Small Scale Structure in Vector Dark Matter

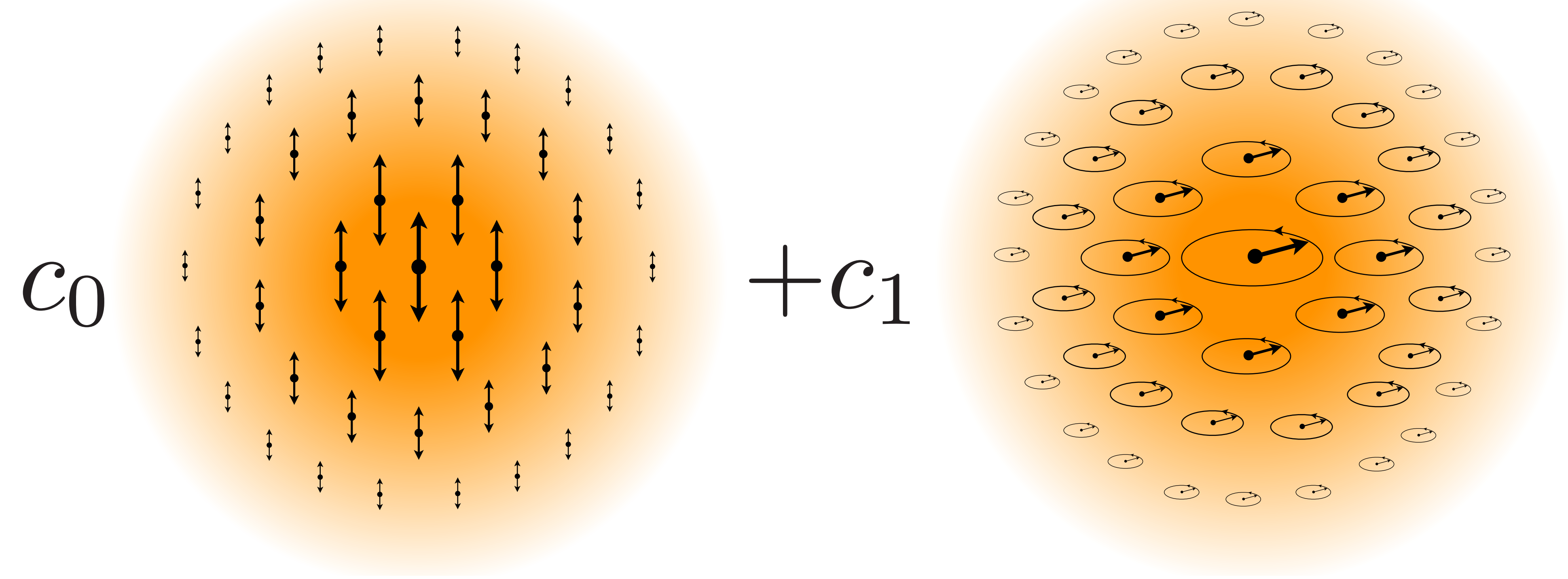
Mustafa A. Amin RICE

M. Jain R. Karur P. Mocz H. Zhang

arXiv: 2111.08700
arXiv: 2109.04892
arXiv: 2203.11935

distinct phenomenology in (ultra)light vector dark matter

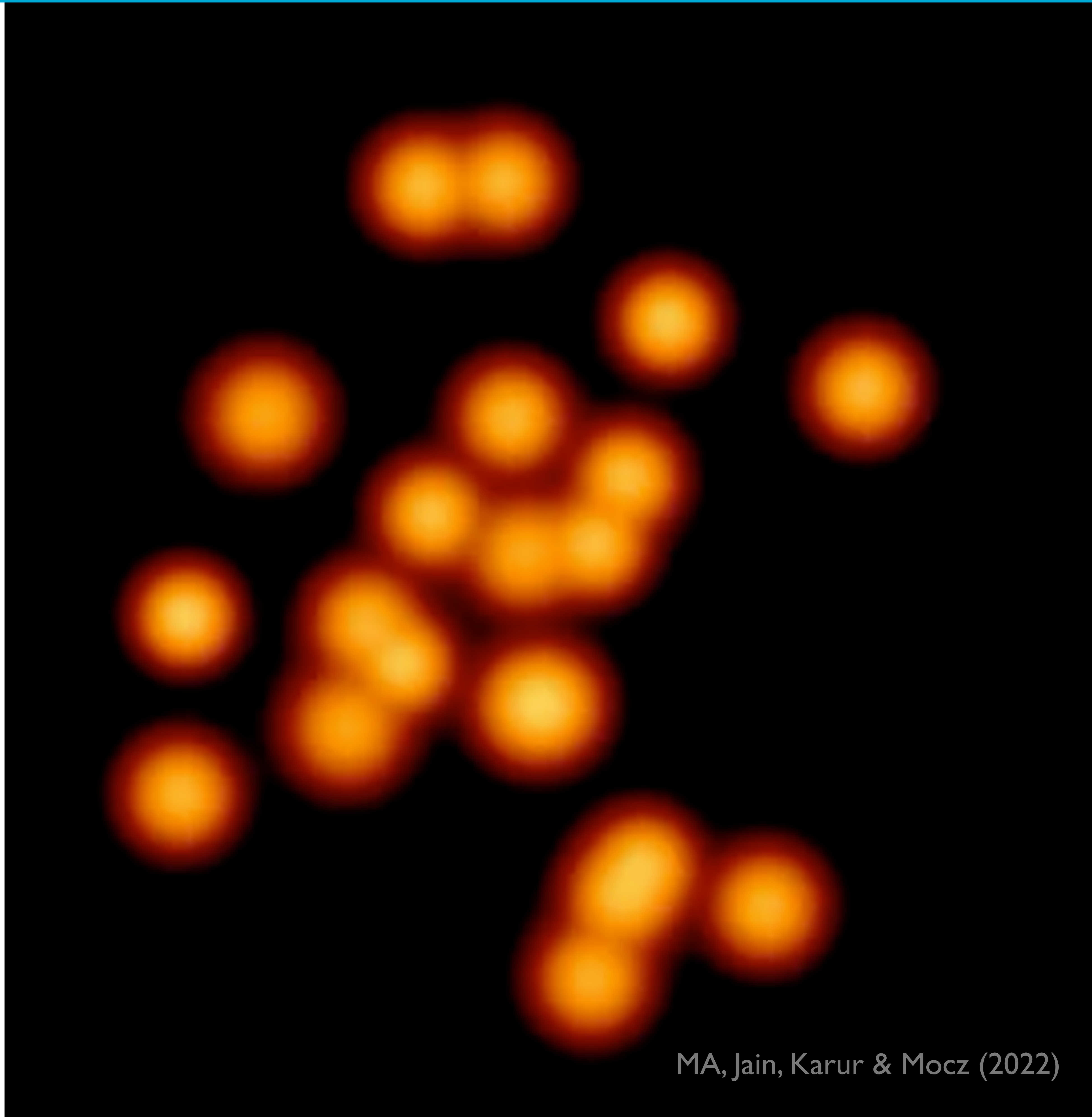
- new class of polarized vector solitons with macroscopic spin



Adshead & Lozanov (2021) Jain and MA (2021) Zhang, Jain MA (2021)

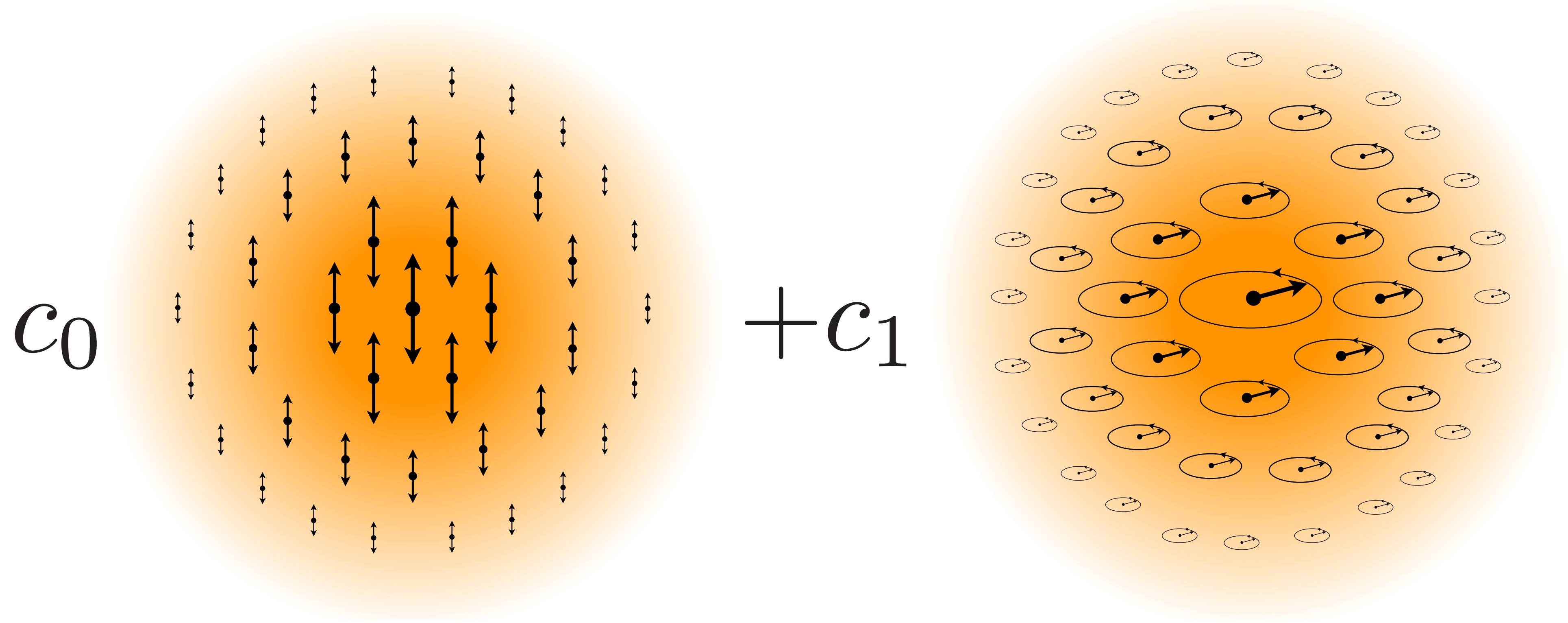
- interference patterns, and halo density profiles

MA, Jain, Karur & Mocz (2022)



distinct phenomenology in (ultra)light vector dark matter

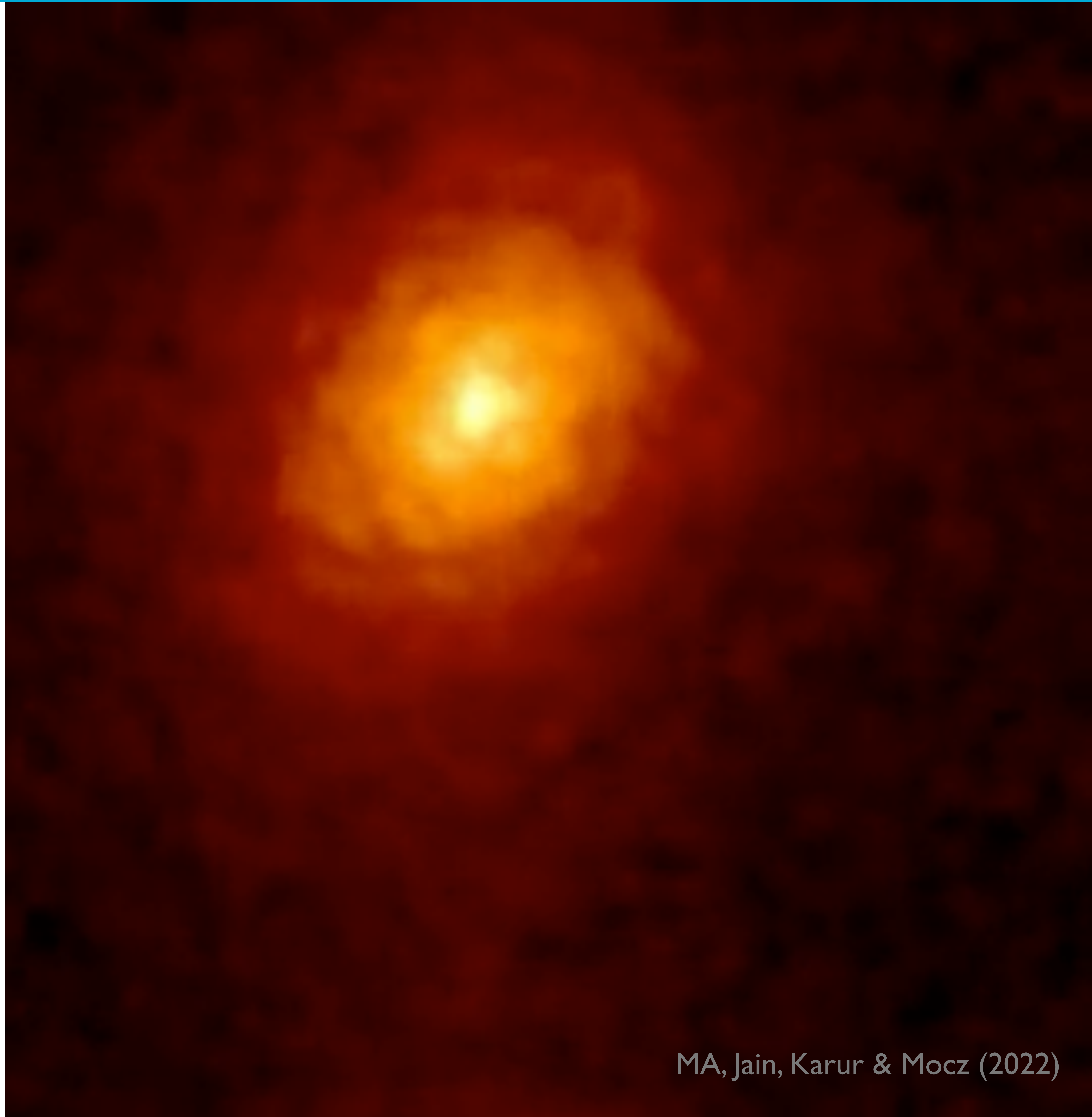
- new class of polarized vector solitons with macroscopic spin



