A Spin on Wave Dark Matter

Mustafa A. Amin

Small Scale Structure in Vector Dark Matter

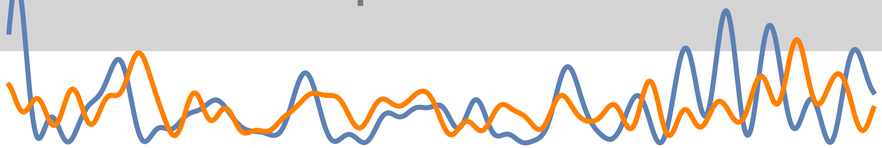
Mustafa A. Amin

generalization 3 —> $(2s+1)$

main results

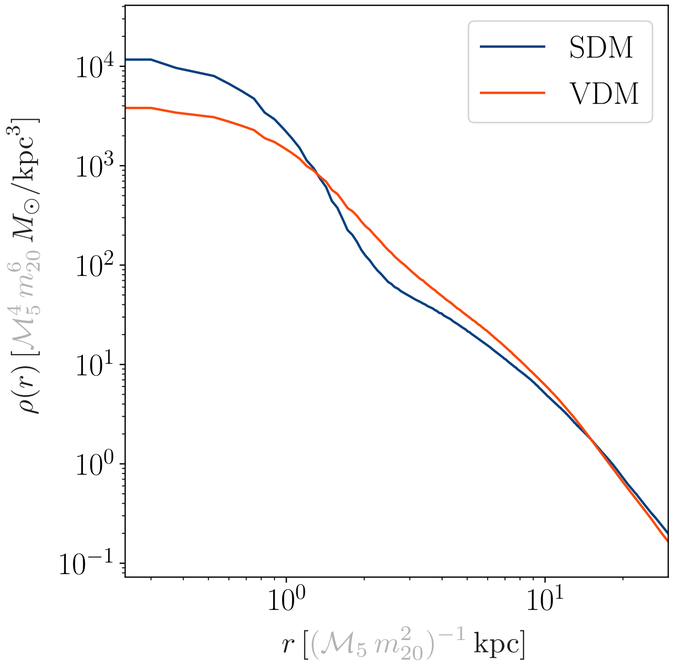
distinct phenomenology in (ultra)light vector dark matter

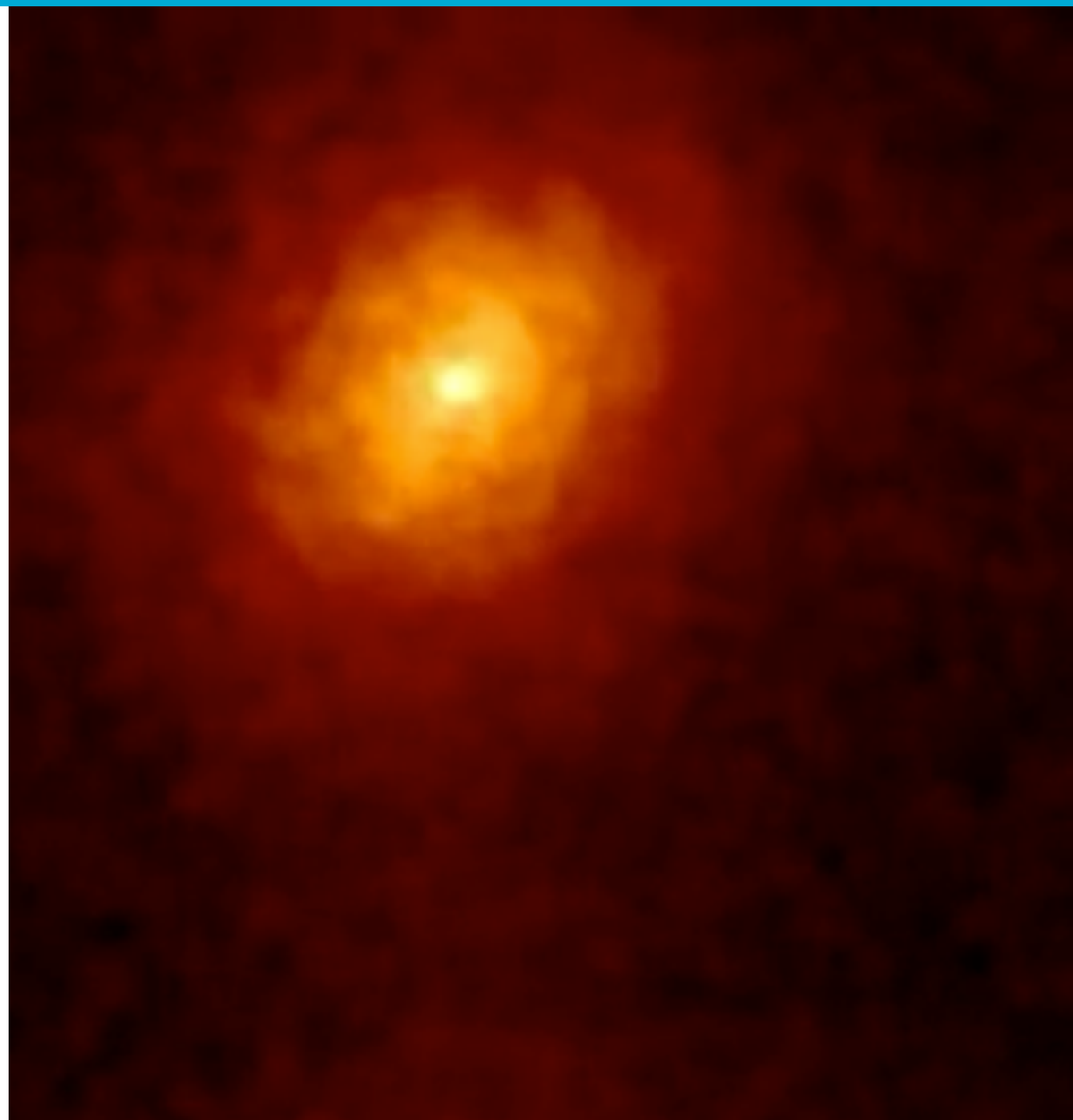
- interference patterns, and halo density profiles



$(\delta\rho/\rho)_{\rm VDM} \approx (\delta\rho/\rho)_{\rm SDM}/\sqrt{3}$

$\lambda_{\rm dB}({\rm VDM}) \approx \lambda_{\rm dB}({\rm SDM})$

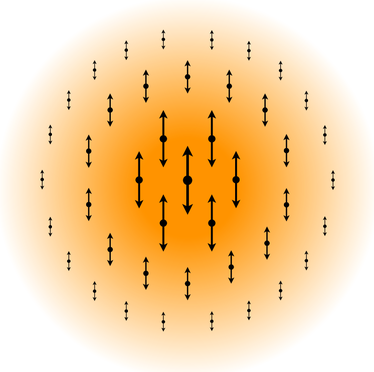
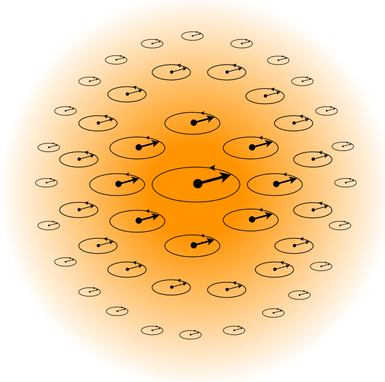




distinct phenomenology in (ultra)light vector dark matter

- interference patterns, and halo density profiles

- new class of polarized vector solitons with macroscopic spin

c_0  $+c_1$ 

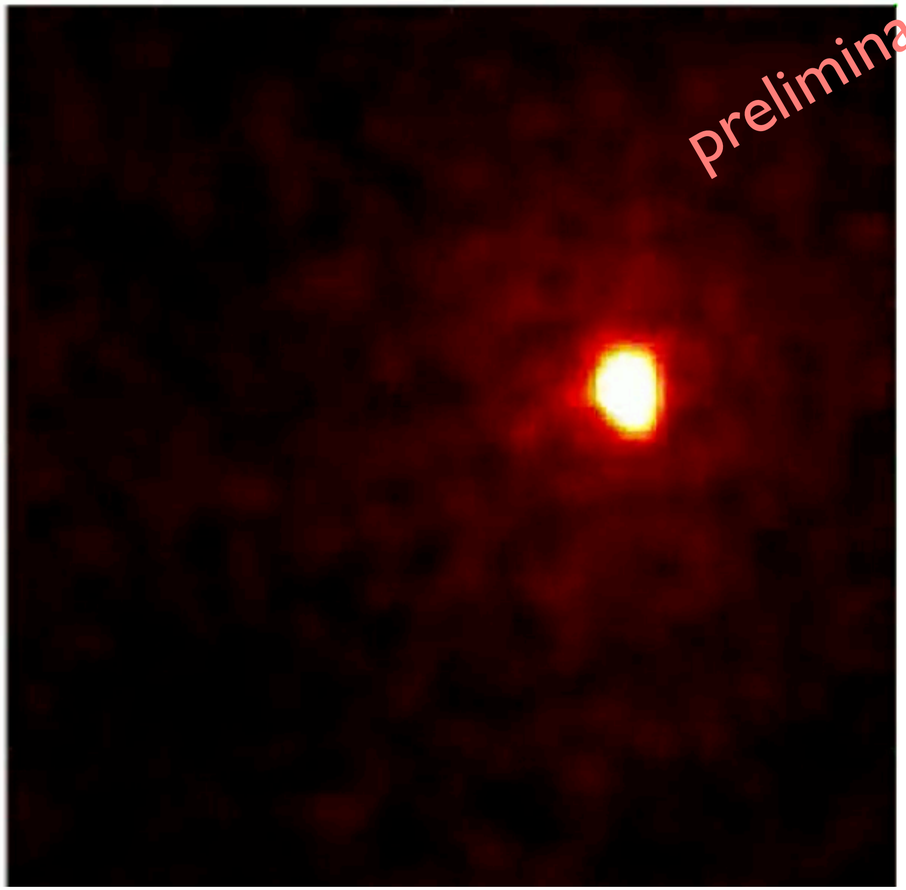
$$0 \leq |\boldsymbol{S}_{\rm tot}| \leq \frac{M_{\rm sol}}{m}\hbar$$

distinct phenomenology in (ultra)light vector dark matter

- interference patterns, and halo density profiles

- new class of polarized vector solitons with macroscopic spin

- VDM formation and soliton nucleation



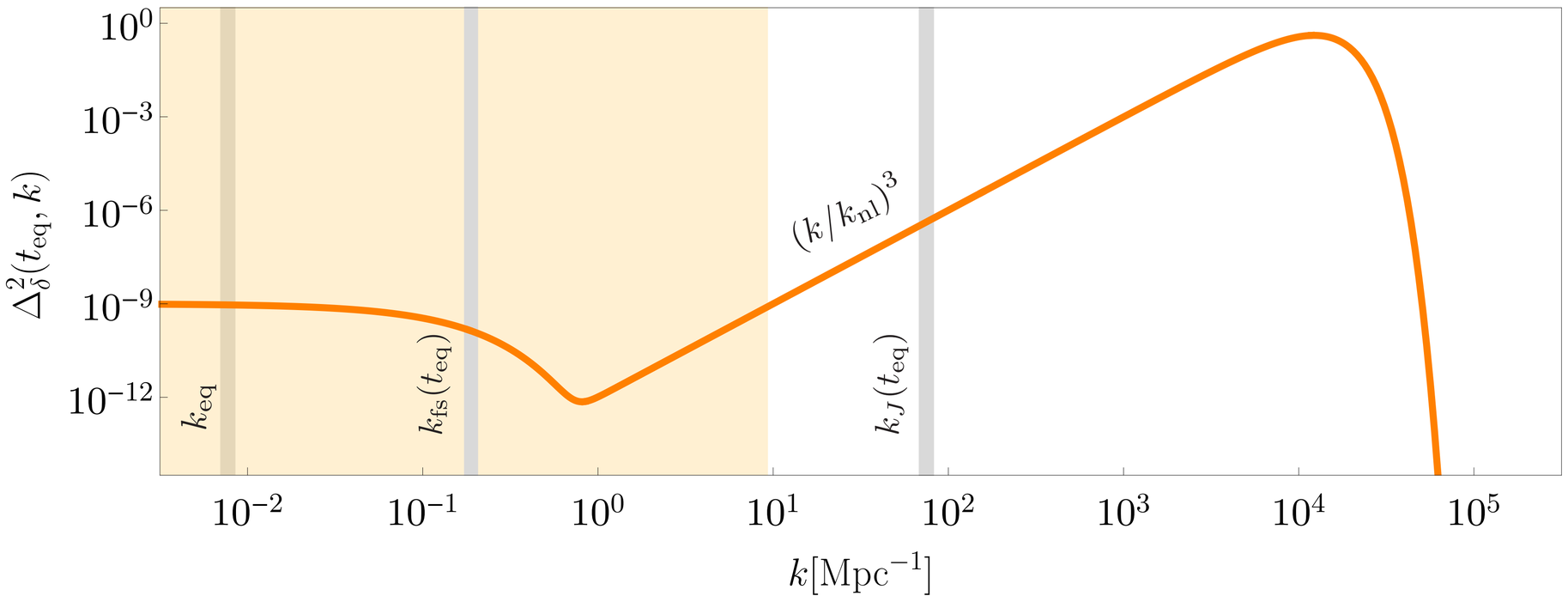
I may not have time for …

- post-inflationary formation of any dark matter

MA & Mirbabayi (2022)

$$m \gtrsim 10^{-18}\,\mathrm{eV}$$

lower bound on the mass



motivation & introduction

dark matter mass ?

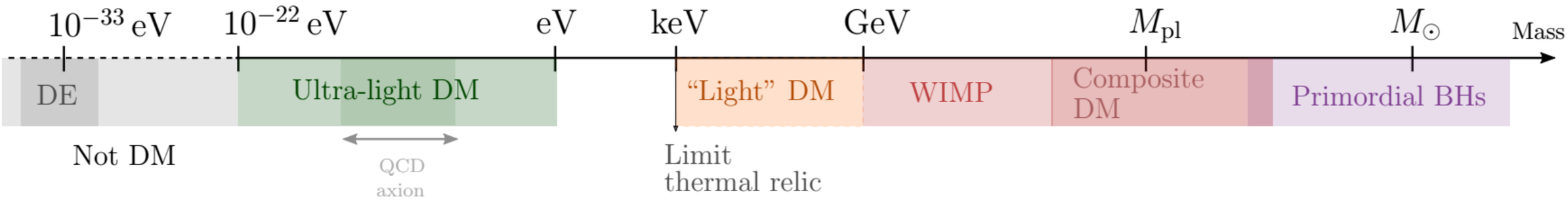
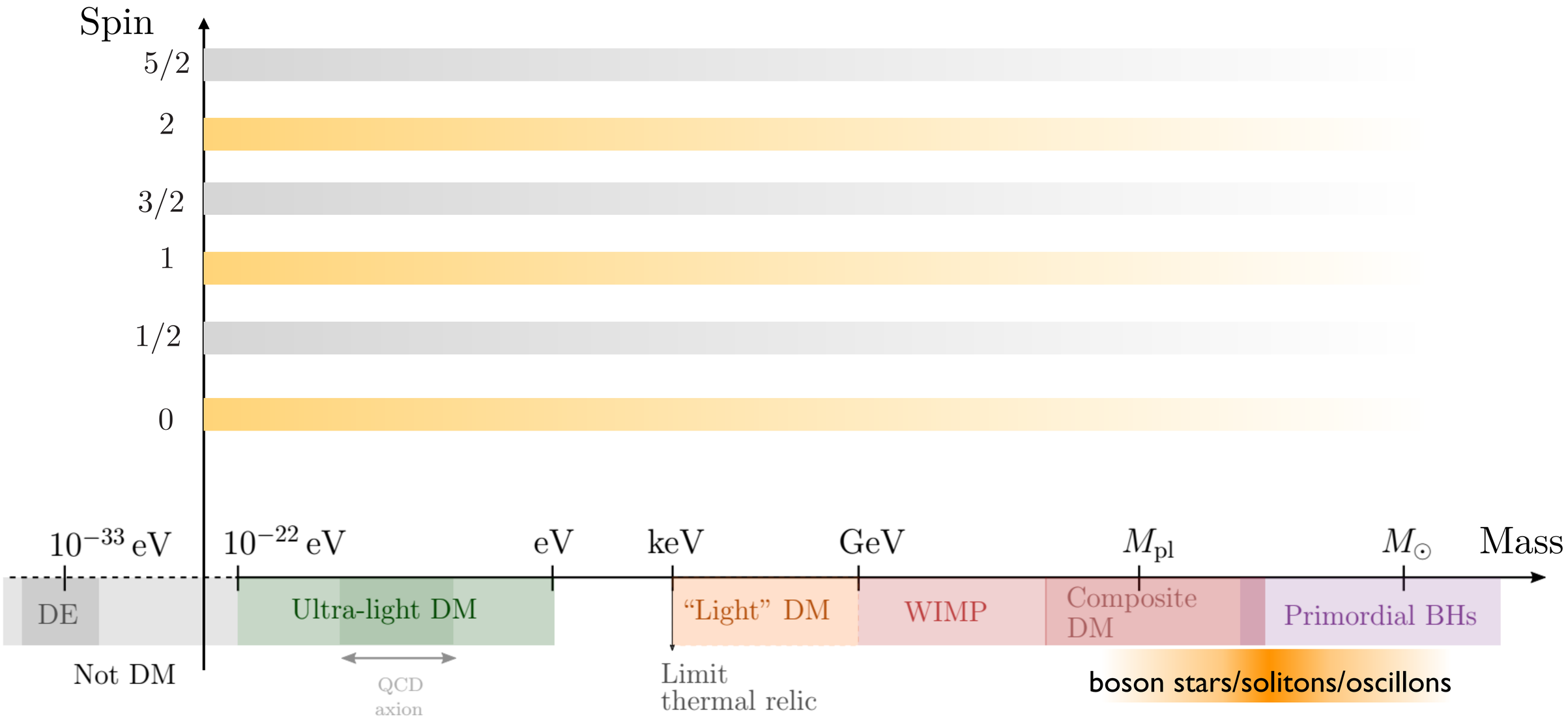
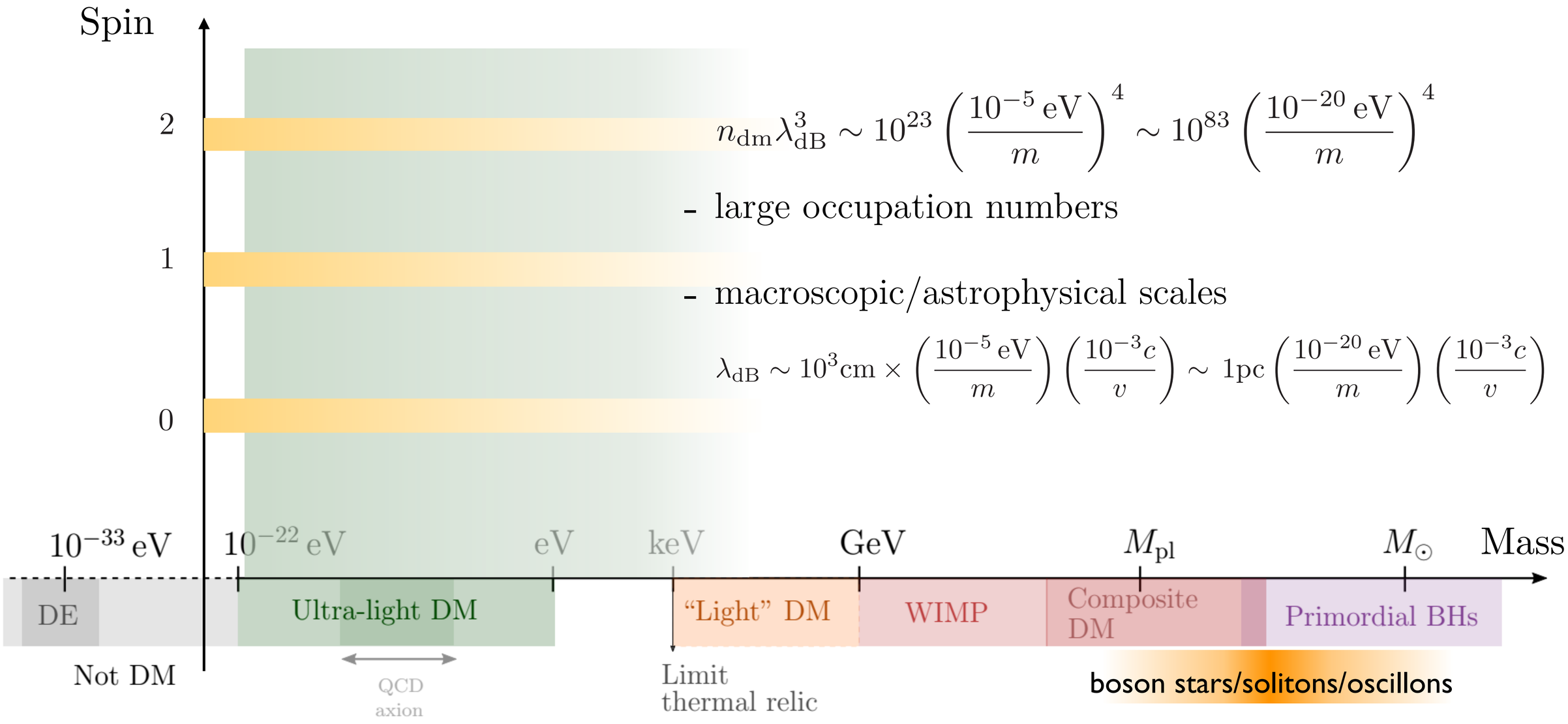


