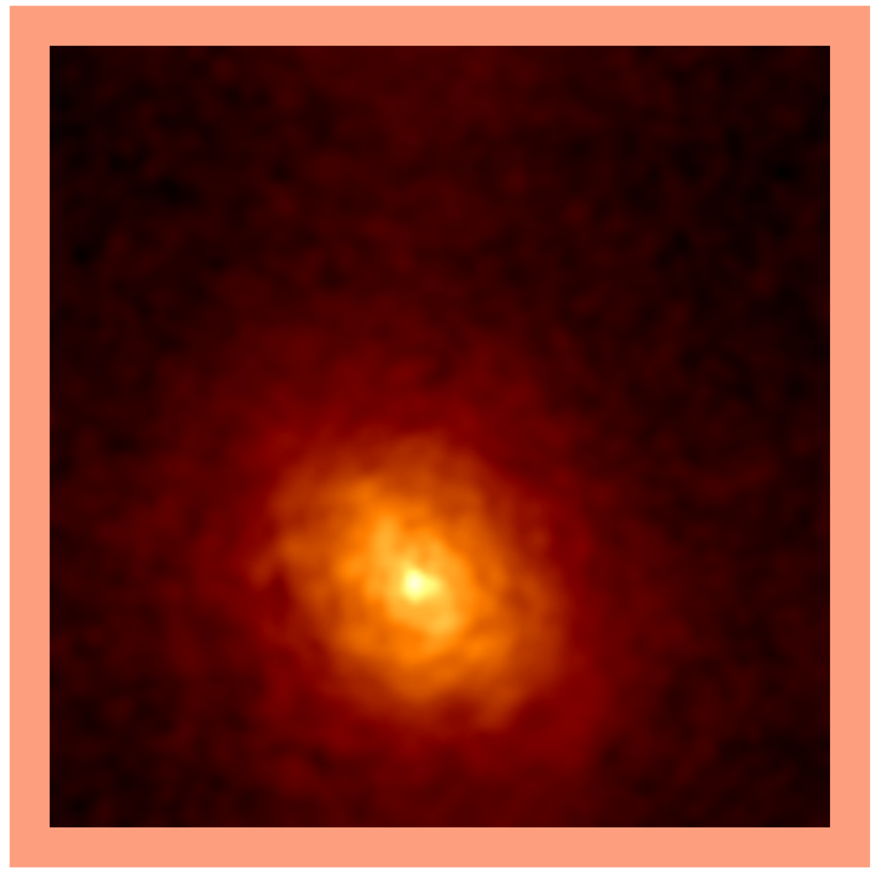
radial density profiles

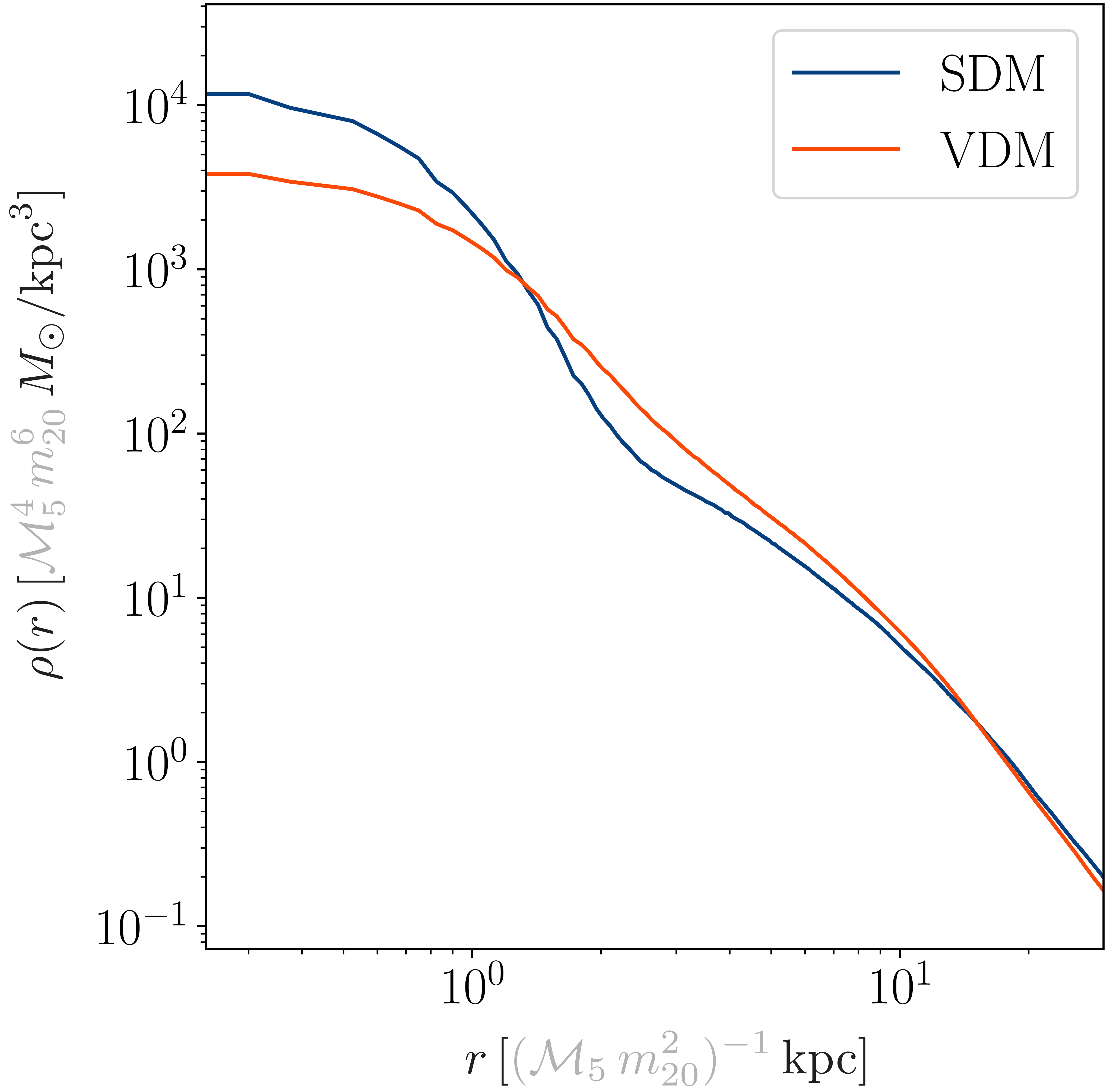


scalar vs. vector dark matter

- less dense & broader core
- smoother transition to $r^{-(2\text{–}3)}$ tail







condensation in the kinetic regime

with M. Jain, J. Thomas, Wanichwecharungruang (2023)

condensation in the kinetic regime

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condensation in the kinetic regime

with M. Jain, J. Thomas, Wanichwecharungruang (2023)

condensation in the kinetic regime

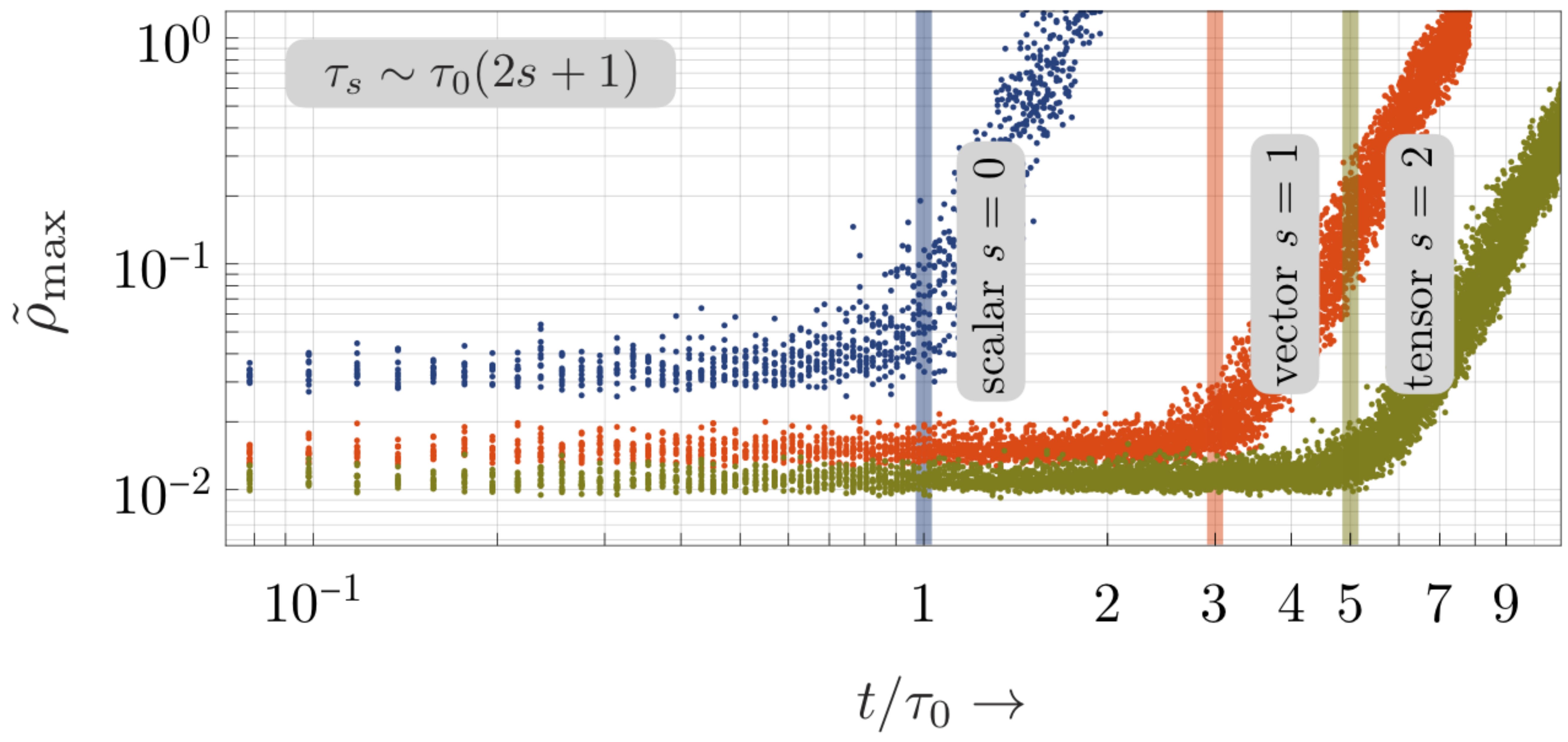
- nucleation time scale

$$\tau_s \sim (2s+1)\tau_{s=0}$$

$$\tau_{s=0} = \left[n\sigma_{gr} v \mathcal{N}\right]^{-1}$$

$$\sigma_{gr} \sim (Gm/v^2)^2, \qquad \mathcal{N} \sim n\lambda_{dB}^3$$

with M. Jain, J. Thomas, Wanichwecharungruang (2023)



$$\tau_0 \sim \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^3 \left(\frac{\sigma}{100\,\mathrm{km}\,s^{-1}}\right)^6 \left(\frac{10^8 M_\odot \mathrm{kpc}^{-3}}{\bar{\rho}^3}\right)^2 \times 10\mathrm{Gyrs}$$

see Levkov et. al (2018) for scalar case

what are these blobs?

with M. Jain, J. Thomas, Wanichwecharungruang (2023)

soliton ?

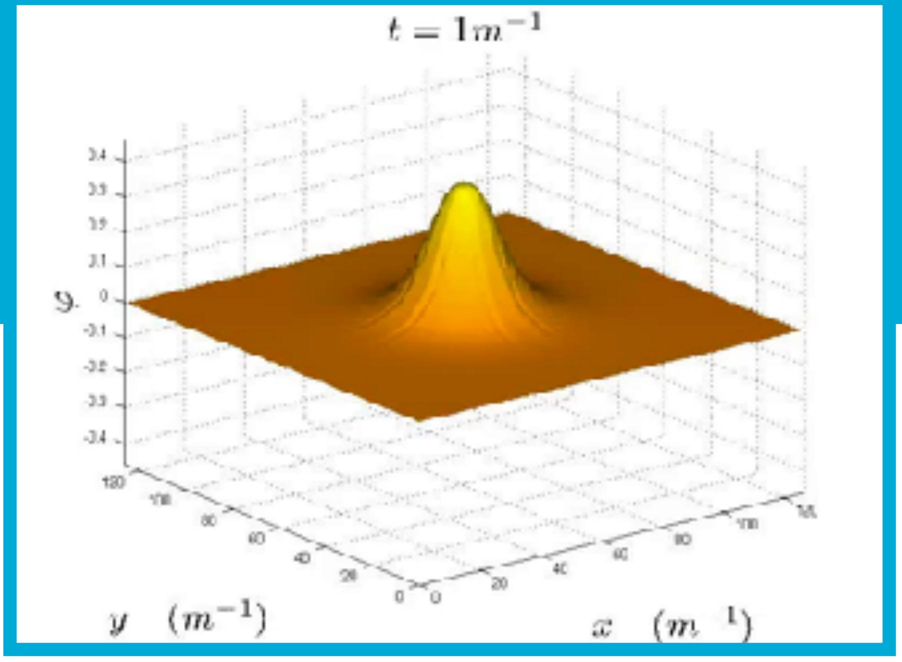
Image Credit: Heriot-Watt University

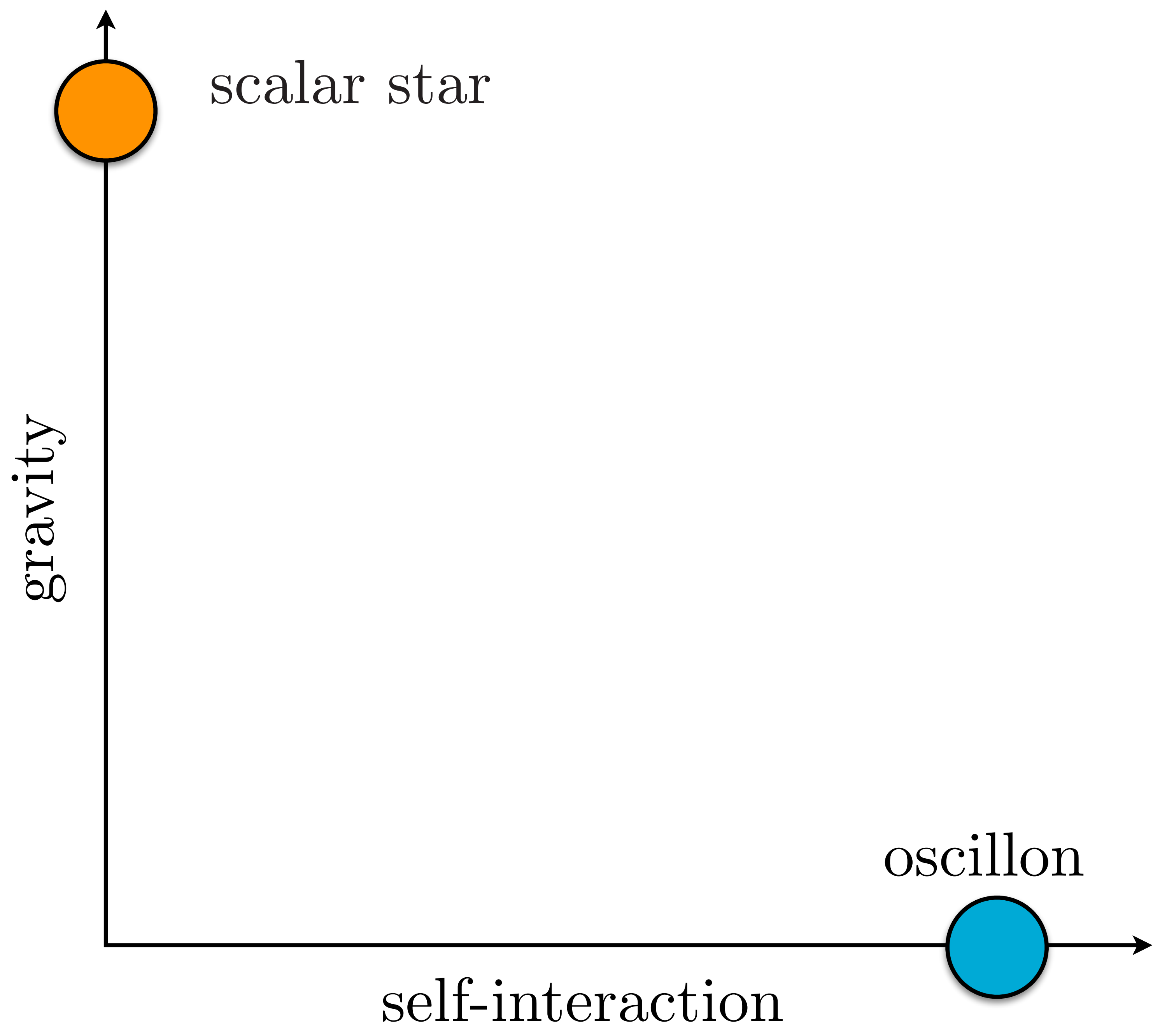
very-long lived, dynamical excitation in a field, with nonlinearities balancing dispersion

- discovered in nonlinear waves in water in canals (John Scott Russell, 1834)
- optics, hydrodynamics, BECs, high energy physics, and cosmology

non-topological “solitons” (real-valued)

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

exceptionally long-lived

Makhanakov, Bolglubovsky, Kruskal & Seagur

Seidel & Sun …

Gleiser, Copeland, Muller, Graham …

Hindmarsh, Salmi…

Kasuya, Kawasaki, Takahashi, …

MA & Shirokoff

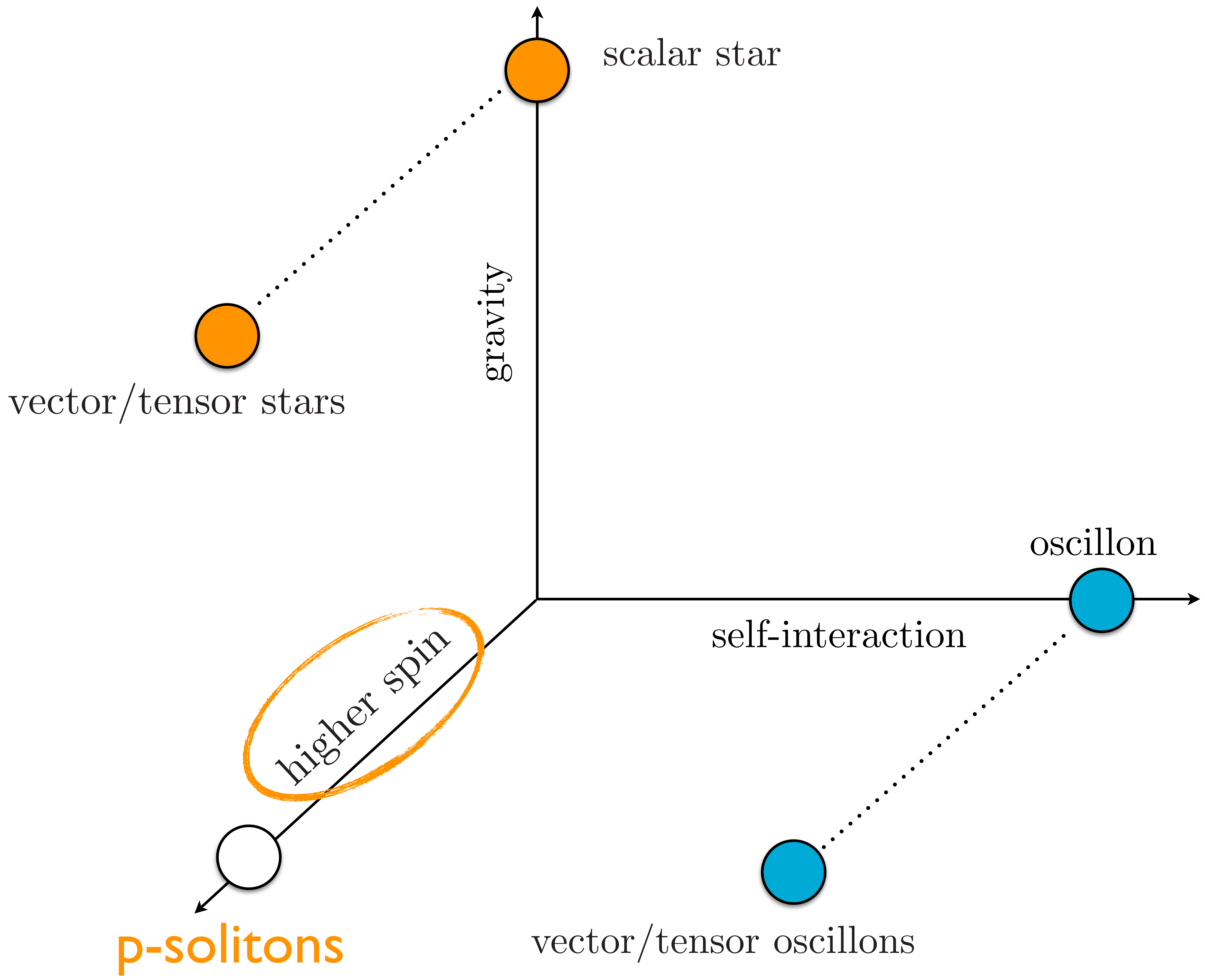
Mukaida, Takimoto, Yamada

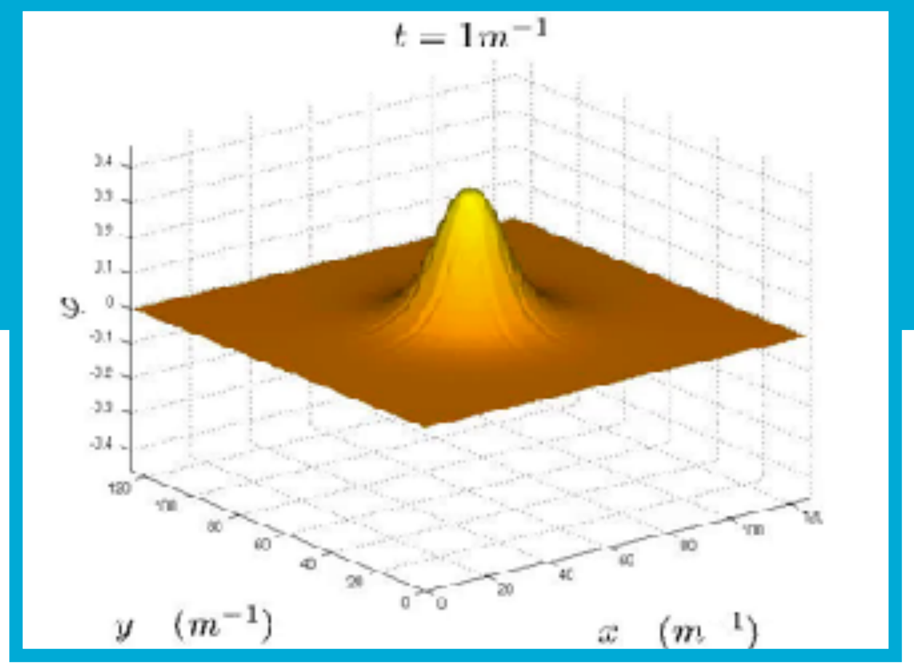
Zhang, MA, Copeland, Lozanov & Saffin

For complex-valued fields, see Q-ball lit (ask V. Takhistov about it)

non-topological solitons

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

exceptionally long-lived


Jain & MA (2021)