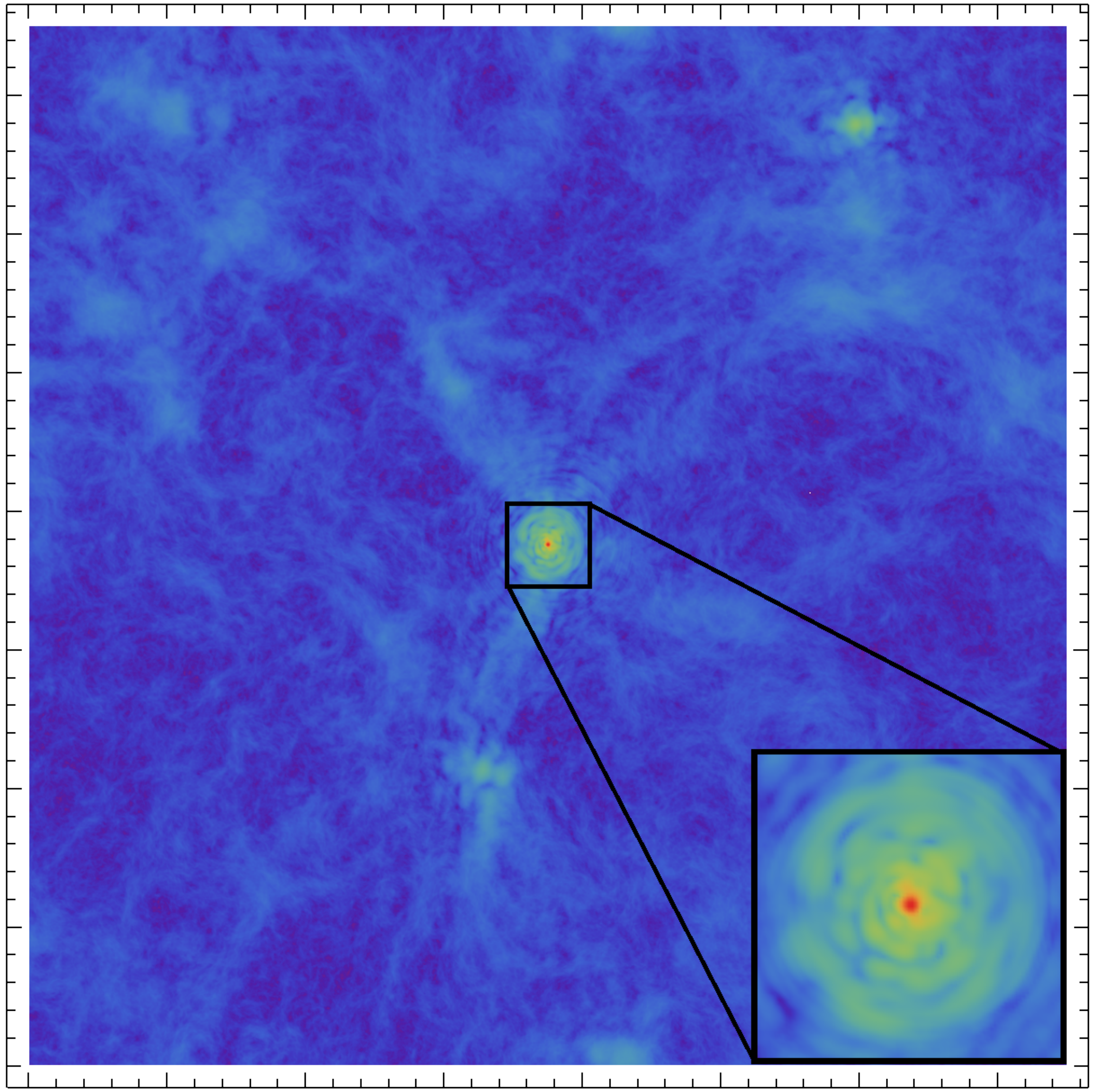
Adshead & Lozanov (2021) Jain and MA (2021) Zhang, Jain MA (2021)

- interference patterns, and halo density profiles

MA, Jain, Karur & Mocz (2022)

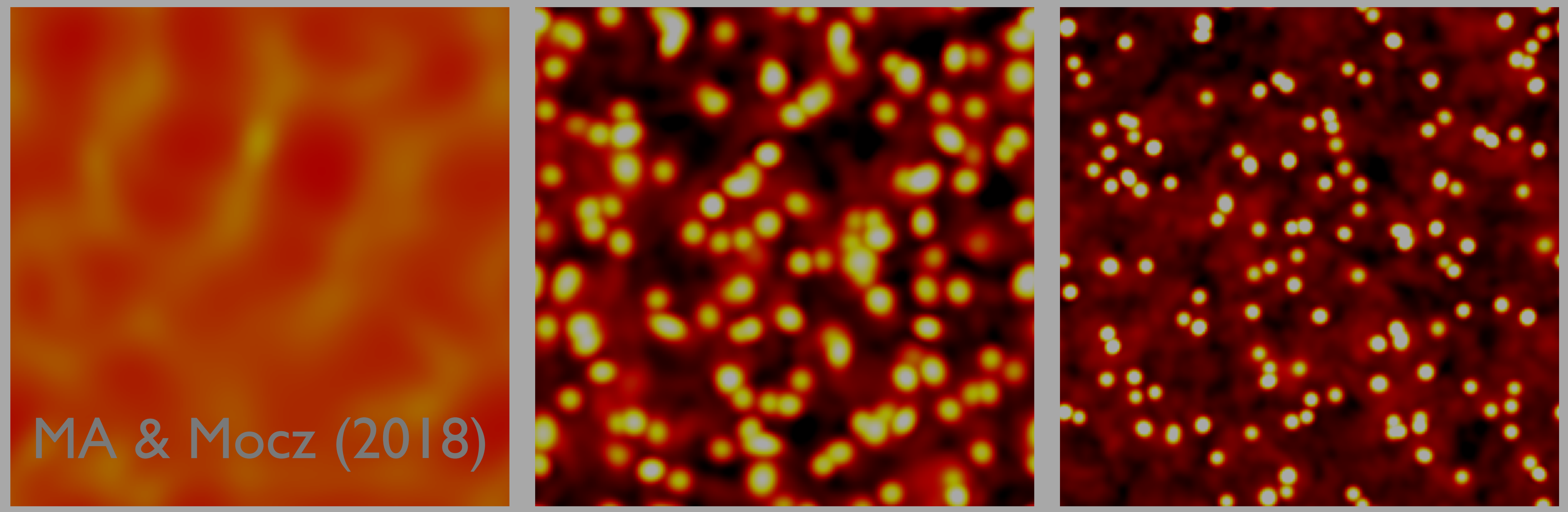


formation mechanisms?



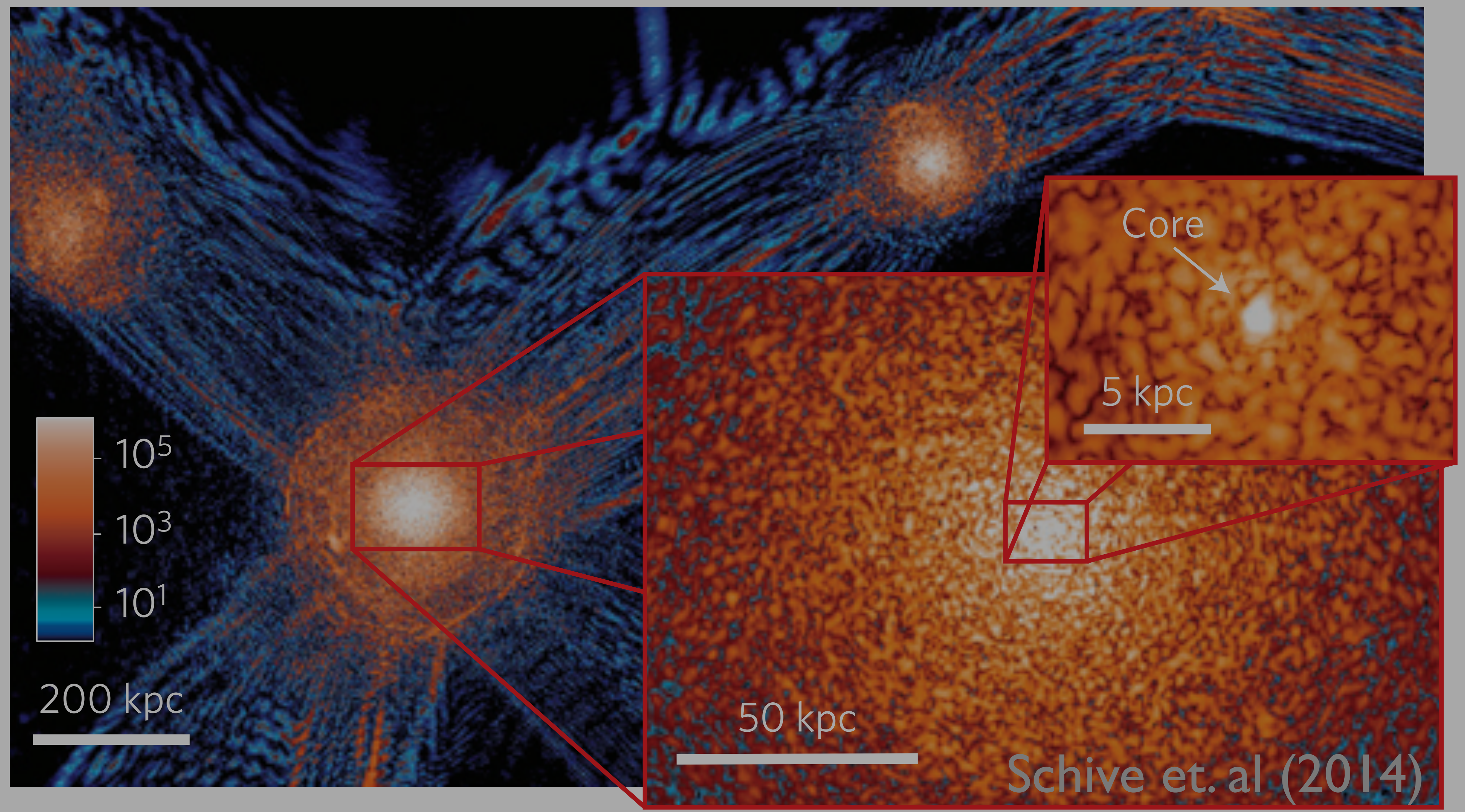
Gorghetto, Hardy, March-Russell, Song & West (2022)

misaligned scalar + self-interactions?



in progress

gravitational in late universe



motivation & introduction

dark matter mass ?

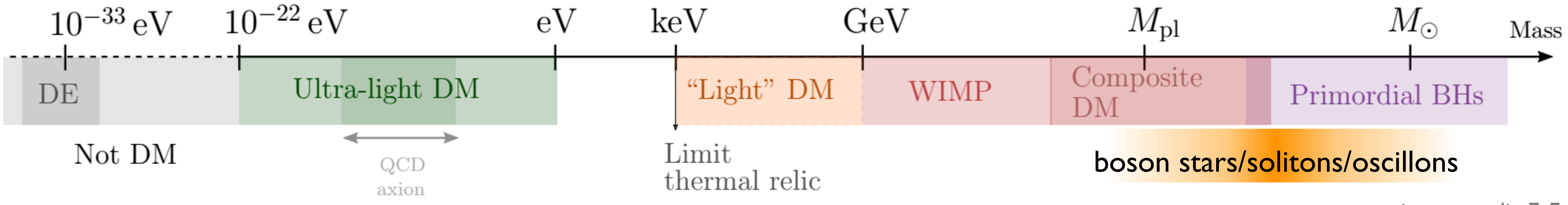
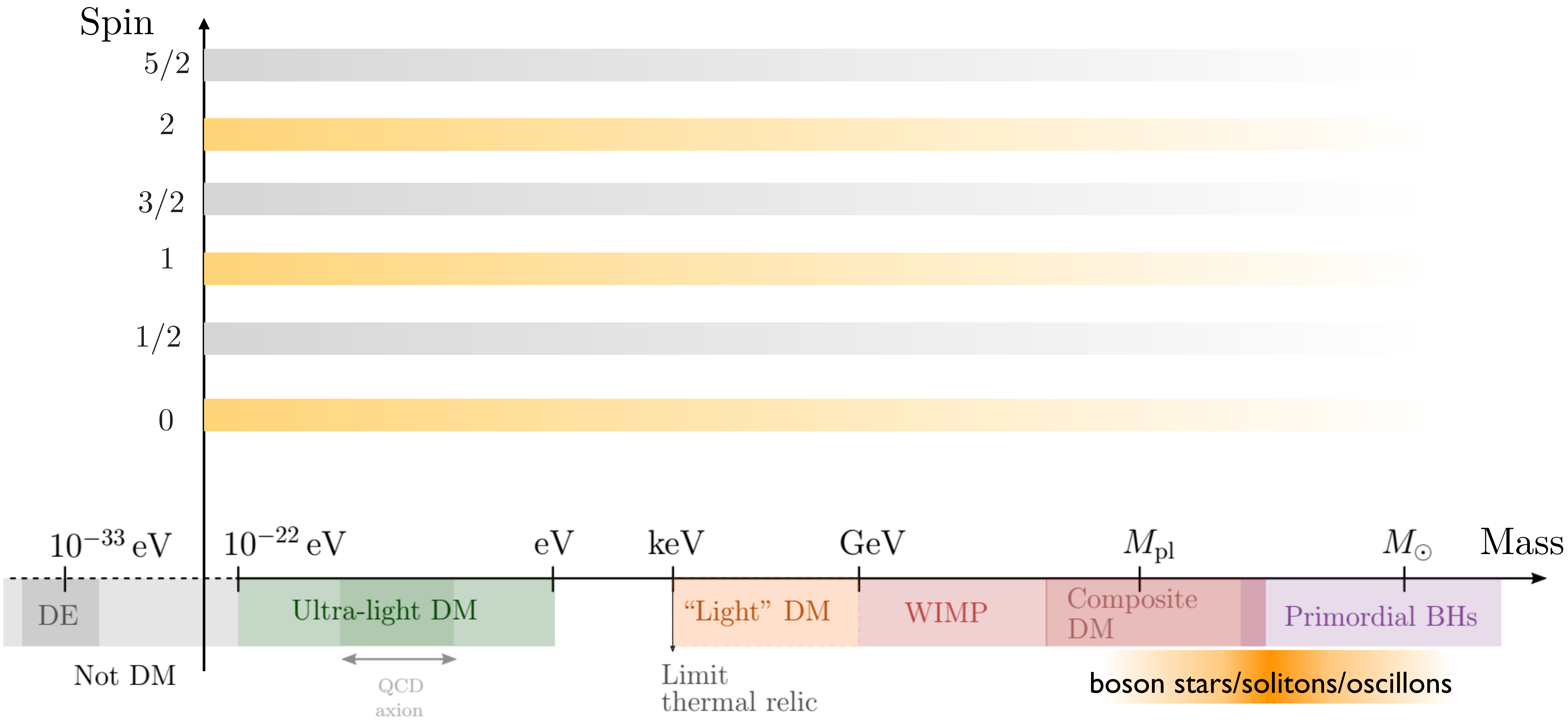
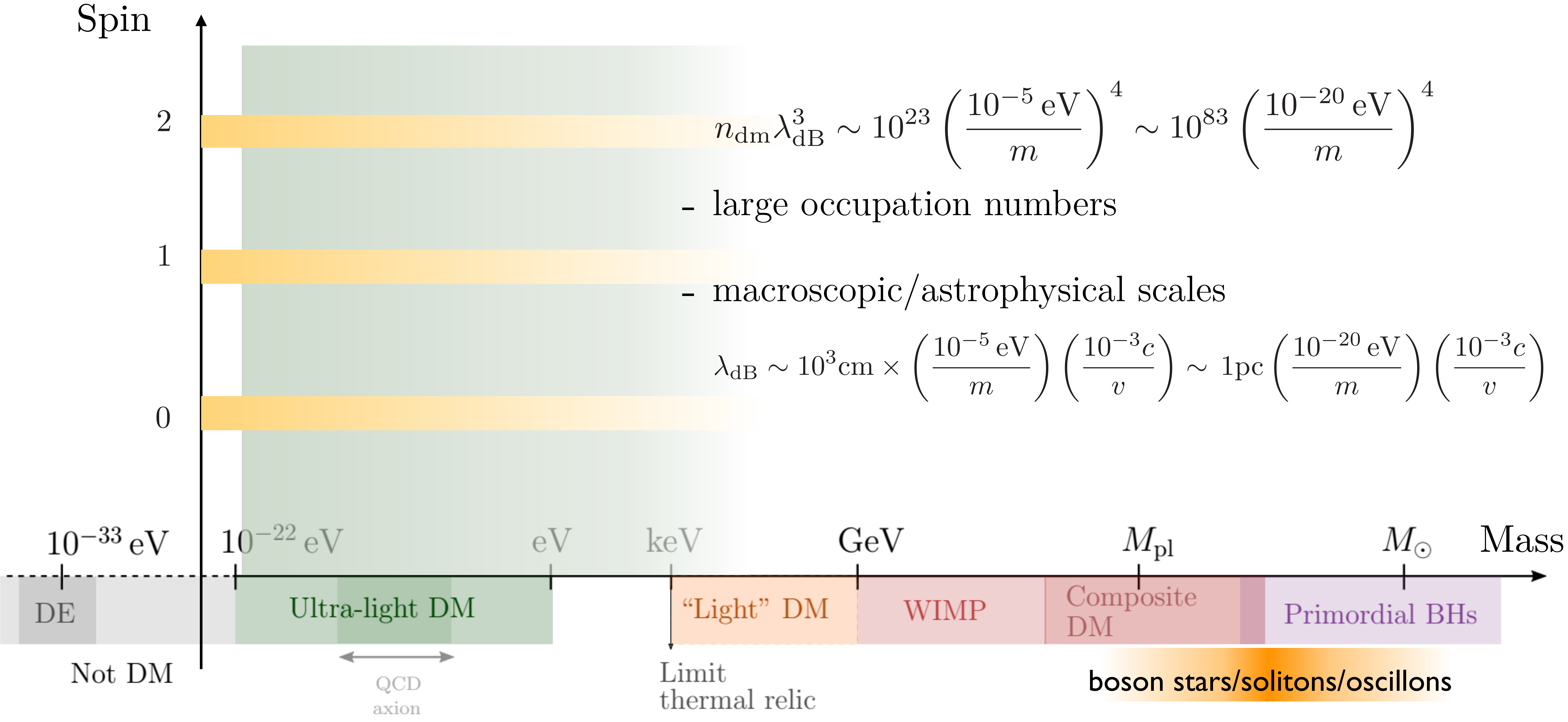