image credit: E. Ferreira

dark matter spin ?



light, bosonic wave dark matter

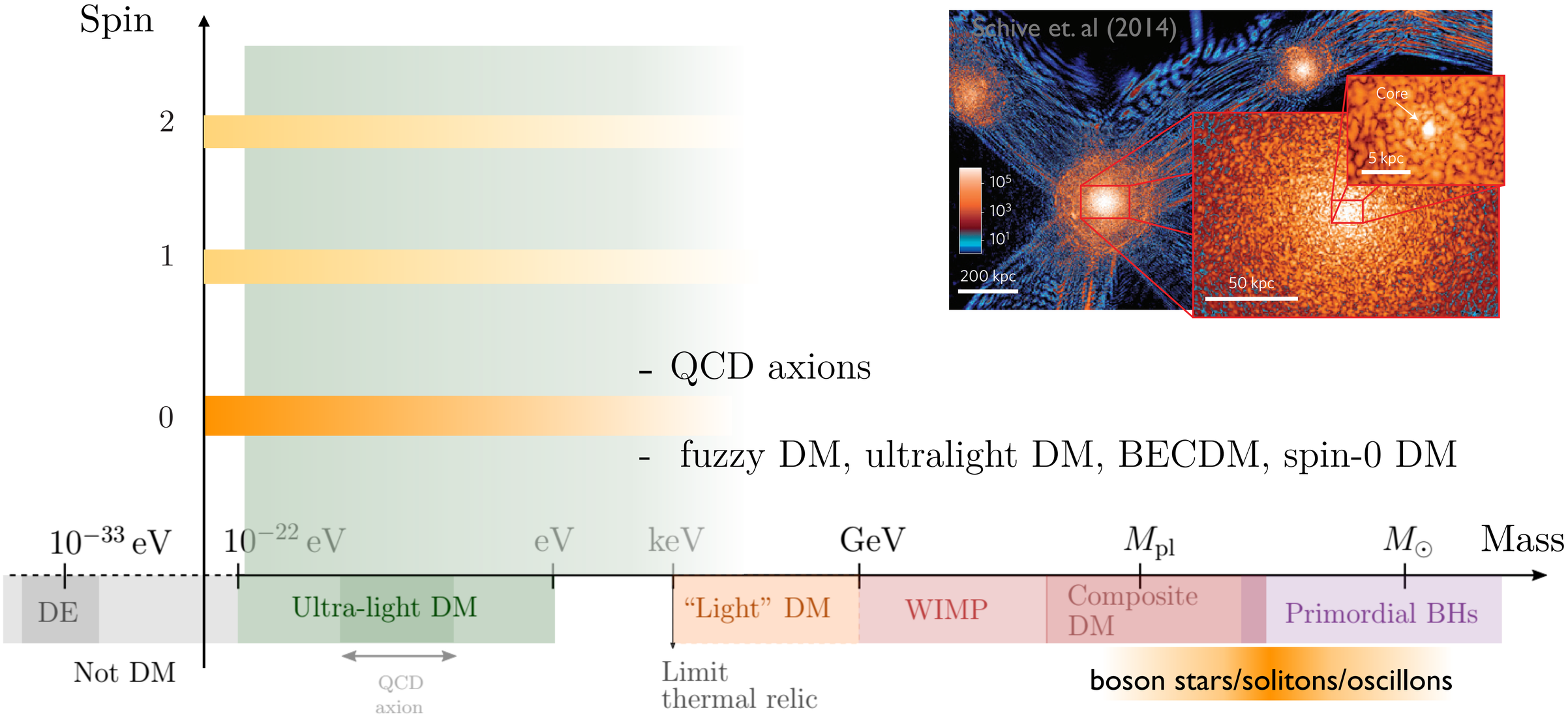


$$n_{\rm dm}\lambda_{\rm dB}^3 \sim 10^{23}\left(\frac{10^{-5}\,{\rm eV}}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)^4$$

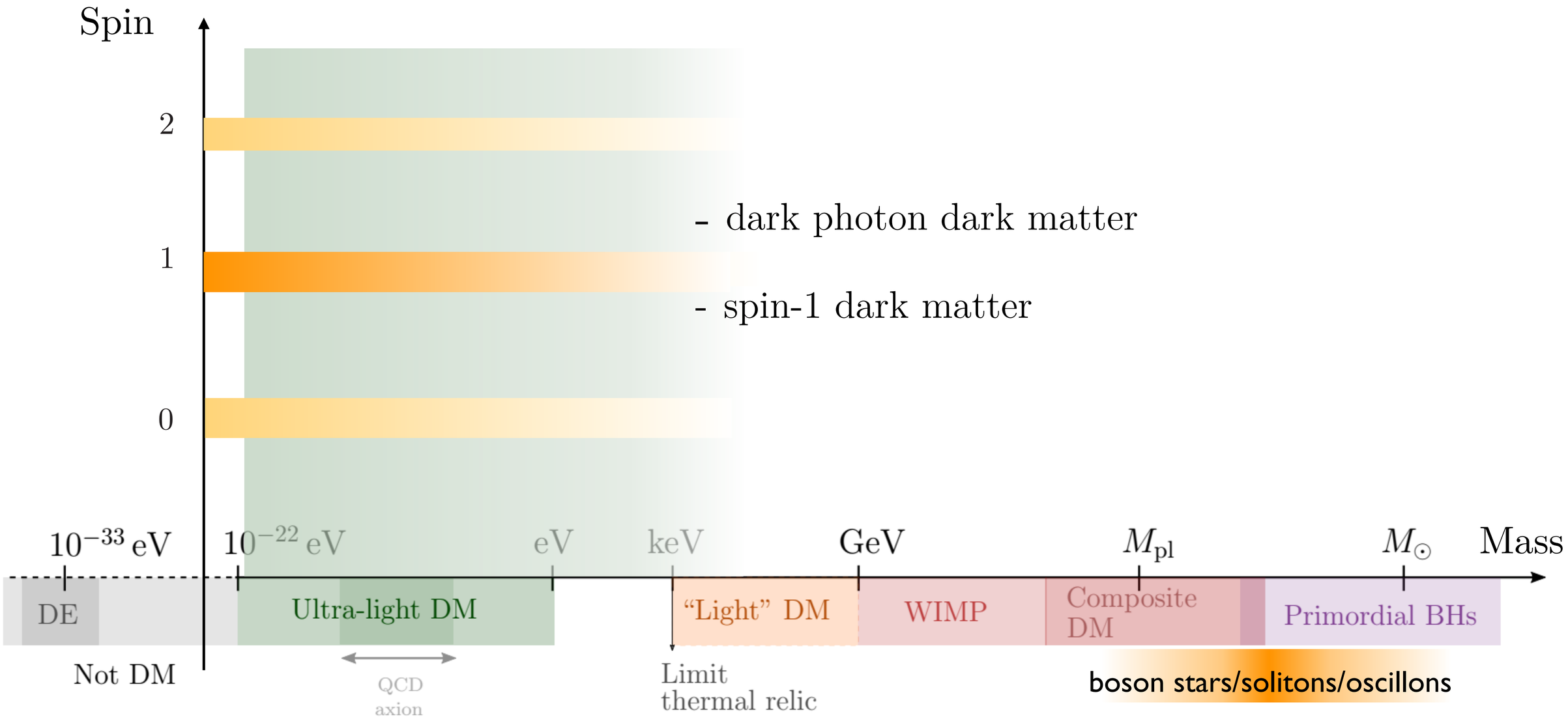
- large occupation numbers
- macroscopic/astrophysical scales

$$\lambda_{\rm dB} \sim 10^3{\rm cm}\times\left(\frac{10^{-5}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right) \sim 1{\rm pc}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

scalar dark matter



vector dark matter

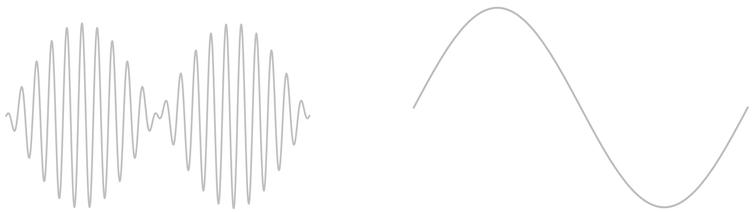


models

non-relativistic limit = multicomponent Schrödinger-Poisson

$$S = \int \mathrm{d}^4x \sqrt{-g}\Big[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\,\mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\,g^{\mu\nu}W_\mu W_\nu + \frac{c^3}{16\pi G}R + \ldots\Big]$$ + non-grav, interactions

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



split in "fast" and "slow" parts

non-relativistic limit

$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\mathcal{S}_{nr} = \int \mathrm{d}t\mathrm{d}^3x\left[\frac{i\hbar}{2}\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}} + \text{c.c.} - \frac{\hbar^2}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} + \frac{1}{8\pi G}\,\Phi\nabla^2\Phi - m\,\Phi\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}\right]$$

Adshead & Lozanov (2021)
Jain & MA (2022)

non-relativistic limit = multicomponent Schrödinger-Poisson

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

$$i\hbar \frac{\partial}{\partial t} \mathbf{\Psi} = -\frac{\hbar^2}{2m} \nabla^2 \mathbf{\Psi} + m\, \Phi\, \mathbf{\Psi}\,, \qquad \nabla^2 \Phi = 4\pi G m\, \mathbf{\Psi}^\dagger \mathbf{\Psi}$$

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1$ scalar case

lead by Mudit Jain

arXiv: 2109.04892

generalization to spin -s field, Jain & MA (2022)

at this level this is just $2s{+}1$ scalar fields

conserved quantities

$$[\boldsymbol{\Psi}]_i = \psi_i \text{ with } i = 1, 2, 3$$

$$N = \int \mathrm{d}^3x \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}, \quad \text{and} \quad M = mN, \qquad \text{(particle number and rest mass)}$$

$$E = \int \mathrm{d}^3x \left[\frac{\hbar^2}{2m} \nabla \boldsymbol{\Psi}^\dagger \cdot \nabla \boldsymbol{\Psi} - \frac{Gm^2}{2} \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi} \int \frac{\mathrm{d}^3y}{4\pi |\boldsymbol{x} - \boldsymbol{y}|} \boldsymbol{\Psi}^\dagger(\boldsymbol{y}) \boldsymbol{\Psi}(\boldsymbol{y}) \right], \qquad \text{(energy)}$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger, \qquad \text{(spin angular momentum)}$$

$$\boldsymbol{L} = \hbar \int \mathrm{d}^3x \, \Re \left(i \, \boldsymbol{\Psi}^\dagger \nabla \boldsymbol{\Psi} \times \boldsymbol{x} \right). \qquad \text{(orbital angular momentum)}$$

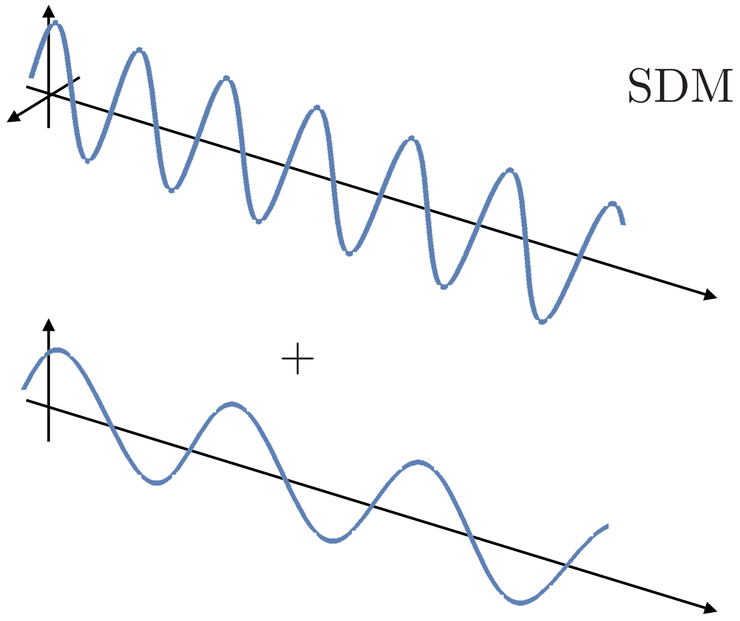
vector vs. scalar DM: two key differences

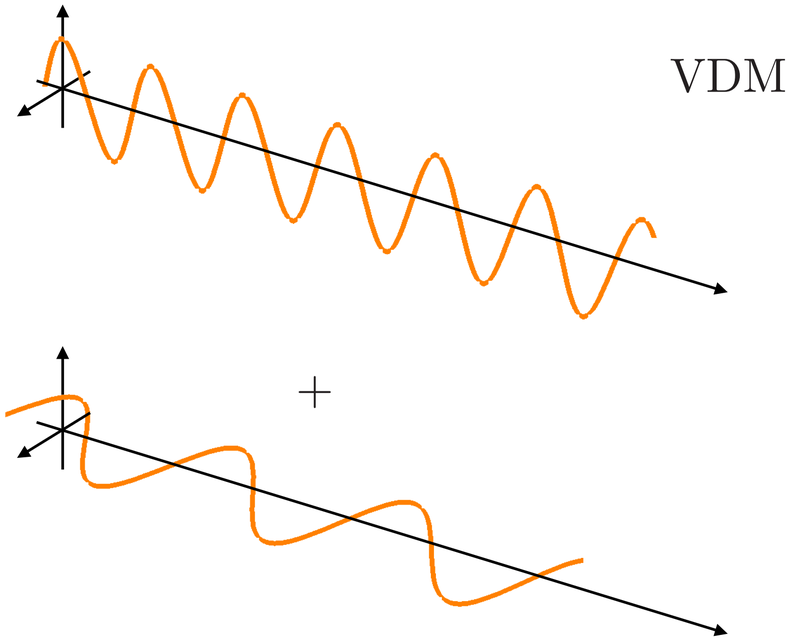
interference

polarized solitons

analytics

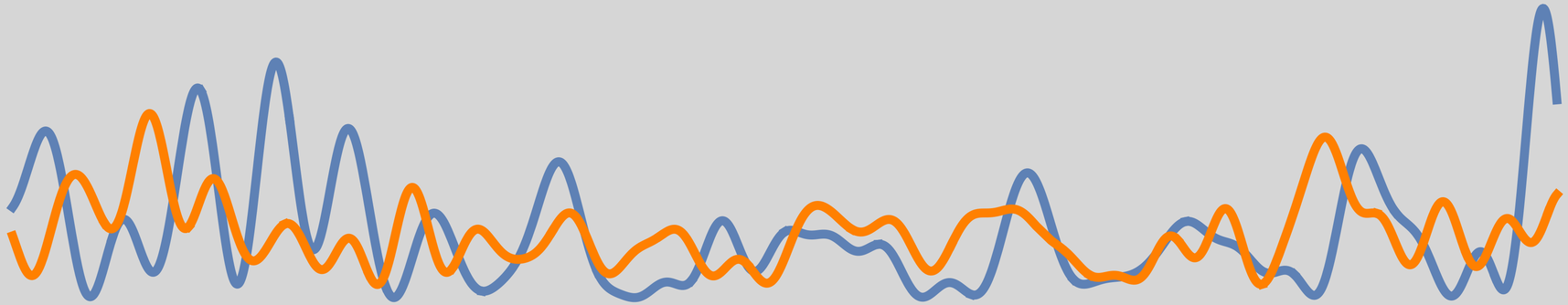
wave interference





$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 \neq |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$