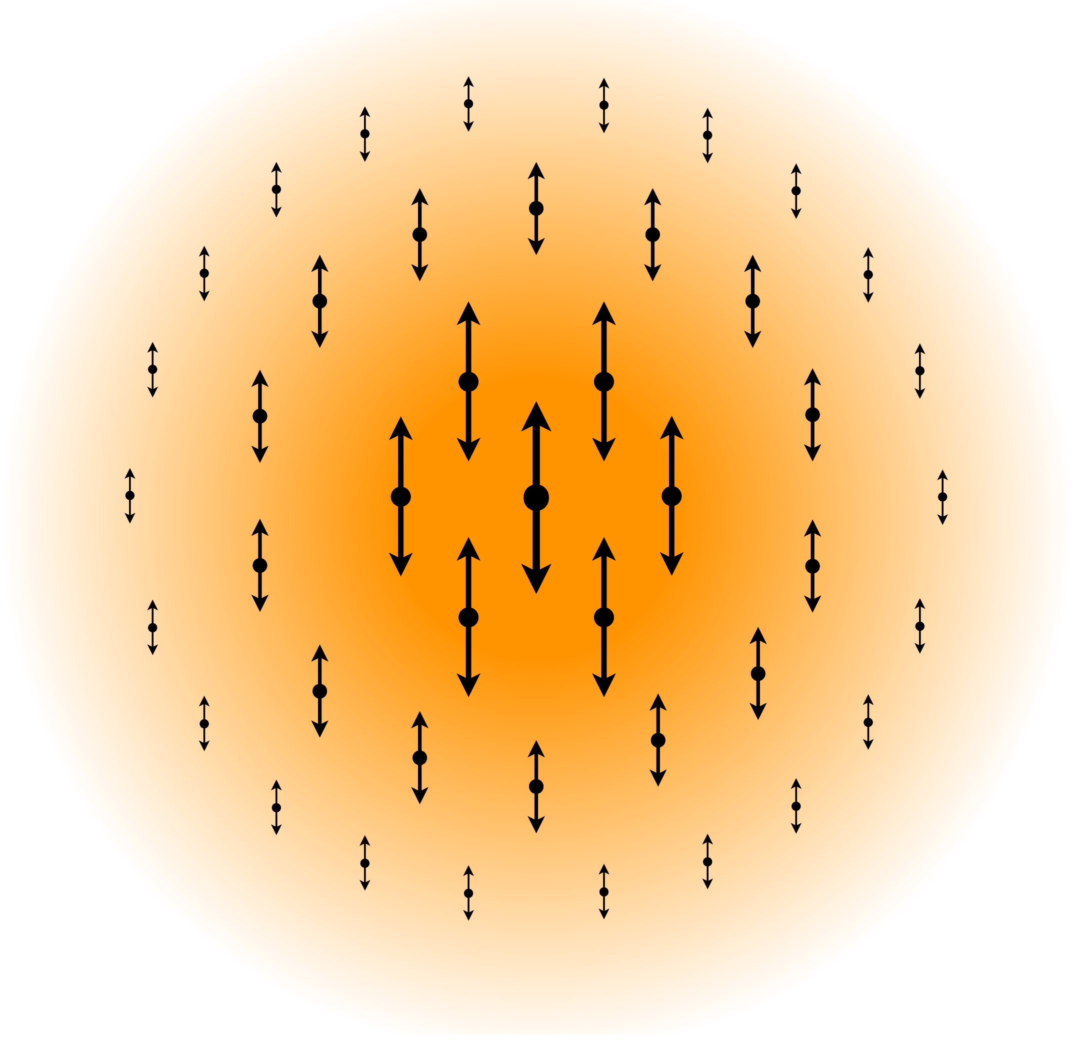
Zhang, Jain & MA (2021)

"polarized" vector solitons (with macroscopic spin)

$$\boldsymbol{S}_{\rm sol} = i(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^{\dagger})\frac{M_{\rm sol}}{m}\hbar$$

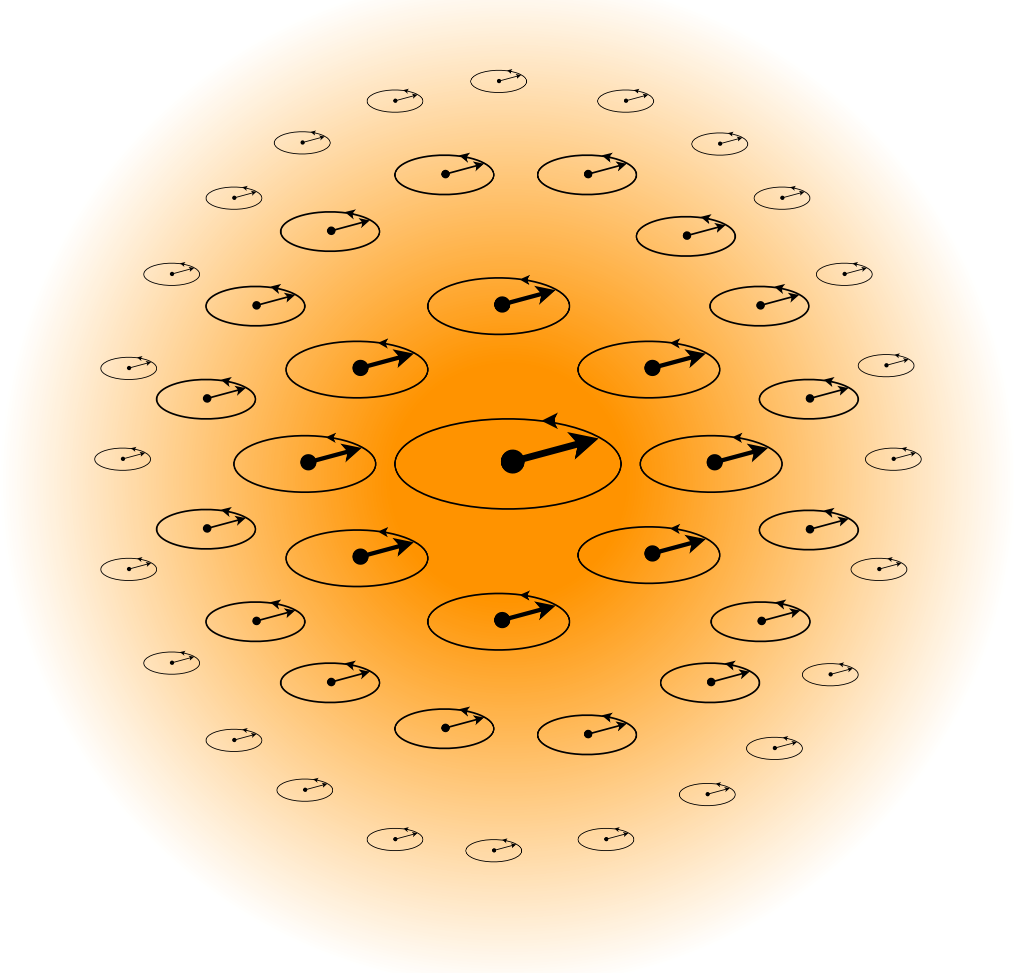
macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

$N =$ # of particles in soliton



$$\boldsymbol{\epsilon}^{(0)}_{1,\hat{z}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\boldsymbol{S}_{\rm tot} = 0\hat{z}$$



$$\boldsymbol{\epsilon}^{(\pm 1)}_{1,\hat{z}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

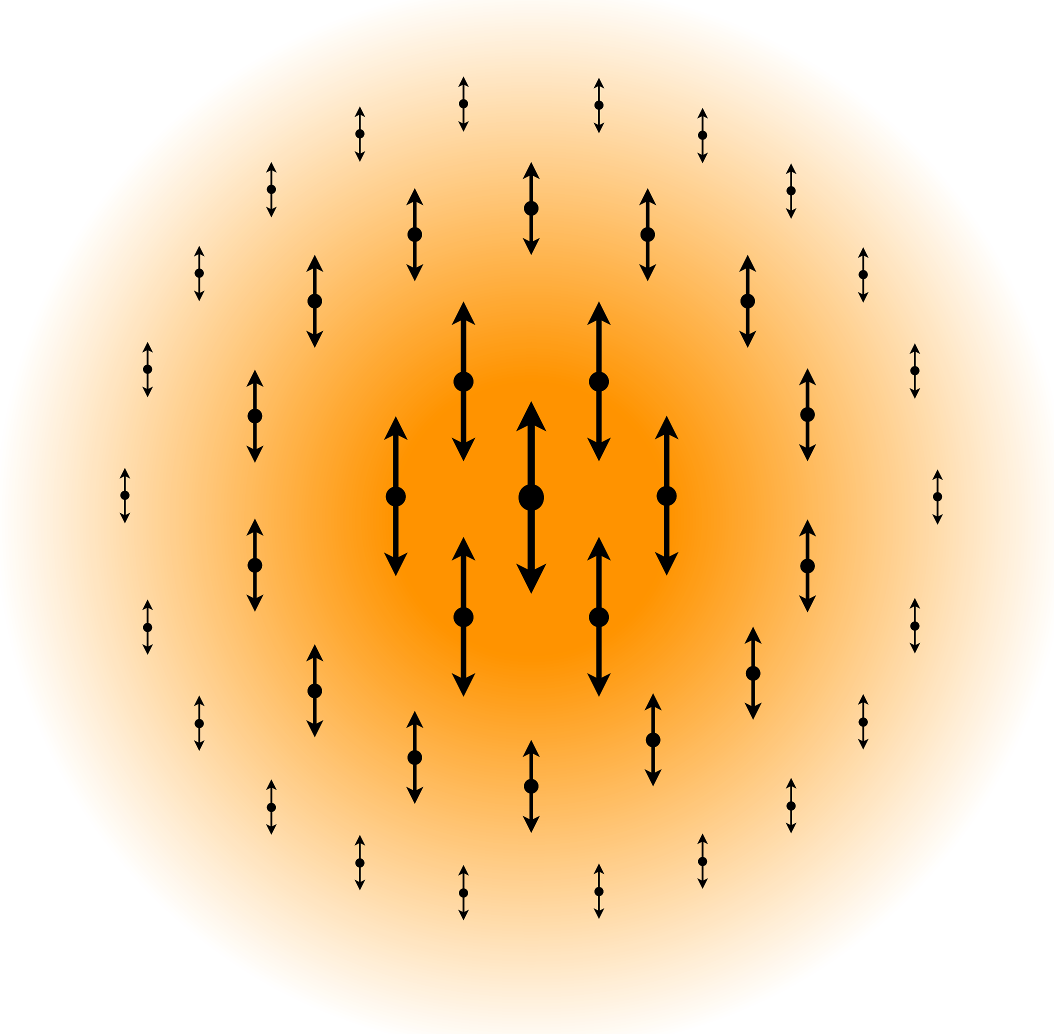
Jain & MA (2021)

"polarized" vector solitons

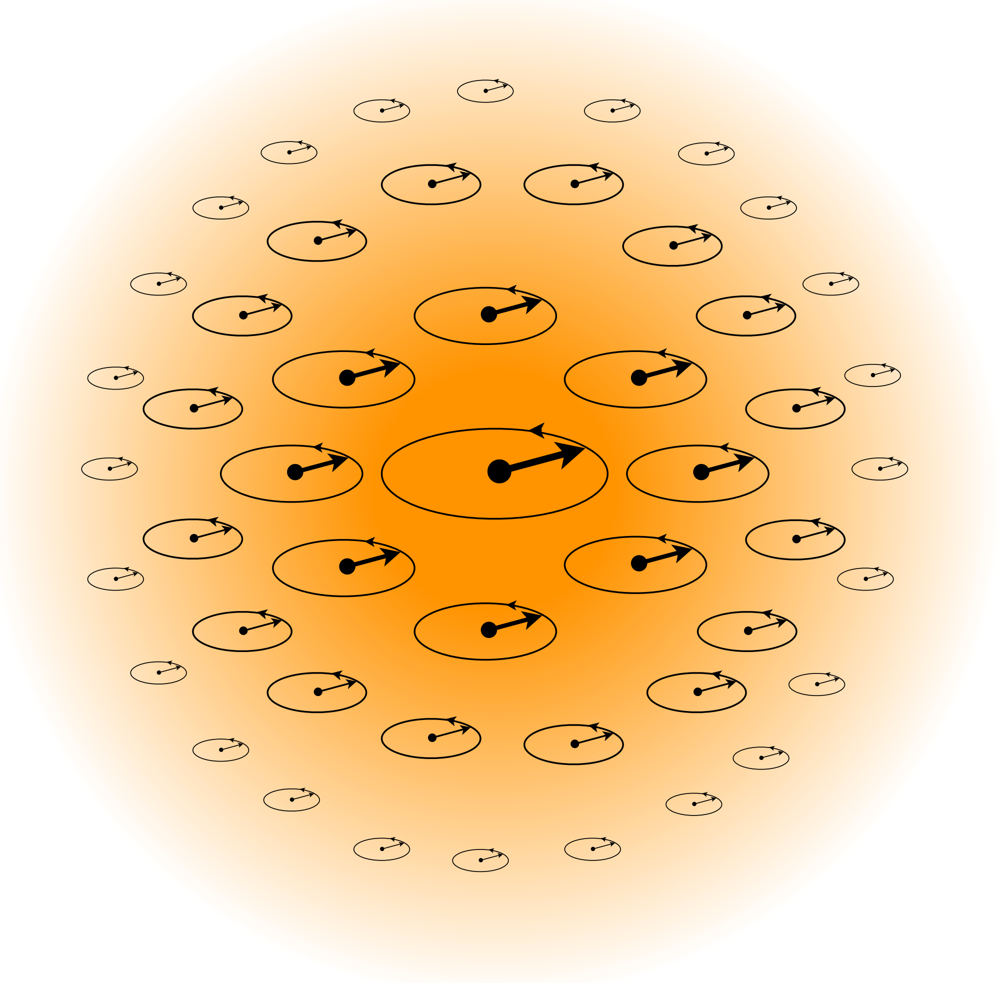
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

macroscopic spin $\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$

- all lowest energy for fixed M
- bases for partially-polarized solitons

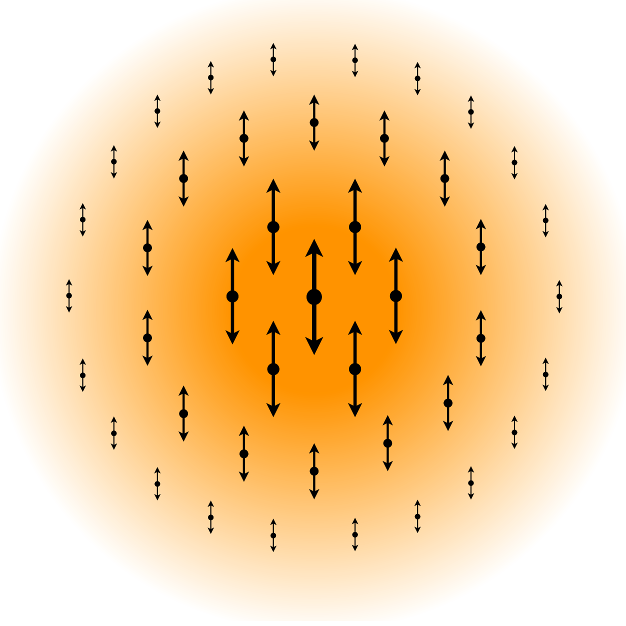
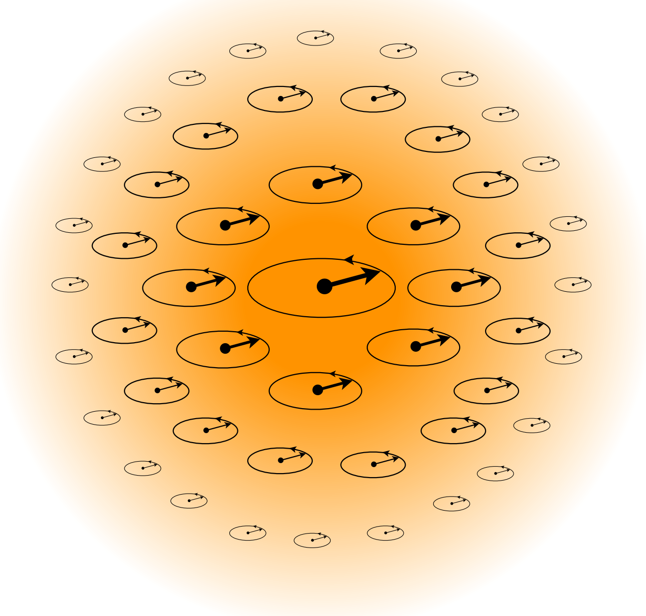
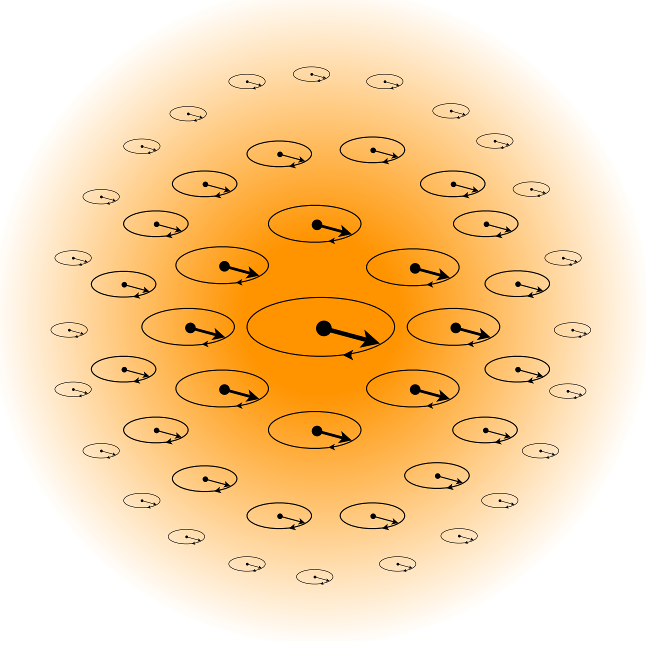


$$\boldsymbol{S}_{\rm tot} = 0\hat{z}$$



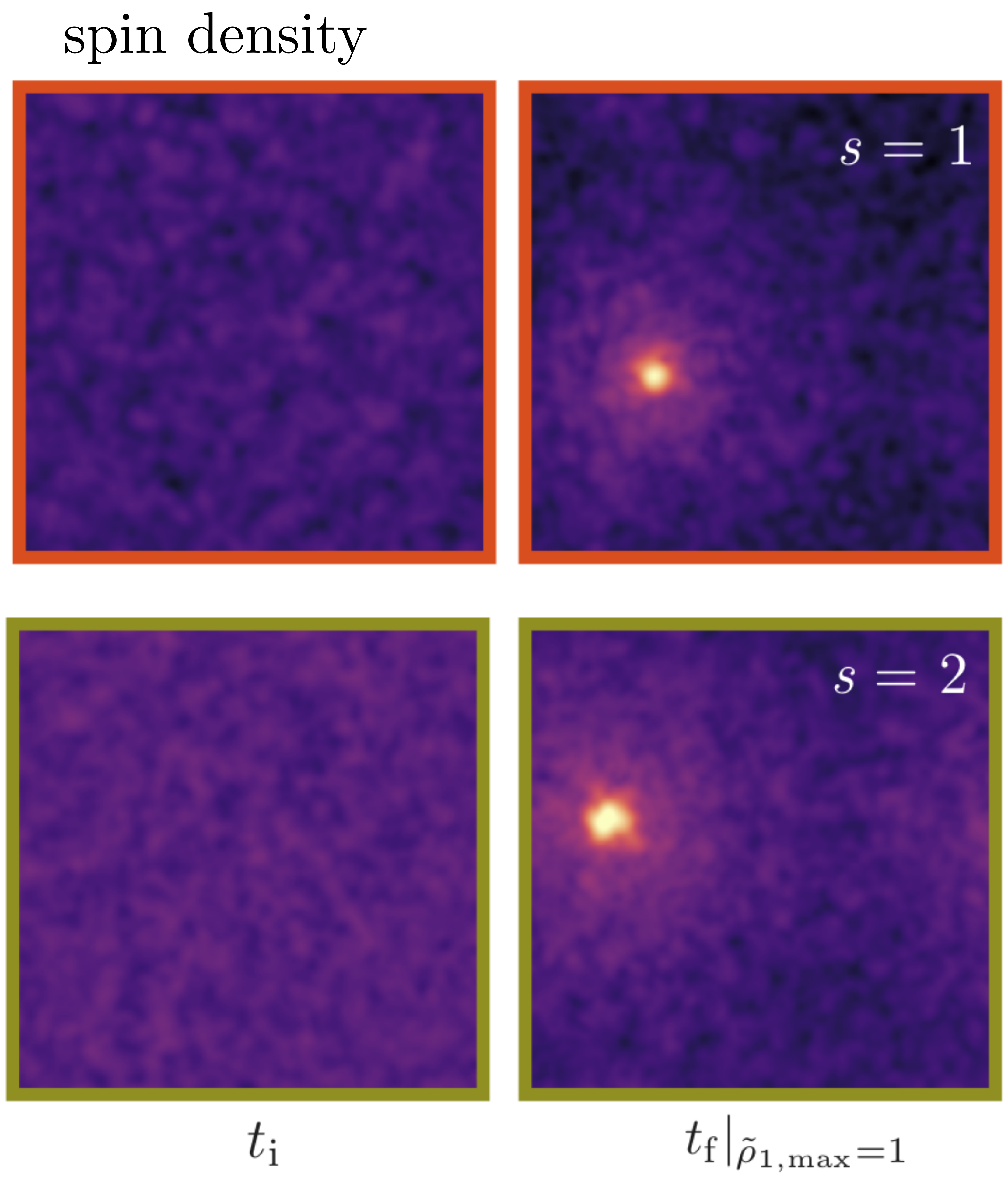
$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