image credit: E. Ferreira

dark matter spin ?



light, bosonic wave dark matter



$$n_{\rm dm}\lambda_{\rm dB}^3 \sim 10^{23}\left(\frac{10^{-5}\,{\rm eV}}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)^4$$

- large occupation numbers

- macroscopic/astrophysical scales

$$\lambda_{\rm dB} \sim 10^3{\rm cm}\times\left(\frac{10^{-5}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right) \sim 1{\rm pc}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

models

vector dark matter

$$S = \int \mathrm{d}^4x \, \sqrt{-g}\Big[-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} \, \mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2} \, g^{\mu\nu} W_\mu W_\nu + \frac{c^3}{16\pi G} R + ...\Big]$$

vector case

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

$$S = \int d^4x \, \sqrt{-g} \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2 + \frac{c^3}{16\pi G}R + \ldots\right]$$

scalar case

non-relativistic limit = multicomponent Schrödinger-Poisson

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\,\mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\,g^{\mu\nu}W_\mu W_\nu + \frac{c^3}{16\pi G}R + \ldots\Big]$$

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

non-relativistic limit

split in "fast" and "slow" parts

$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\mathcal{S}_{nr} = \int \mathrm{d}t\mathrm{d}^3x\left[\frac{i\hbar}{2}\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}} + \mathrm{c.c.} - \frac{\hbar^2}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} + \frac{1}{8\pi G}\,\Phi\nabla^2\Phi - m\,\Phi\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}\right]$$

Adshead & Lozanov (2021)
Jain & MA (2021)

non-relativistic limit = multicomponent Schrödinger-Poisson

$[\boldsymbol{\Psi}]_i = \psi_i \text{ with } i = 1, 2, 3$ vector case

$$i\hbar \frac{\partial}{\partial t}\boldsymbol{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi}\,, \qquad \nabla^2\Phi = 4\pi G m\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$$

$[\boldsymbol{\Psi}]_i = \psi_i \text{ with } i = 1$ scalar case

generalization to s spin-s field field in **Jain & MA (2021)**

conserved quantities

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$

$$N = \int \mathrm{d}^3x \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}, \quad \text{and} \quad M = mN, \qquad \text{(particle number and rest mass)}$$

$$E = \int \mathrm{d}^3x \Big[\frac{\hbar^2}{2m} \nabla \boldsymbol{\Psi}^\dagger \cdot \nabla \boldsymbol{\Psi} - \frac{Gm^2}{2} \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi} \int \frac{\mathrm{d}^3 y}{4\pi |\boldsymbol{x} - \boldsymbol{y}|} \boldsymbol{\Psi}^\dagger(\boldsymbol{y}) \boldsymbol{\Psi}(\boldsymbol{y}) \Big], \qquad \text{(energy)}$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger, \qquad \text{(spin angular momentum)}$$

$$\boldsymbol{L} = \hbar \int \mathrm{d}^3x \, \Re \left(i \, \boldsymbol{\Psi}^\dagger \nabla \boldsymbol{\Psi} \times \boldsymbol{x} \right). \qquad \text{(orbital angular momentum)}$$

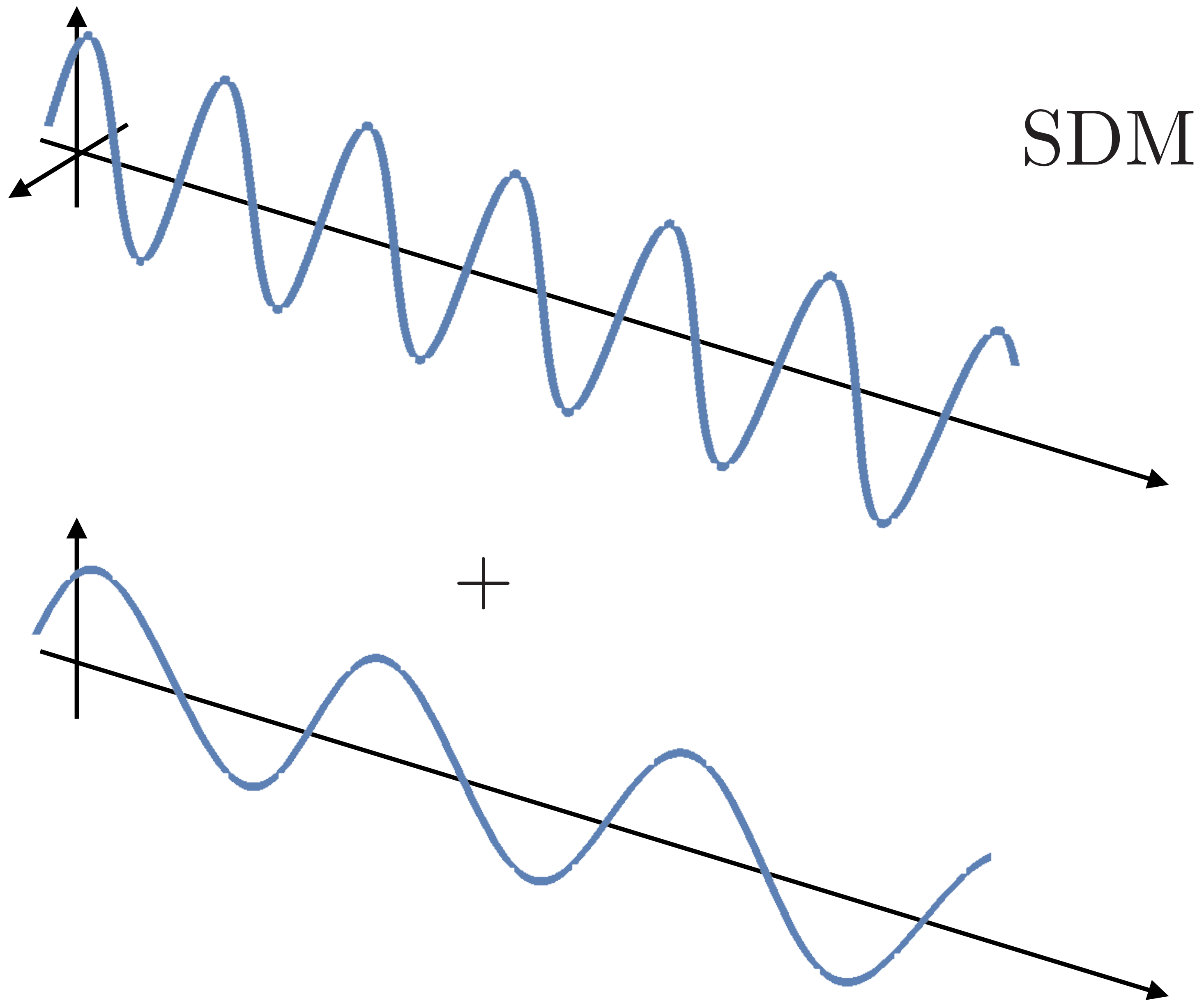
vector vs. scalar DM:
two key differences

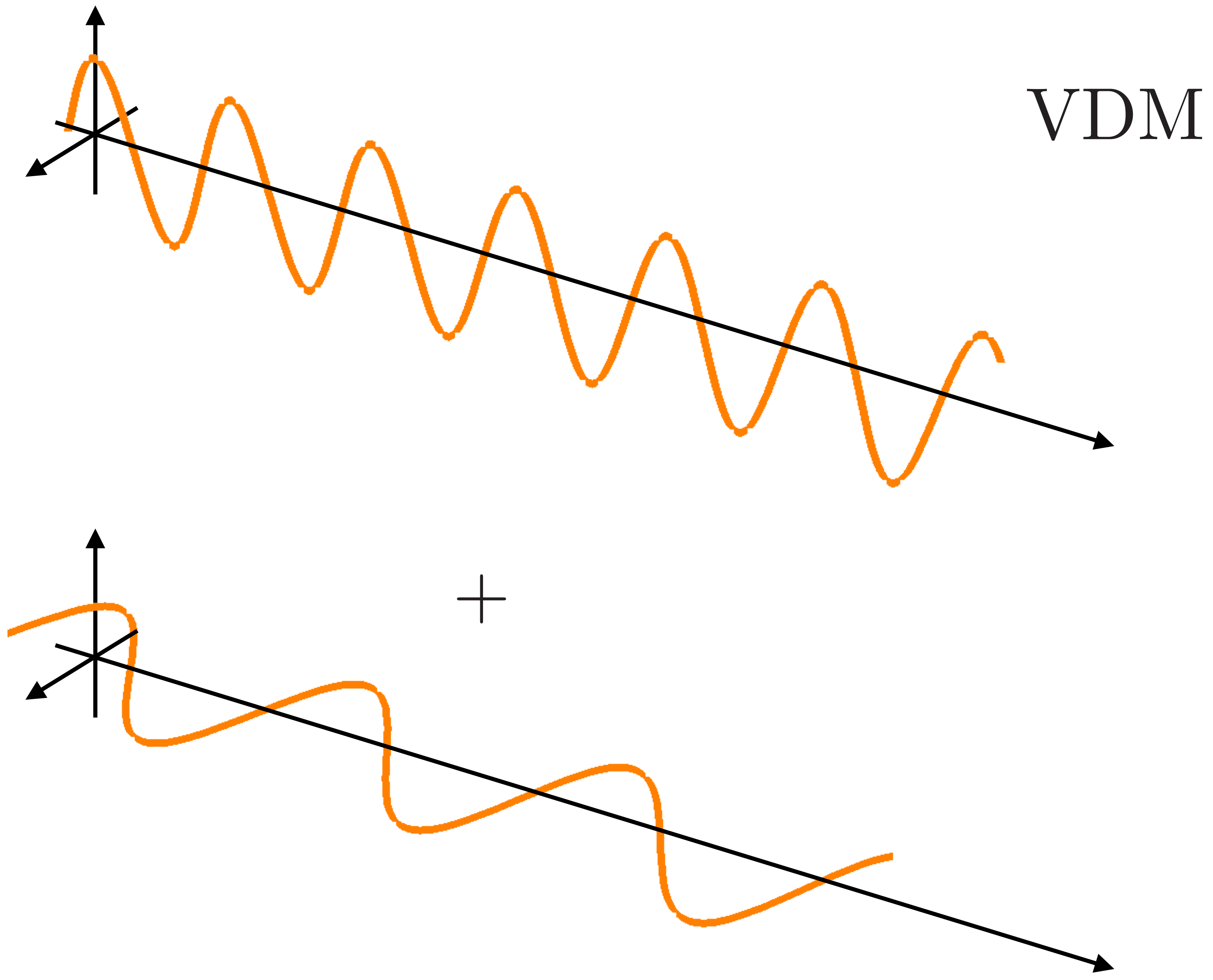
interference

polarized solitons

analytics

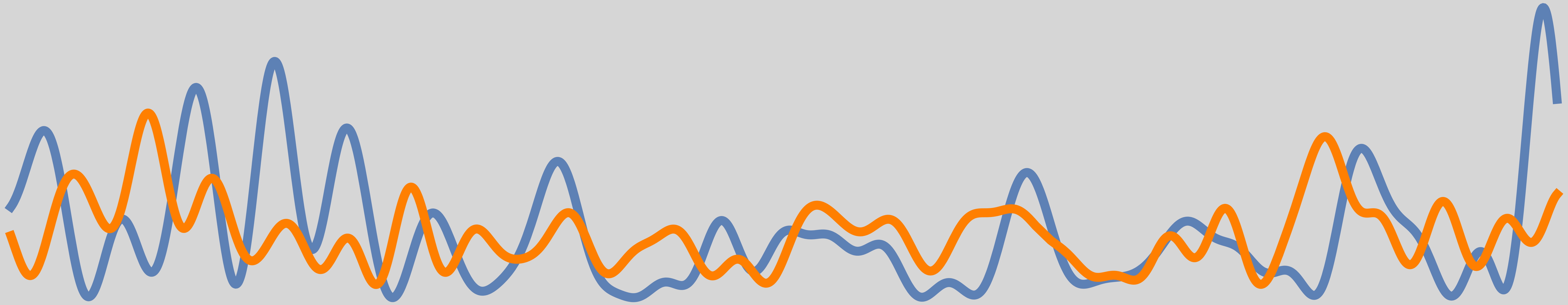
wave interference





$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 \neq |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$