$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 = |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$



reduced wave interference in VDM

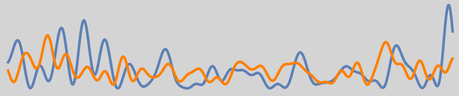
$s = 0$ for SDM and $s = 1$ for VDM

$$\boldsymbol{\Psi}_a(\boldsymbol{x}) = \boldsymbol{\epsilon}_a^{(s)} e^{i\boldsymbol{k}_a \cdot \boldsymbol{x}}$$

$$|\boldsymbol{\Psi}_a(\boldsymbol{x}) + \boldsymbol{\Psi}_b(\boldsymbol{x})|^2 = 2\left(1 + \Re\left[\boldsymbol{\epsilon}_a^{(s)\dagger} \cdot \boldsymbol{\epsilon}_a^{(s)} e^{-i(\boldsymbol{k}_a - \boldsymbol{k}_b)\cdot \boldsymbol{x}}\right]\right) = 2\left(1 + \mathrm{int}_{(s)}\right)$$

$$\sqrt{\langle \mathrm{int}_{(s)}^2 \rangle} = \frac{1}{\sqrt{2(2s+1)}}$$

$$\frac{\sqrt{\langle \mathrm{int}_{(1)}^2 \rangle}}{\sqrt{\langle \mathrm{int}_{(0)}^2 \rangle}} = \frac{1}{\sqrt{3}}$$

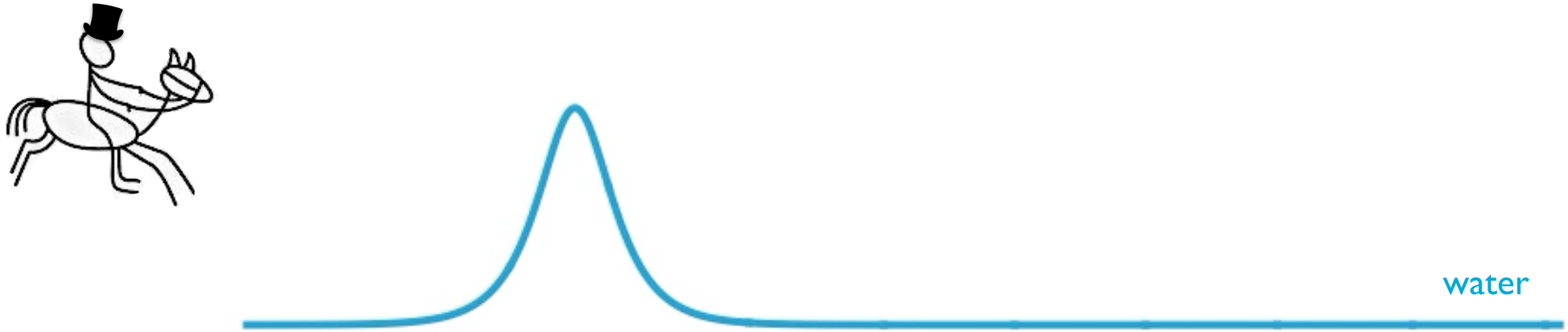


MA, Jain, Karur & Mocz (2022)

soliton ?

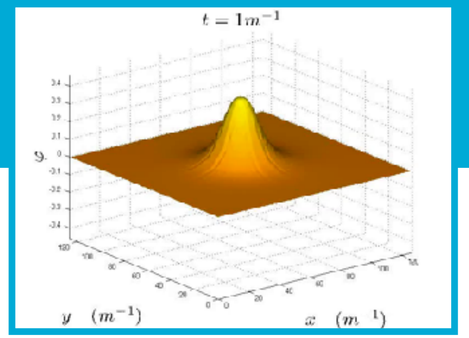
Image Credit: Heriot-Watt University

very-long lived, dynamical excitation in a field, with nonlinearities balancing dispersion



- discovered in nonlinear waves in water in canals (John Scott Russell, 1834)
- optics, hydrodynamics, BECs, high energy physics, and cosmology

solitons in massive spin-0 (scalar fields)



$$\phi(t, \boldsymbol{x}) = \frac{\hbar}{\sqrt{2mc}} \Re[\Psi_{\rm sol}(\mu, \boldsymbol{x}) e^{imc^2 t/\hbar}]$$

$$\Psi_{\rm sol}(t, \boldsymbol{x}) = \psi_{\rm sol}(\mu, \boldsymbol{x}) e^{i\mu c^2 t/\hbar}$$

$$M_{\rm sol} \approx 60.7 \frac{m_{\rm pl}^2}{m} \sqrt{\frac{\mu}{m}}$$

$$R_{\rm sol} \sim \sqrt{\frac{m}{\mu}} \frac{\hbar}{mc}$$

lowest energy solution for fixed total *Mass (in non-relativistic limit)*

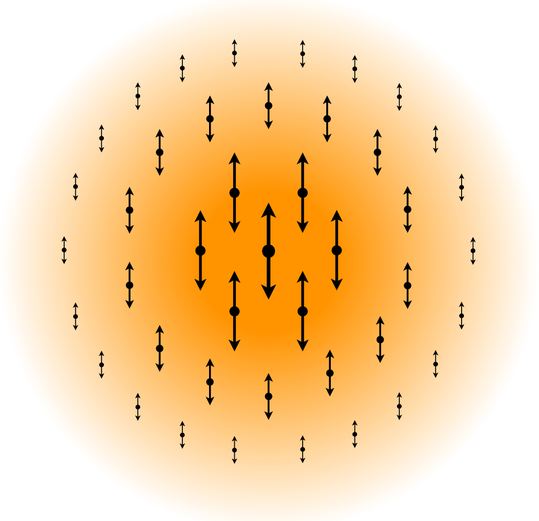
$\mu/m \ll 1$

“polarized” vector solitons

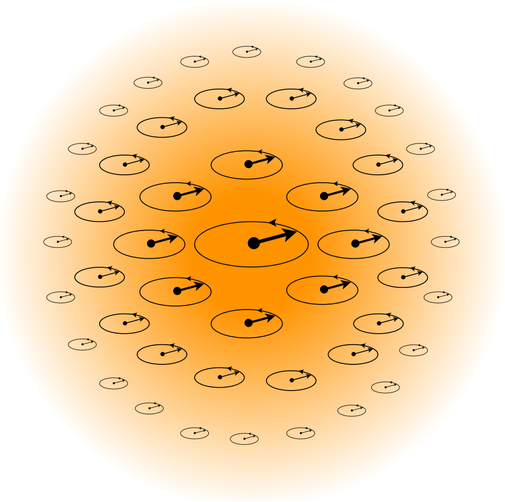
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2}mc}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon}$$

$$\boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$



$$\boldsymbol{\epsilon}^{(0)}_{1,\hat{z}} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$



$$\boldsymbol{\epsilon}^{(\pm1)}_{1,\hat{z}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\pm i\\0\end{pmatrix}$$

“polarized” vector solitons

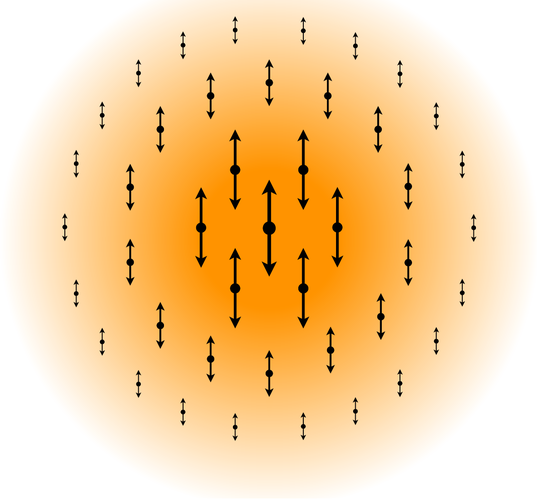
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

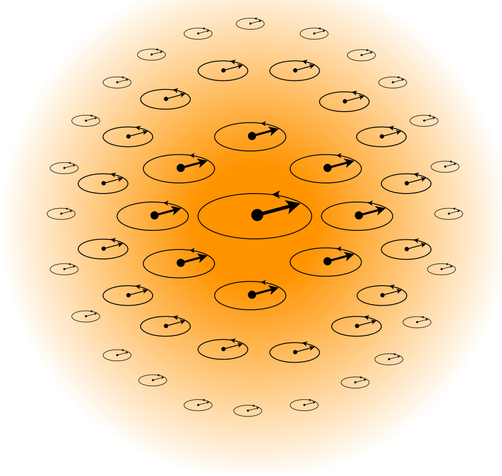
$$\boldsymbol{S}_{\rm sol} = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton



$$\boldsymbol{S}_{\rm tot} = 0\hat{z}$$



$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

lead by Mudit Jain

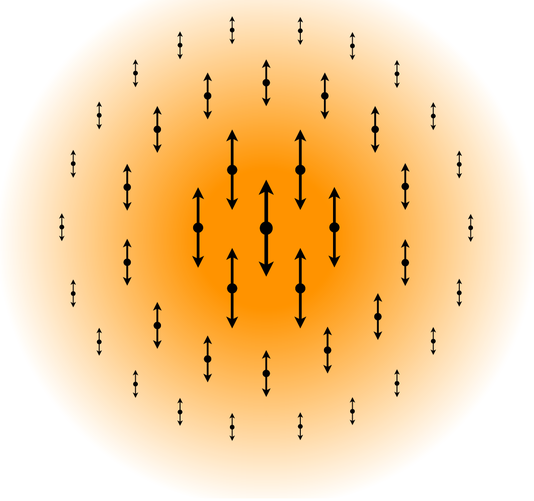
arXiv: 2109.04892

“polarized” vector solitons

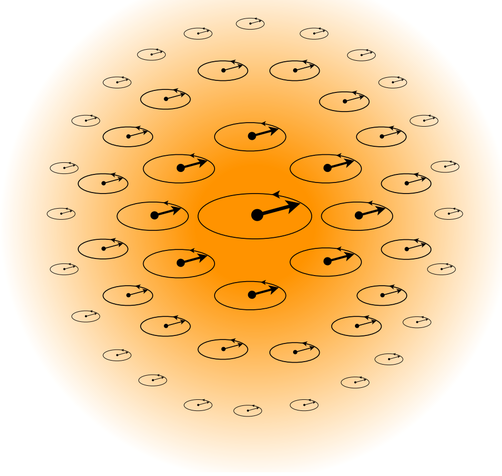
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$\boldsymbol{S}_{\rm sol} = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$



$$\boldsymbol{S}_{\rm tot} = 0\hat{z}$$

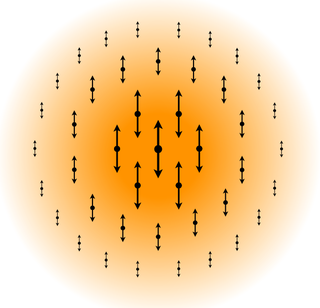
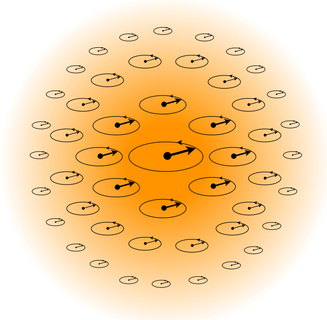
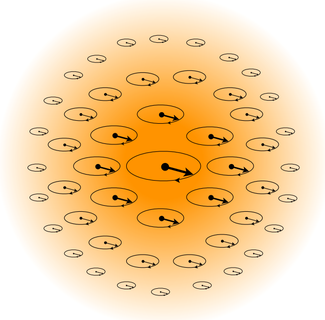


$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton

- all lowest energy for fixed M
- bases for partially-polarized solitons

c_0  $+c_1$  $+c_{-1}$ 

$$0 \le |\boldsymbol{S}_{\rm tot}| \le \frac{M_{\rm sol}}{m}\hbar$$

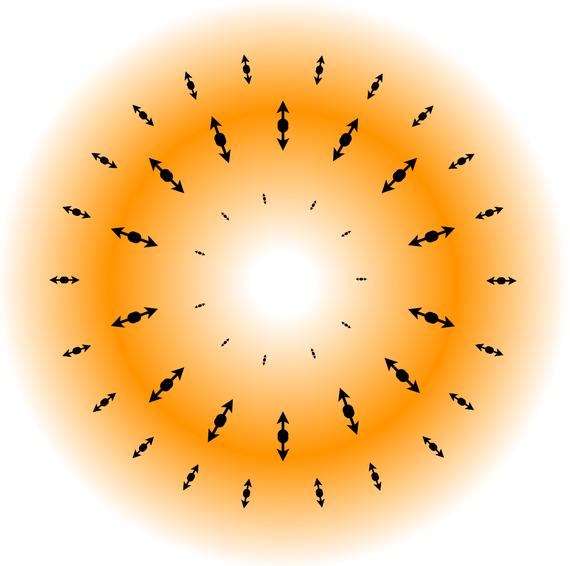
a different higher energy soliton: the “hedgehogs”

earlier literature

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

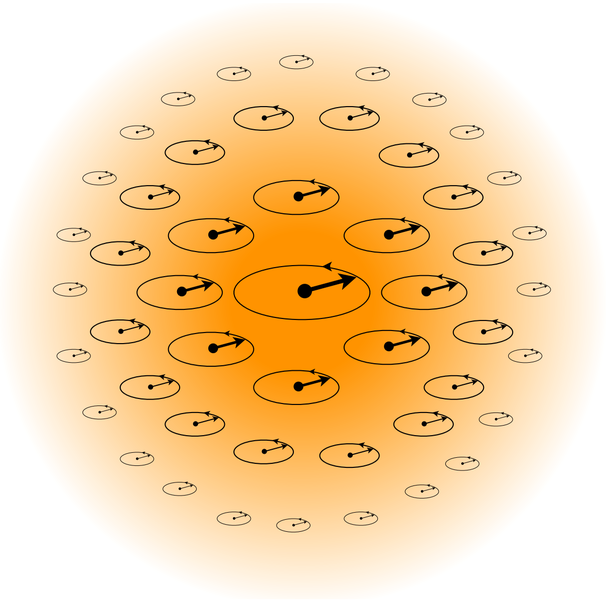
hedgehogs
not ground states

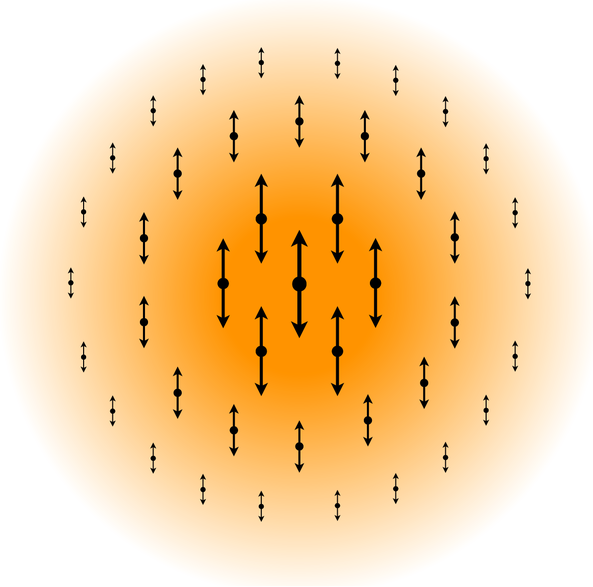
at least when non-relativistic
Lozanov & Adshead (2021)