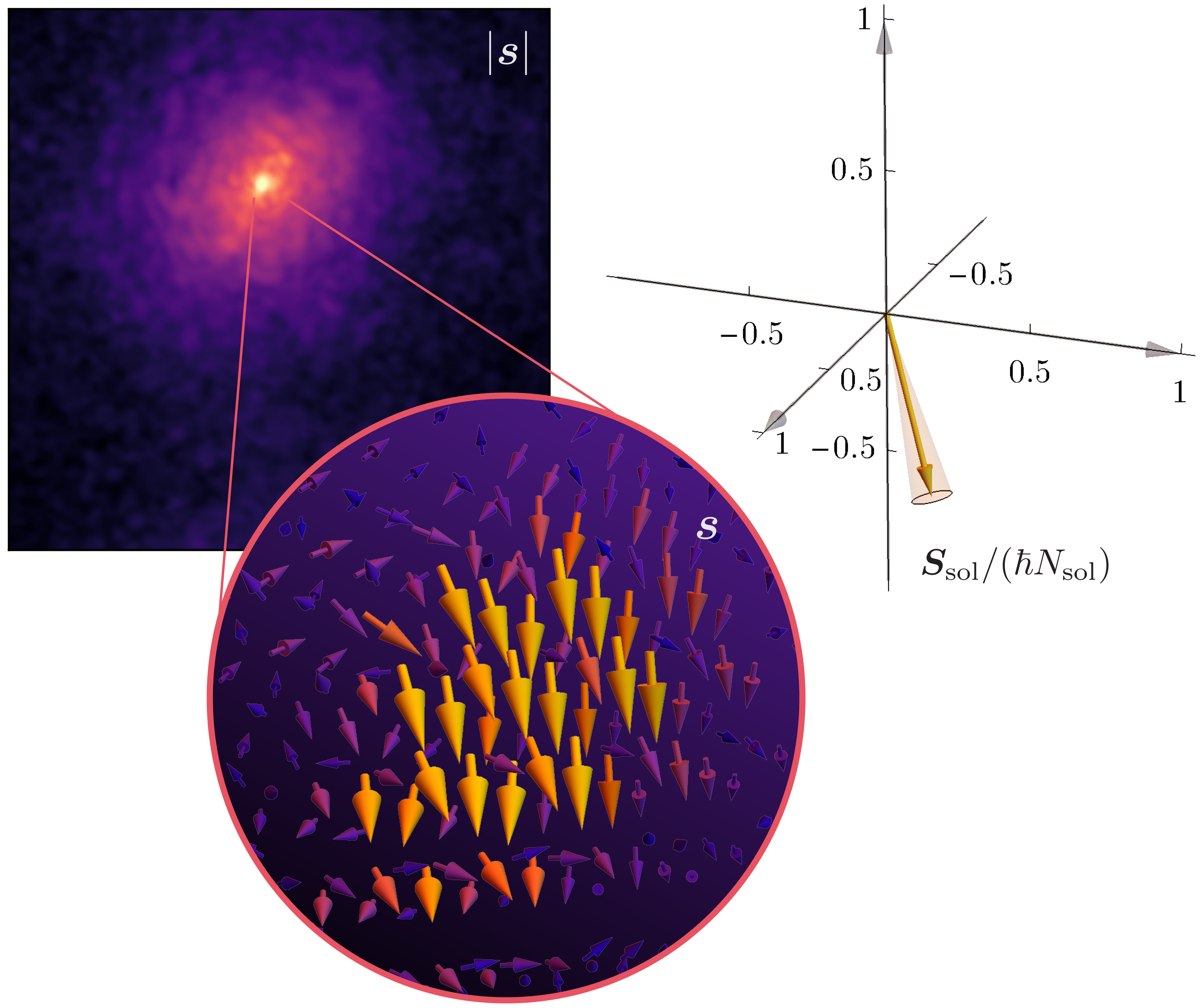
$$0 \le |\boldsymbol{S}_{\rm tot}| \le \frac{M_{\rm sol}}{m}\hbar$$

c_0  $+c_1$  $+c_{-1}$ 

Also see: Aoki et. al (2017 for massive tensors geons), Adshead & Lozanov (2021), Jain & MA (2021)

born to spin





$$S_{\mathrm{core}} \sim \hbar \frac{M_{\mathrm{core}}}{m}$$

Even when initial total spin is negligible

MA, Jain, Karur & Mocz(2022)

Jain, MA, Thomas, Wanichwecharungruang (2023)

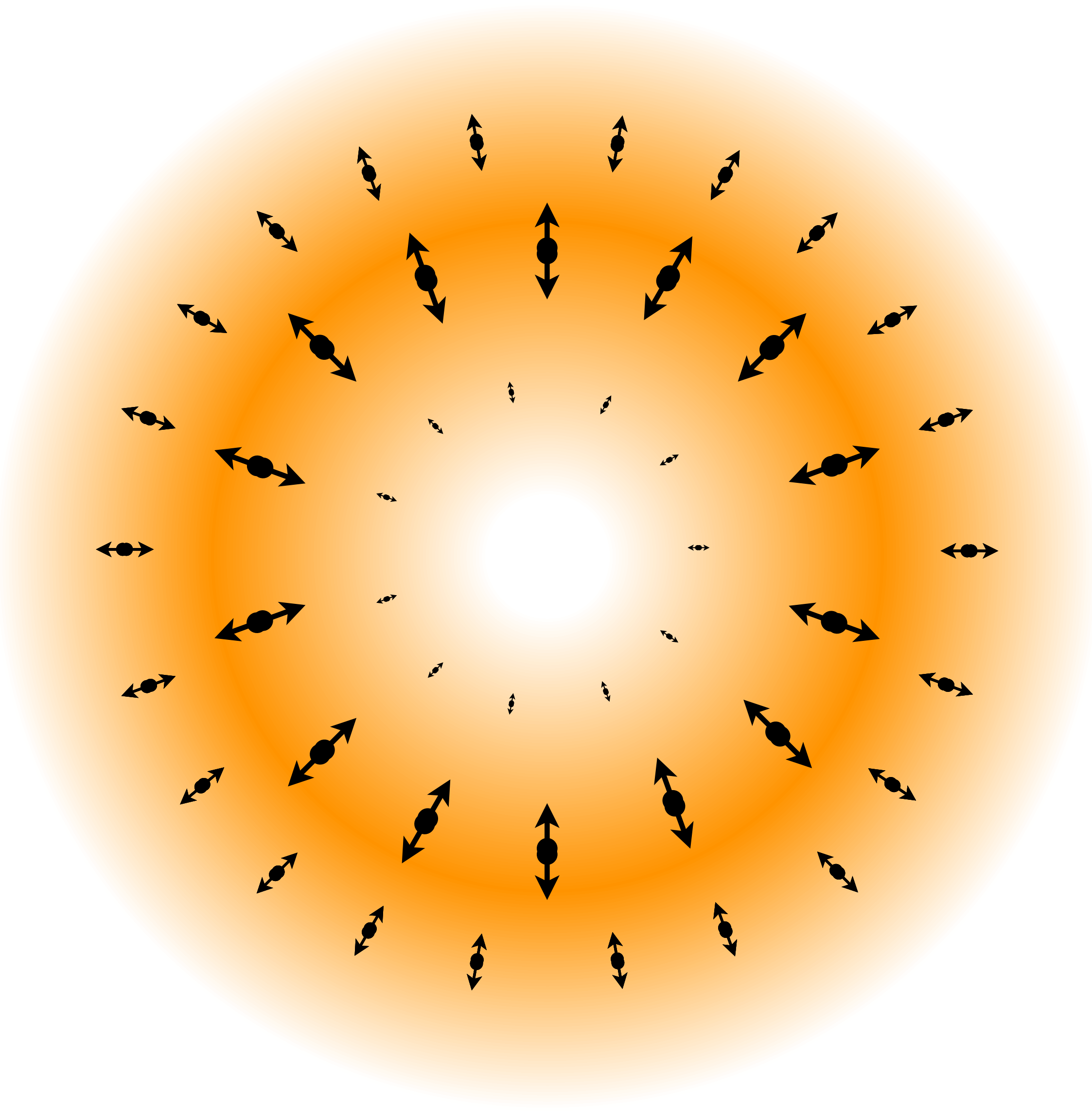
a different higher energy soliton: the "hedgehogs"

earlier literature

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

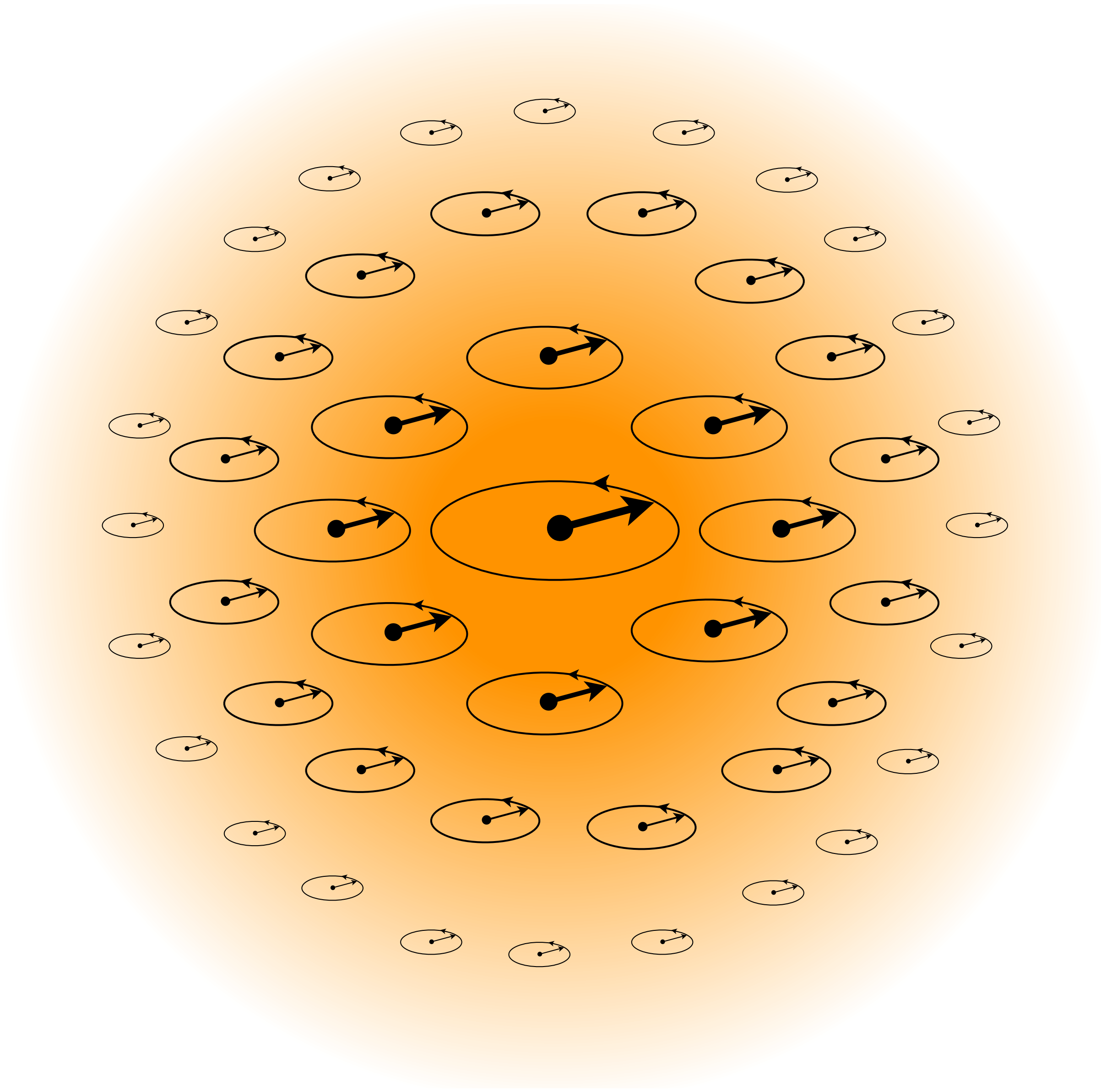
hedgehogs
not ground states

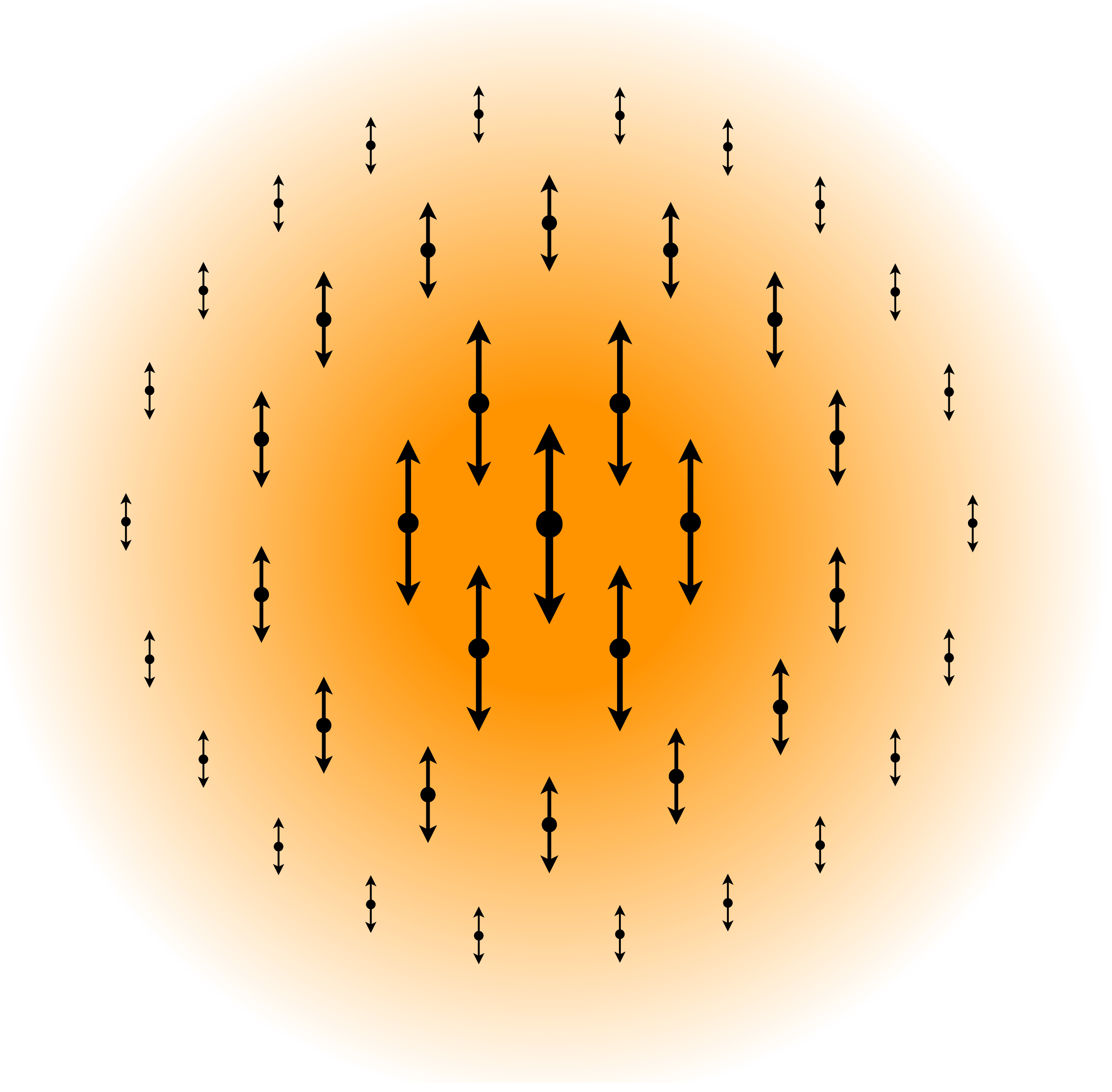
at least when non-relativistic
Lozanov & Adshead (2021)

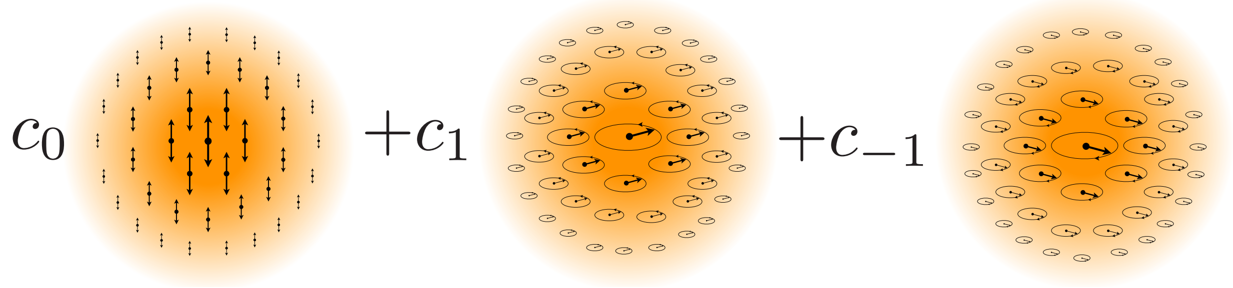


$$E^s_{\mathrm{hh}} > E$$

$$E^{s=1}_{\mathrm{hh}} \approx 0.33E \; < 0$$





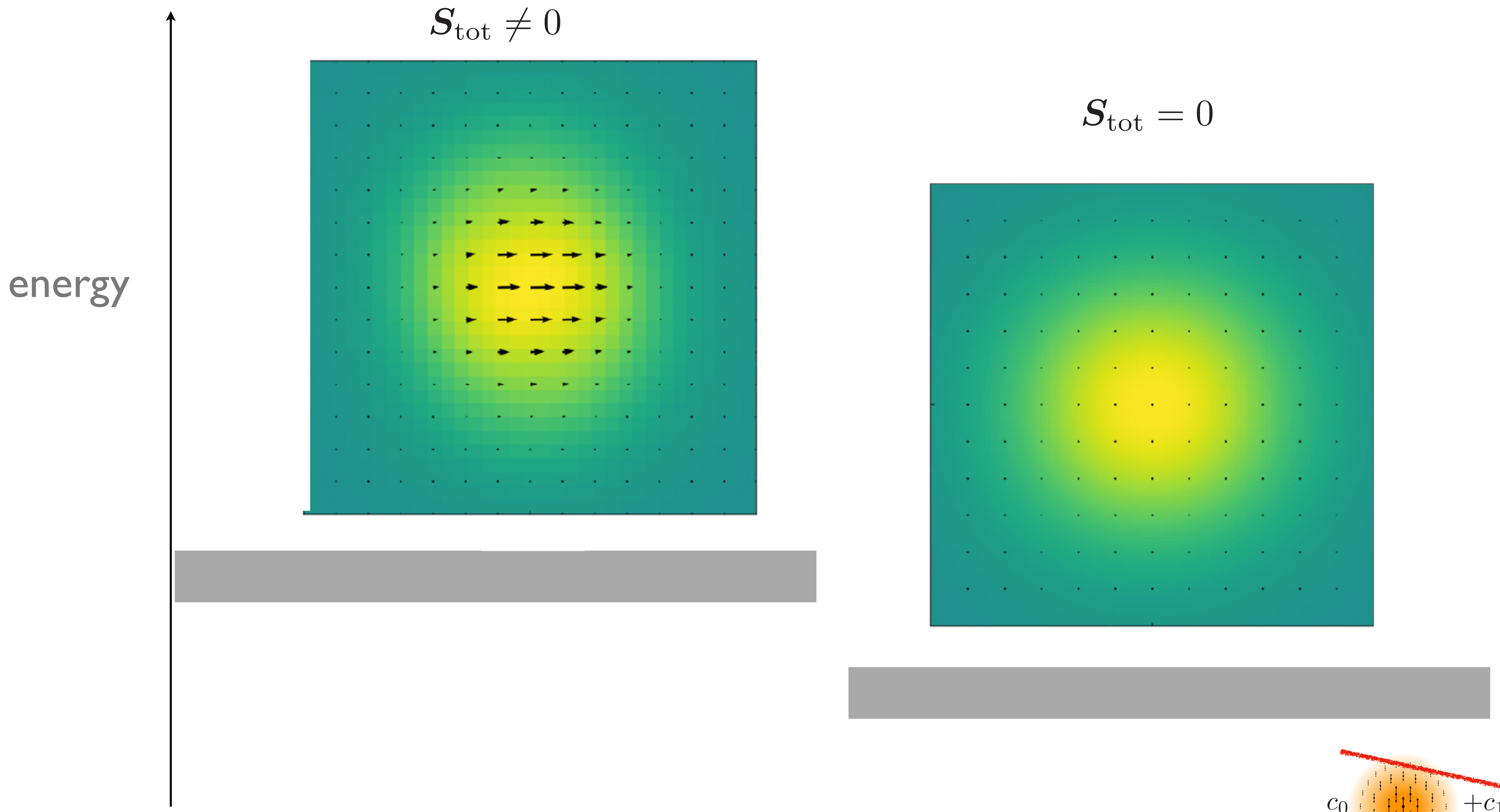


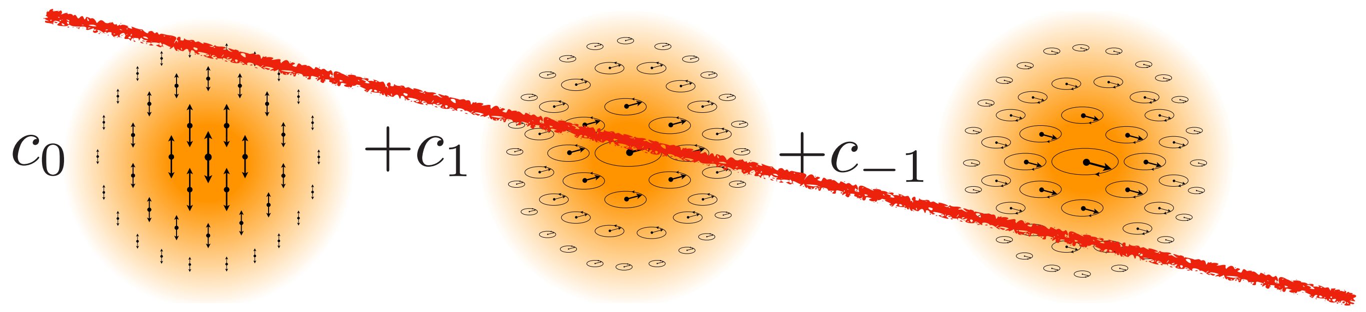
attractive non gravitational self-interactions