$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 = |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$



reduced wave interference in VDM

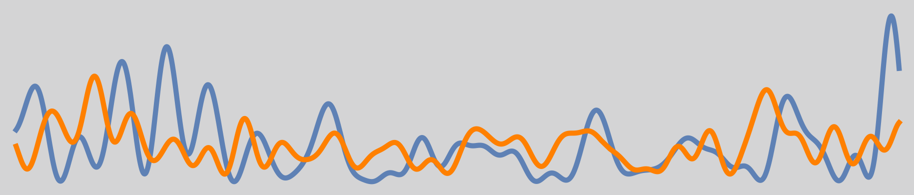
$s = 0$ for SDM and $s = 1$ for VDM

$$\boldsymbol{\Psi}_a(\boldsymbol{x}) = \boldsymbol{\epsilon}_a^{(s)} e^{i\boldsymbol{k}_a \cdot \boldsymbol{x}},$$

$$|\boldsymbol{\Psi}_a(\boldsymbol{x}) + \boldsymbol{\Psi}_b(\boldsymbol{x})|^2 = 2\left(1 + \Re\left[\boldsymbol{\epsilon}_a^{(s)\dagger} \cdot \boldsymbol{\epsilon}_a^{(s)} e^{-i(\boldsymbol{k}_a - \boldsymbol{k}_b)\cdot \boldsymbol{x}}\right]\right) = 2\left(1 + \mathrm{int}_{(s)}\right)$$

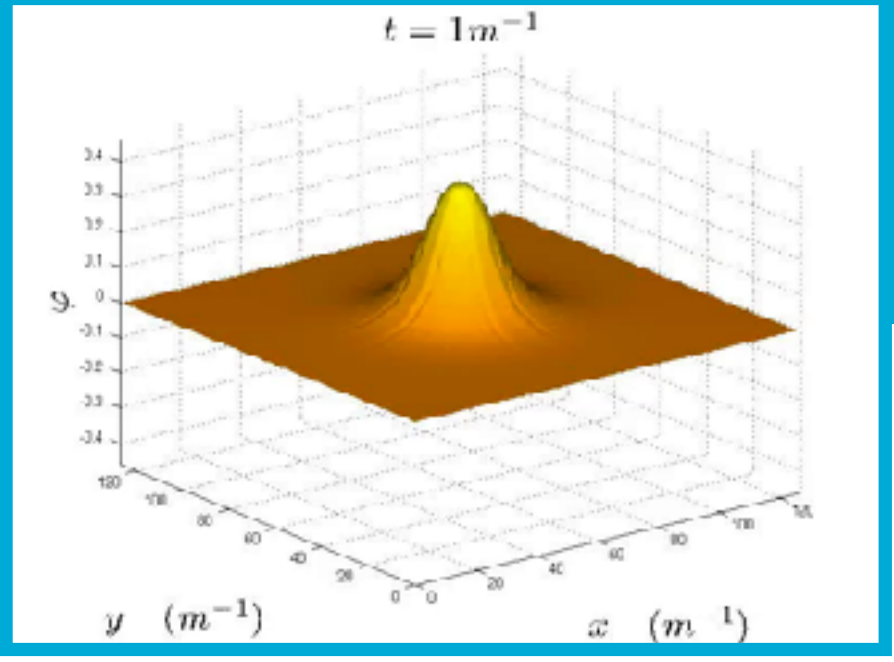
$$\sqrt{\langle \mathrm{int}^2_{(s)} \rangle} = \frac{1}{\sqrt{2(2s+1)}}$$

$$\frac{\sqrt{\langle \mathrm{int}^2_{(1)} \rangle}}{\sqrt{\langle \mathrm{int}^2_{(0)} \rangle}} = \frac{1}{\sqrt{3}}$$



MA, Jain, Karur & Mocz (2022)

solitons in massive spin-0 (scalar fields)



$$\phi(t, \boldsymbol{x}) = \frac{\hbar}{\sqrt{2mc}} \Re[\Psi_{\rm sol}(\mu, \boldsymbol{x}) e^{imc^2 t/\hbar}]$$

$$\Psi_{\rm sol}(t, \boldsymbol{x}) = \psi_{\rm sol}(\mu, \boldsymbol{x}) e^{i\mu c^2 t/\hbar}$$

$$M_{\rm sol} \approx 60.7 \frac{m_{\rm pl}^2}{m} \sqrt{\frac{\mu}{m}}$$

$$R_{\rm sol} \sim \sqrt{\frac{m}{\mu}} \frac{\hbar}{mc}$$

lowest energy state for fixed N

$N =$ # of particles in soliton

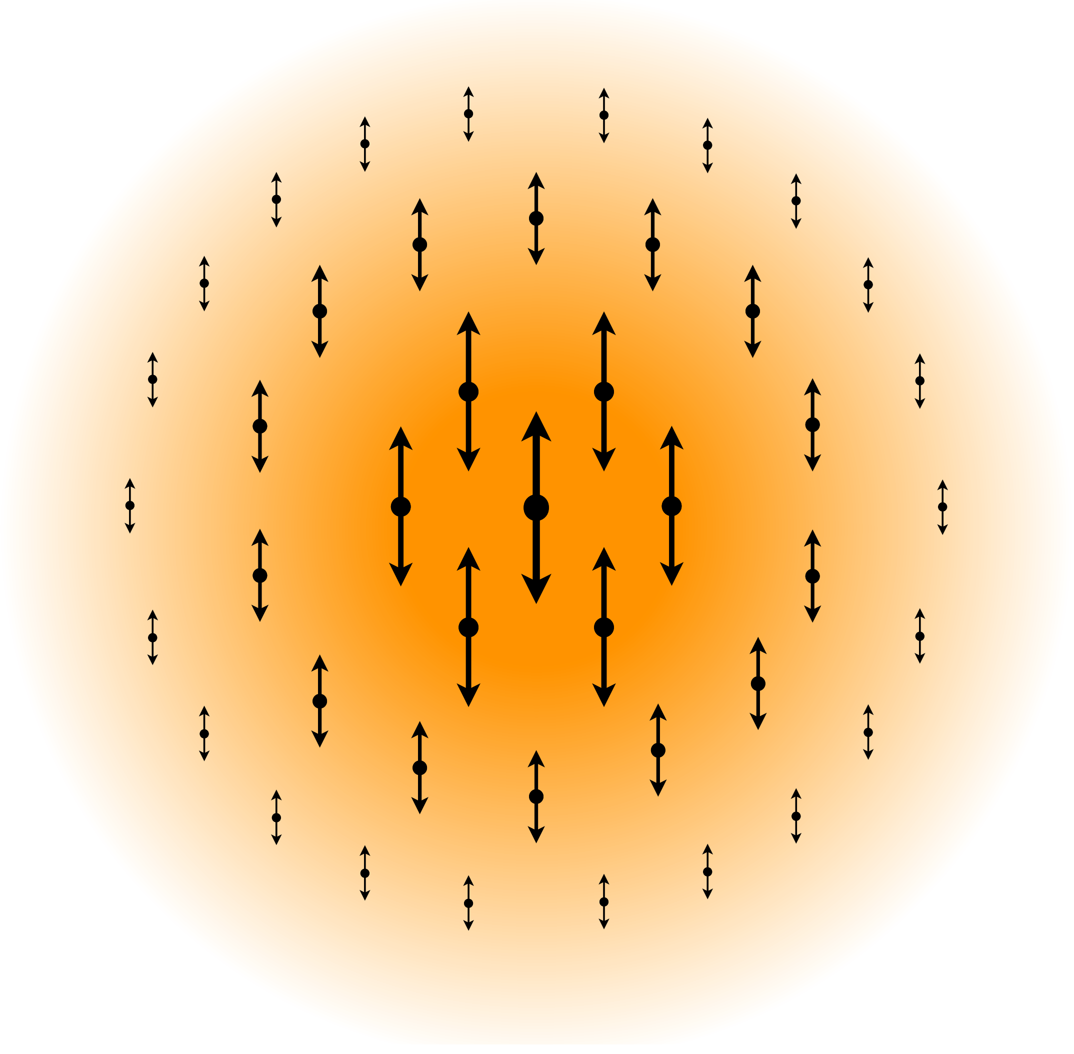
$\mu/m \ll 1$

"polarized" vector solitons

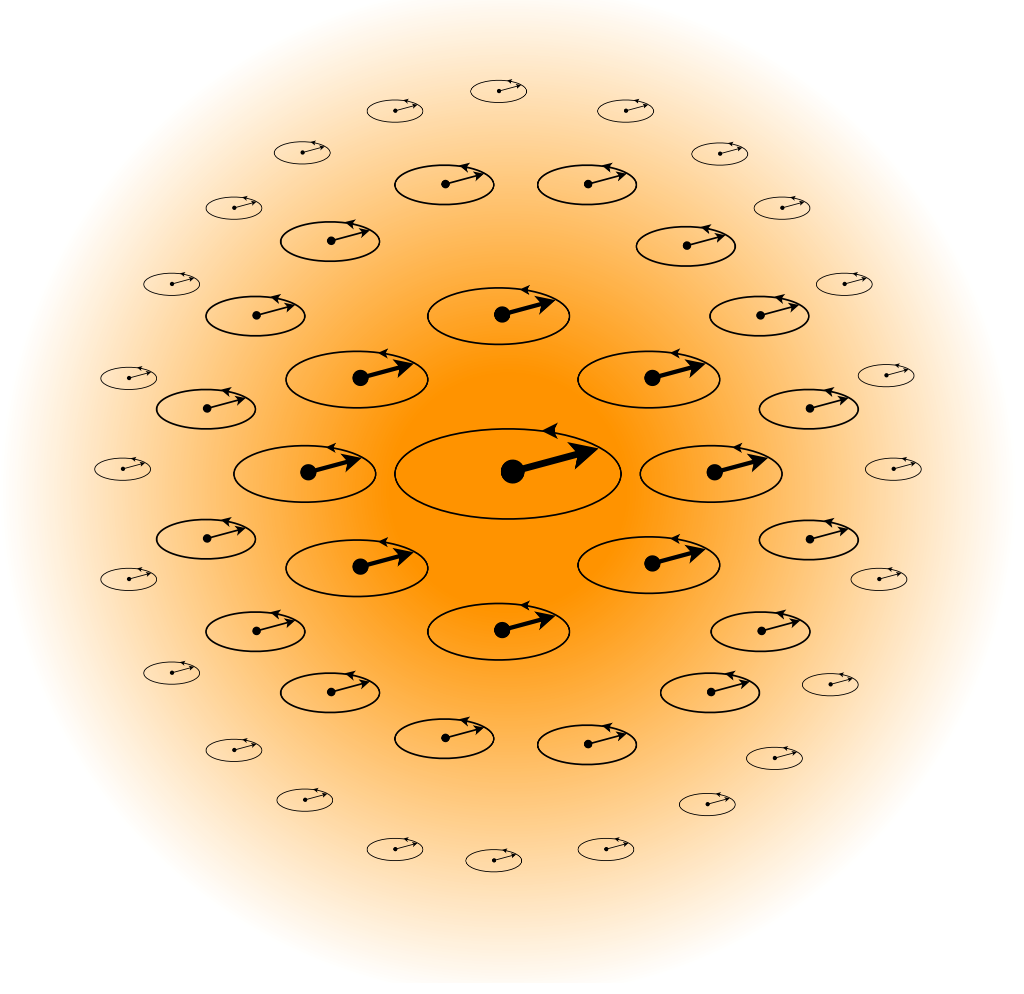
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}} \Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x}) e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\mathrm{sol}}(t,\boldsymbol{x}) = \psi_{\mathrm{sol}}(\mu, r) e^{i\mu c^2 t/\hbar} \boldsymbol{\epsilon}$$

$$\boldsymbol{\epsilon}^\dagger \boldsymbol{\epsilon} = 1$$



$$\boldsymbol{\epsilon}^{(0)}_{1,\hat{z}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$\boldsymbol{\epsilon}^{(\pm 1)}_{1,\hat{z}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

“polarized” vector solitons

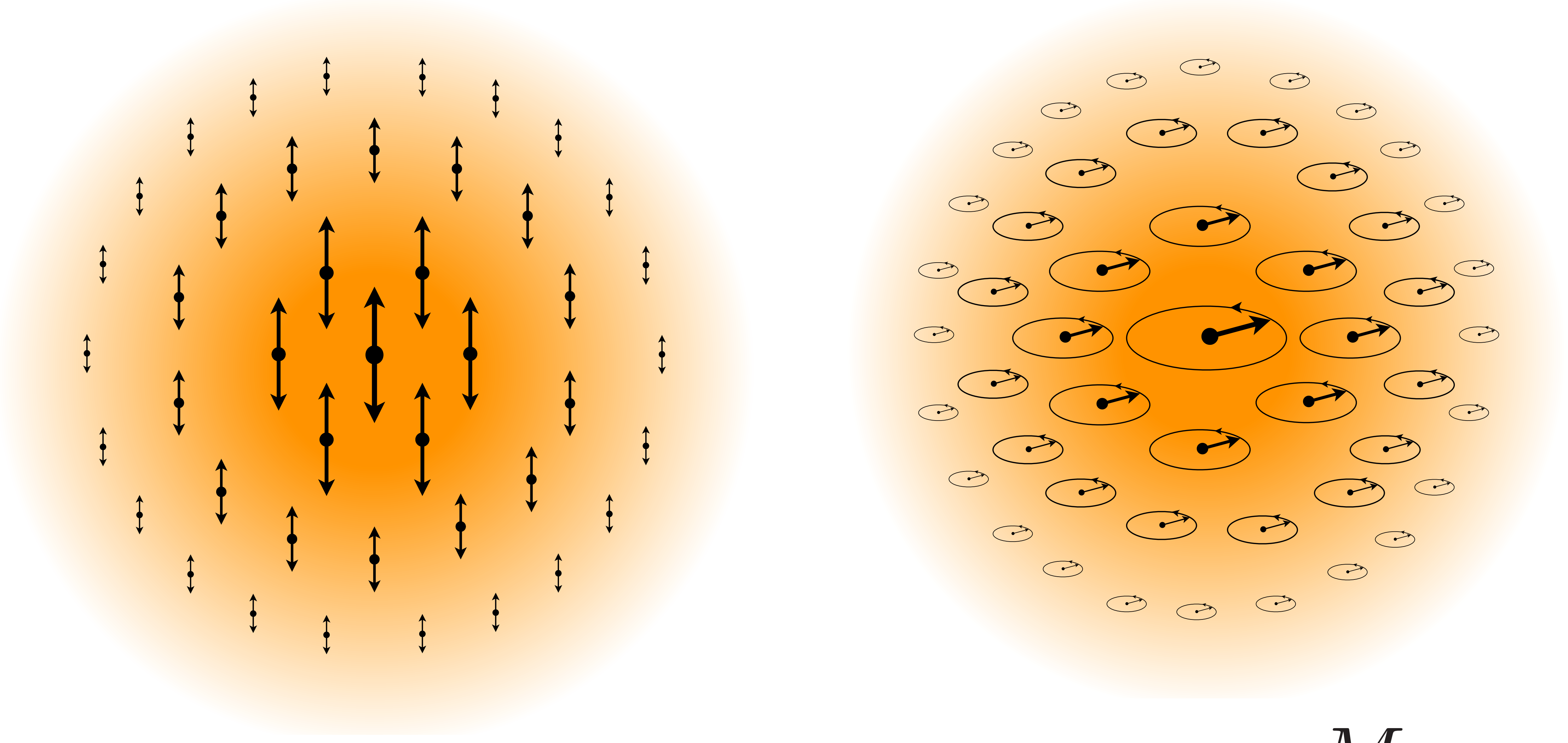
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$\boldsymbol{S}_{\rm sol} \approx i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)60.7\frac{m_{\rm pl}^2}{m^2}\sqrt{\frac{\mu}{m}}\hbar \quad = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$

macroscopic spin $\qquad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton



$\boldsymbol{S}_{\rm tot} = 0\hat{z}$ $\qquad$ $\boldsymbol{S}_{\rm tot} = \hbar\dfrac{M_{\rm sol}}{m}\hat{z}$