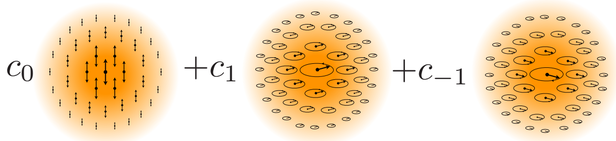


$$E^{s}_{\rm hh} > E$$

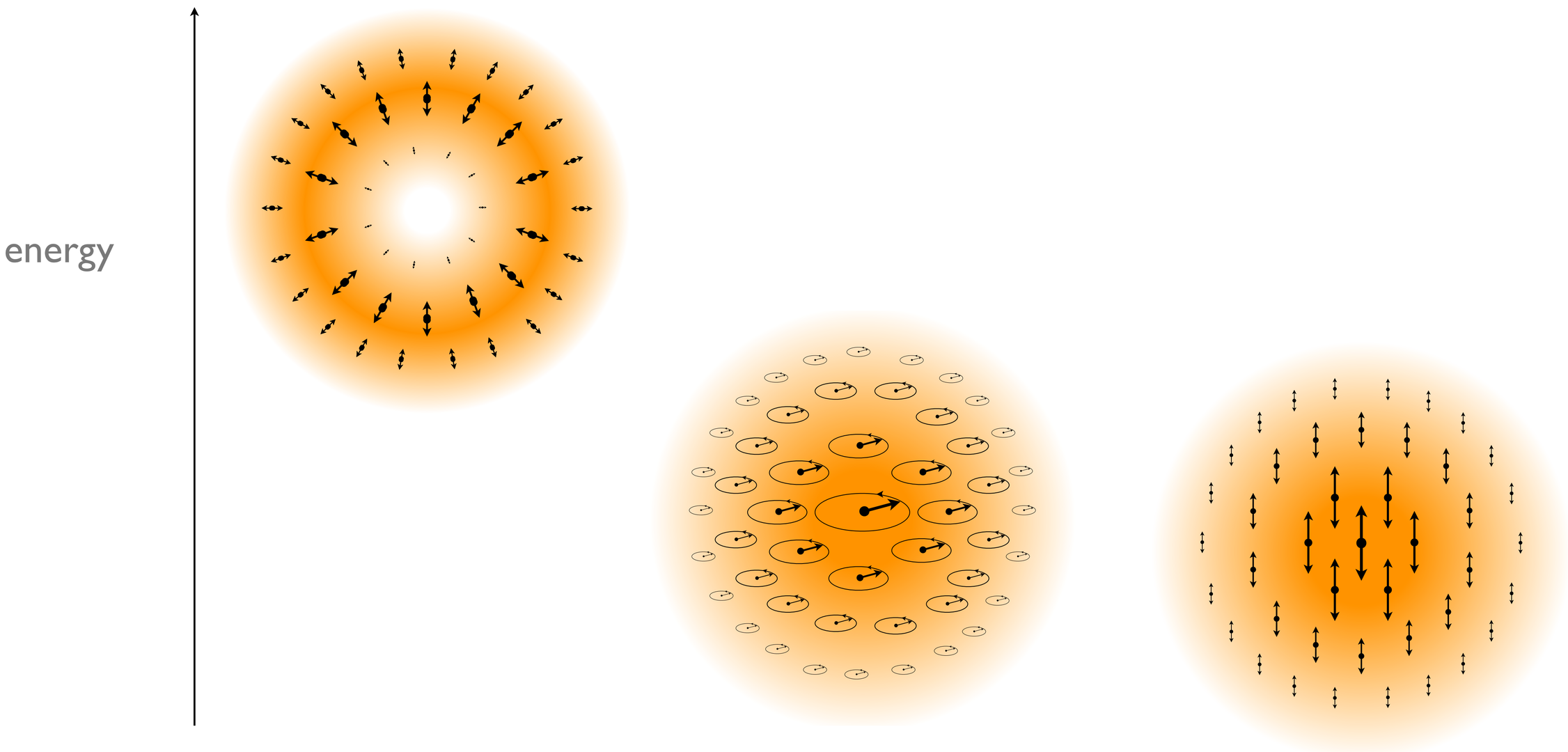
$$E^{s=1}_{\rm hh} \approx 0.33E < 0$$







gravity alone



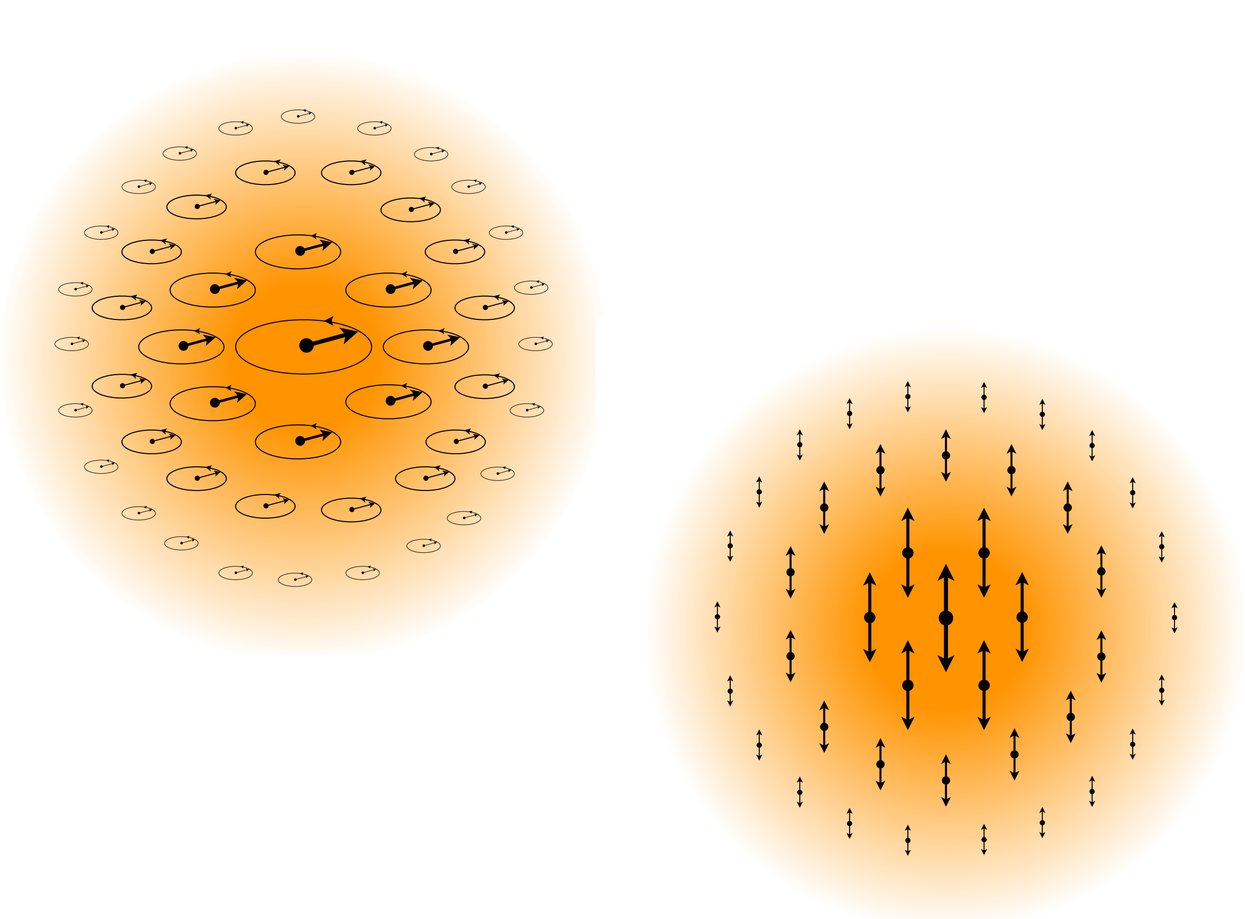
gravity + (attractive self-interactions)

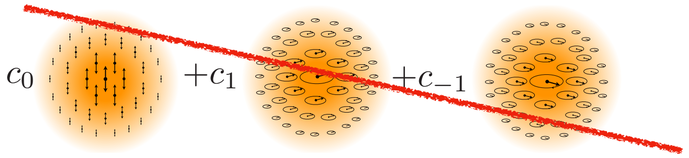
Zhang, Jain & MA (2022)

Jain (2022)

lead by HongYi Zhang

energy



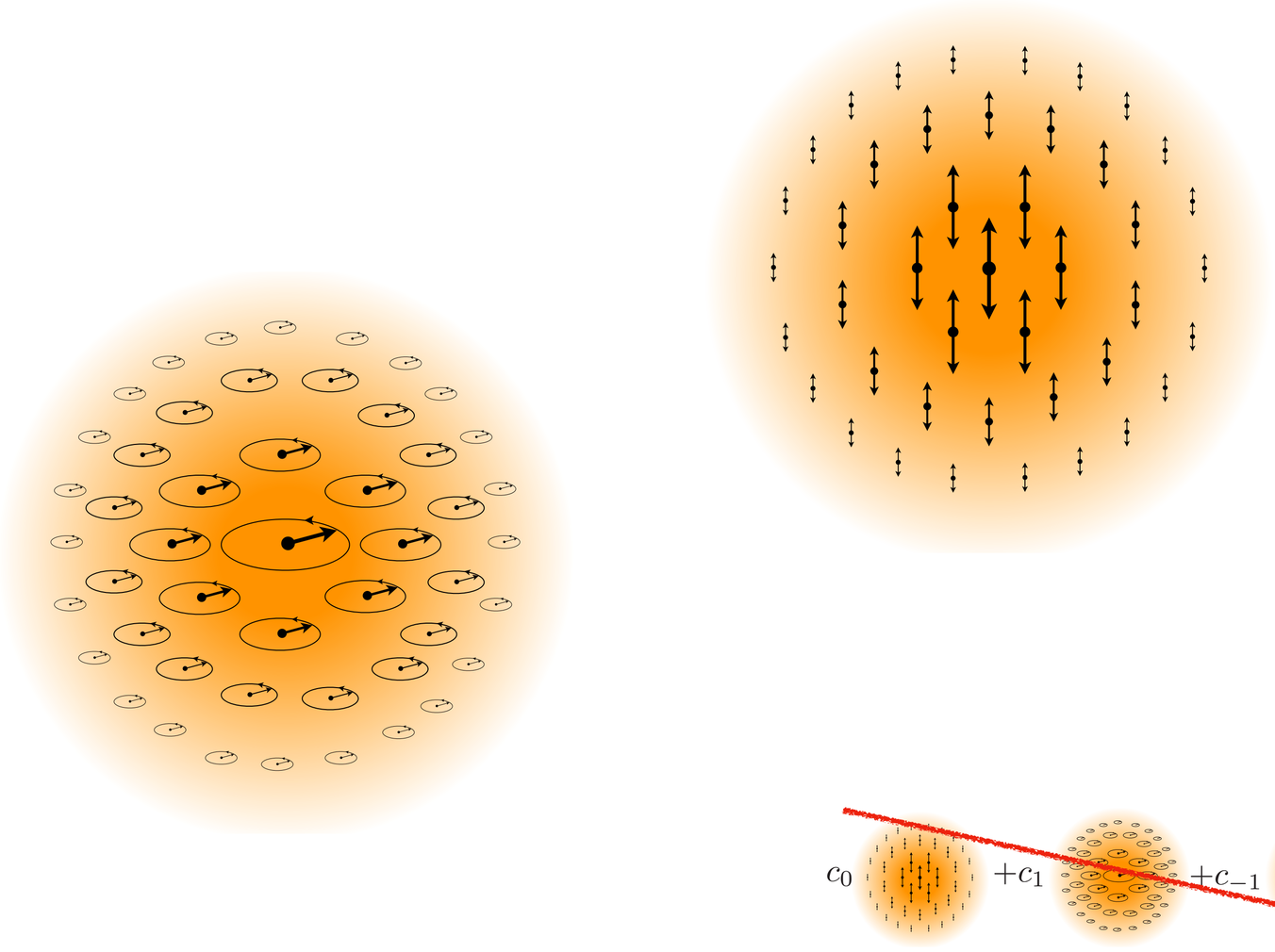


gravity + repulsive self-interactions

Zhang, Jain & MA (2022)

Jain (2022)

energy

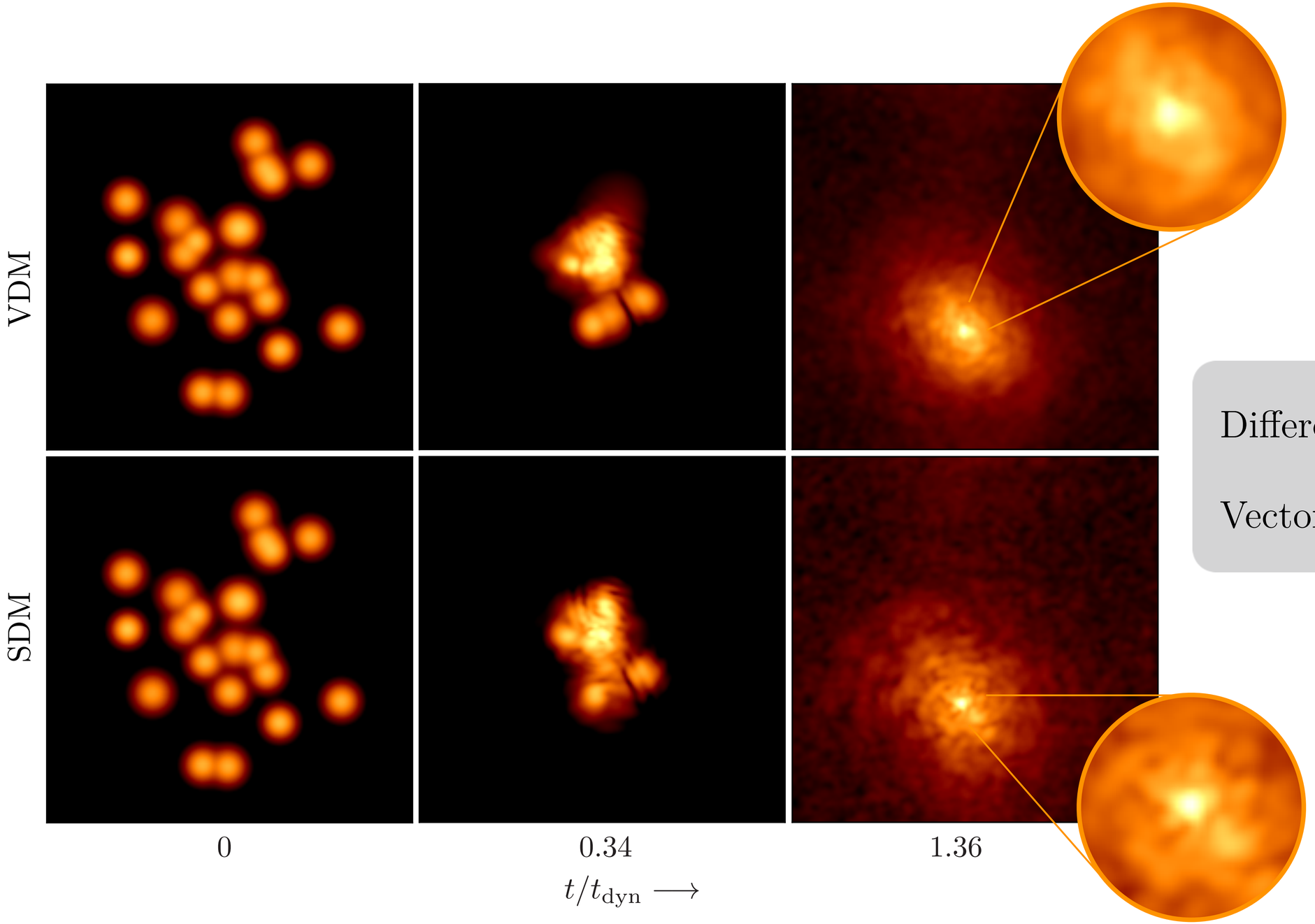


vector vs. scalar DM: two key differences

interference

polarized solitons

3+1 dimensional simulations



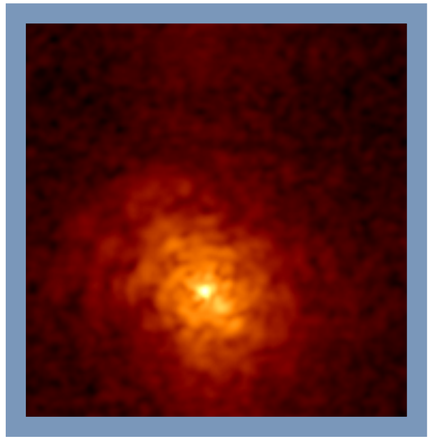
Difference between

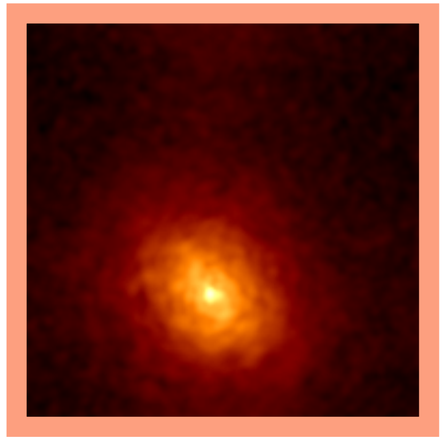
Vector & Scalar Dark Matter

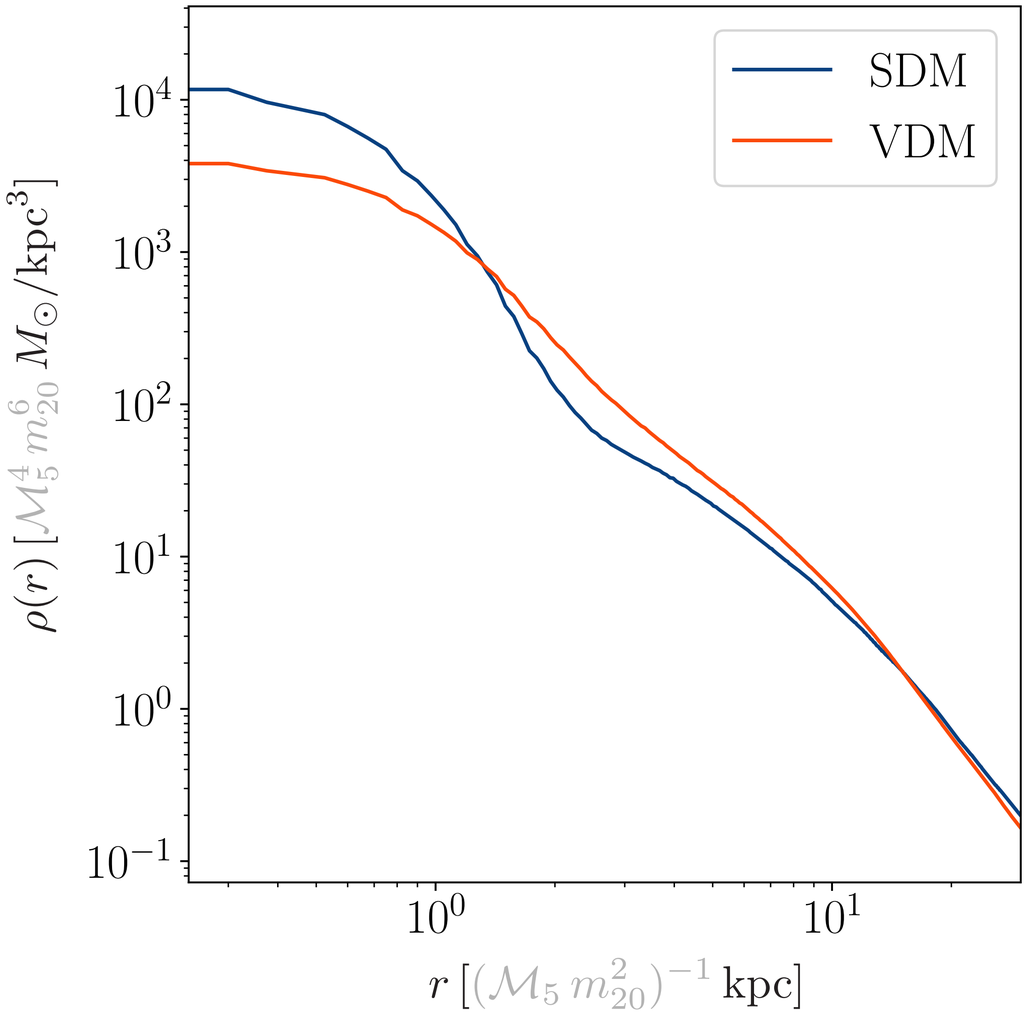
radial density profiles

scalar vs. vector dark matter

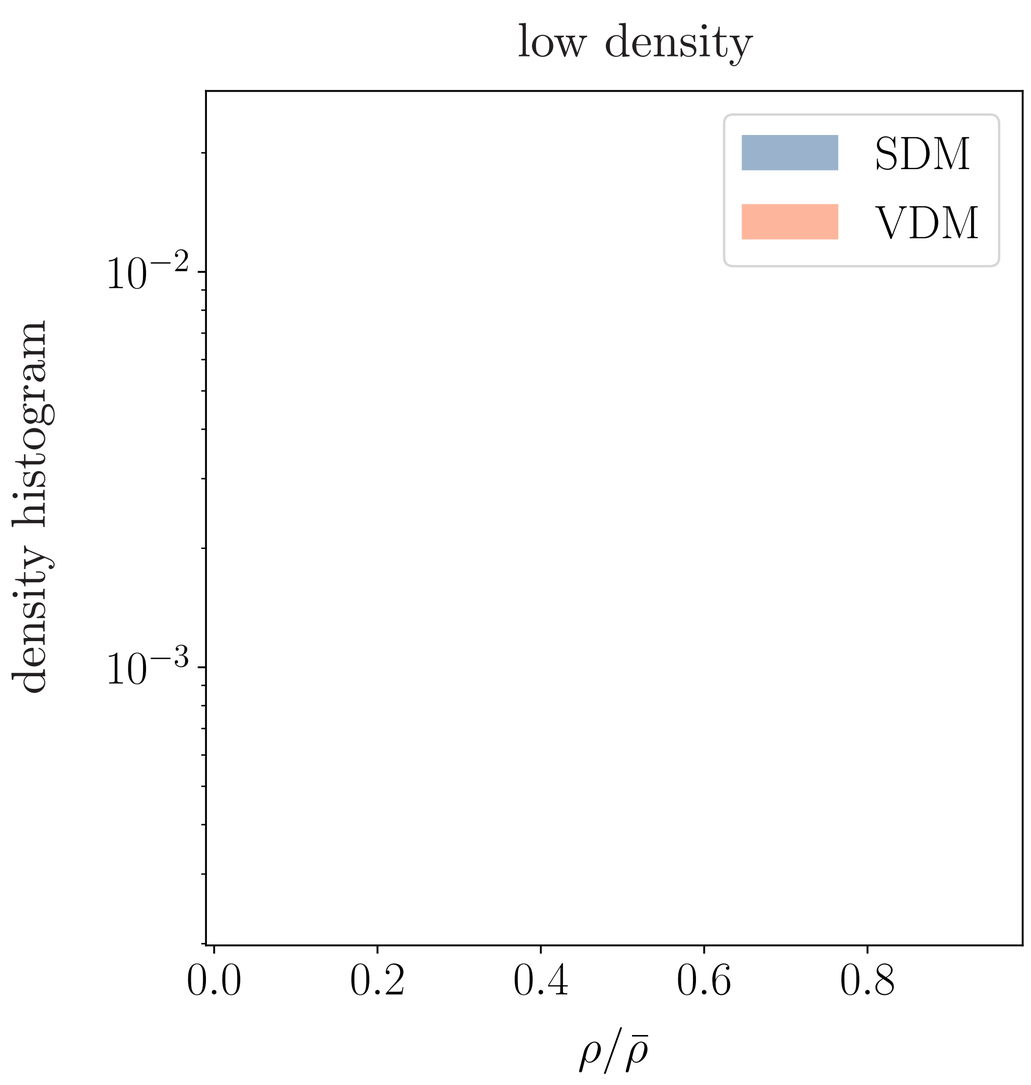
- less dense & broader core
- smoother transition to r^{-3} tail