Zhang, Jain & MA (2022)

lead by HongYi Zhang

Also see Zhang & Ling (2023)





Also Jain (2021)

i-SPin: An integrator for multicomponent Schrodinger-Poisson systems with self-interactions

arXiv: 2211.08433

Mudit Jain & Mustafa Amin

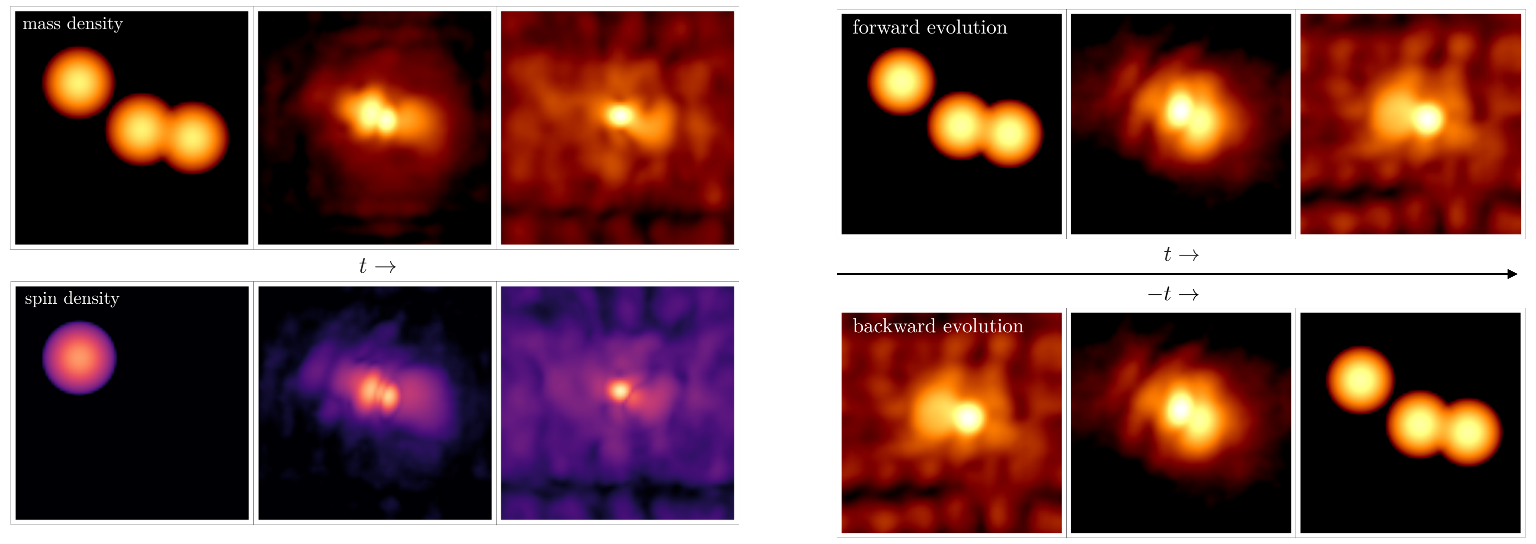
i-SPin: An algorithm (and publicly available code) to numerically evolve multicomponent Schrodinger-Poisson (SP) systems, including attractive/repulsive self-interactions + gravity

problem: If SP system represents the non-relativistic limit of a massive vector field, non-gravitational self-interactions (in particular, *spin-spin* type interactions) introduce new challenges related to mass and spin conservation which are not present in purely gravitational systems.

solution: Above challenges addressed with a novel analytical solution for the non-trivial 'kick' step in the algorithm (sec 4.3.2)

features: (i) second order accurate evolution (ii) spin and mass conserved to machine precision (iii) reversible

generalizations: n-component fields with SO(n) symmetry, an expanding universe relevant for cosmology, and the inclusion of external potentials relevant for laboratory settings

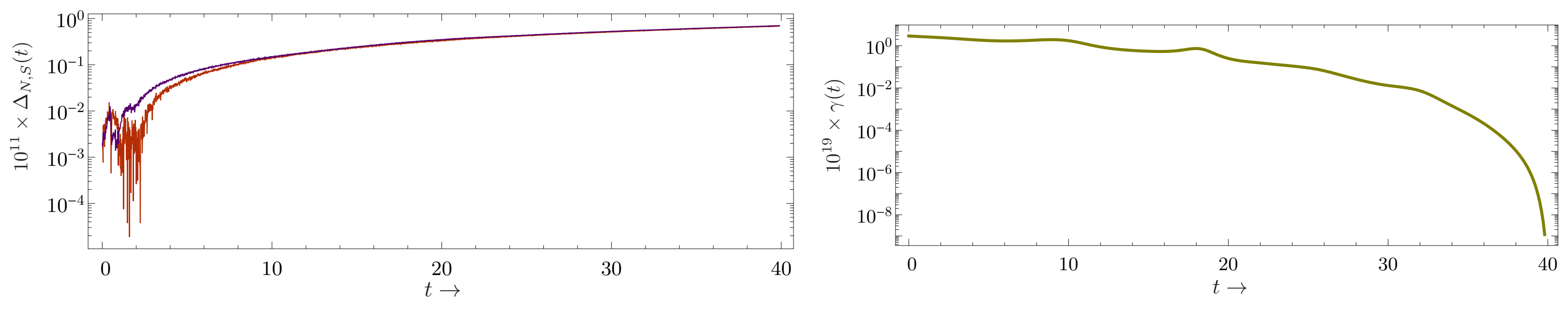


$$i\hbar\,\partial_t \boldsymbol{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi} - \frac{\lambda(\hbar c)^3}{4(mc^2)^2}\left[(\boldsymbol{\Psi}\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}^\dagger + 2\,(\boldsymbol{\Psi}^\dagger\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}\right]$$

$$V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) = -\frac{\lambda(\hbar c)^3}{8(mc^2)^2}\left[3\rho^2 - \frac{(\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{S}})}{\hbar^2}\right]$$

number density $\rho = \boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$

spin density $\boldsymbol{\mathcal{S}} = i\hbar\boldsymbol{\Psi}\times\boldsymbol{\Psi}^\dagger$



i-SPin 2: An integrator for general spin-s Gross-Pitaevskii systems

arXiv: 2305.01675

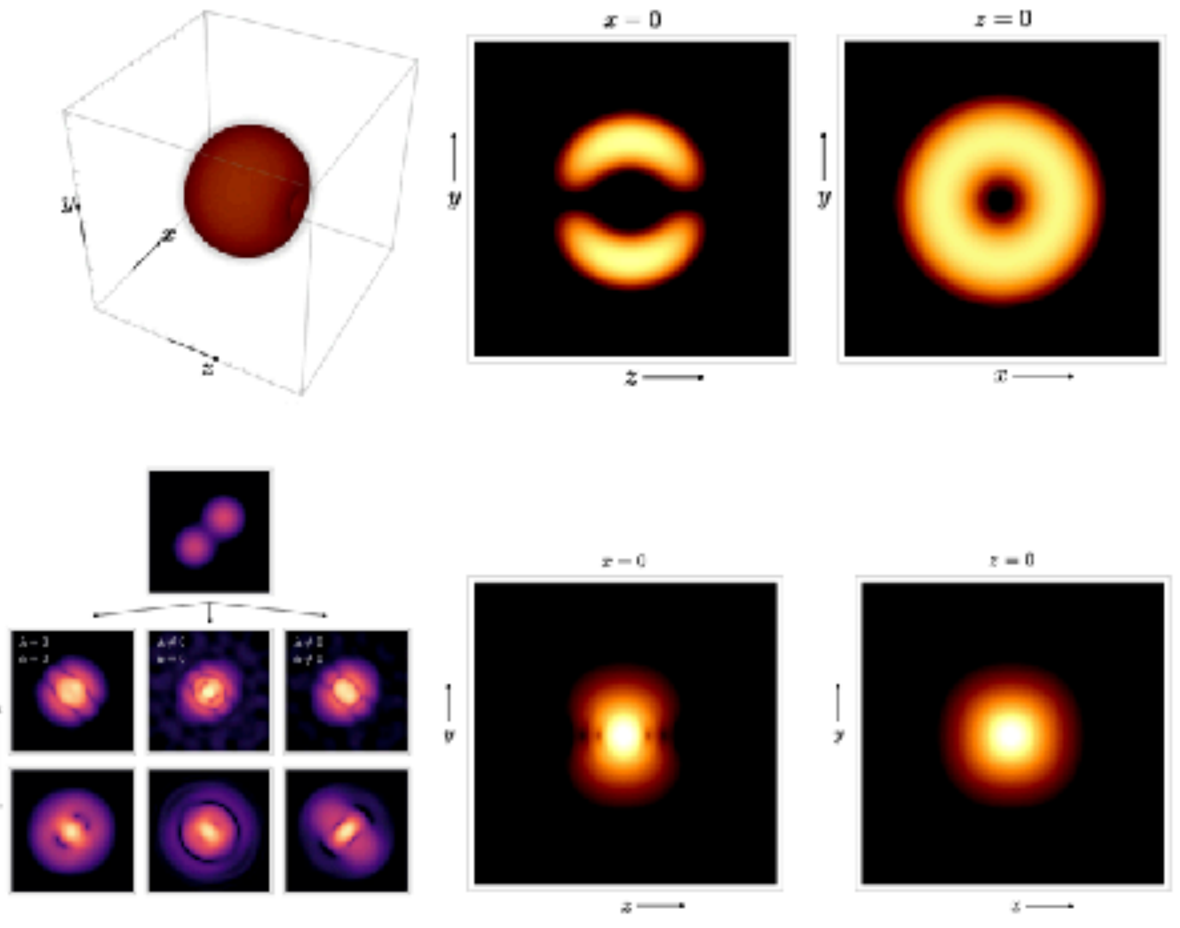
Mudit Jain, Mustafa Amin & H. Pu

i-Spin 2: An algorithm for evolving general spin-s Gross-Pitaevskii / non-linear Schrodinger systems carrying a variety of interactions, where the $2s+1$ components of the 'spinor' field represent the different spin-multiplicity states.

Allowed interactions: Nonrelativistic interactions up to quartic order in the Schrodinger field (both short and long-range, and spin-dependent and spin-independent interactions), including explicit spin-orbit couplings. The algorithm allows for spatially varying external and/ or self-generated vector potentials that couple to the spin density of the field.

Applications: (a) Laboratory systems such as spinor Bose-Einstein condensates (BECs). (b) Cosmological/astrophysical systems such as self-interacting bosonic dark matter.

Numerical features: Our symplectic algorithm is second-order accurate in time, and is extensible to the known higher-order accurate methods.



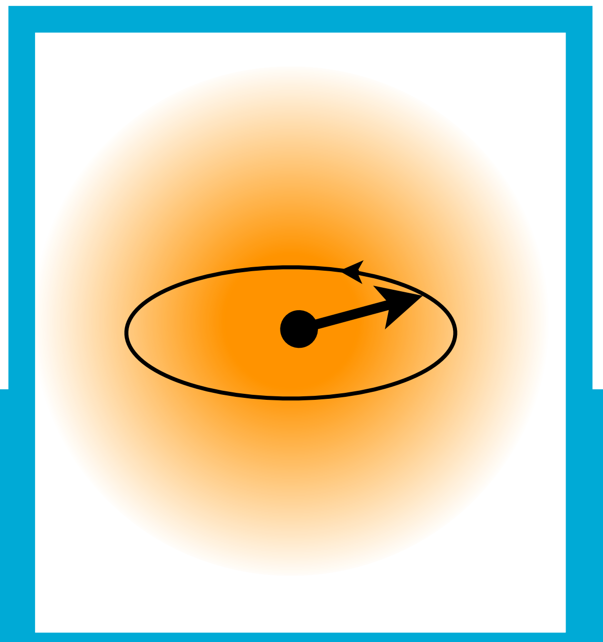
$$\begin{aligned}\mathcal{S}_{\rm nr} = \int {\rm d}t\,{\rm d}^3x\bigg[&\frac{i}{2}\psi_n^\dagger\dot{\psi}_n + {\rm c.c.} - \frac{1}{2\mu}\nabla\psi_n^\dagger\cdot\nabla\psi_n \\ &-\mu\rho V(\boldsymbol{x}) - \gamma\,\boldsymbol{\mathcal{S}}\cdot\bar{\boldsymbol{B}}(\boldsymbol{x},t) - V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) \\ &-\frac{\xi}{2}\frac{1}{(2s+1)}|\psi_n\,\hat{A}_{nn'}\psi_{n'}|^2 \\ &+ i\,g_{ij}\,\psi_n^\dagger\,[\hat{S}_i]_{nn'}\,\nabla_j\,\psi_n\bigg],\end{aligned}$$

with $\bar{\boldsymbol{B}}(\boldsymbol{x},t) = f(t)\boldsymbol{B}(\boldsymbol{x})$, and

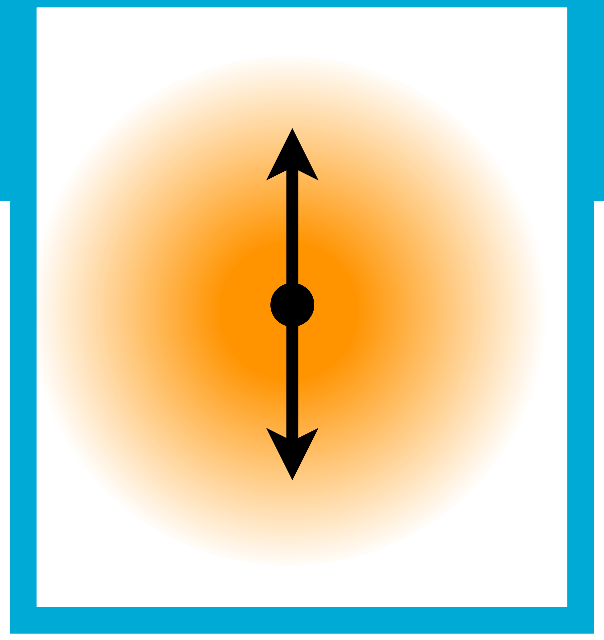
$$V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) = -\frac{1}{2\mu^2}\left[\lambda\rho^2 + \alpha\,(\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{S}})\right].$$

number density $\rho = \psi_n^\dagger\psi_n$

spin density $\boldsymbol{\mathcal{S}} = \psi_n^*\,\hat{\boldsymbol{S}}_{nn'}\,\psi_{n'}$



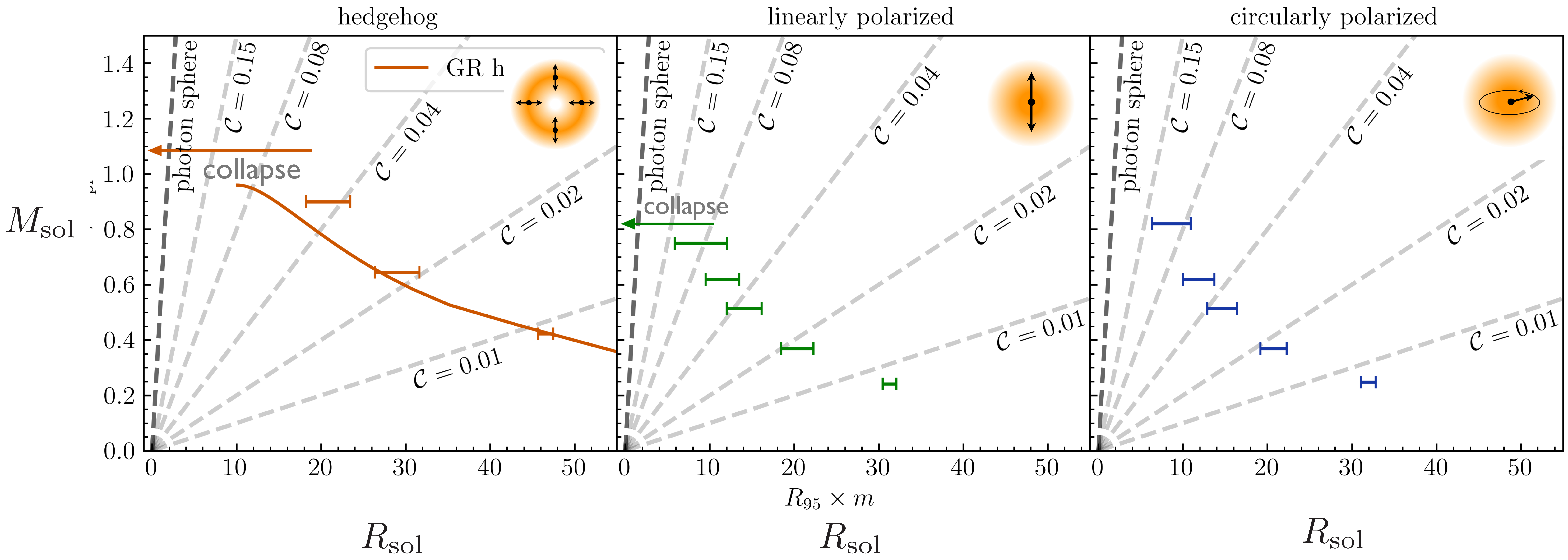
intrinsic spin



compactness of polarized solitons

more resistance to collapse to BH for circularly polarized stars

$$\mathcal{C}_{\text{hedgehog}} < \mathcal{C}_{\text{linear}} < \mathcal{C}_{\text{circular}}$$