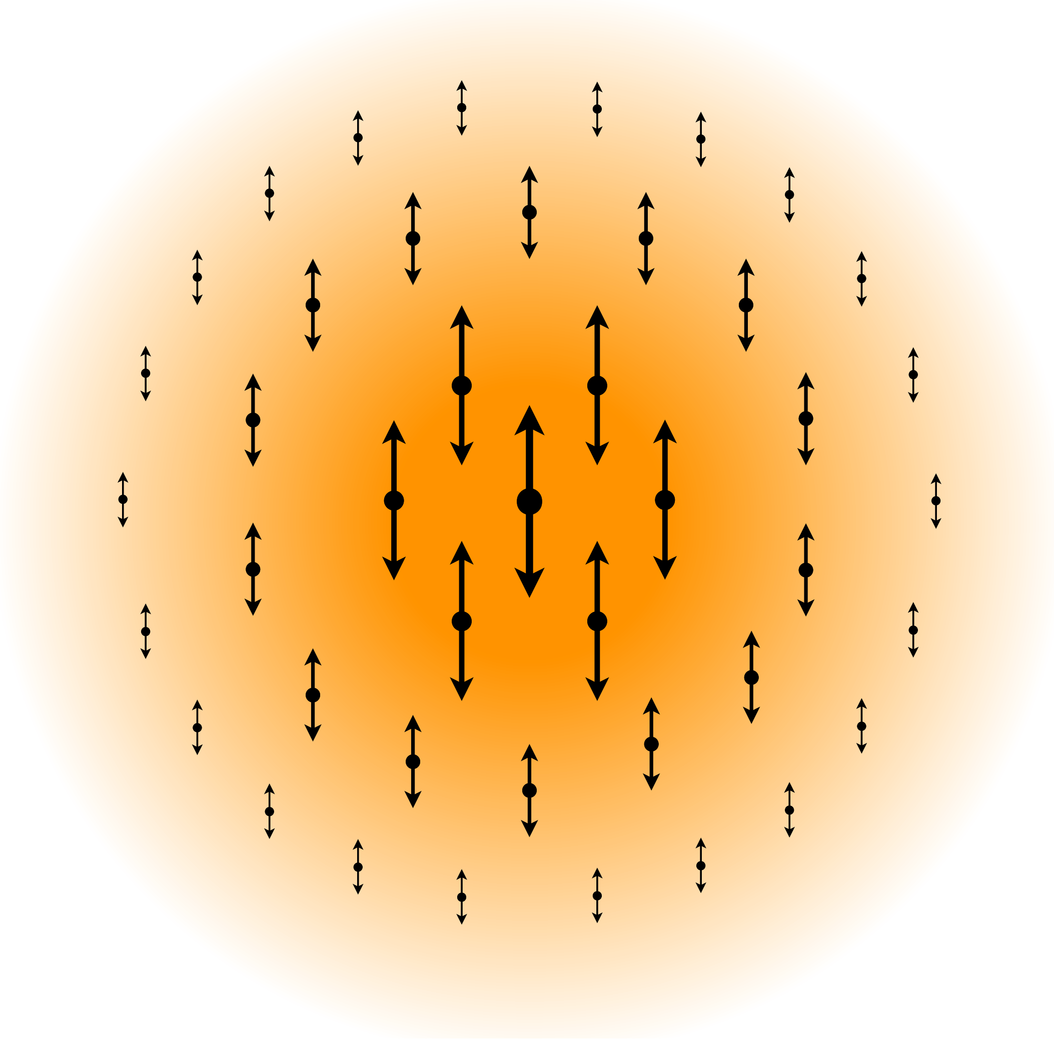
Jain & MA (2021)

“polarized” vector solitons

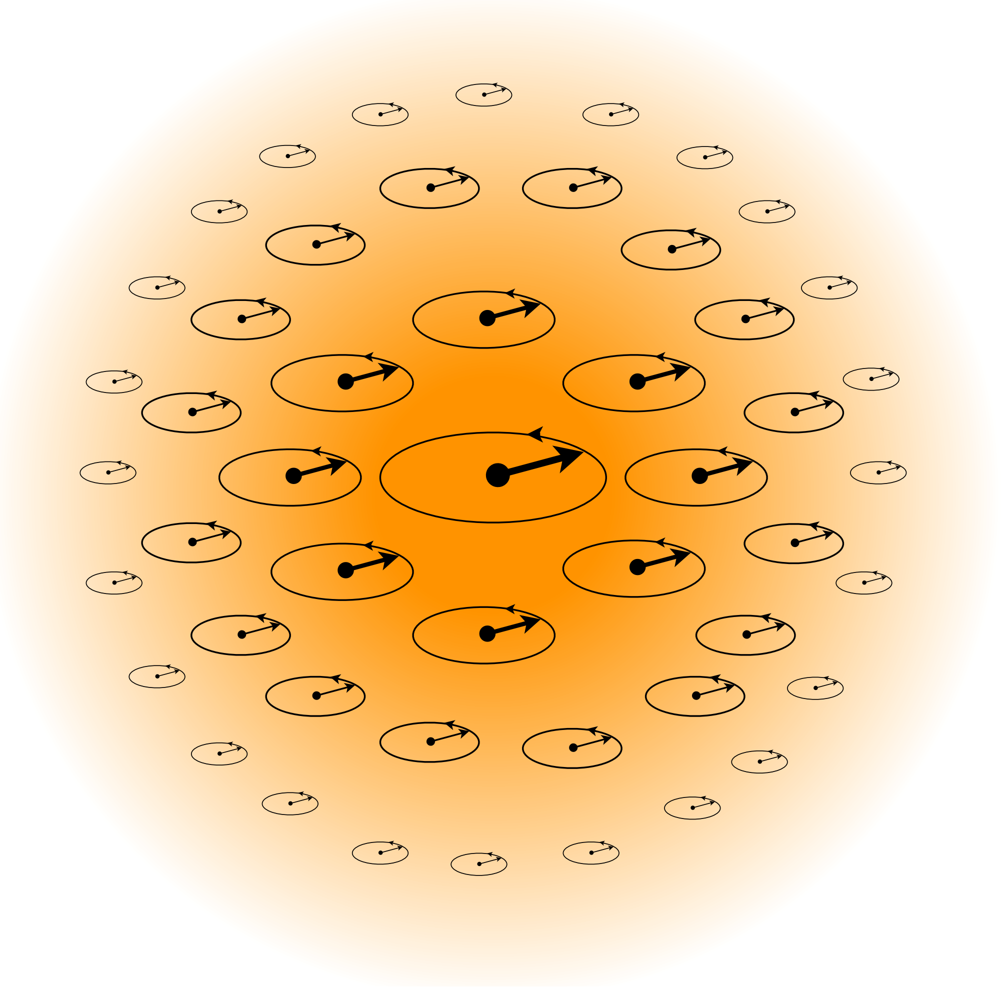
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$\boldsymbol{S}_{\rm sol} \approx i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)60.7\frac{m_{\rm pl}^2}{m^2}\sqrt{\frac{\mu}{m}}\hbar \quad = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$



$$\boldsymbol{S}_{\rm tot} = 0\hat{z}$$

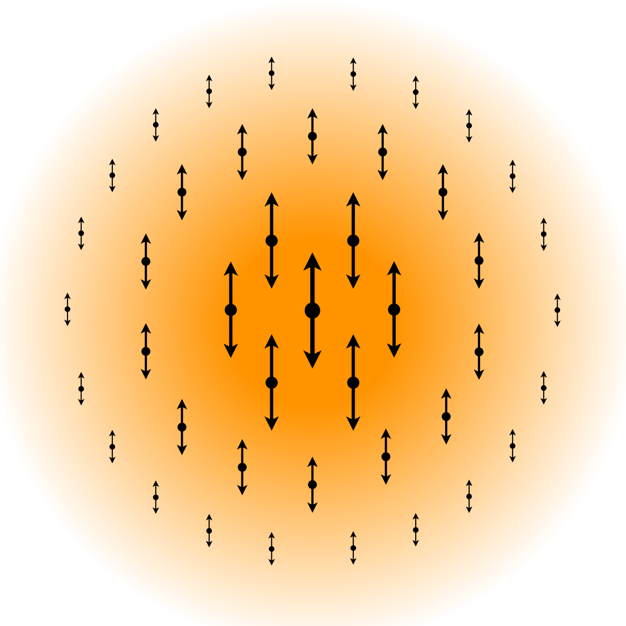
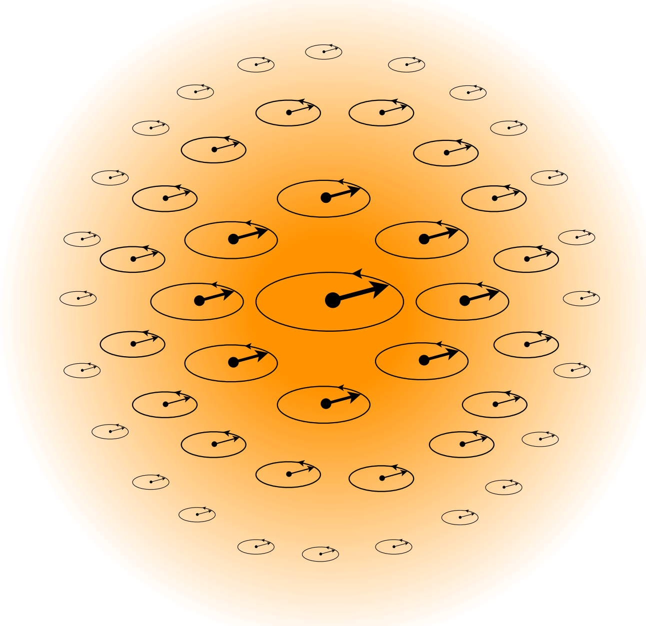
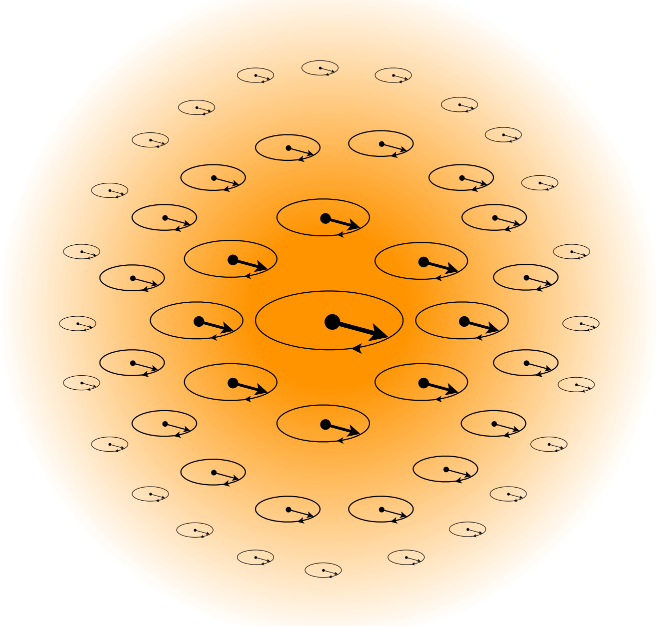