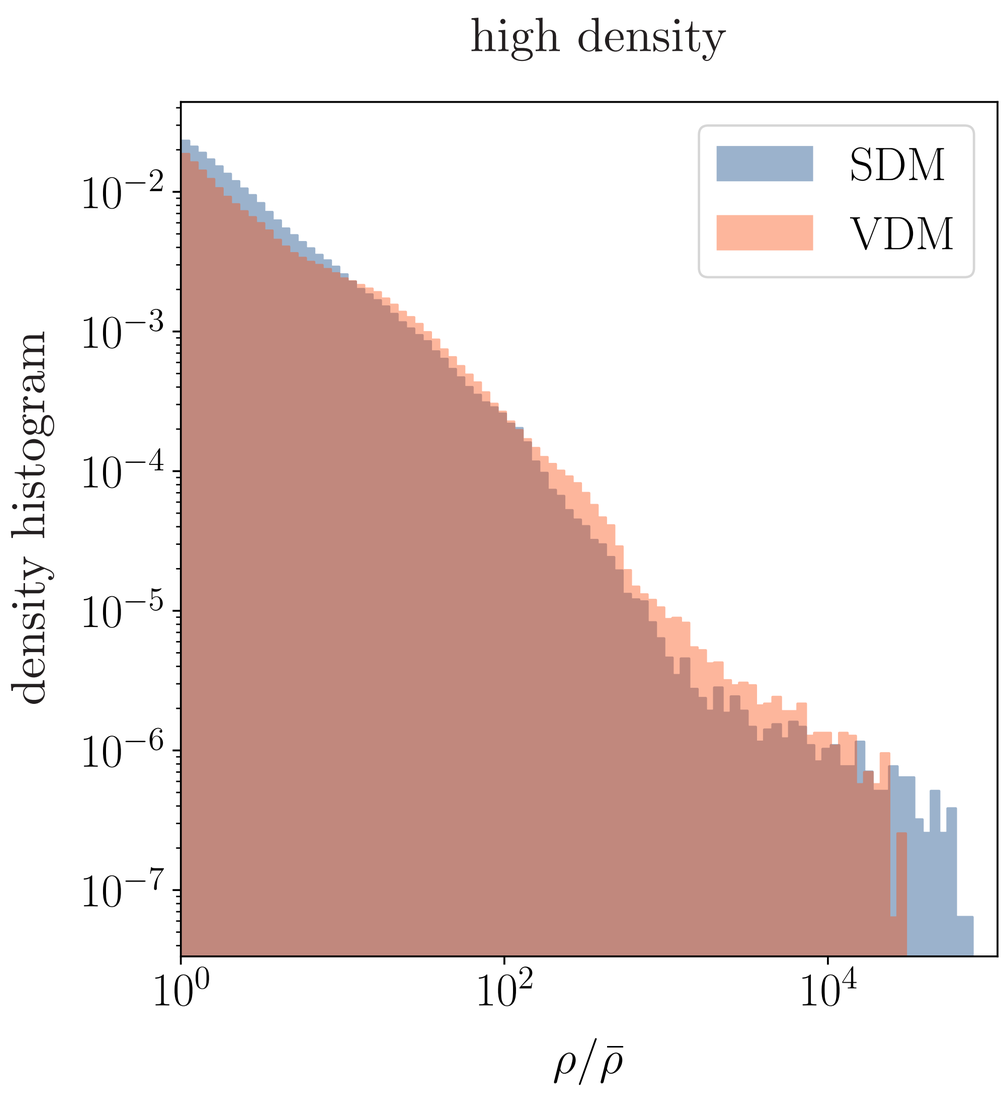






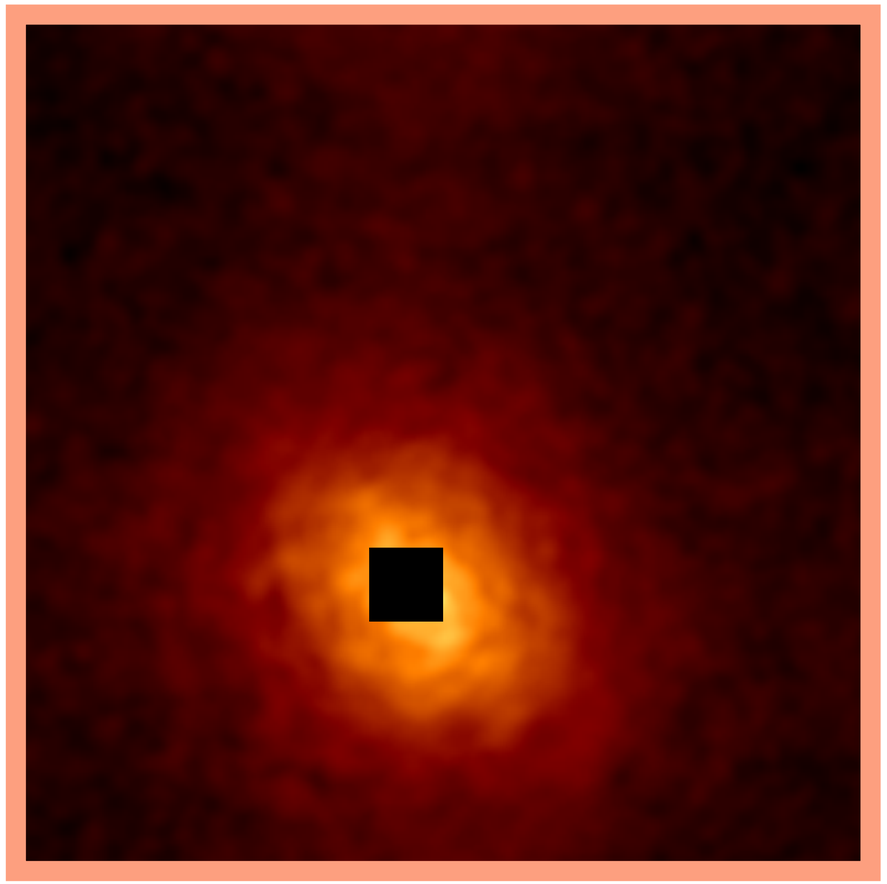
interference and density pdf

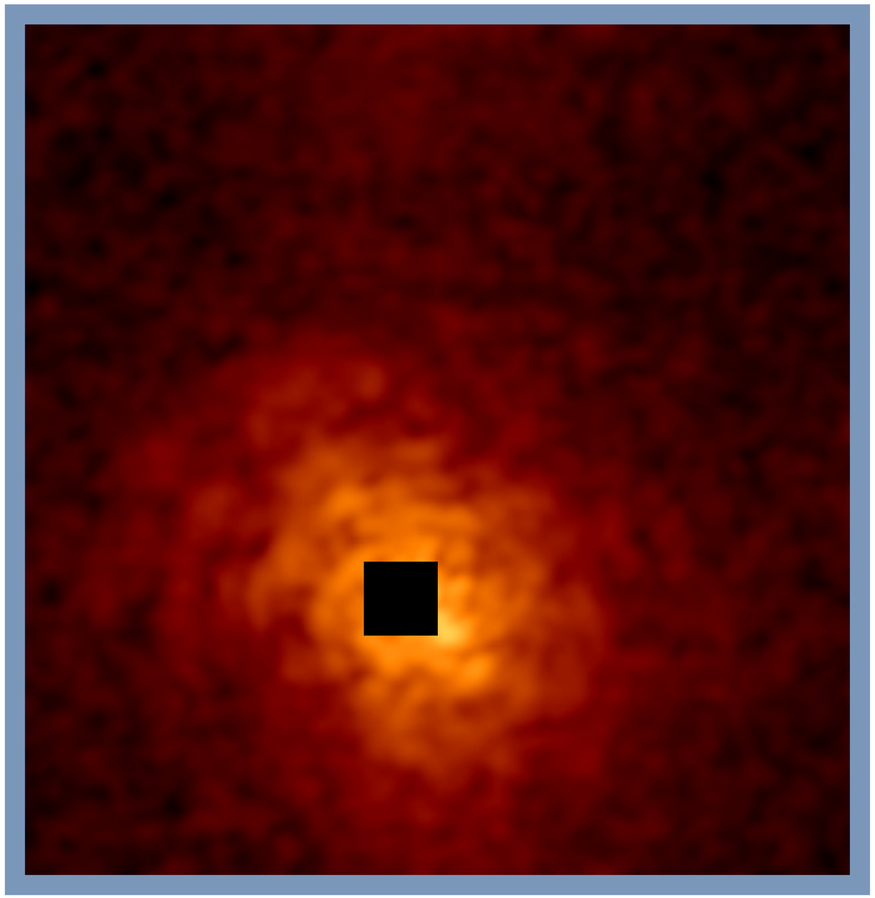




same length scale of interference patterns

$$\lambda_{\mathrm{dB}}(\mathrm{VDM}) \approx \lambda_{\mathrm{dB}}(\mathrm{SDM})$$

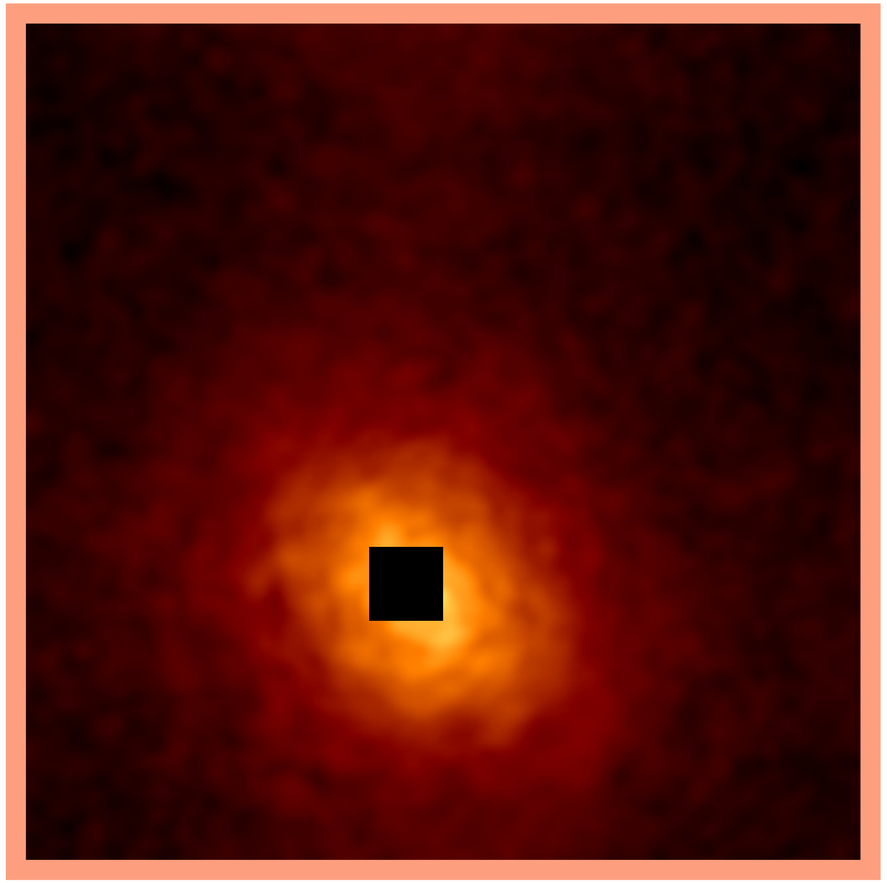


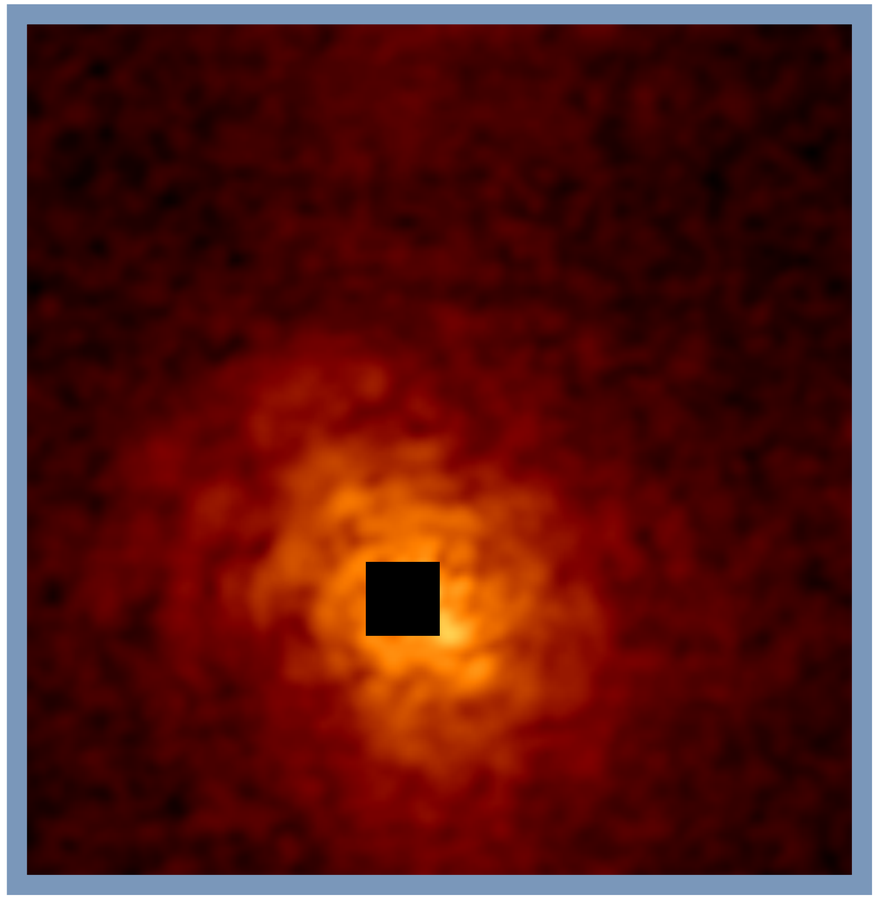


but different amplitude of density fluctuations

$$(\delta\rho/\rho)_{\rm VDM} \approx (\delta\rho/\rho)_{\rm SDM}/\sqrt{3}$$

generalization 3 —> $(2s+1)$





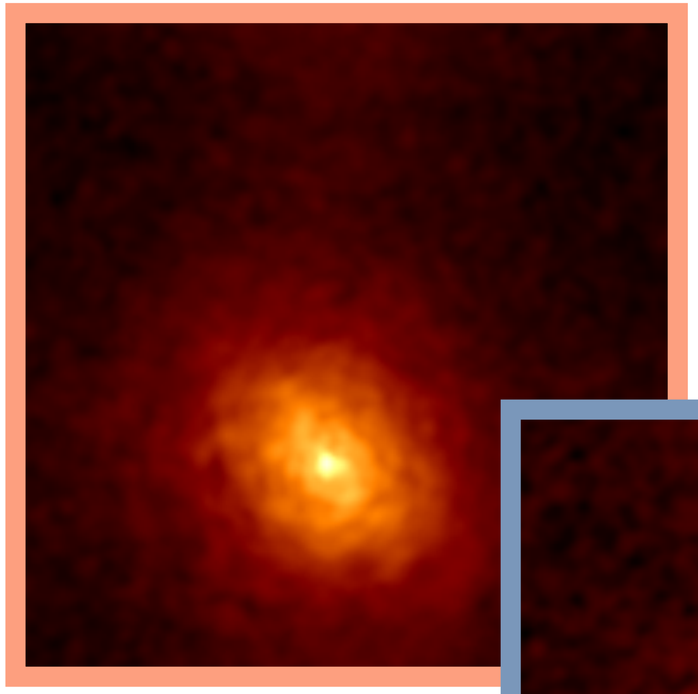
also see similar result for multiple scalar fields case: Gosenca et. al (2023)