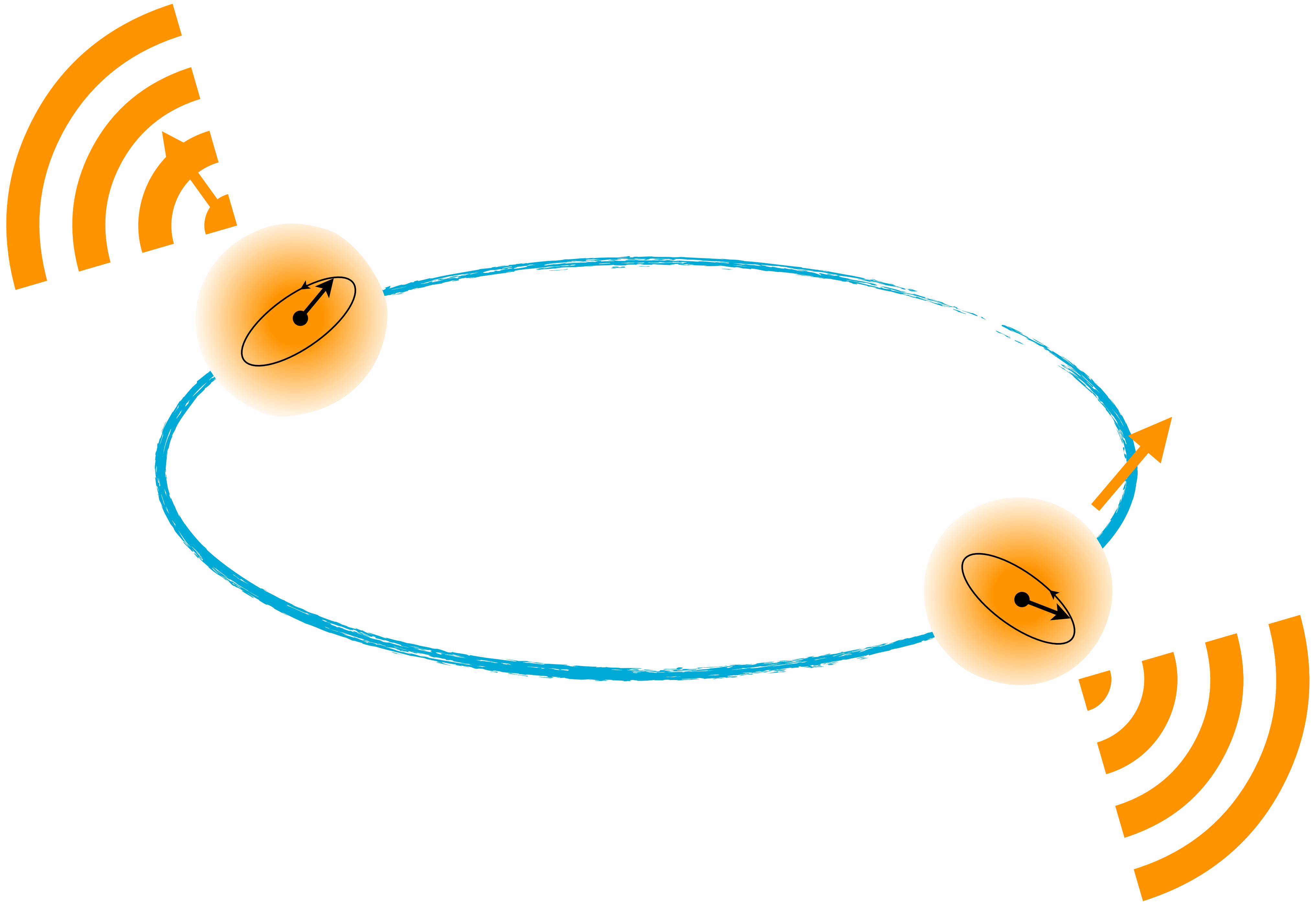


$\mathcal{C} = GM/Rc^2$

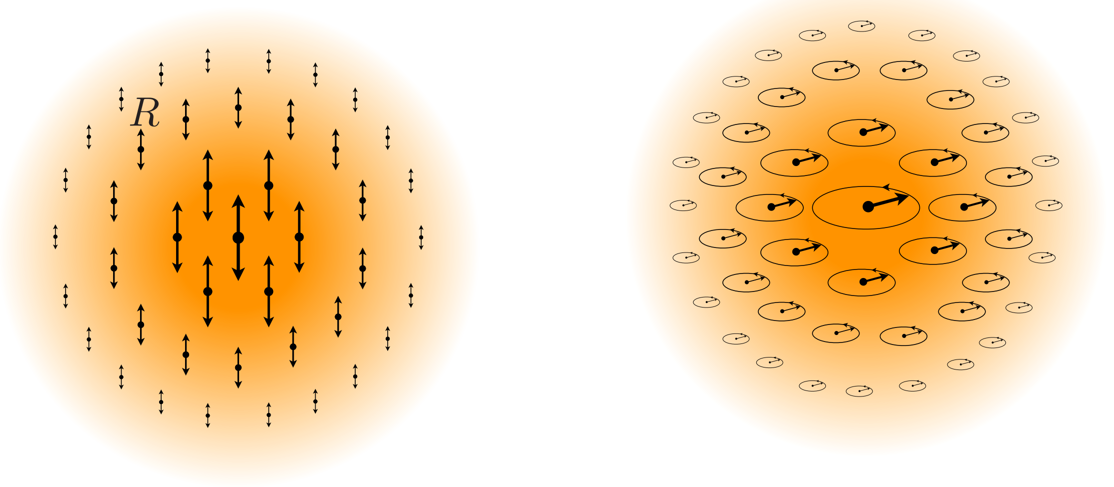
with Thomas Helfer & Zipeng Wang (soon, 2023)

gravitational waves and spin

$$V = -\frac{GM_1M_2}{r}\left[1 + \mathcal{O}(v^2/c^2) - \frac{2}{rc}[\hat{\boldsymbol{r}} \times (\boldsymbol{v}_1 - \boldsymbol{v_2})] \cdot \sum_{a=1}^{2} \frac{\boldsymbol{S}_a}{M_a}\right.$$
$$\left. + \frac{1}{r^2c^2}\left\{\frac{\boldsymbol{S}_1}{M_1} \cdot \frac{\boldsymbol{S}_2}{M_2} - 3\left(\frac{\boldsymbol{S}_1}{M_1} \cdot \hat{\boldsymbol{r}}\right)\left(\frac{\boldsymbol{S}_2}{M_2} \cdot \hat{\boldsymbol{r}}\right) + \sum_{a=1}^{2} \frac{C_{ES^2}^{(a)}}{2M_1M_2}\left[S_a^2 - 3(\boldsymbol{S}_a \cdot \hat{\boldsymbol{r}})^2\right]\right\} + \dots\right]$$



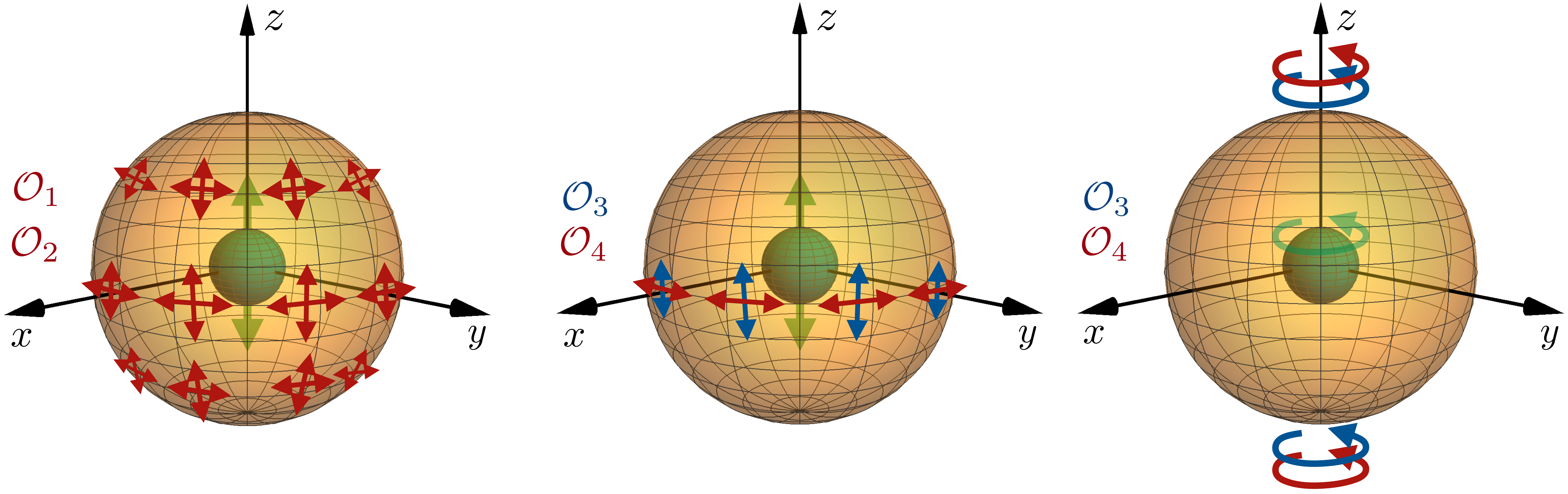
spin of soliton & polarization of photons



$$\mathcal{O}_1 = -\tfrac{1}{2} F_{\mu\nu}\tilde{F}^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_2 = -\tfrac{1}{2} F_{\mu\nu}F^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho}X^{\mu}X_{\nu}$$
$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^{\mu}X_{\nu}$$

explosive photon production (under certain conditions)

$$\mu R \gtrsim 1, \qquad \mu \sim g^2 X^2 m$$



with Schiappacasse & Long (2022)

early universe formation mechanism:

initial power spectrum — nonlinear structure

gravitational particle production to nonlinear structures

cannot easily do ultralight dark photons

$$\Omega_{\rm vdm} \sim 0.3 \left(\frac{m}{10^{-5}\,{\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^4$$

Graham, Mardon, Rajendran (2016)

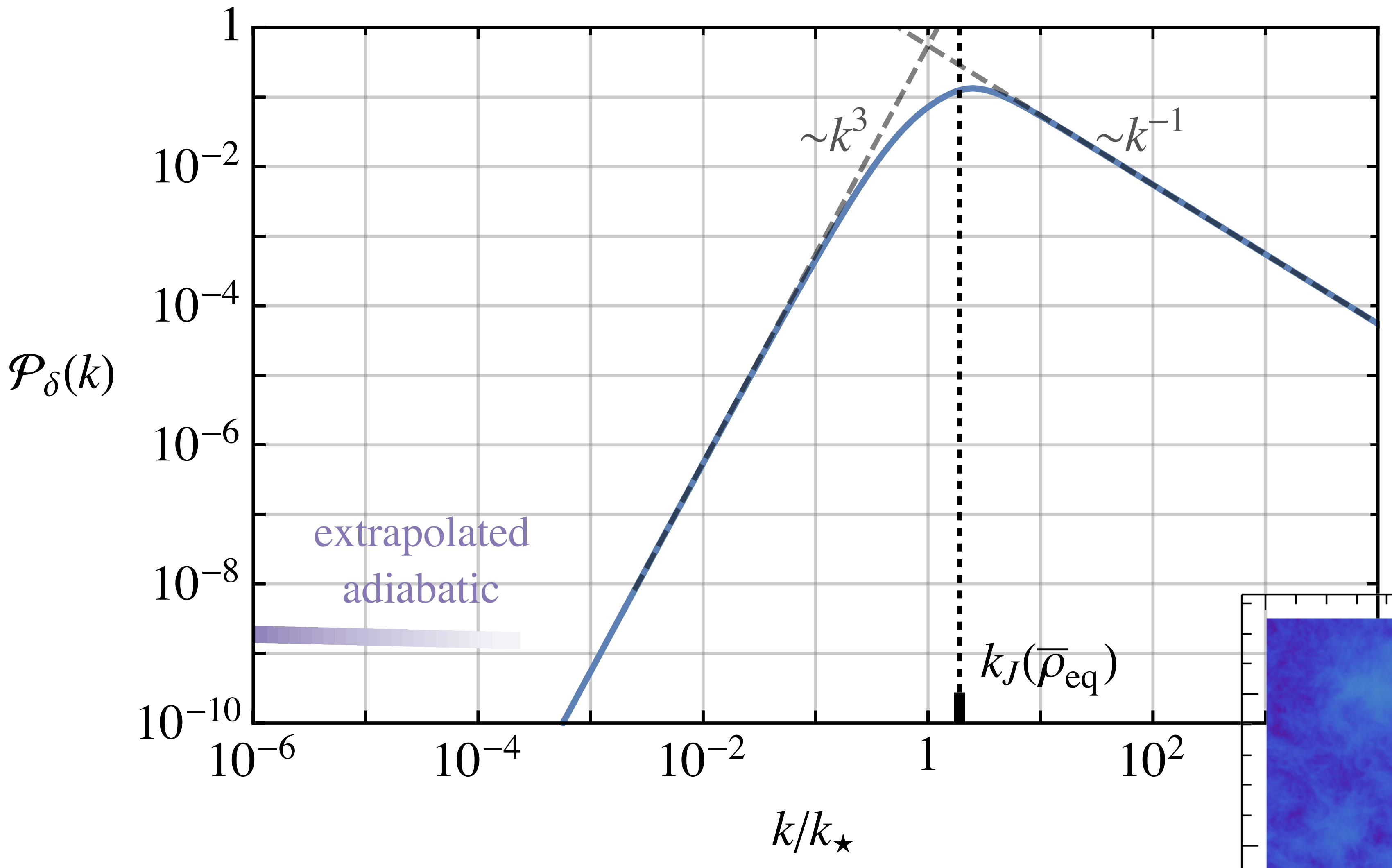
Ahmed, Grzadkowski, Socha (2020)

Kolb & Long (2020)

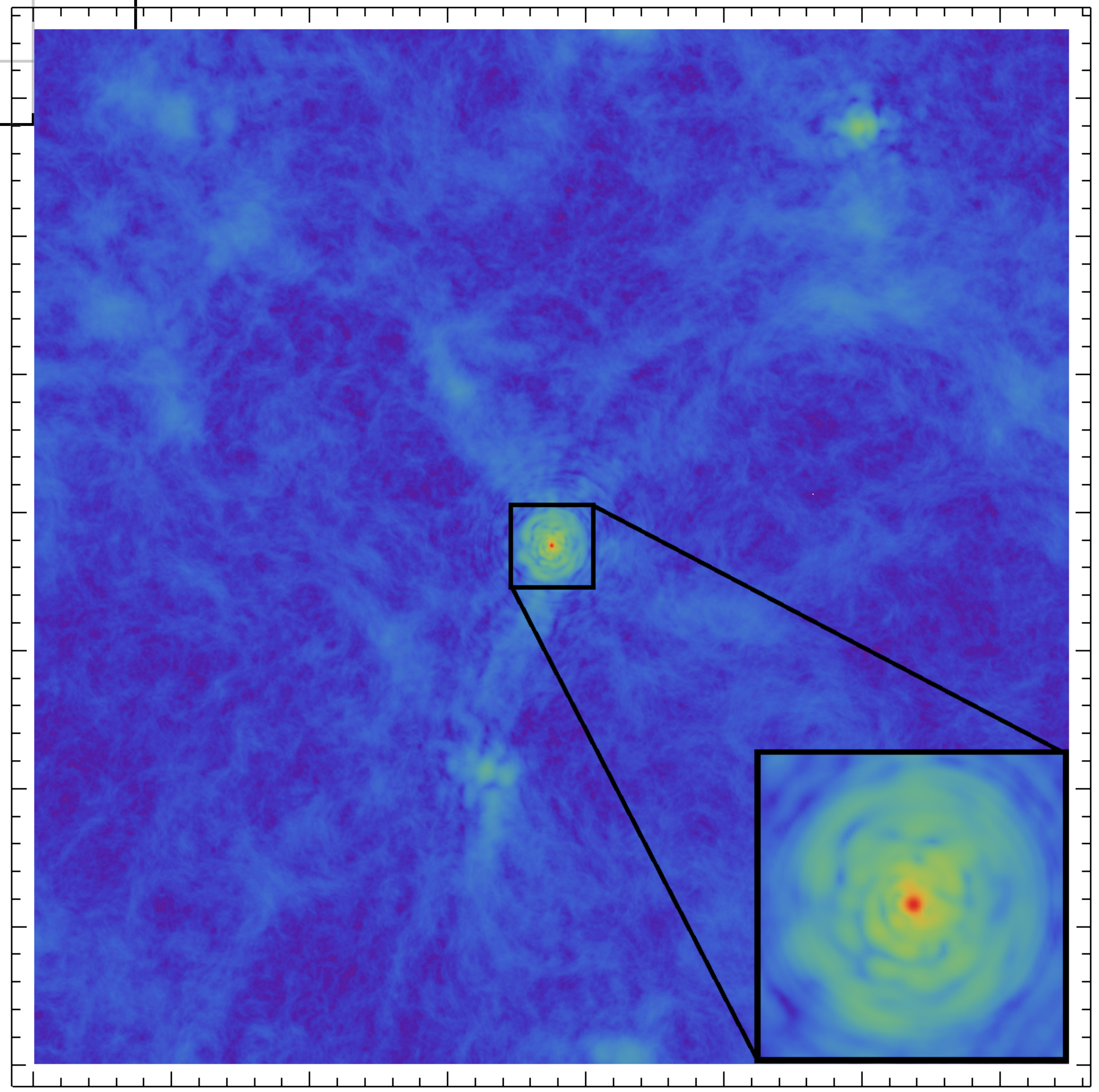
early derivation of action for longitudinal mode: Himmetoglu, Contaldi & Peloso (2008)

$$M_{\rm sol}(a) \sim 10^{-23} M_\odot \left(\frac{a_{\rm eq}}{a}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{3/2}$$

$$R_{\rm sol}(a) \sim 10^4\,{\rm km} \left(\frac{a}{a_{\rm eq}}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{1/2}$$



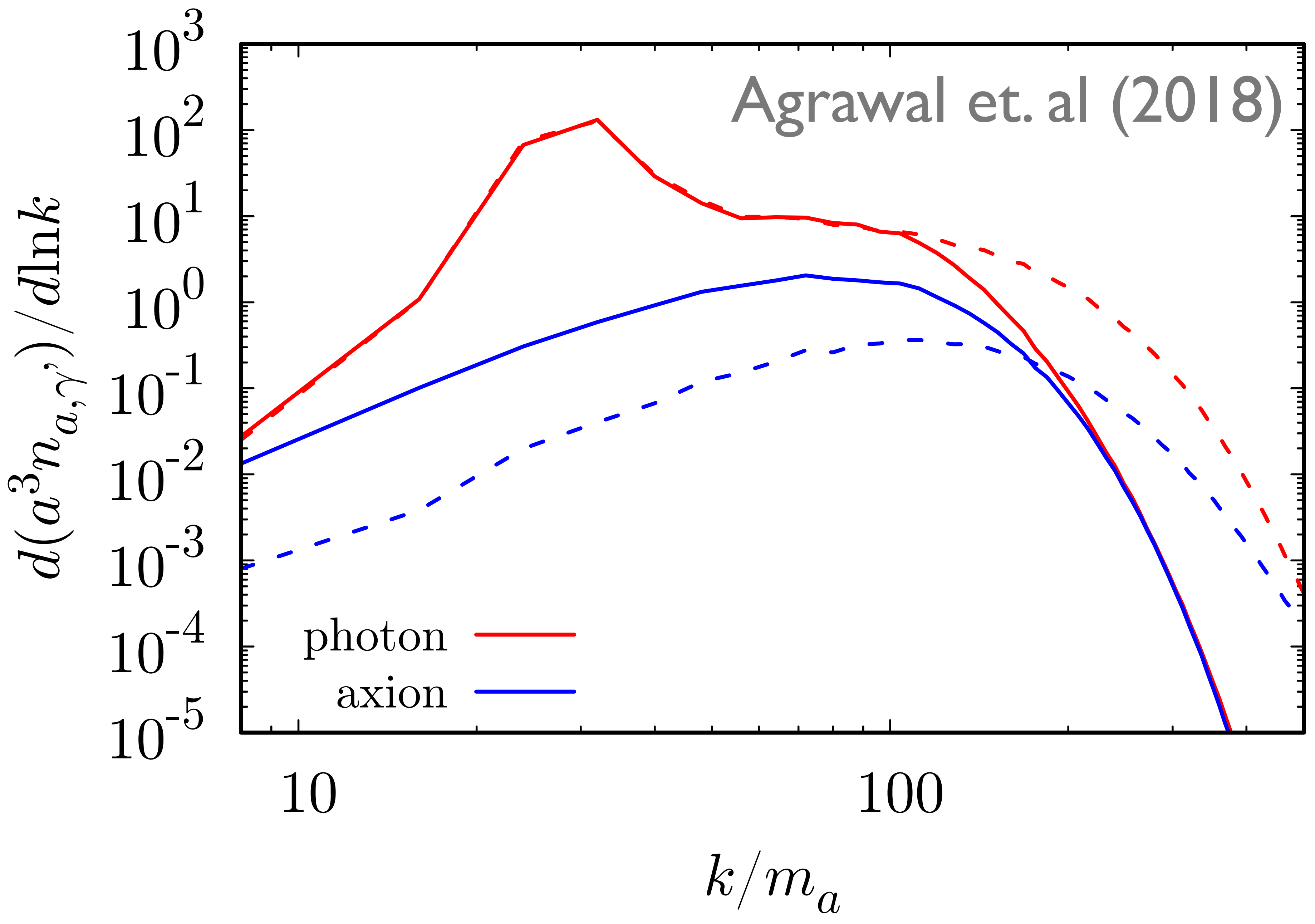
$$k_J \sim \sqrt{m H_{\rm eq}}$$



Gorghetto et. al (2022)

non-gravitational post-inflationary dark production?

$$\ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$



lower bound on mass ?

can do ultralight dark photons

Also see: Adshead, Lozanov and Weiner (2023)

Nakai, Namba and Obata (2023)

Co, Pierce, Zhang, and Zhao (2018)
Dror, Harigaya, and Narayan (2018)
Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018)

Also see: Long & Wang for production from strings and Co et. al for production from axion rotations

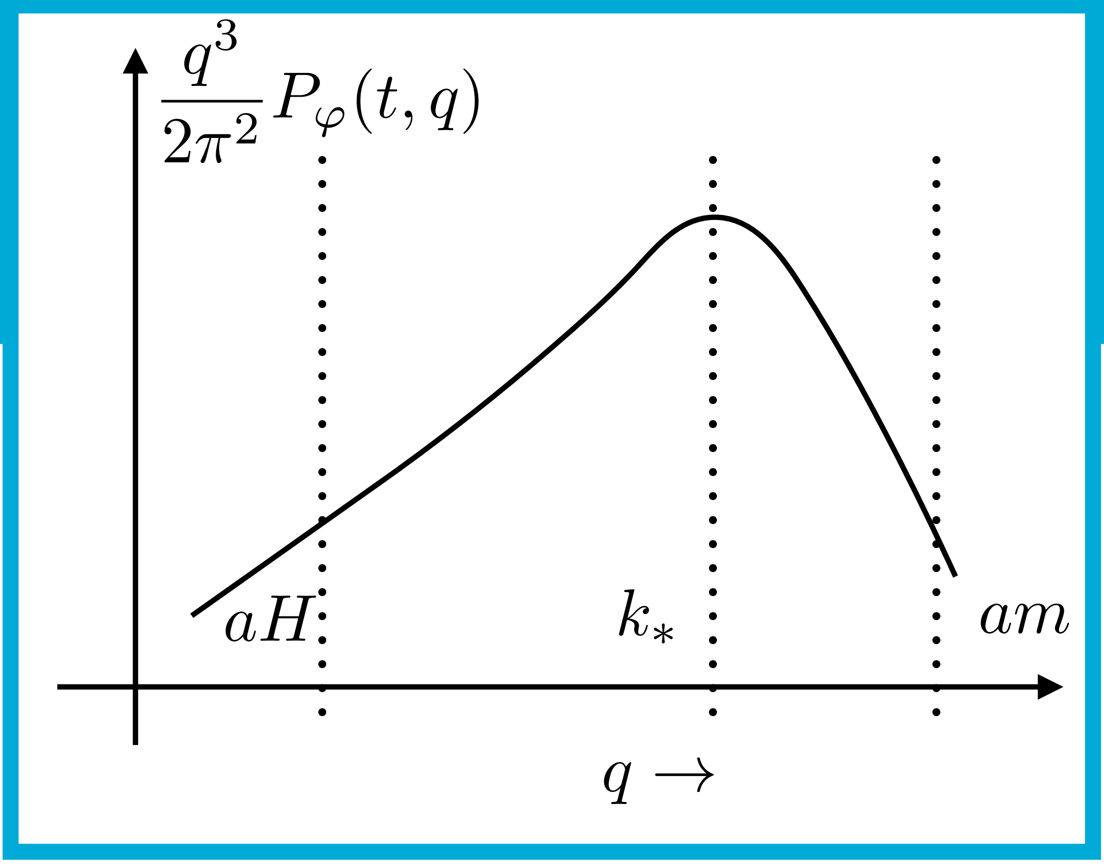
A lower bound on dark matter mass

Mustafa A. Amin RICE

with Mehrdad Mirbabayi (ICTP Trieste)

arXiv:2211.09775

a lower bound on dark matter mass



Dark matter density dominated by sub-Hubble field modes

$$\implies m \gtrsim 10^{-18}\,\mathrm{eV}$$

MA & Mirbabayi (2022)

our argument

Dark matter density dominated by sub-Hubble field modes

1. white-noise isocurvature excess in isocurvature density pert.
2. free-streaming suppression in adiabatic density pert.