$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton

- all lowest energy for fixed N
- bases for partially-polarized solitons

c_0  $+c_1$  $+c_{-1}$ 

$$0 \le |\boldsymbol{S}_{\rm tot}| \le \frac{M_{\rm sol}}{m}\hbar$$

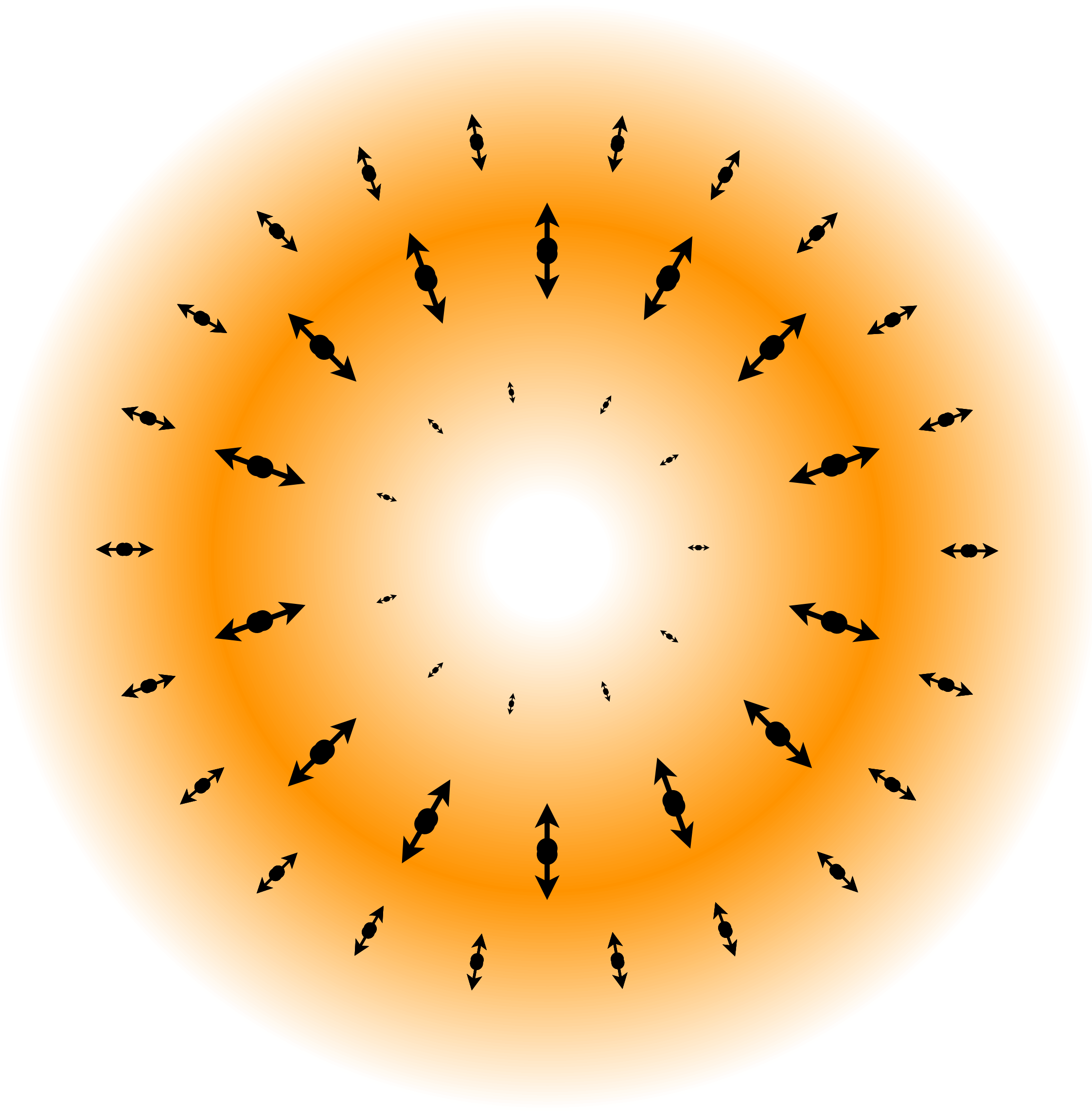
a different higher energy soliton: the “hedgehogs”

earlier literature

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

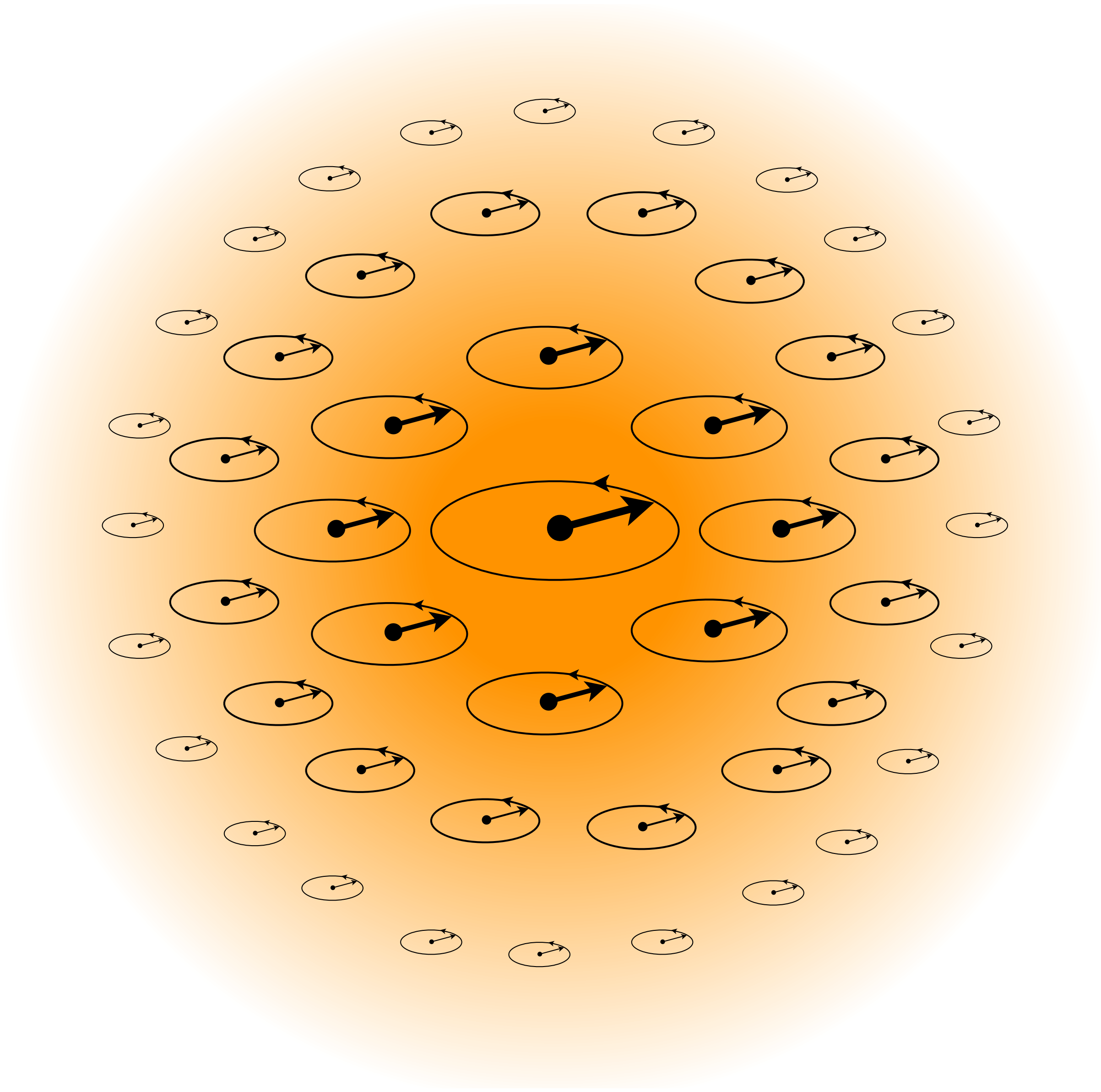
hedgehogs

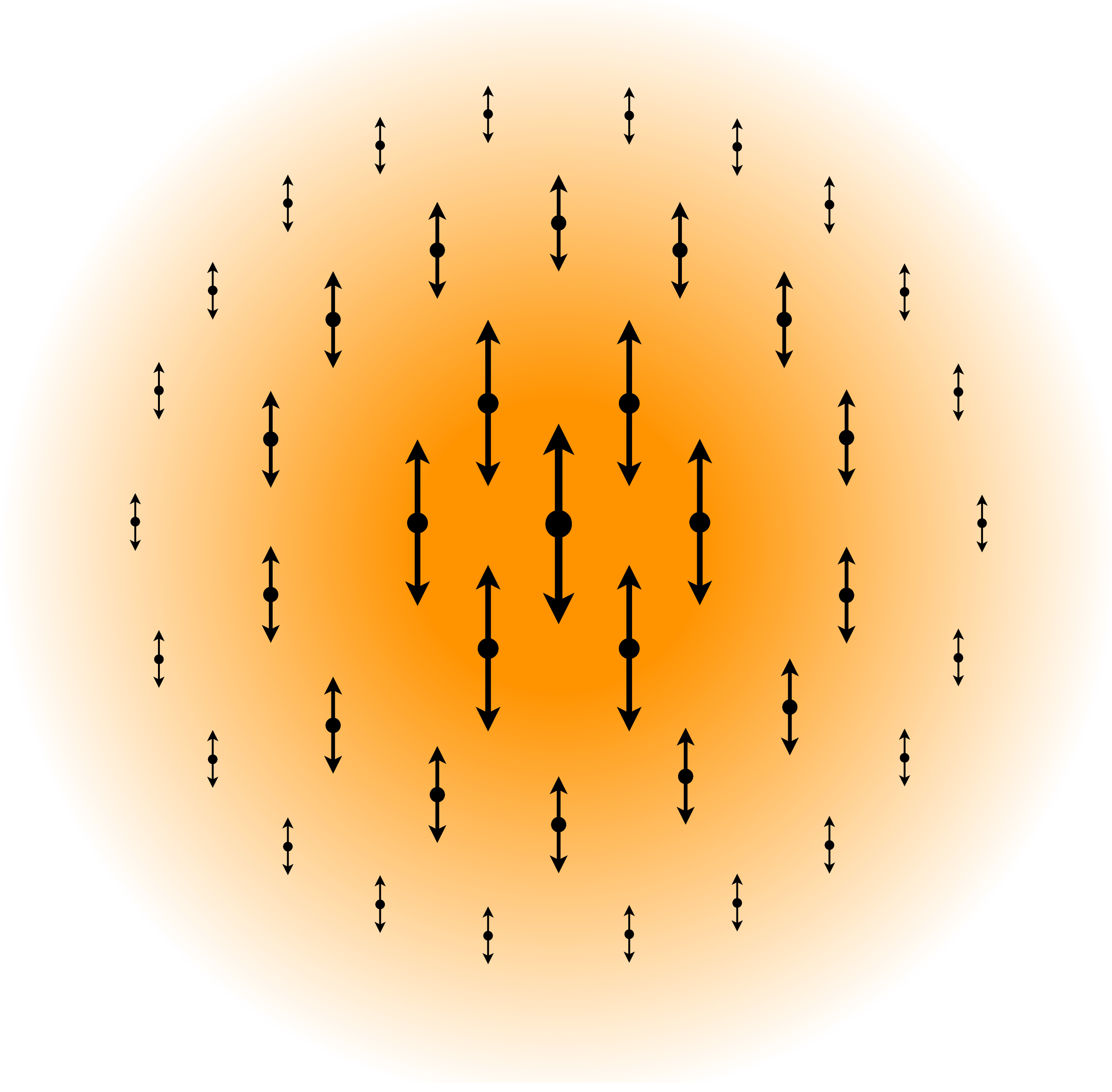
not ground states

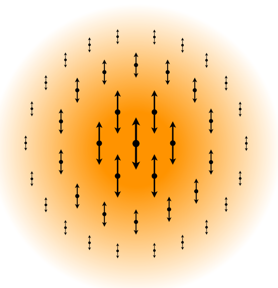
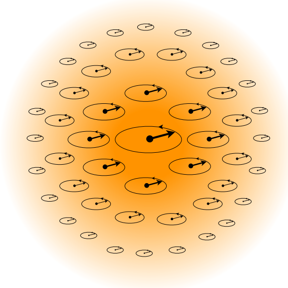
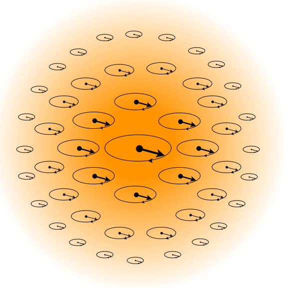