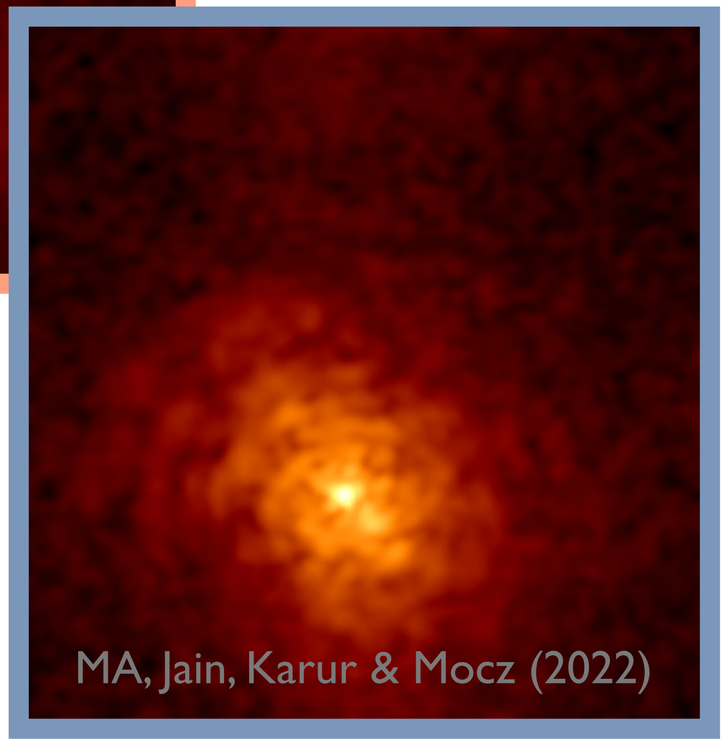
gravitational implications (examples)

- dynamical heating of stars

$$m \gtrsim \frac{1}{(2s+1)^{1/3}} \left[3 \times 10^{-19}\mathrm{eV}\right]$$

Dalal & Kratsov (2022)





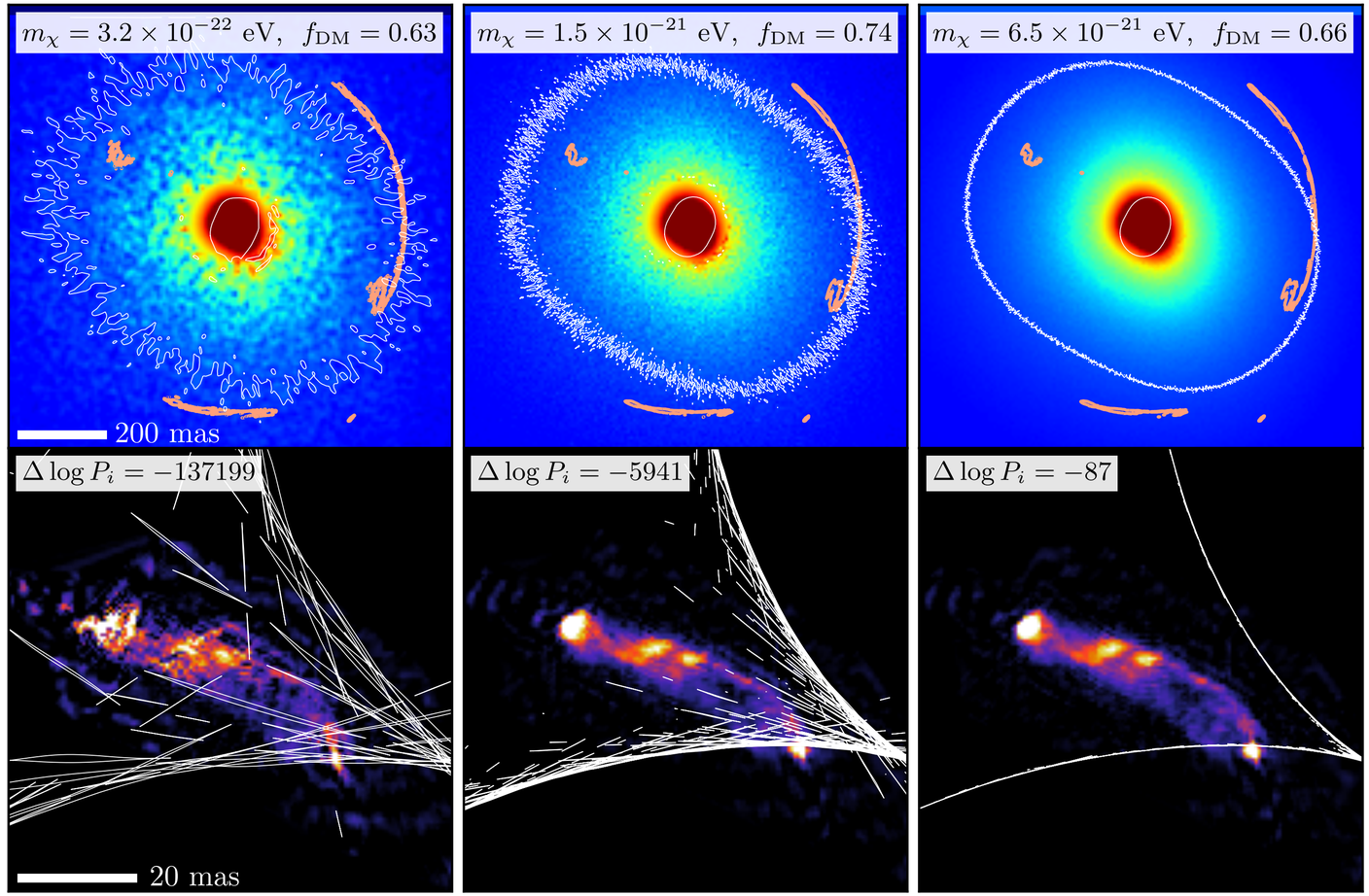
MA, Jain, Karur & Mocz (2022)

gravitational implications (examples)

- lensing

$$m \gtrsim \frac{1}{(2s+1)} \left[4.4 \times 10^{-21}\,\mathrm{eV}\right]$$

Powell et. al (2023)



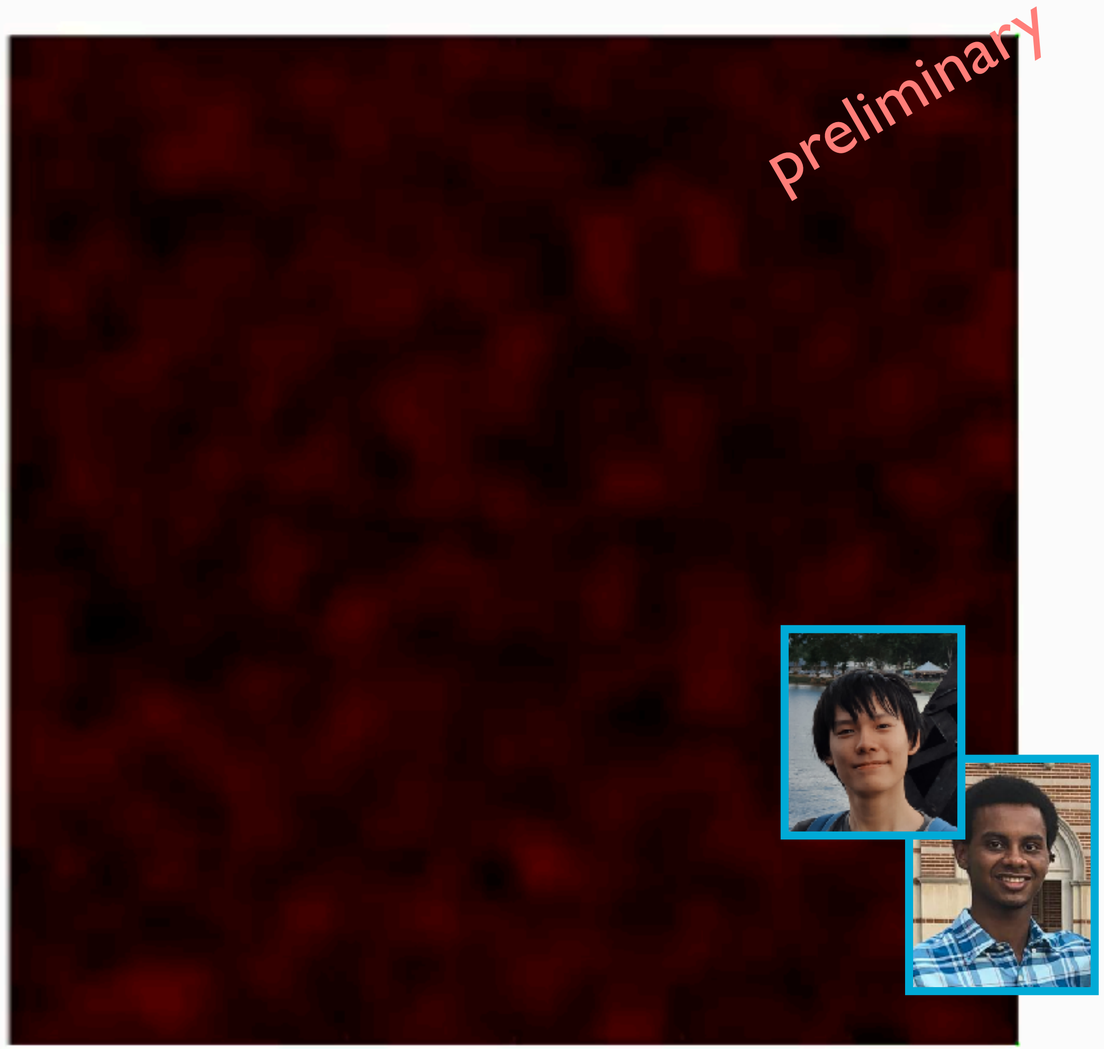
kinetic nucleation time scales

- nucleation time scale

$$\tau_{\rm s} \sim (2s+1)\tau_{\rm s=0}$$

$$\tau_{\rm s=0} \sim n\sigma_{\rm gr} v \mathcal{N}$$ Levkov et. al (2018)

$$\sigma_{\rm gr} \sim (Gm/v^2)^2, \qquad \mathcal{N} \sim n\lambda_{\rm dB}^3$$



with M. Jain, J. Thomas, Wanichwecharungruang

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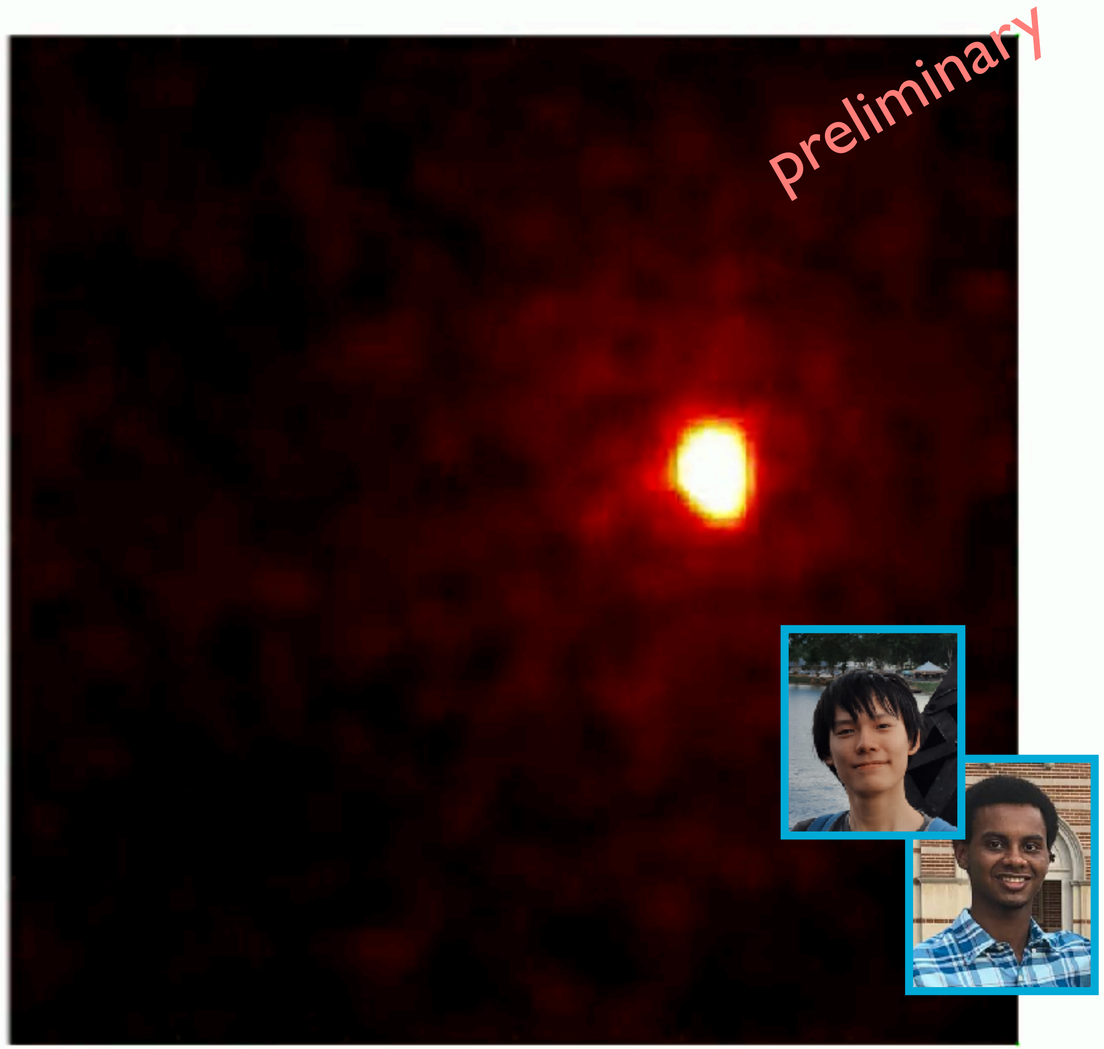

with M. Jain, J. Thomas, Wanichwecharungruang

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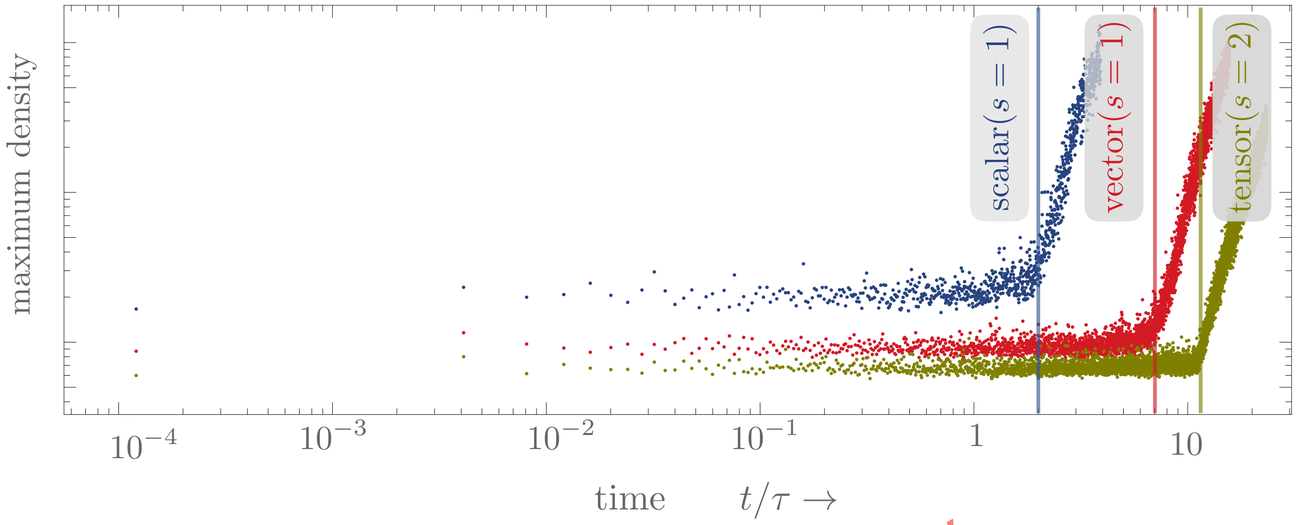
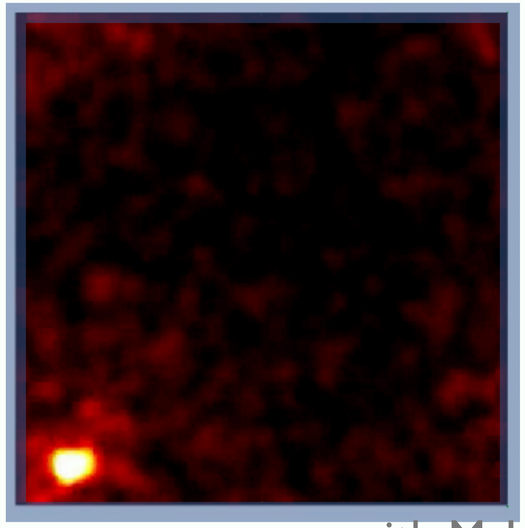
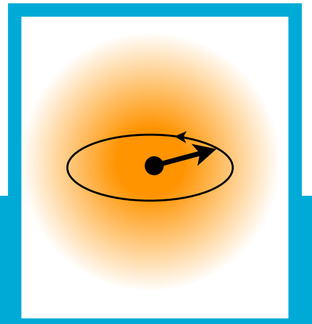
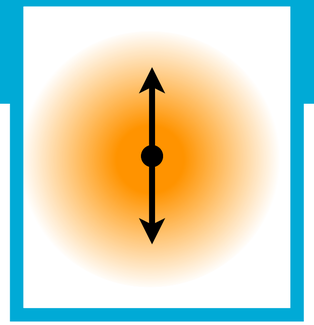




with M. Jain, J. Thomas, Wanichwecharungruang



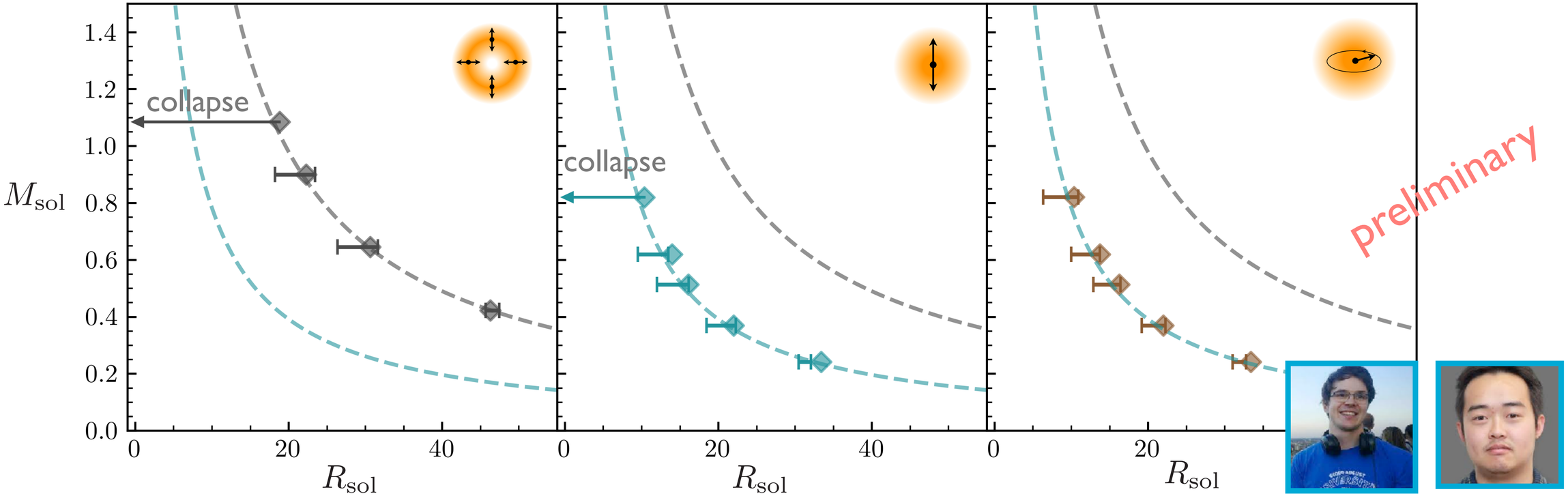
intrinsic spin



implications of non-zero spin solitons?