1. and 2. not seen for $k < k_{\rm obs} \sim 10\,\mathrm{Mpc}^{-1}$

$$m \gtrsim 10^{-18}\,\mathrm{eV}$$

MA & Mirbabayi (2022)

strengths

"model independent" -- applies to all gravitationally interacting,

non-relativistic fields (scalar, vector, tensor ...)

"**loophole**" — inflationary production with infrared spectra (not sub-Hubble)

for vectors (and tensors?), even inflationary production leads to sub-Hubble spectra

$k_{\rm fs} \ll k_J \sim a\sqrt{mH} \implies$ stronger bound

$m_{\rm bound} \propto k_{\rm obs}^2 \implies$ look at MW satellites

with Nadler and Wechsler

comparison with literature

$m \gtrsim 2 \times 10^{-21}\,\mathrm{eV}$	Irsic et. al (2017) — Lyα
$m \gtrsim 3 \times 10^{-21}\,\mathrm{eV}$	Nadler et. al (2021) — MW satellites
$m \gtrsim 3 \times 10^{-19}\,\mathrm{eV}$	Dalal & Kravtsov (2022) — dynamical heating of stars
$m \gtrsim 4 \times 10^{-21}\,\mathrm{eV}$	Powell et. al (2023) — lensing
$m \gtrsim 10^{-18}\,\mathrm{eV}$	MA & Mirbabayi (2022)

*Above are model independent constraints, stronger constraints exist for particular models (Irsic, Xiao & McQuinn, 2020)

some details

*to us, results were “intuitively convincing” but quantitative calculation is non-trivial

*analytic calculation of density spectra, see appendix of MA & Mirbabayi (2022)

average density from field

$\varphi(t, \boldsymbol{x})$

light, but non-relativistic scalar field during rad. dom.

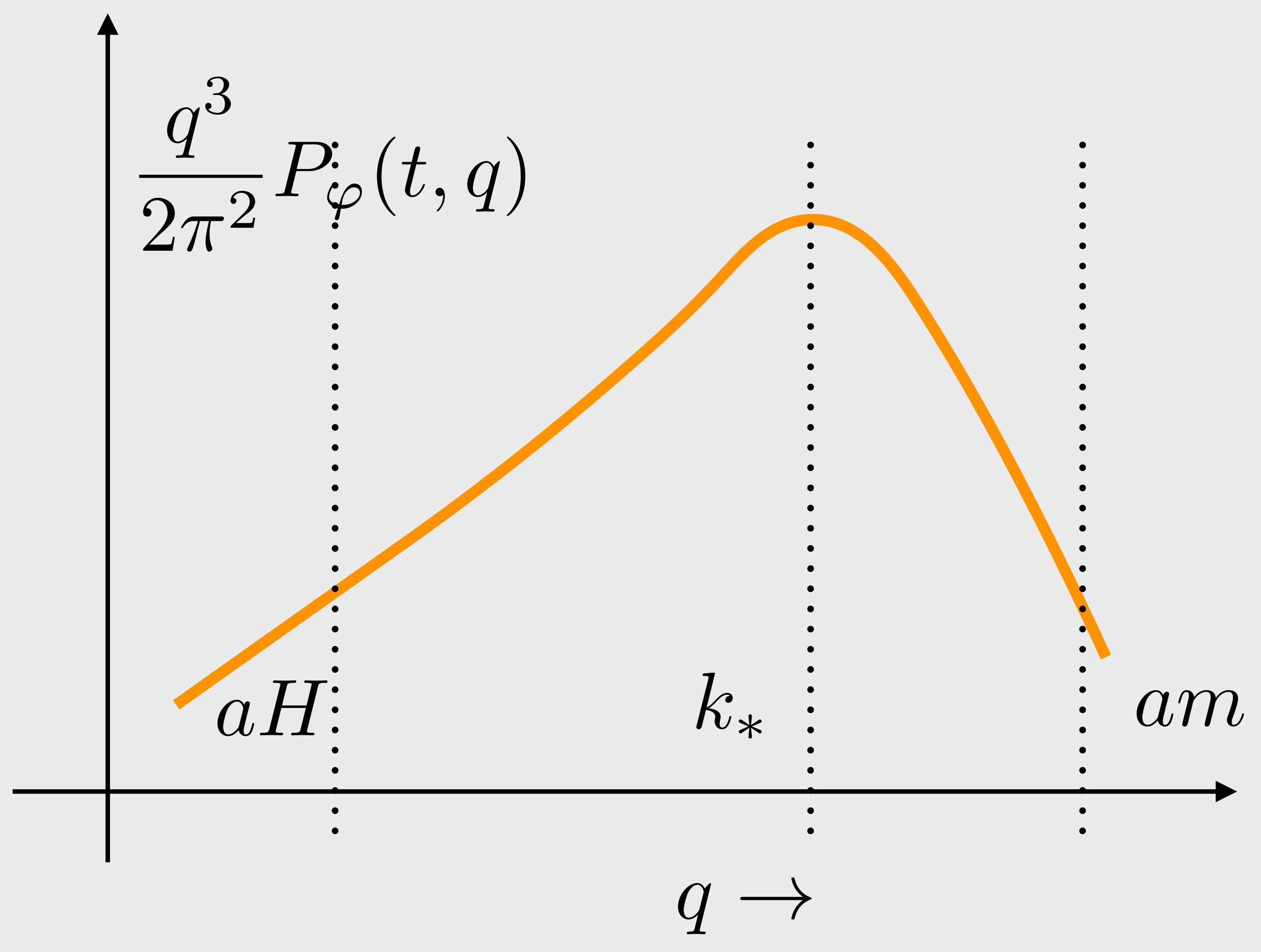
average density from field

$\varphi(t, \boldsymbol{x})$

light, but non-relativistic scalar field during rad. dom.

$$\bar{\rho}(t) \approx m^2 \int d\ln q \, \frac{q^3}{2\pi^2} P_\varphi(t, q)$$

dark matter density close to matter radiation eq.



average density from field

$\varphi(t, \boldsymbol{x})$

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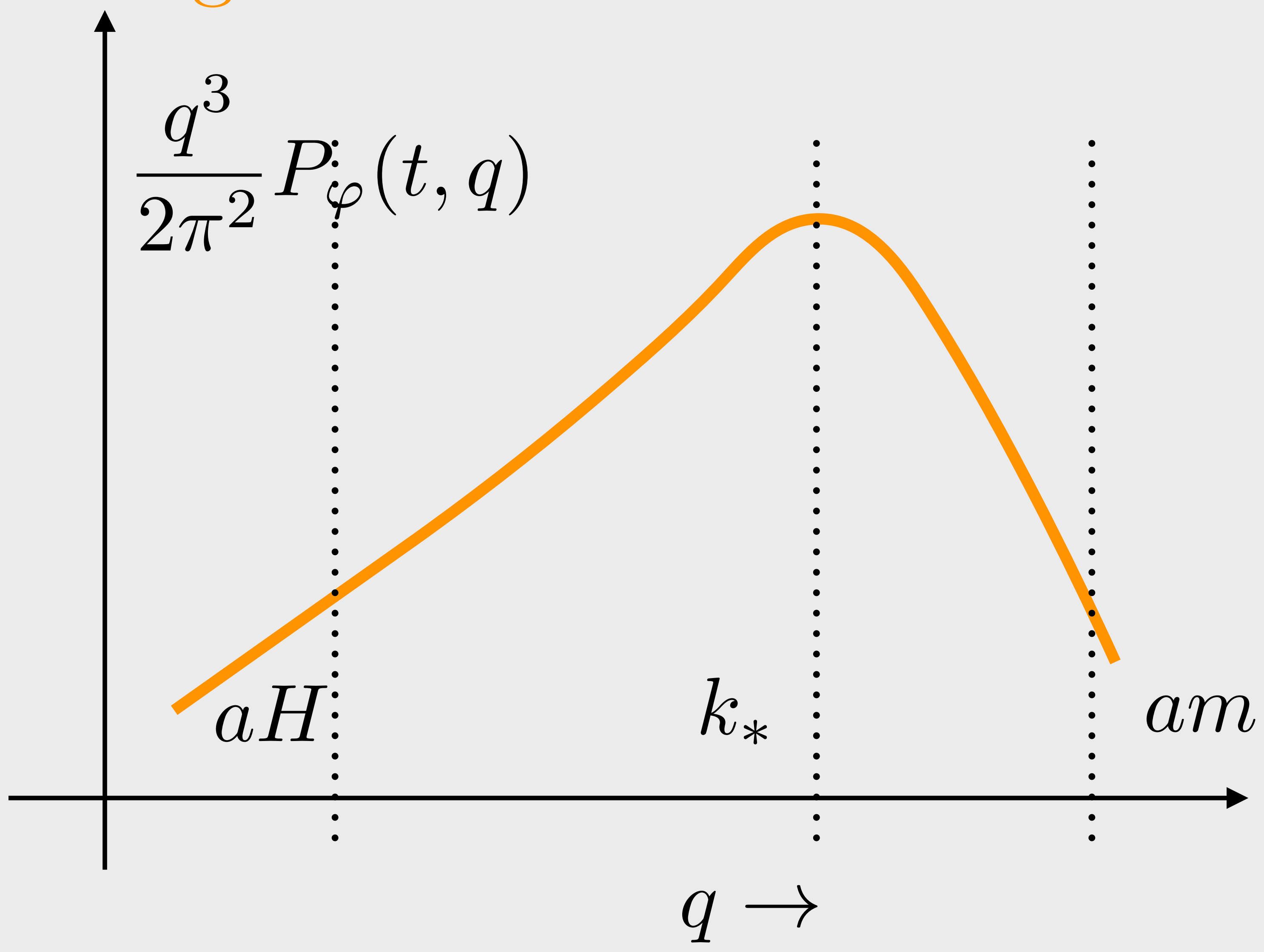
dark matter density close to matter radiation eq.

no significant zero mode of the field



$$\frac{q^3}{2\pi^2} P_\varphi(t, q)$$

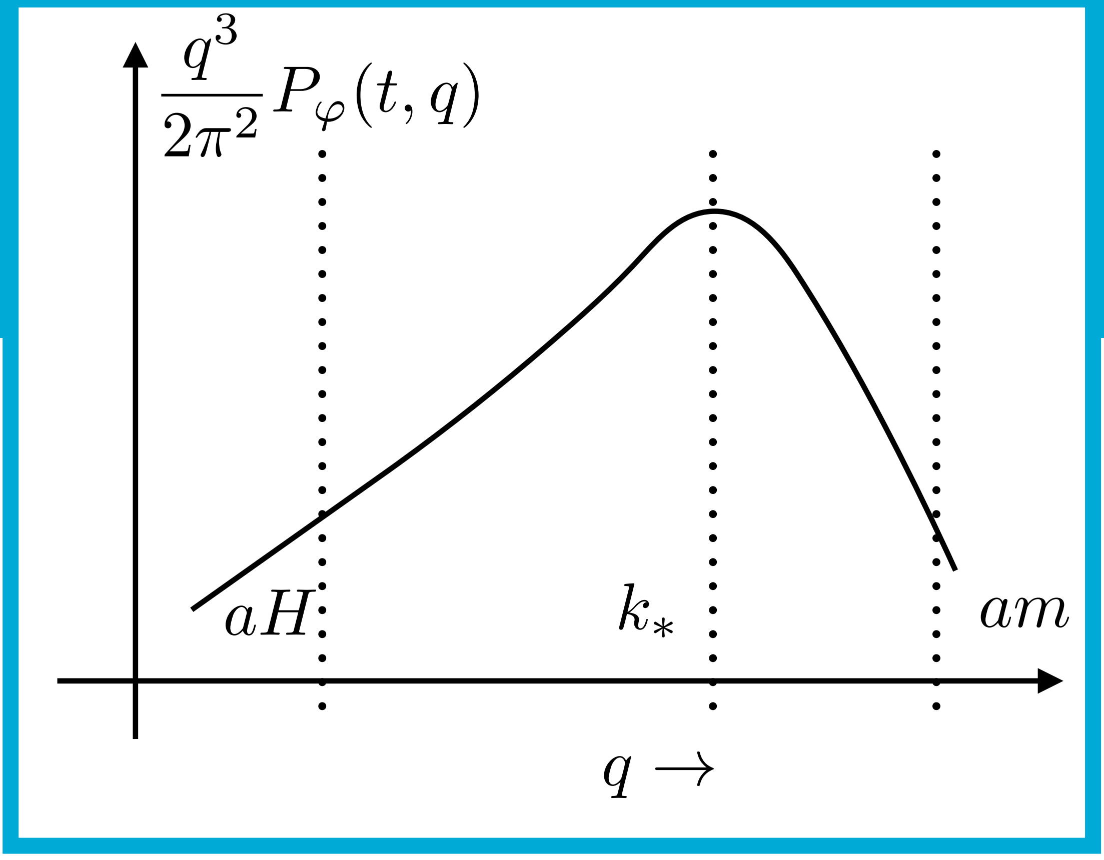
power spectrum of field, peaked at k_*

$a(t)H(t) \ll k_*$ holds for field produced after inflation

$k_* \ll a(t)m$ eventually non-relativistic to be DM

Such spectra are seen in Graham, Mardon & Rajendran (2015); Agrawal et.al (2018); Adshead, **Lozanov** & Weiner (2023); Nakai, Namba & **Obata** (2023) and others ... generally true for "causal" production mechanisms [early examples include axions with PQ breaking after inflation]

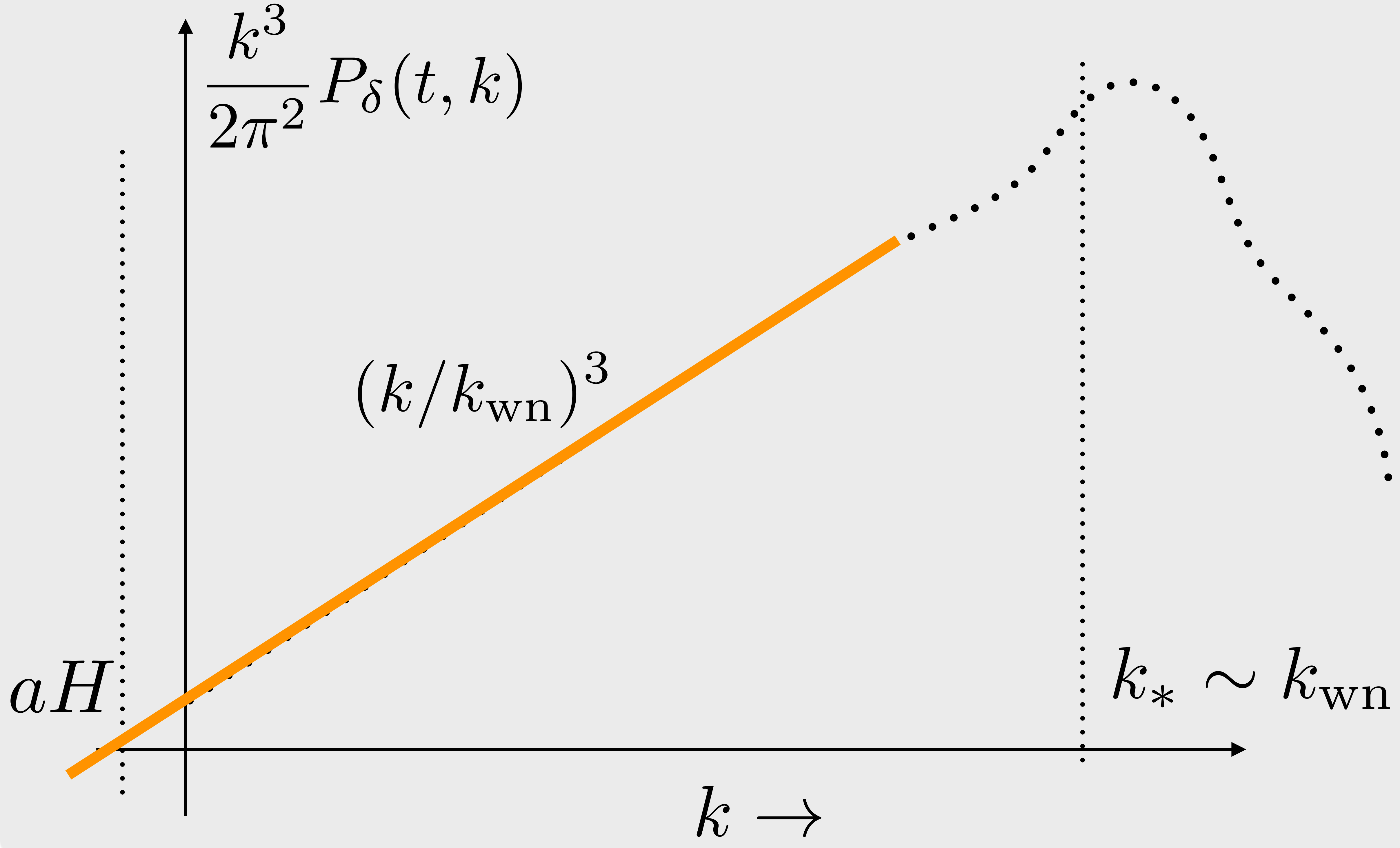
density power spectrum (isocurvature)



$$P_\delta^{(\mathrm{iso})}(t,k) \approx \frac{m^4}{\bar{\rho}^2(t)} \int d\ln q \, \frac{q^3}{2\pi^2} \left[P_\varphi(q,t)\right]^2 \equiv \frac{2\pi^2}{k_{\mathrm{wn}}^3}$$

independendent of k for $k \ll k_*$

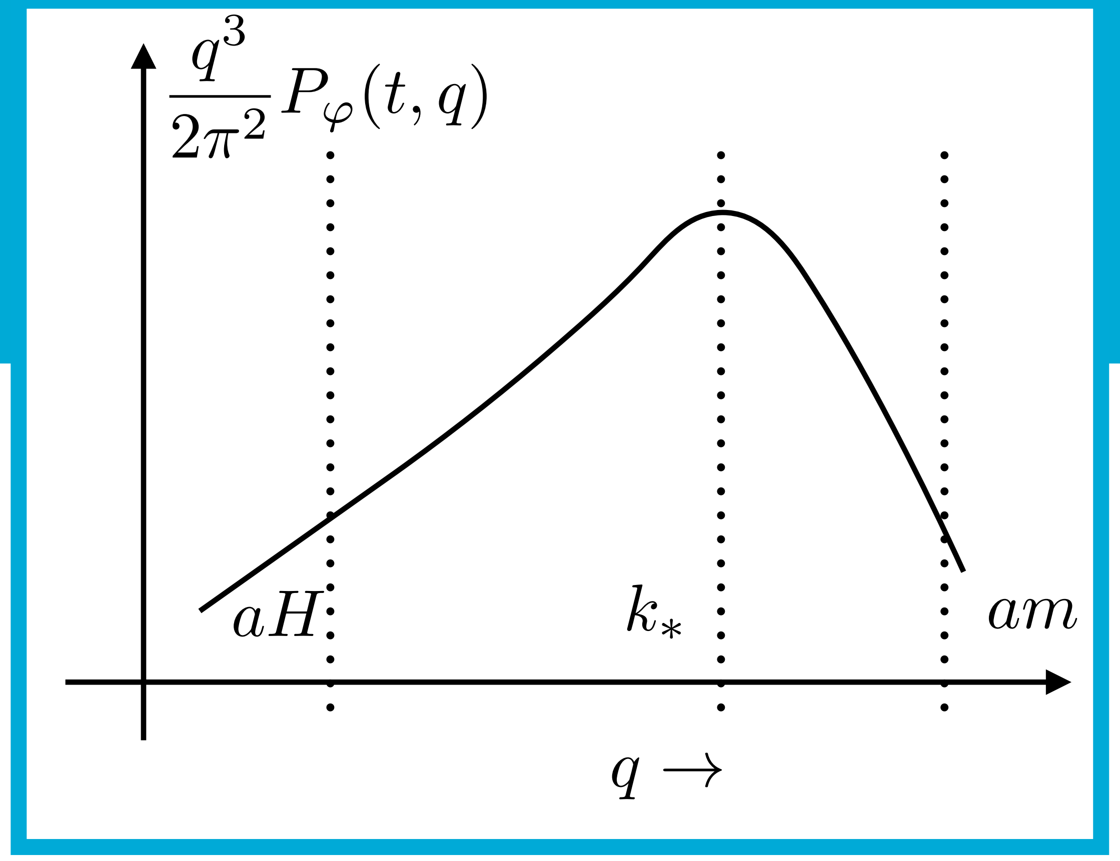
k_{wn} is defined by the above relation



*ignore gravitational potentials on these scales during radiation domination

density power spectrum (adiabatic)

density perturbations in DM sourced by gravitational potentials in rad.



free streaming !

$$\text{field power at } k_* \Longrightarrow k_{\rm fs}(t) \approx \frac{a^2 H m}{k_* \ln(2am/k_*)}$$

our argument — quantitative

Dark matter density dominated by $\boxed{\text{sub-Hubble}}$ field modes

1. $\boxed{\text{white-noise}}$ isocurvature excess in isocurvature density pert. $k_{\rm dev} \approx 10^{-3} k_*$
2. $\boxed{\text{free-streaming}}$ suppression in adiabatic density pert. $k_{\rm fs}(t) \approx \dfrac{a^2 H m}{k_* \ln(2am/k_*)}$

1. and 2. not seen for $k < k_{\rm obs} \sim 10\,{\rm Mpc}^{-1}$ e.g. [Lyα]

$$\boxed{k_{\rm dev}, k_{\rm fs} \gtrsim k_{\rm obs}}$$

$$m \gtrsim 10^{-18}\,{\rm eV}$$

Note that we did not need to know k_*!

strengths

"model independent" -- applies to all gravitationally interacting,

non-relativistic fields (scalar, vector, tensor ...)