$$E^{s}_{\rm hh} > E$$

$$E^{s=1}_{\rm hh} \approx 0.33E \;\; < 0$$





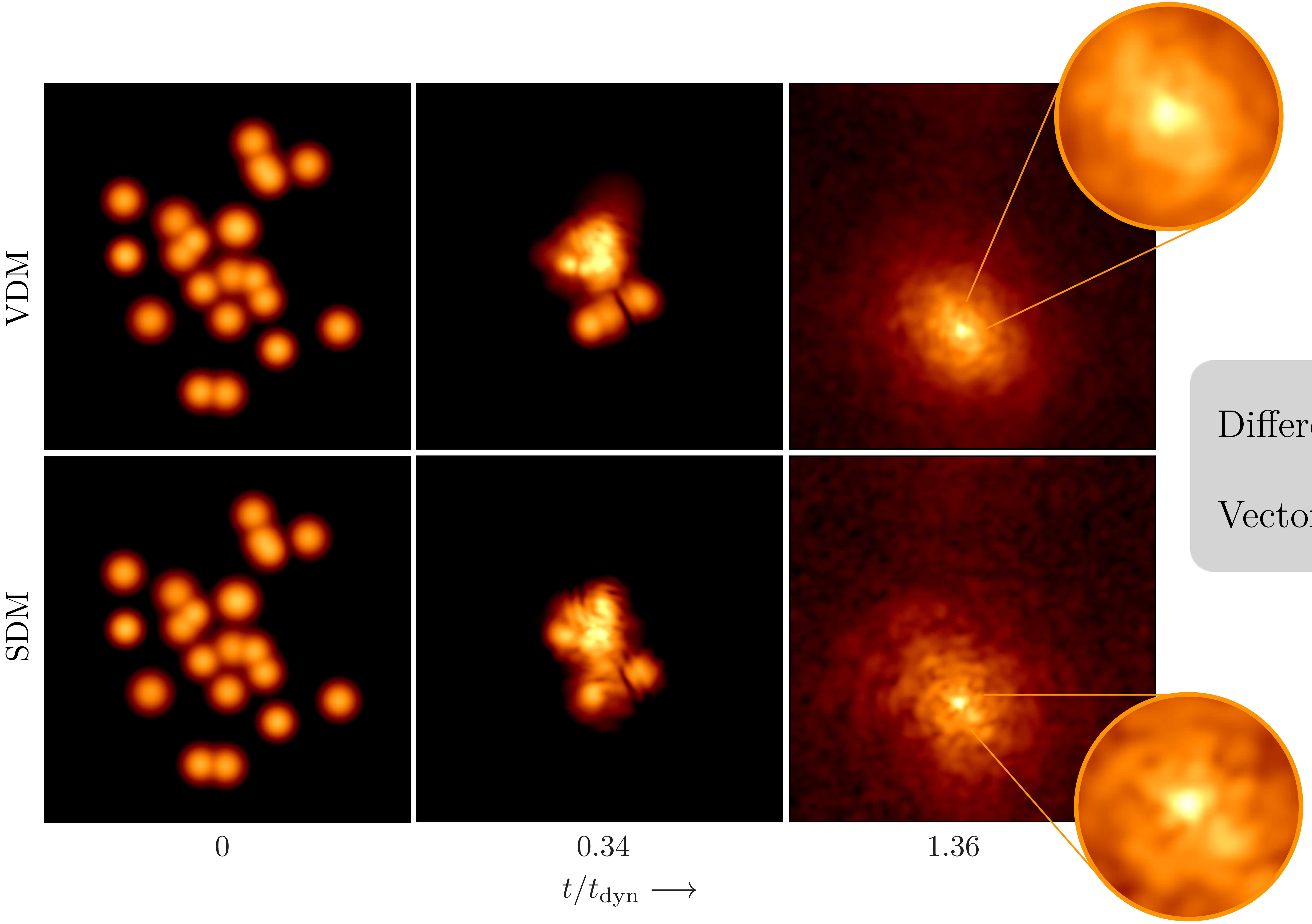
c_0  $+c_1$  $+c_{-1}$ 

vector vs. scalar DM:
two key differences

interference

polarized solitons

3+1 dimensional simulations



Difference between

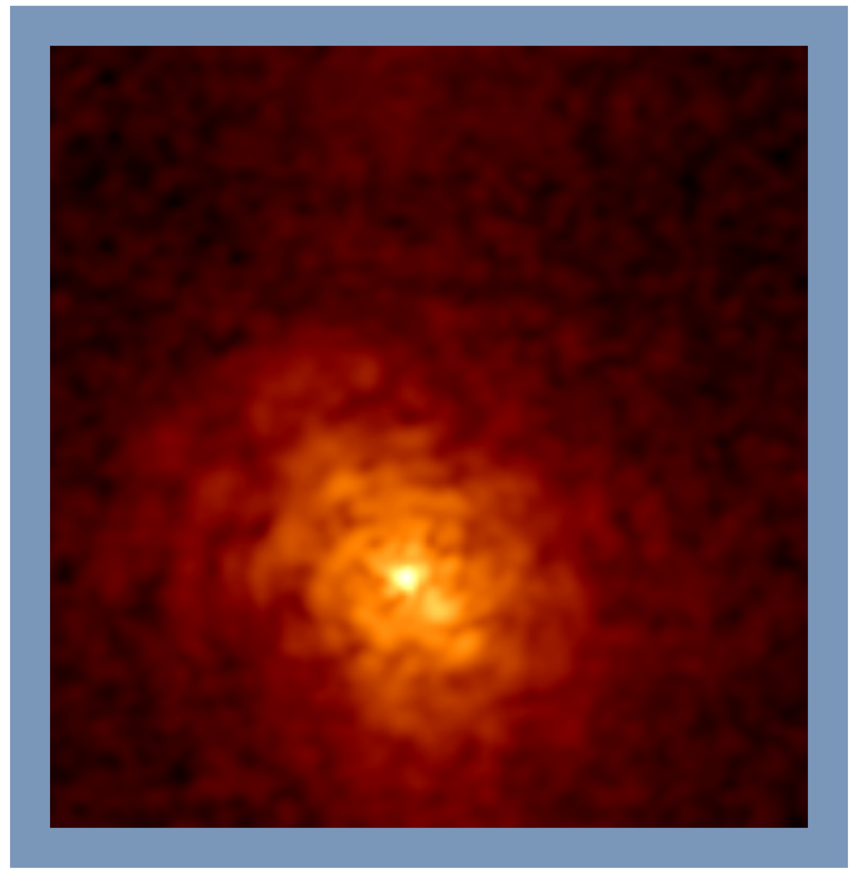
Vector & Scalar Dark Matter

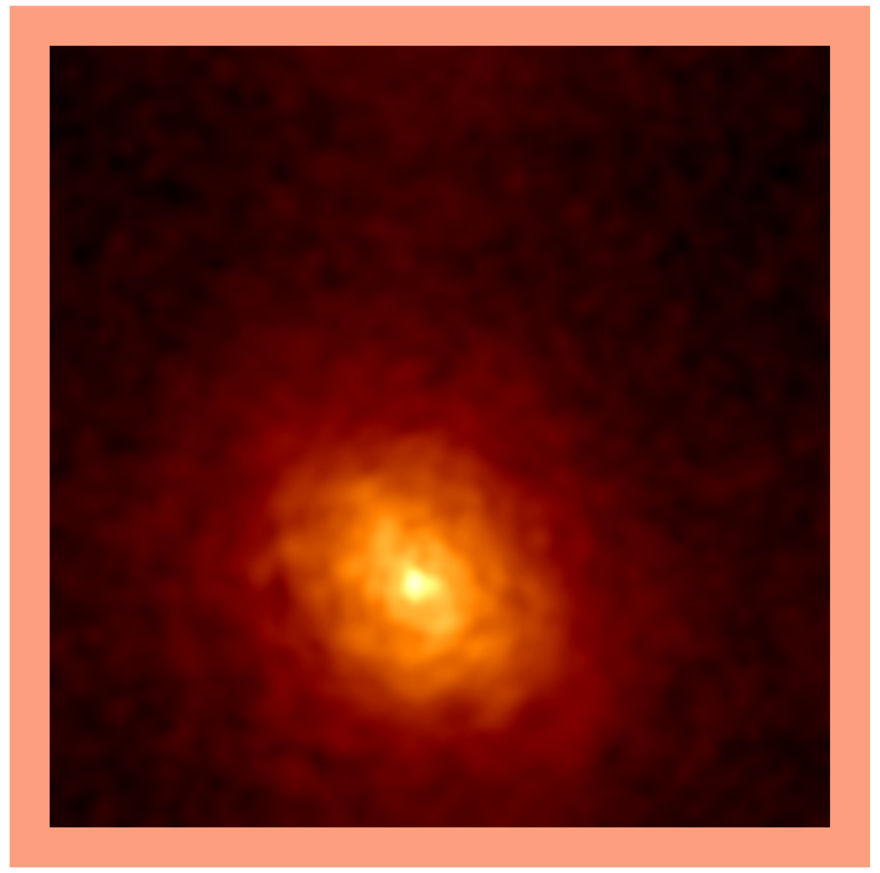
MA, Jain, Karur & Mocz (2022)

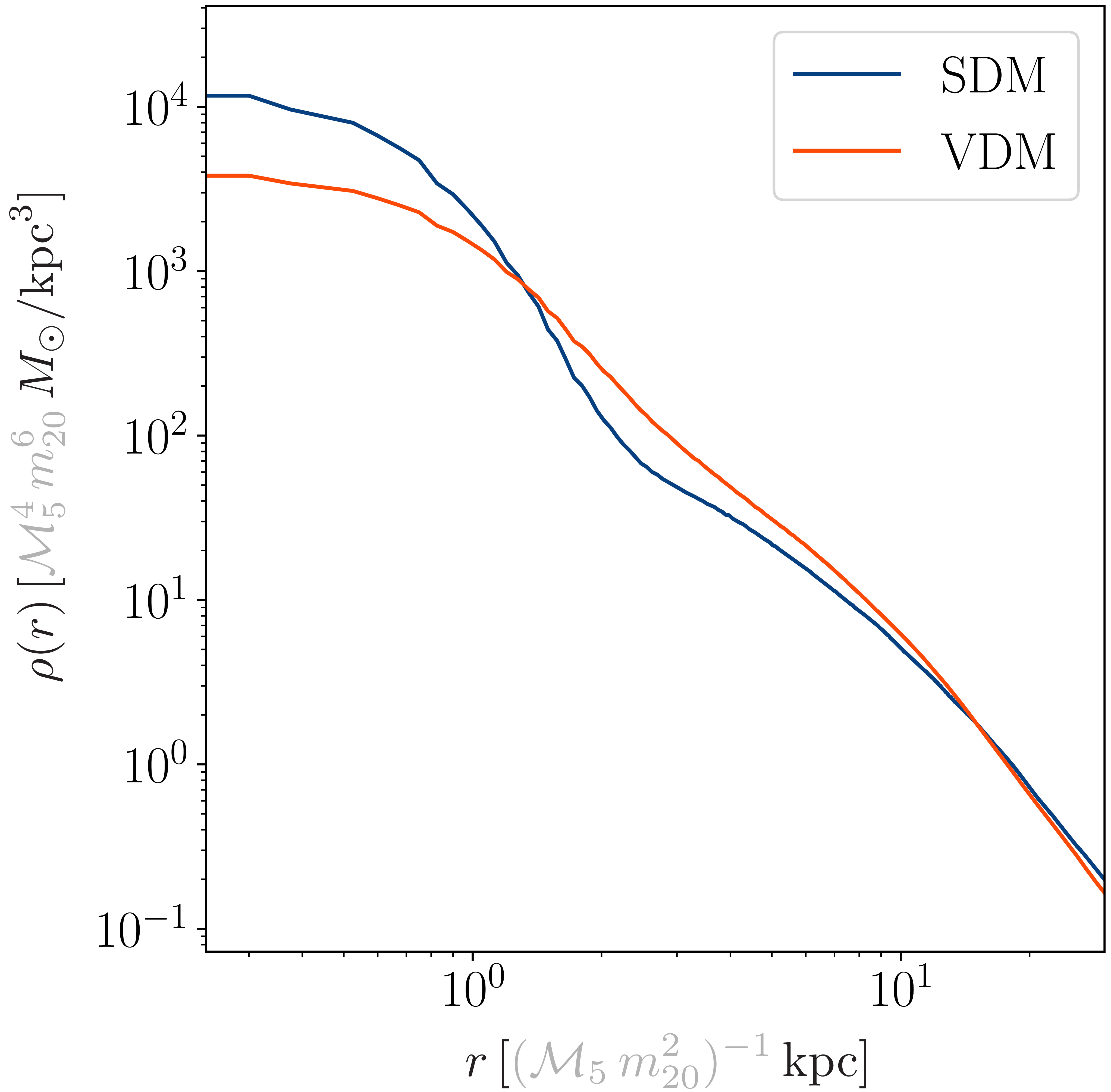
radial density profiles

scalar vs. vector dark matter

- less dense & broader core
- smoother transition to r^{-3} tail

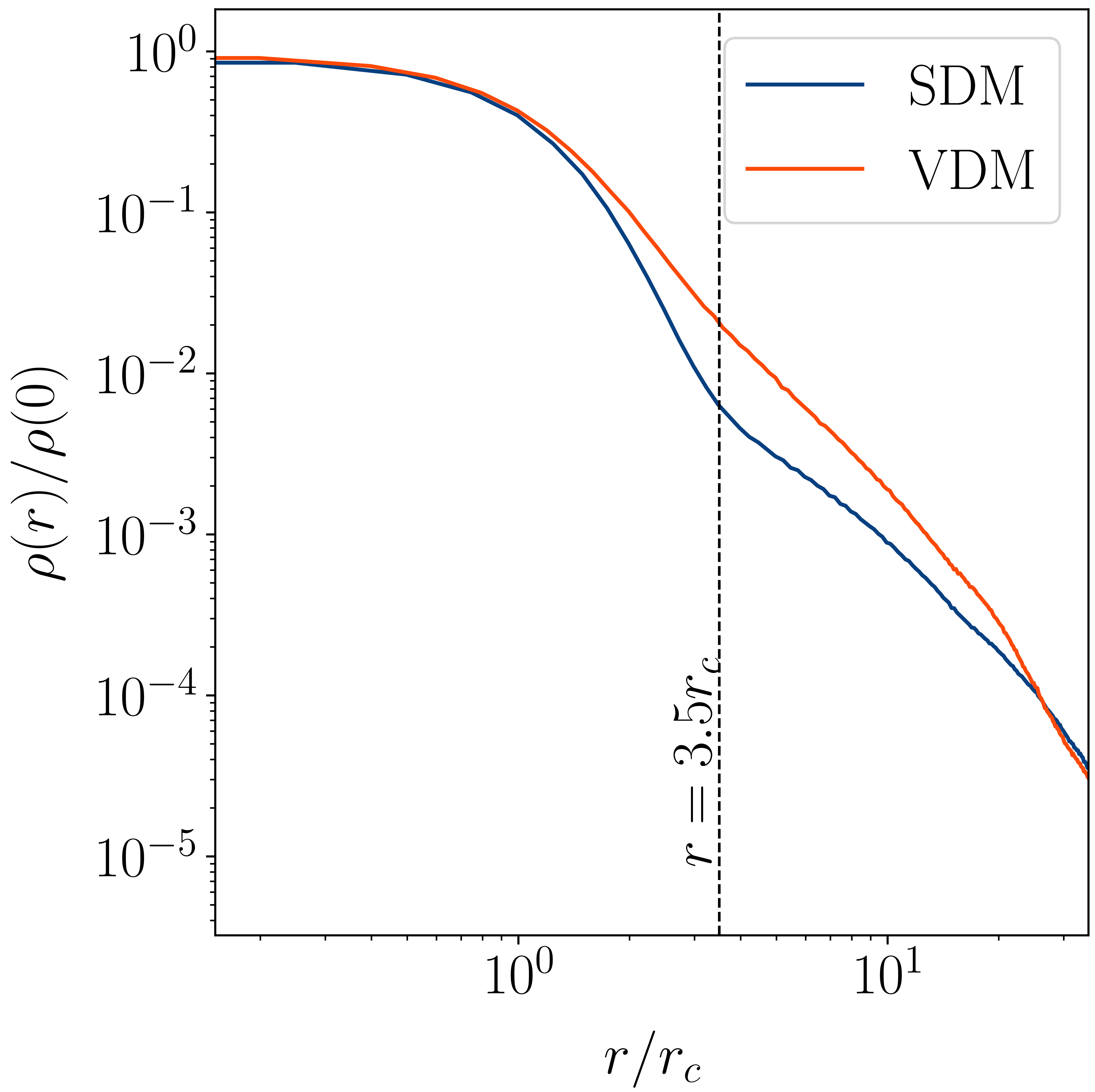






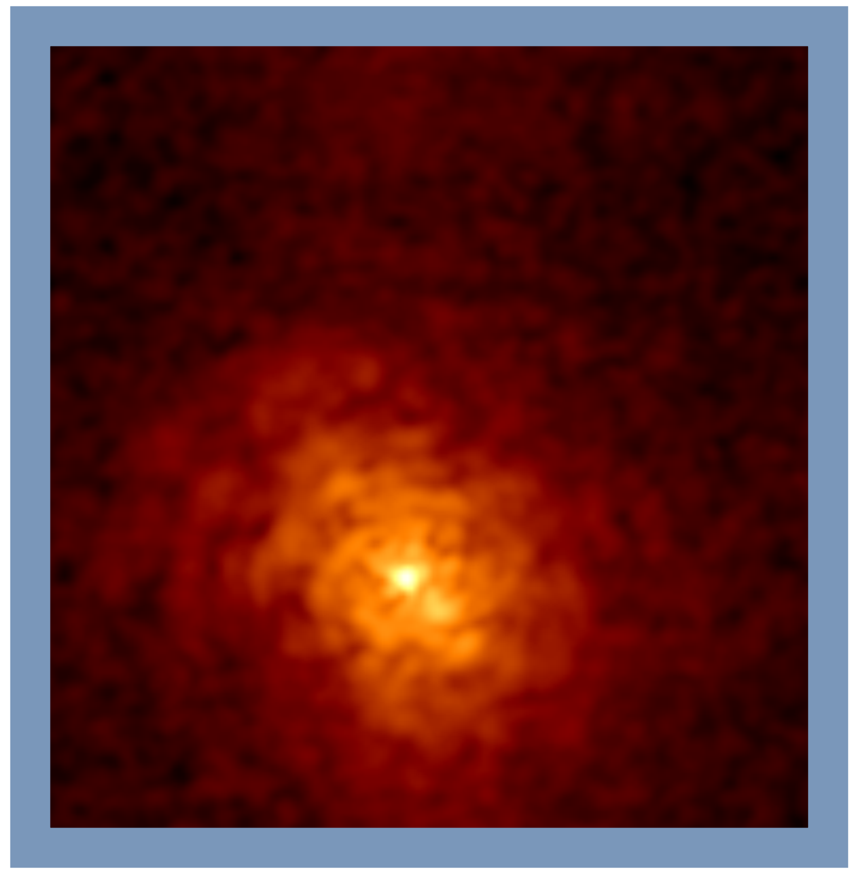
MA, Jain, Karur & Mocz (2022)