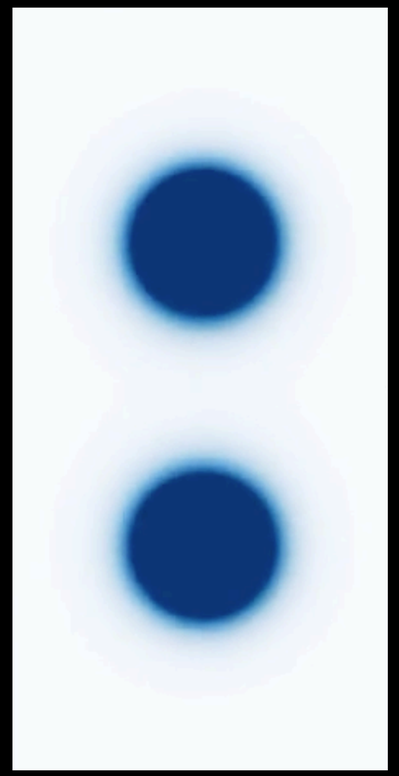
post-Newtonian effects

$$V = -\frac{GM_1M_2}{r}\left[1 + \mathcal{O}(v^2/c^2) - \frac{2}{rc}[\hat{\boldsymbol{r}} \times (\boldsymbol{v}_1 - \boldsymbol{v}_2)] \cdot \sum_{a=1}^{2} \frac{\boldsymbol{S}_a}{M_a} + \frac{1}{r^2c^2}\left\{\frac{\boldsymbol{S}_1}{M_1}\cdot\frac{\boldsymbol{S}_2}{M_2} - 3\left(\frac{\boldsymbol{S}_1}{M_1}\cdot\hat{\boldsymbol{r}}\right)\left(\frac{\boldsymbol{S}_2}{M_2}\cdot\hat{\boldsymbol{r}}\right) + \sum_{a=1}^{2}\frac{C_{ES^2}^{(a)}}{2M_1M_2}\left[S_a^2 - 3(\boldsymbol{S}_a\cdot\hat{\boldsymbol{r}})^2\right]\right\} + \ldots\right]$$



with Thomas Helfer & Zipeng Wang

Photons from Dark Photon Solitons via Parametric Resonance

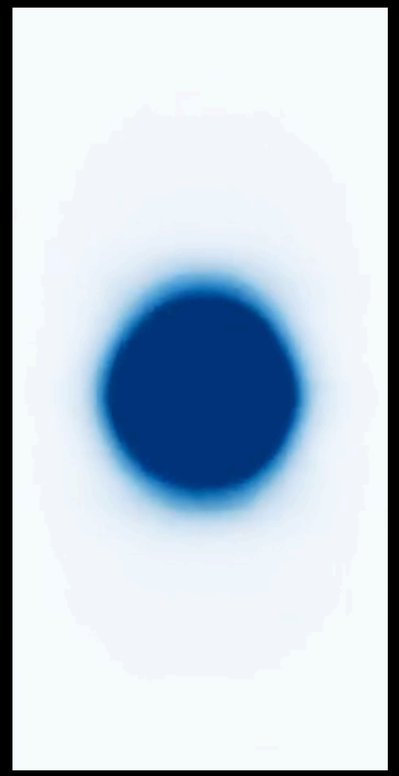


$F_{\mu\nu}G^{\mu\nu}$

$$\mathcal{L}_{\rm int} \sim g^2 W W F F$$

with Schiappacasse & Long (2022)

Photons from Dark Photon Solitons via Parametric Resonance

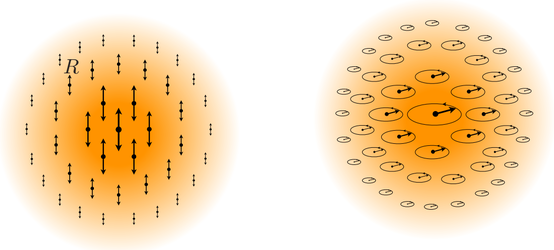


$F_{\mu\nu}G^{\mu\nu}$

$$\mathcal{L}_{\rm int} \sim g^2 W W F F$$

with Schiappacasse & Long (2022)

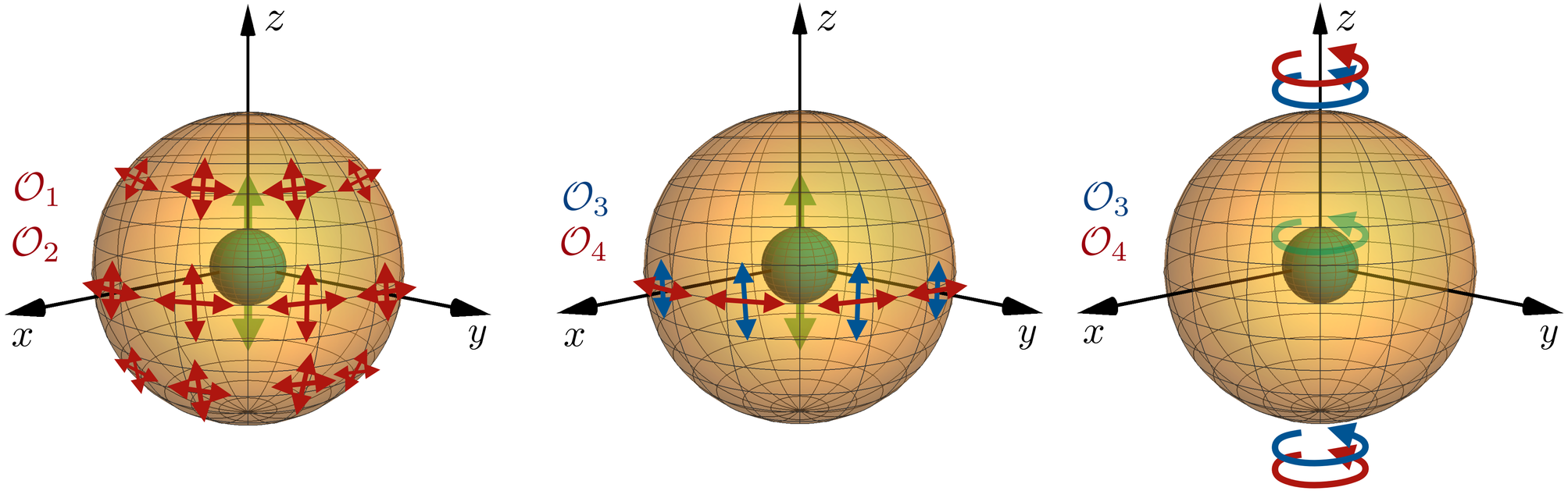
spin of soliton & polarization of photons



$$\mathcal{O}_1 = -\tfrac{1}{2} F_{\mu\nu}\tilde{F}^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_2 = -\tfrac{1}{2} F_{\mu\nu}F^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho}X^{\mu}X_{\nu}$$
$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^{\mu}X_{\nu}$$

explosive photon production (under certain conditions)

$$\mu R \gtrsim 1, \qquad \mu \sim g^2 X^2 m$$



formation mechanism:

initial power spectrum — nonlinear structure

gravitational particle production

$$\Omega_{\rm vdm} \sim 0.3 \left(\frac{m}{10^{-5}\,{\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^{4}$$

Graham, Mardon, Rajendran (2016)

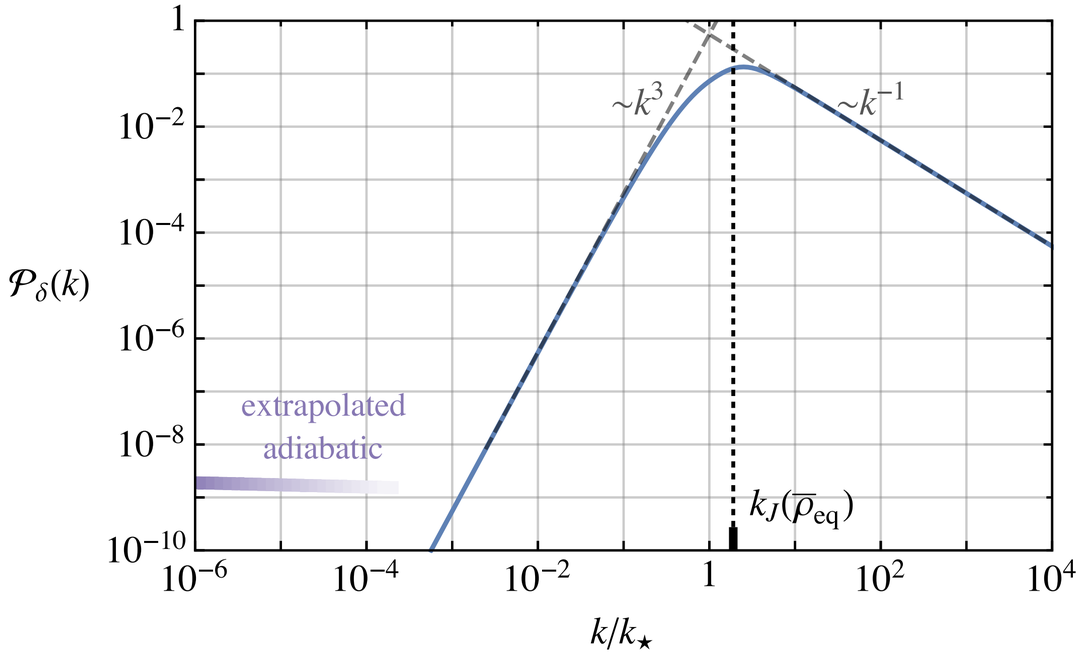
Ahmed, Grzadkowski, Socha (2020)

Kolb & Long (2020)

early derivation of action for longitudinal mode: Himmetoglu, Contaldi & Peloso (2008)

cannot do ultralight dark photons

$$m \gtrsim 10^{-5}\,{\rm eV}$$



$$k_J \sim \sqrt{m H_{\rm eq}}$$

gravitational particle production to nonlinear structures

cannot easily do ultralight dark photons

$$\Omega_{\rm vdm} \sim 0.3 \left(\frac{m}{10^{-5}\,{\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^{4}$$

Graham, Mardon, Rajendran (2016)

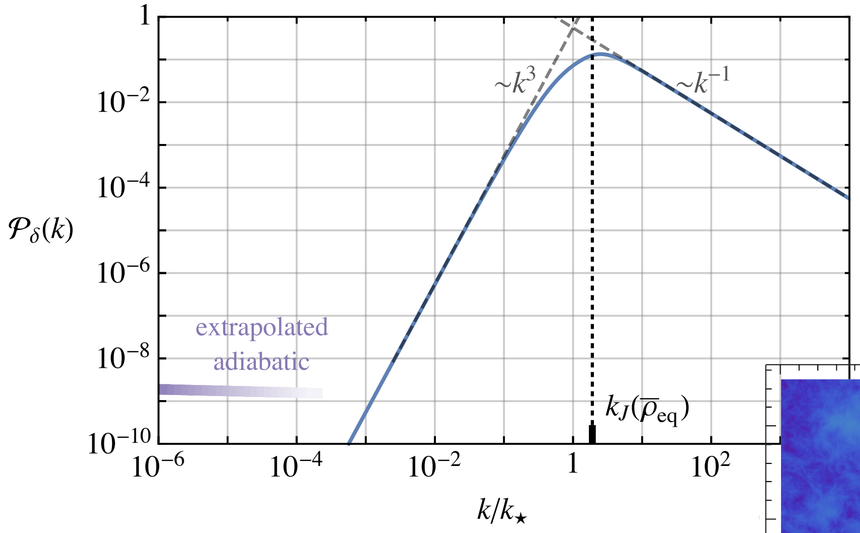
Ahmed, Grzadkowski, Socha (2020)

Kolb & Long (2020)

early derivation of action for longitudinal mode: Himmetoglu, Contaldi & Peloso (2008)

$$M_{\rm sol}(a) \sim 10^{-23} M_\odot \left(\frac{a_{\rm eq}}{a}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{3/2}$$

$$R_{\rm sol}(a) \sim 10^{4}\,{\rm km} \left(\frac{a}{a_{\rm eq}}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{1/2}$$



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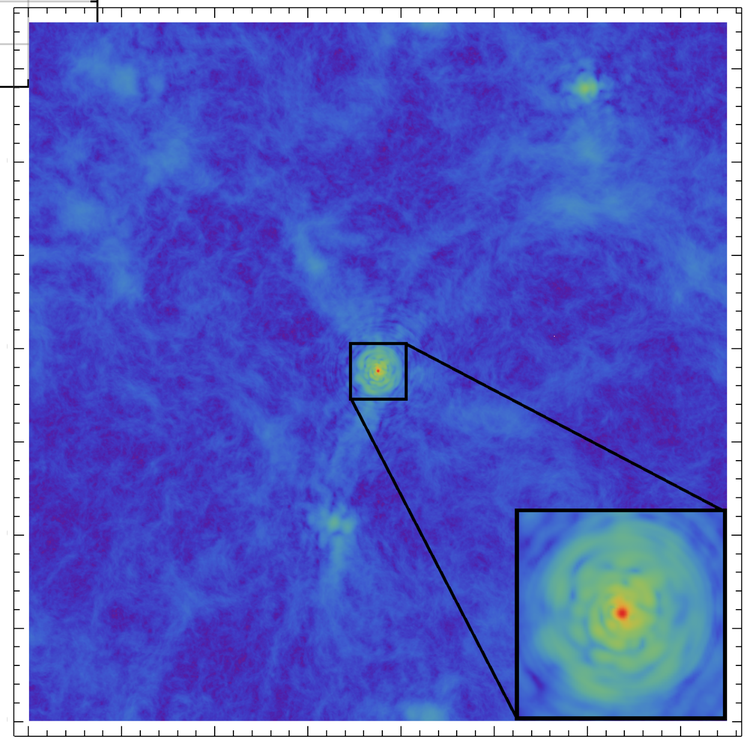
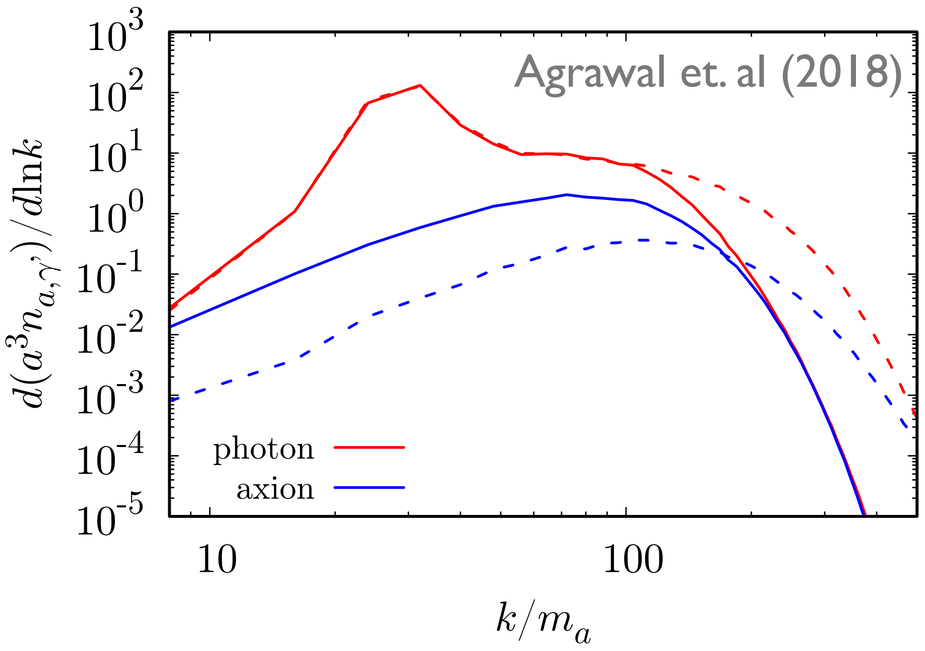


Gorghetto et. al (2022)

non-gravitational post-inflationary dark production?

$$\ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$



can do ultralight dark photons

lower bound on mass ?