"**loophole**" — inflationary production with infrared spectra (not sub-Hubble)

for vectors (and tensors?), even inflationary production leads to sub-Hubble spectra

$$k_{\mathrm{fs}} \ll k_J \sim a\sqrt{mH} \implies \text{stronger bound}$$

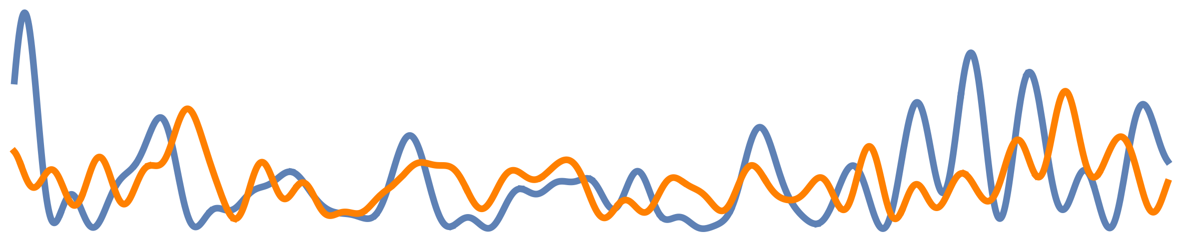
$$m_{\mathrm{bound}} \propto k_{\mathrm{obs}}^2 \implies \text{look at MW satellites}$$

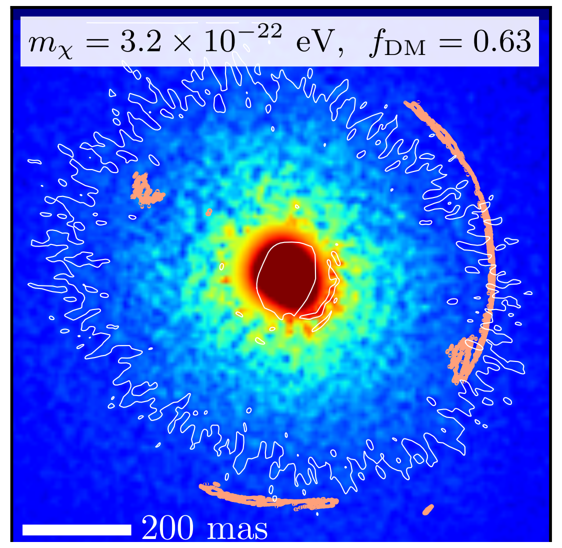
with Nadler and Wechsler

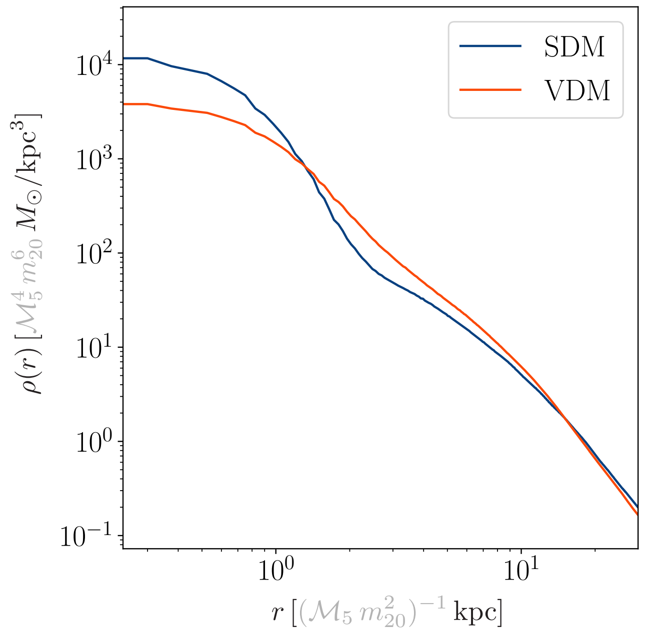
summary

Phenomenology

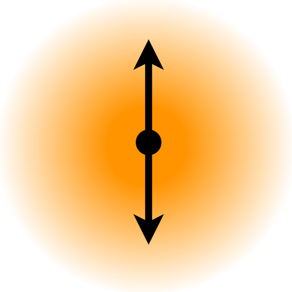
- reduced interference

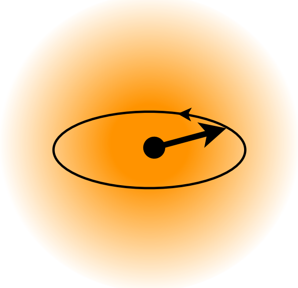


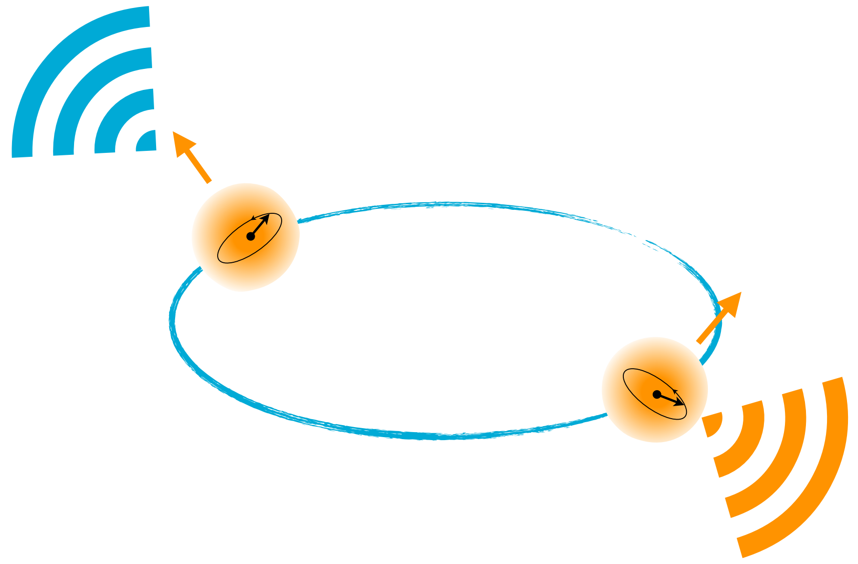




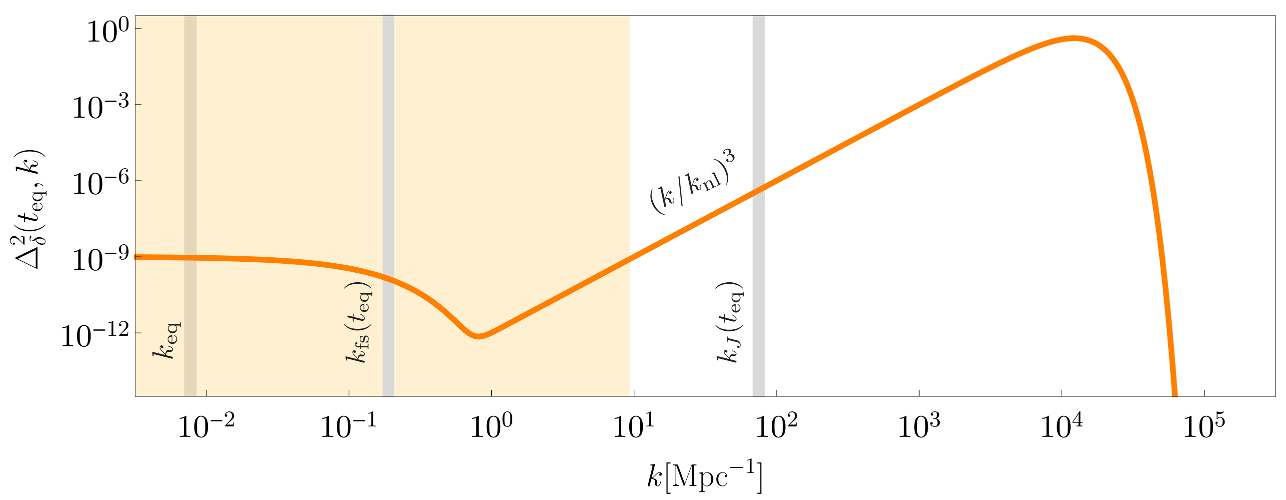
- polarized solitons, with macroscopic spin

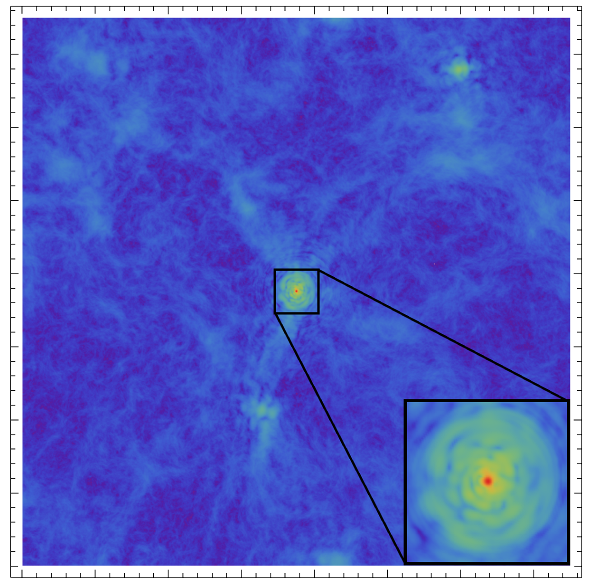


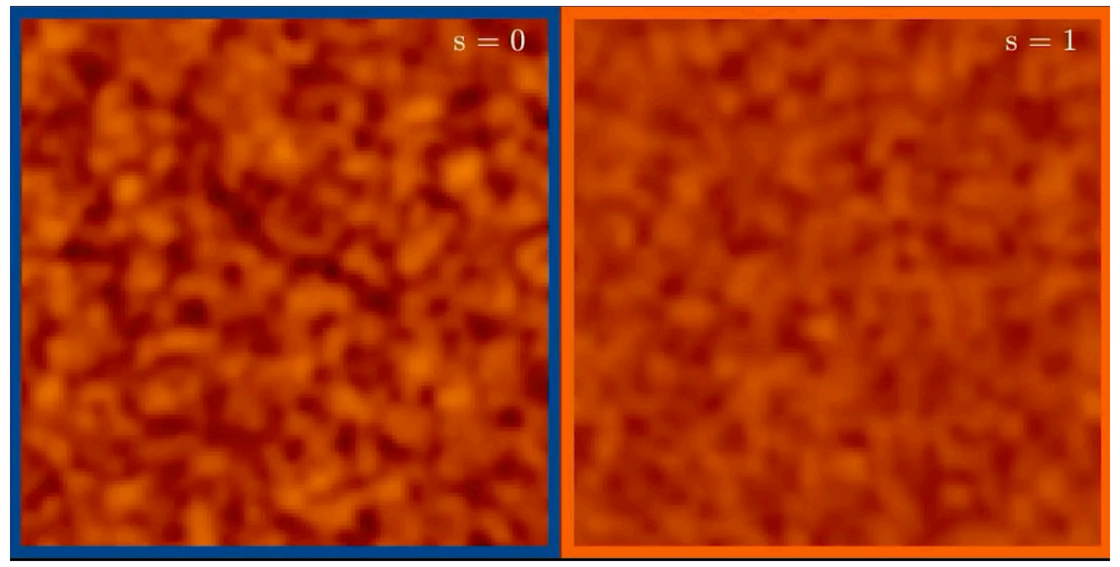


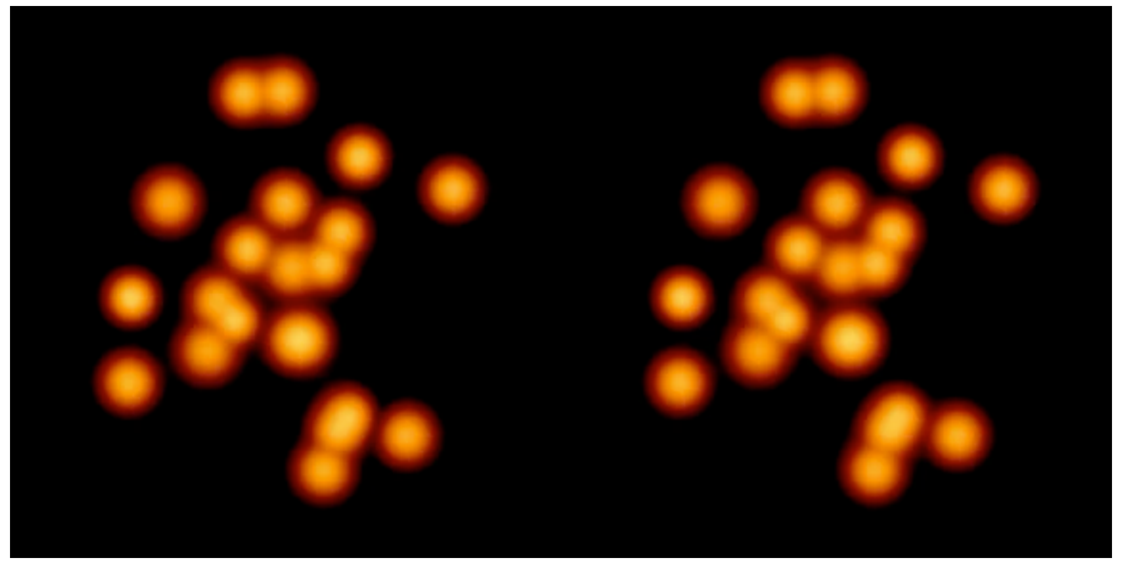


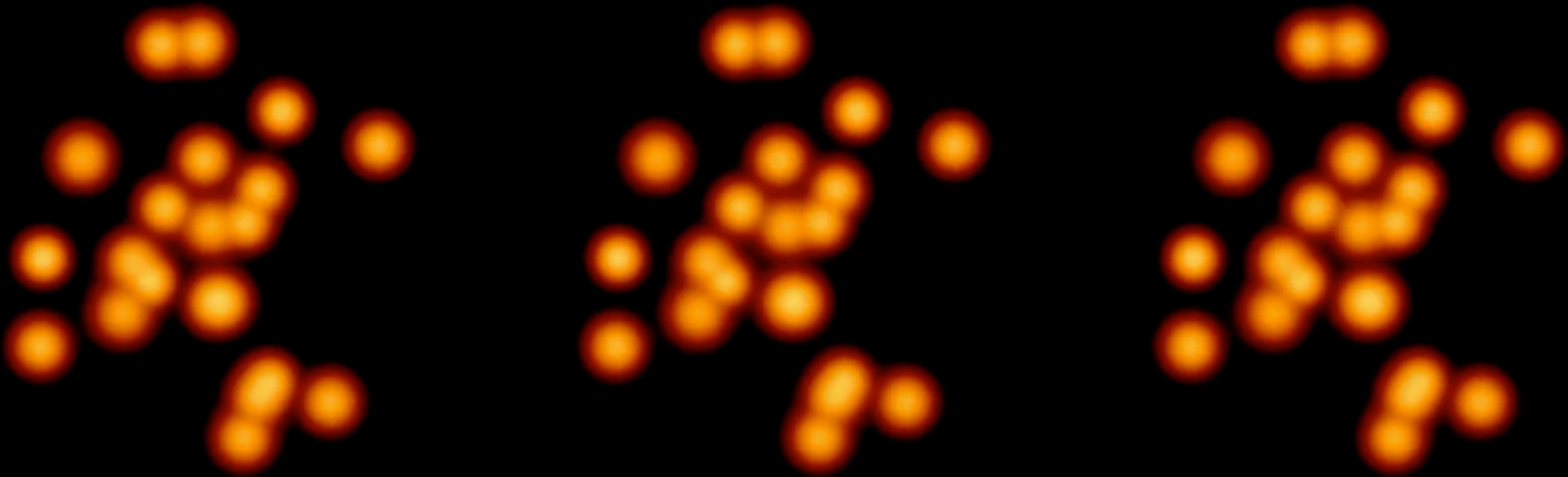
- Mass bound, growth of structure, nucleation time-scales





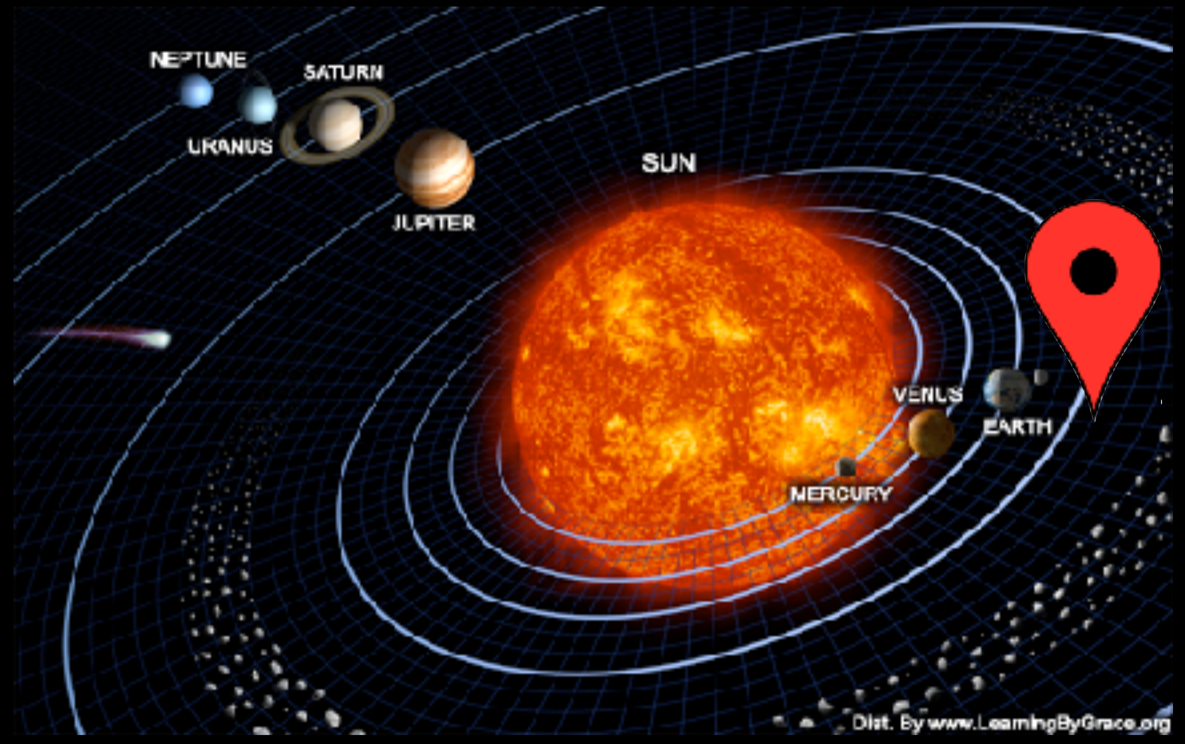






MA, Jain, Karur & Mocz (2022)

RICE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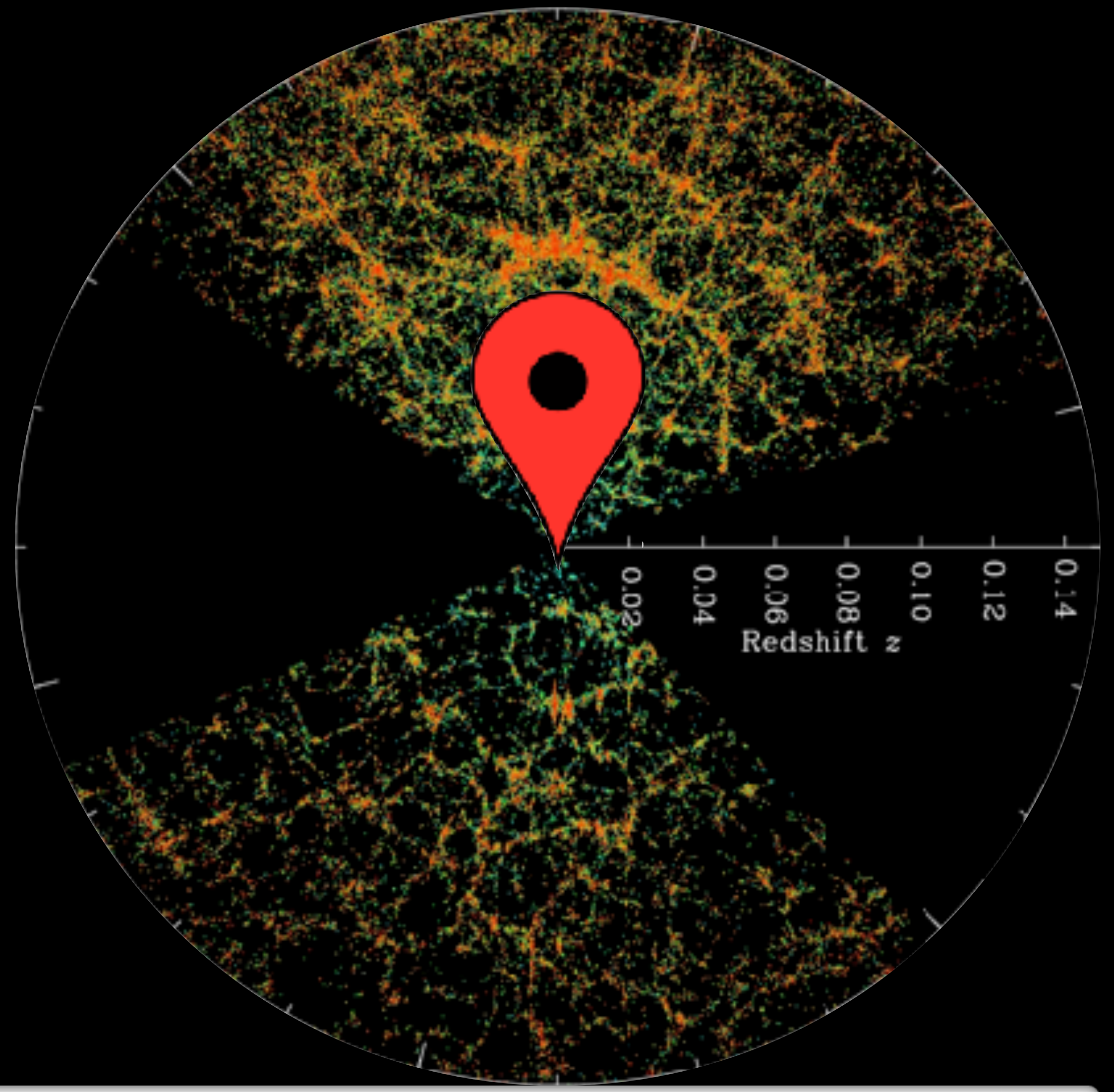
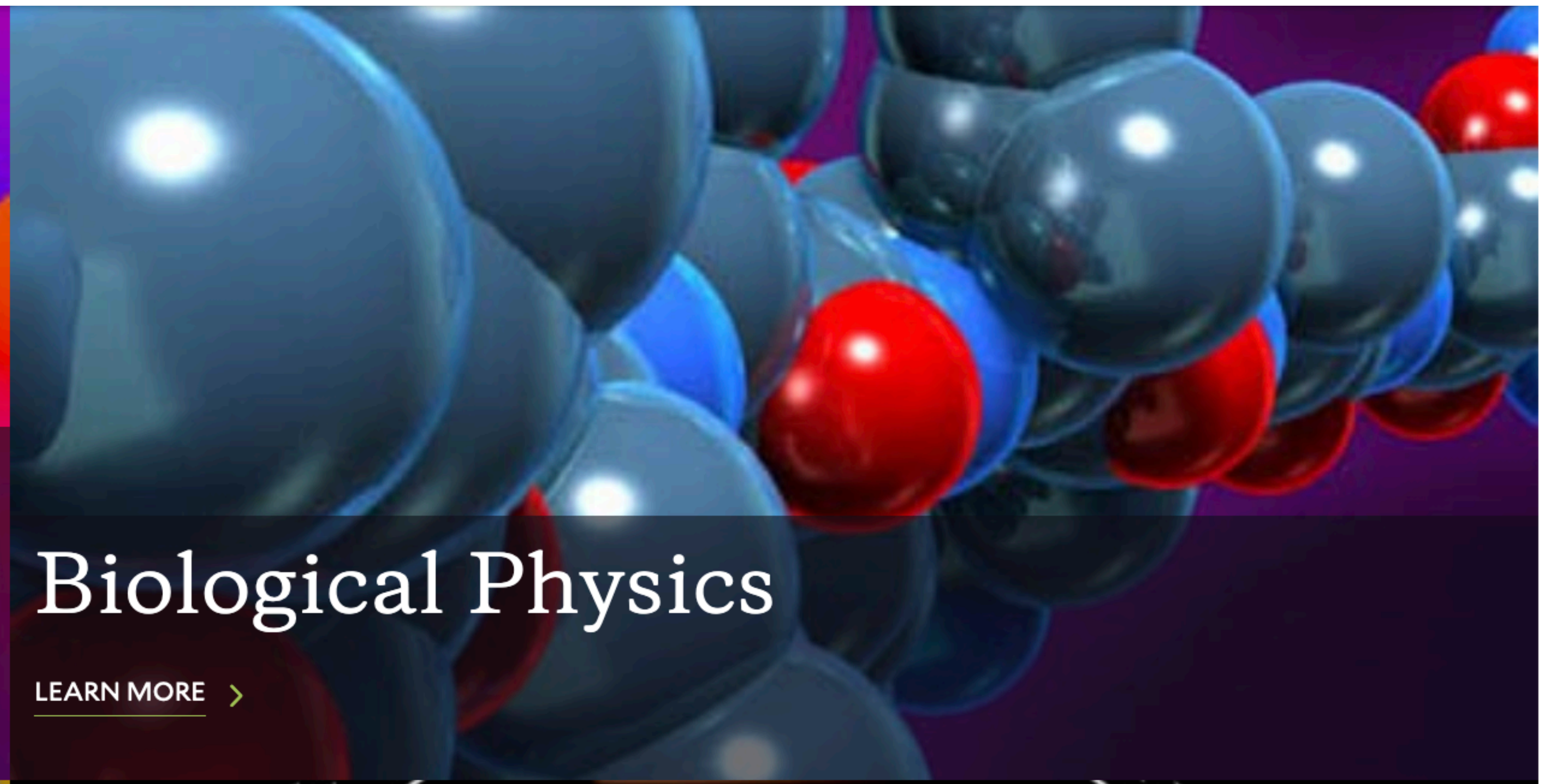
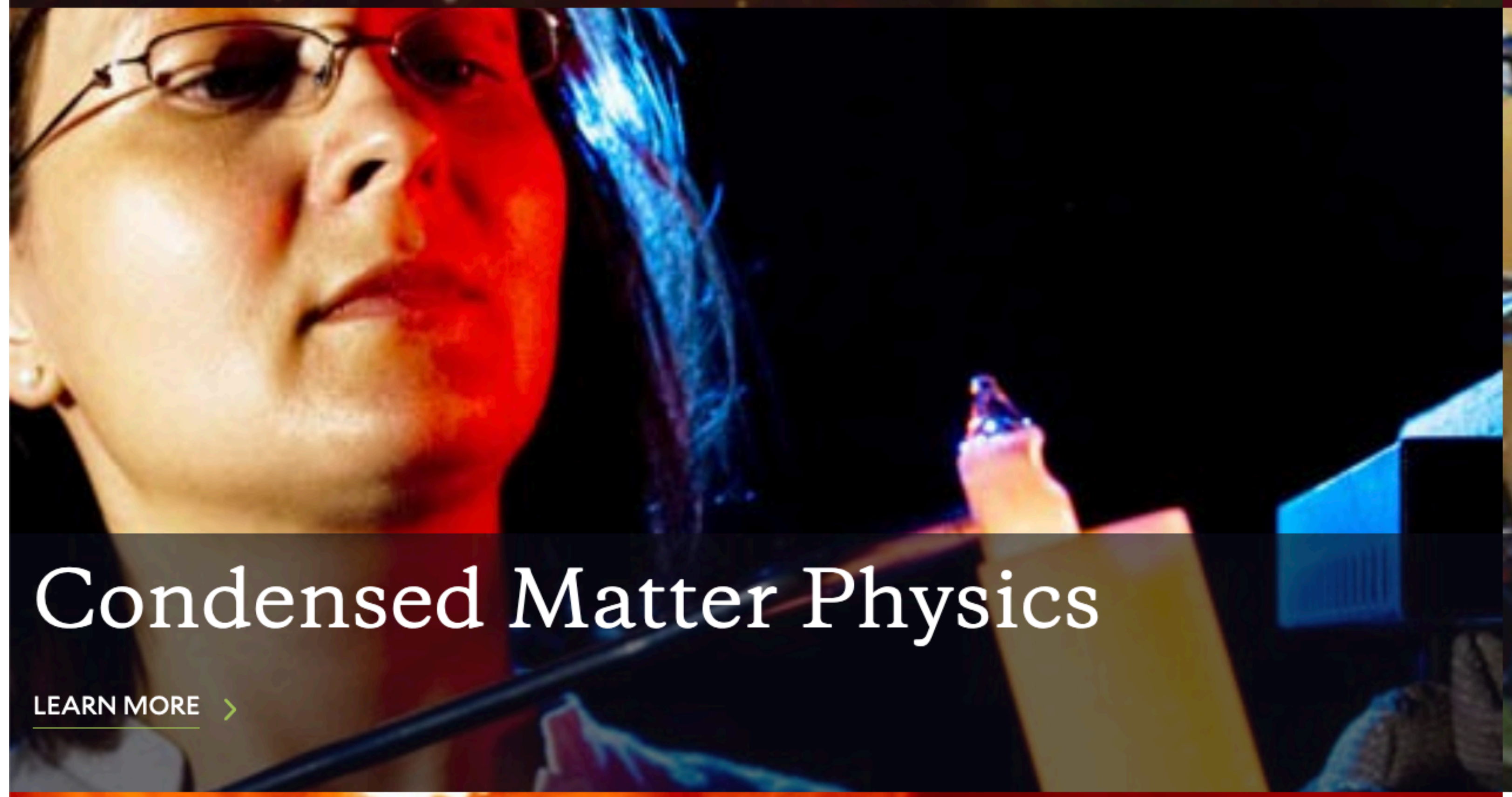
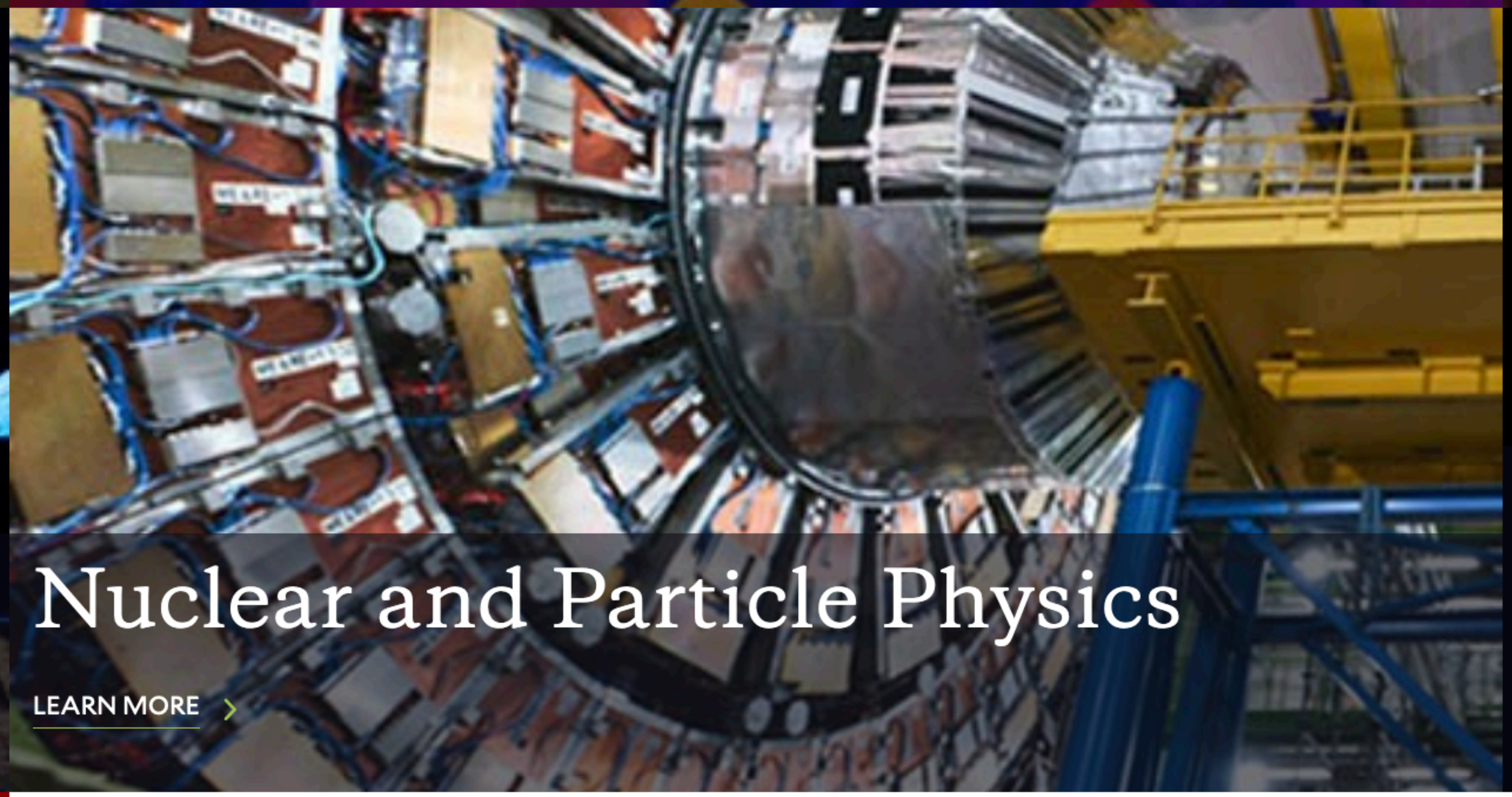
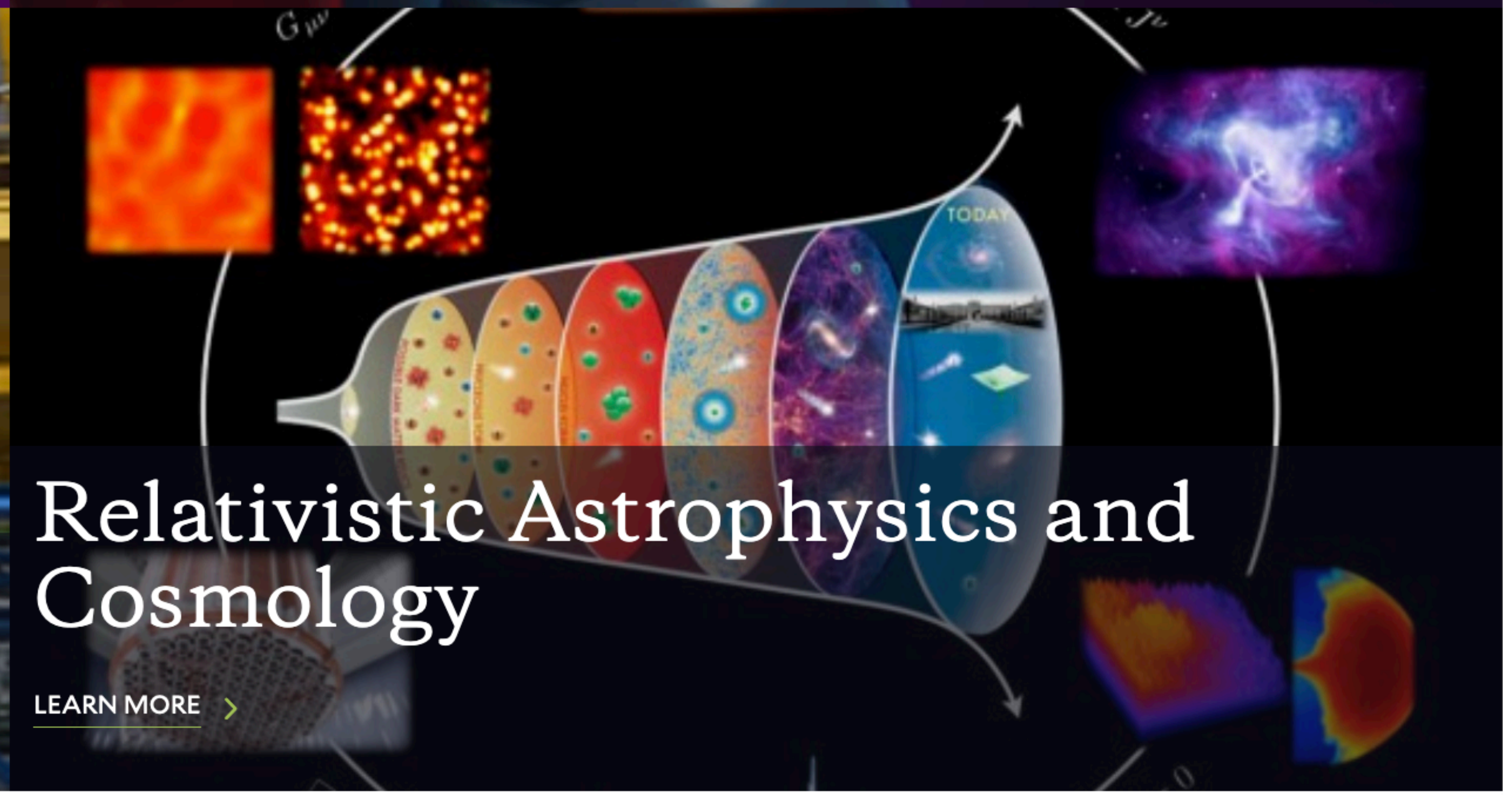
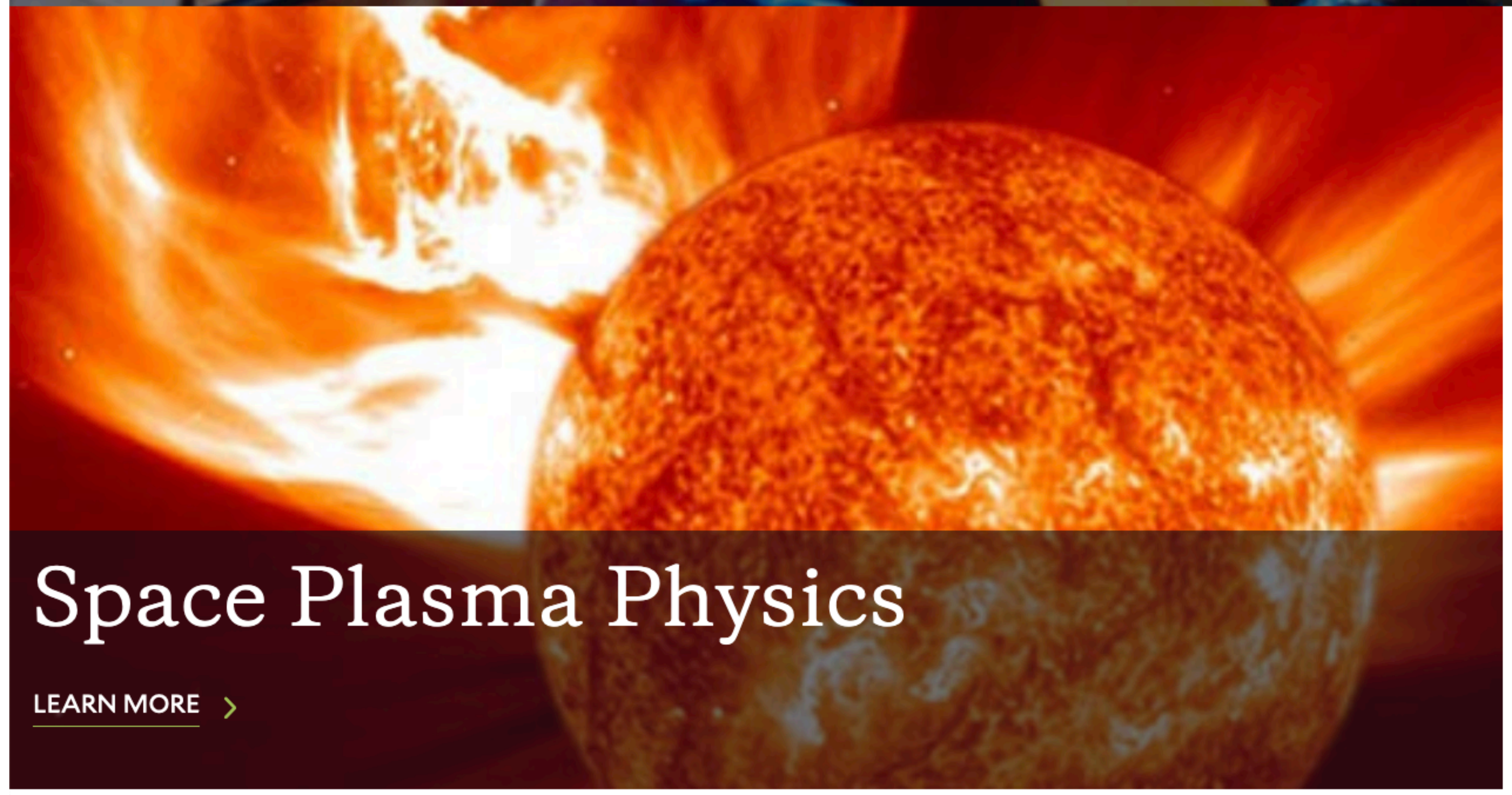

0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z











RICE NATURAL SCIENCES
Department of Physics and Astronomy

~ 40 faculty

~ 25 postdocs

~ 110 grad students