Co, Pierce, Zhang, and Zhao (2018)
Dror, Harigaya, and Narayan (2018)
Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018)

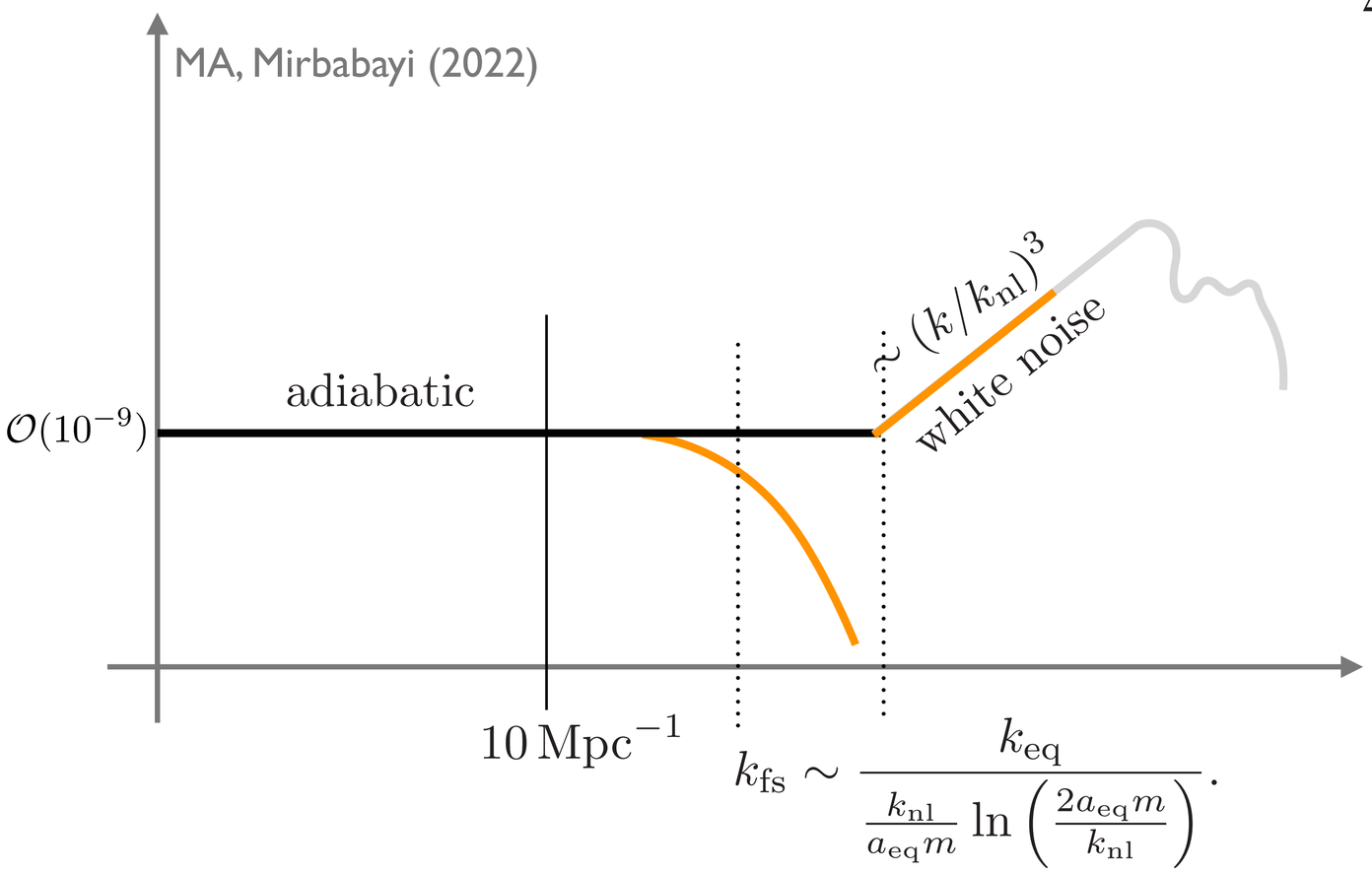
Also see: Long & Wang for production from strings and Co et. al for production from axion rotations

lower bound on ultra-light dark matter mass?

$$\Delta_\delta^2(k, t_{\rm eq}) \sim \Delta_\delta^{2(ad.)}(k, t_{\rm eq}) \times \underline{\Gamma_{\rm fs}(k/k_{\rm fs})} + \underline{\left(\frac{k}{k_{\rm nl}}\right)^3}$$

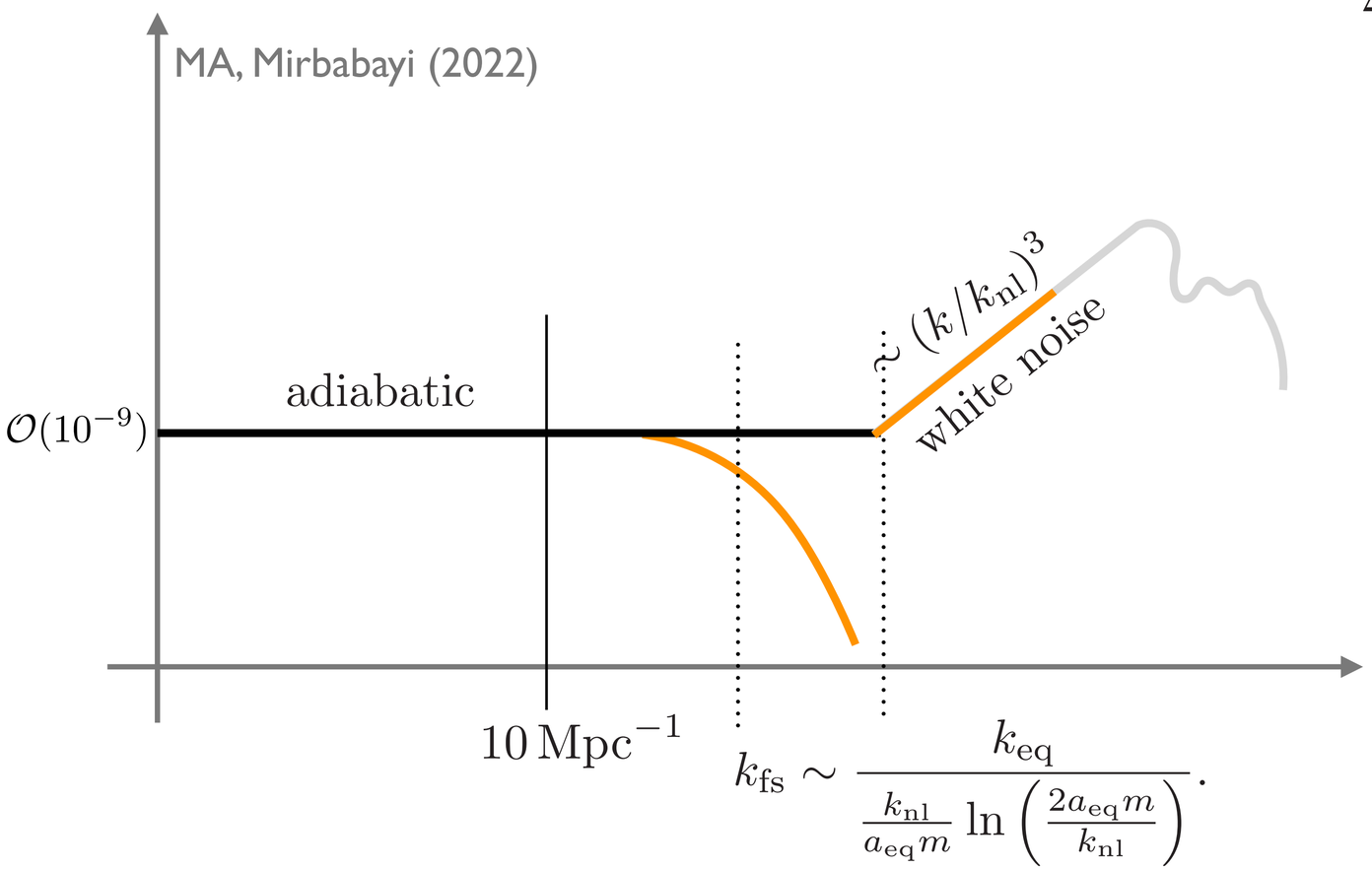
adiabatic * free-streaming

white noise



$k_{\rm nl} \sim$ momentum that dominates the momentum integral for the density of dark matter

lower bound on ultra-light dark matter mass?



$$\Delta_\delta^2(k, t_{\rm eq}) \sim \Delta_\delta^{2(ad.)}(k, t_{\rm eq}) \times \Gamma_{\rm fs}(k/k_{\rm fs}) + \left(\frac{k}{k_{\rm nl}}\right)^3$$

adiabatic * free-streaming white noise

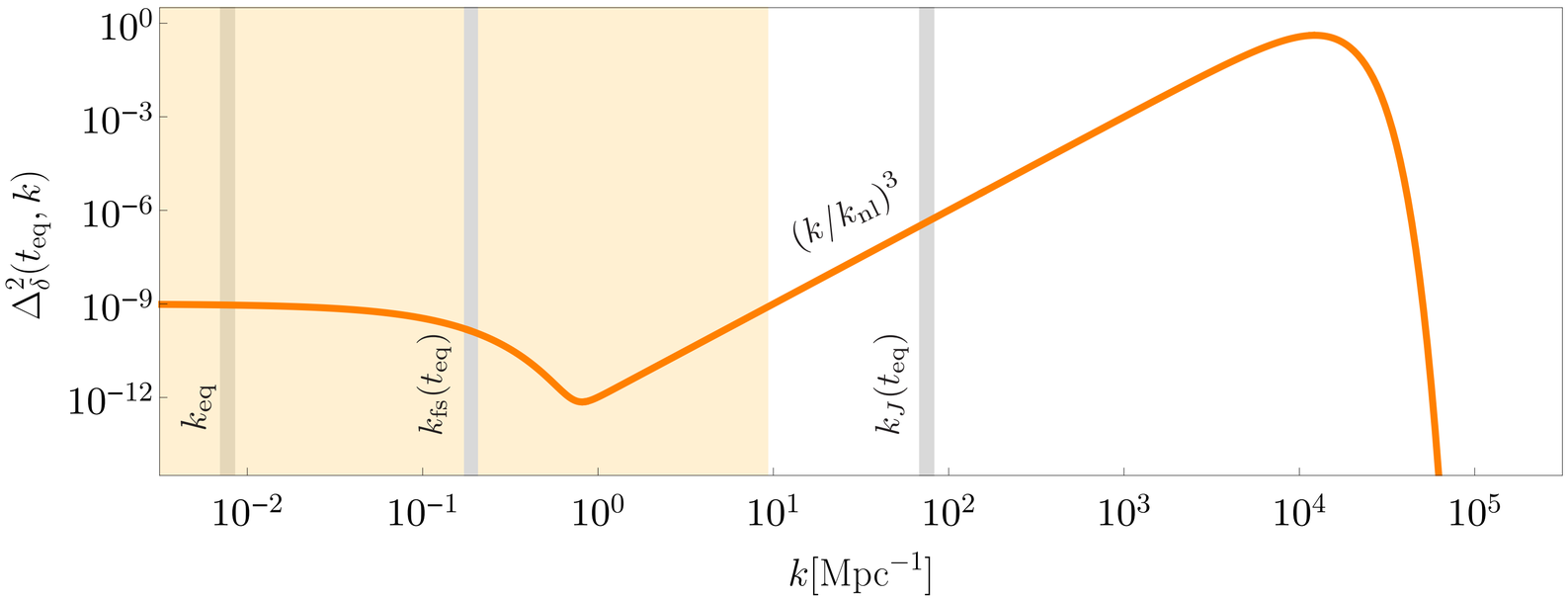
no enhancement or suppression $k \lesssim 10\,\mathrm{Mpc}^{-1}$

$$\Longrightarrow k_{\rm nl} \gtrsim 10^4\,\mathrm{Mpc}^{-1} \quad , \quad k_{\rm fs} \gtrsim 10\,\mathrm{Mpc}^{-1}$$

$$\Longrightarrow m \gtrsim 10^{-18}\,\mathrm{eV}$$

lower bound on ultra-light dark matter mass?

MA, Mirbabayi (2022)



$$\Longrightarrow m \gtrsim 10^{-18}\,{\rm eV}$$

better than that from Jean's scale by >2 orders of magnitude!

applies to all causally produced fields (not just VDM)

generalization to arbitrary spin

spin-s field as dark matter

Einstein

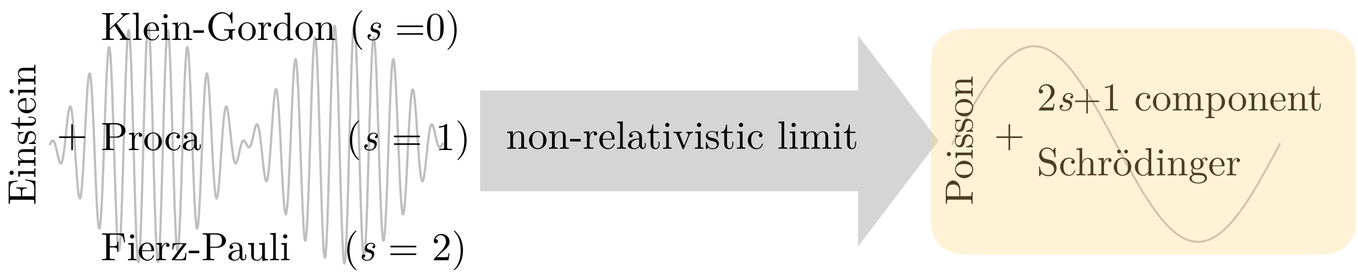
Klein-Gordon (s =0)

+ Proca ($s = 1$)

Fierz-Pauli ($s = 2$)

non-relativistic limit = multicomponent Schrödinger-Poisson

spin-s fields as light dark matter

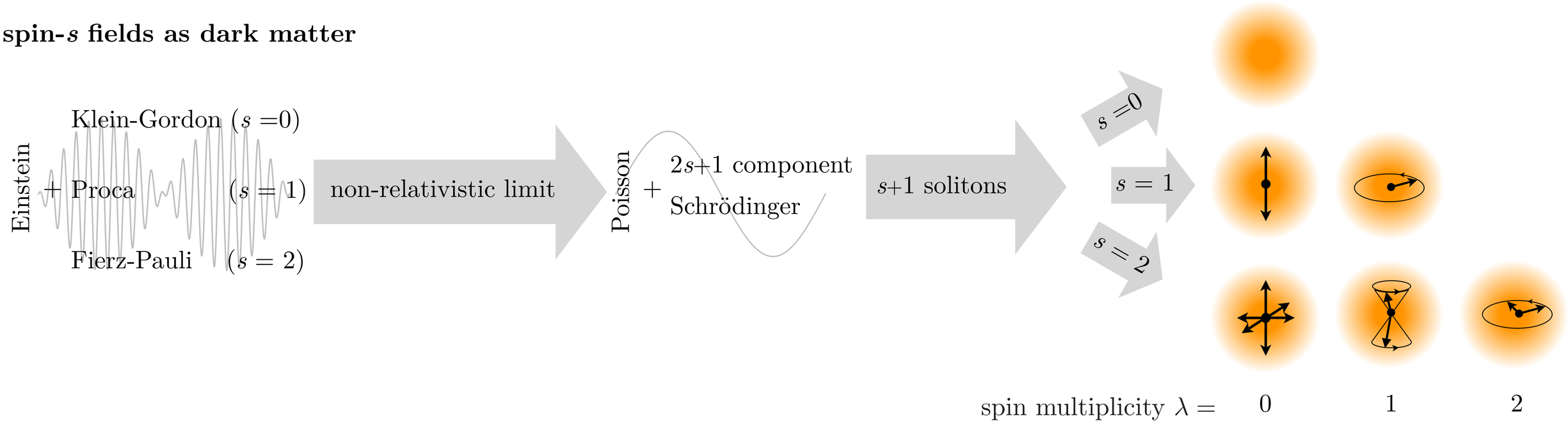


-phenomenology/numerical simulations

- interference ~1/(2s+1)

extremally polarized solitons

spin-s fields as dark matter



macroscopic spin $\boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

$N =$ # of particles in soliton

Jain & MA (2021)

immediate future directions

- cosmological scale simulations remain to be done (with S. May)
- MW sattellite populations related constraints (with R. Wechsler & E. Nadler)
- cold-atom systems (ongoing with H. Pu & M. Jain)

i-SPin: An integrator for multicomponent Schrodinger-Poisson systems with self-interactions

arXiv: 2211.08433
Mudit Jain & Mustafa Amin

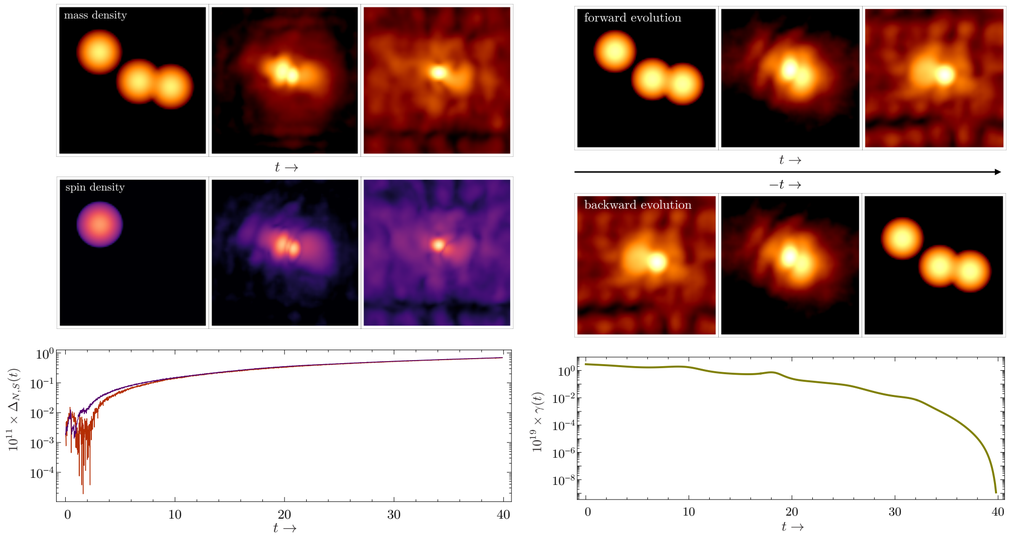
i-SPin: An algorithm (and publicly available code) to numerically evolve multicomponent Schrodinger-Poisson (SP) systems, including attractive/repulsive self-interactions + gravity

problem: If SP system represents the non-relativistic limit of a massive vector field, non-gravitational self-interactions (in particular, *spin-spin* type interactions) introduce new challenges related to mass and spin conservation which are not present in purely gravitational systems.

solution: Above challenges addressed with a novel analytical solution for the non-trivial 'kick' step in the algorithm (sec 4.3.2)

features: (i) second order accurate evolution (ii) spin and mass conserved to machine precision (iii) reversible

generalizations: n-component fields with SO(n) symmetry, an expanding universe relevant for cosmology, and the inclusion of external potentials relevant for laboratory settings



$$i\hbar\,\partial_t \boldsymbol{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi} - \frac{\lambda(\hbar c)^3}{4(mc^2)^2}\Big[(\boldsymbol{\Psi}\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}^\dagger + 2\,(\boldsymbol{\Psi}^\dagger\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}\Big]$$

$$V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) = -\frac{\lambda(\hbar c)^3}{8(mc^2)^2}\left[3\rho^2 - \frac{(\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{S}})}{\hbar^2}\right]$$

number density $\rho = \boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$

spin density $\boldsymbol{\mathcal{S}} = i\hbar\boldsymbol{\Psi}\times\boldsymbol{\Psi}^\dagger$