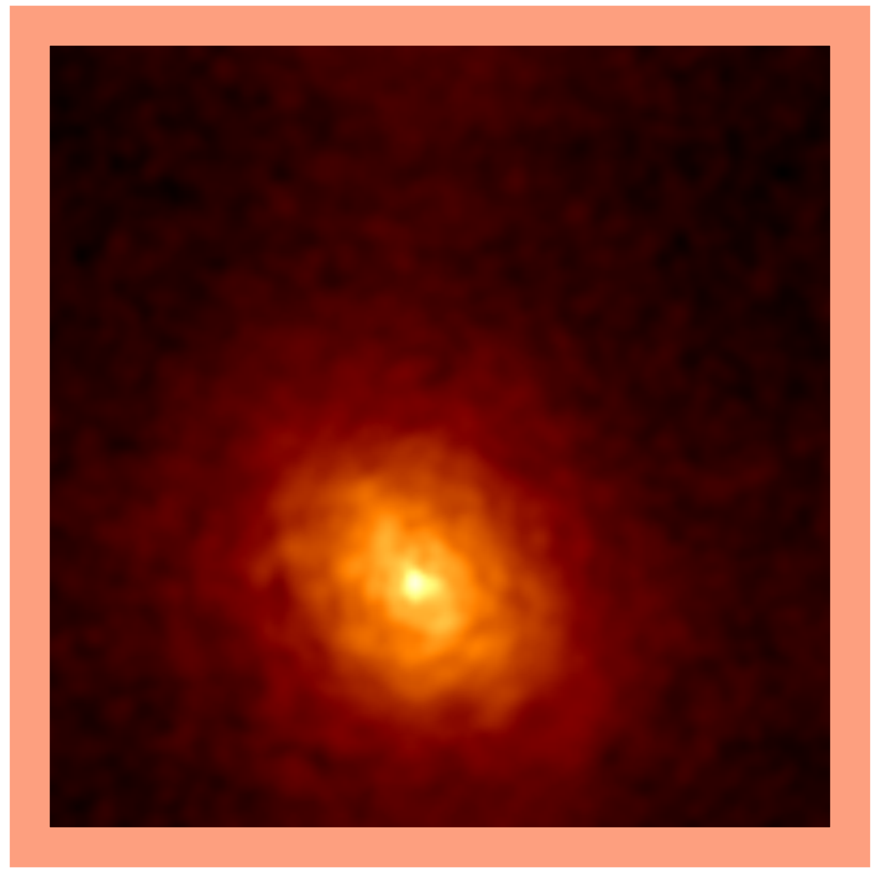
radial density profiles



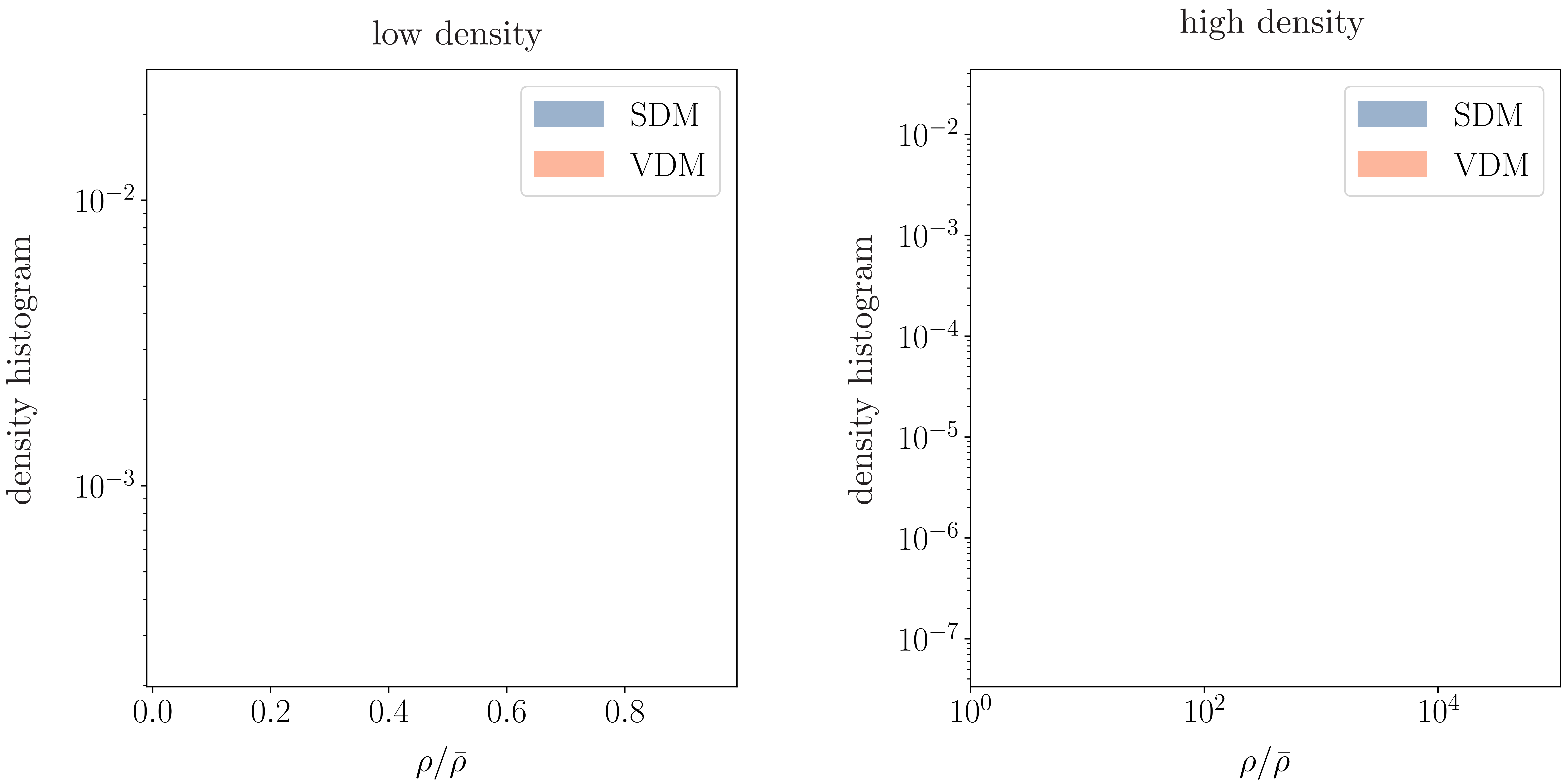
vector vs. scalar dark matter

- *shape* difference
- smoother transition to r^{-3} tail



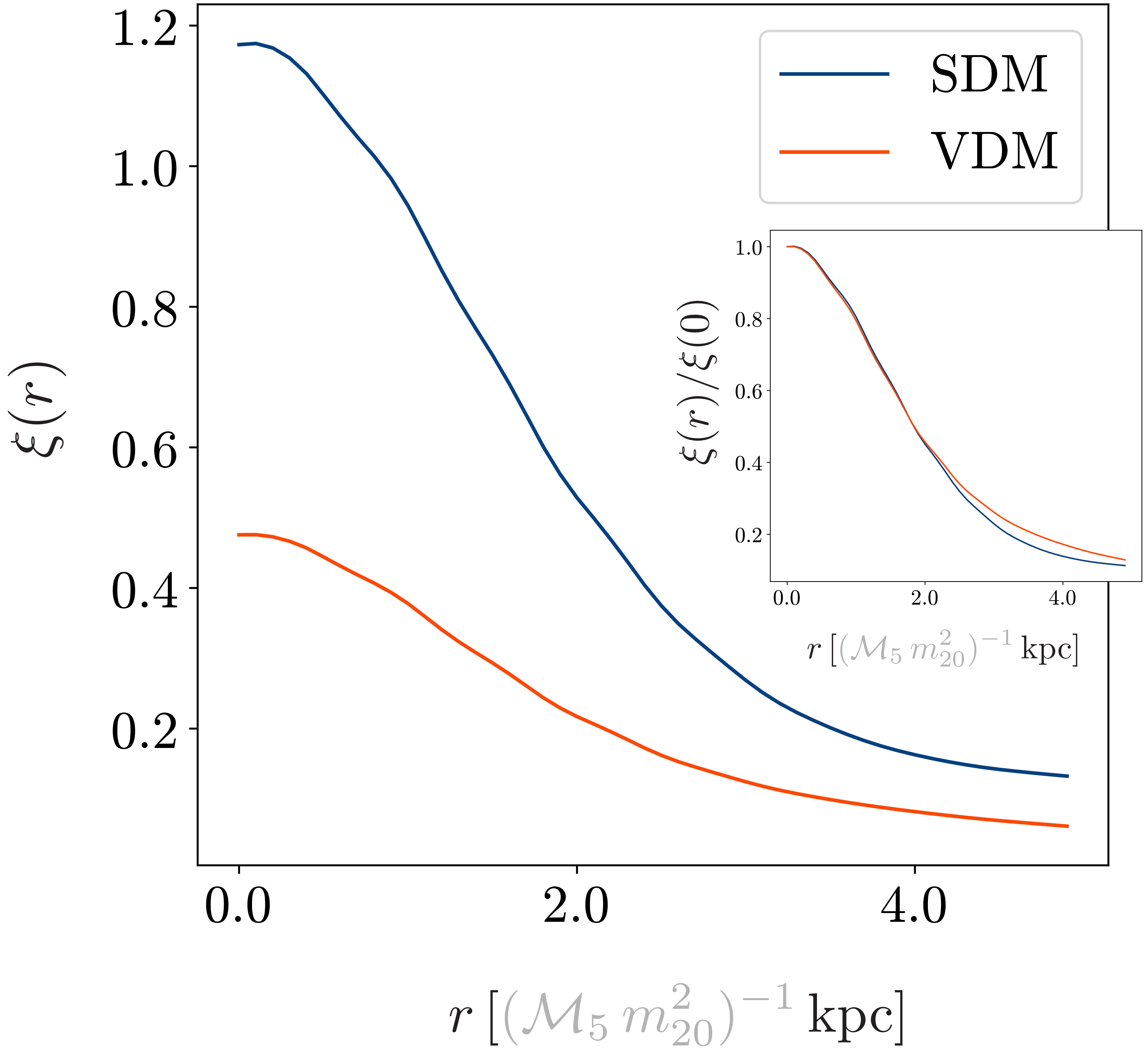


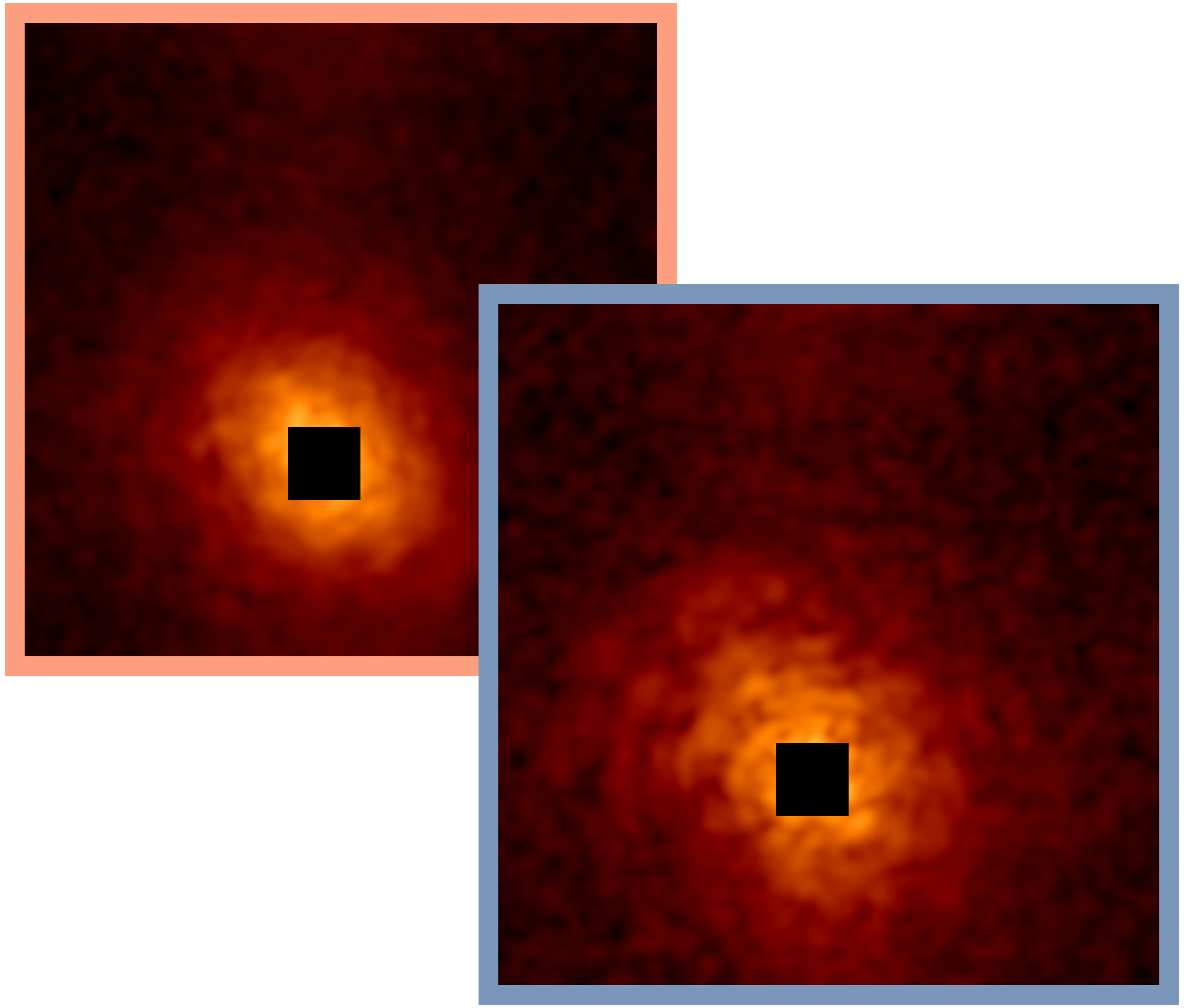
density pdf


low density
SDM
VDM
density histogram
10^{-2}
10^{-3}
0.0
0.2
0.4
0.6
0.8
$\rho/\bar{\rho}$
high density
SDM
VDM
density histogram
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{0}
10^{2}
10^{4}
$\rho/\bar{\rho}$

2-point density correlation

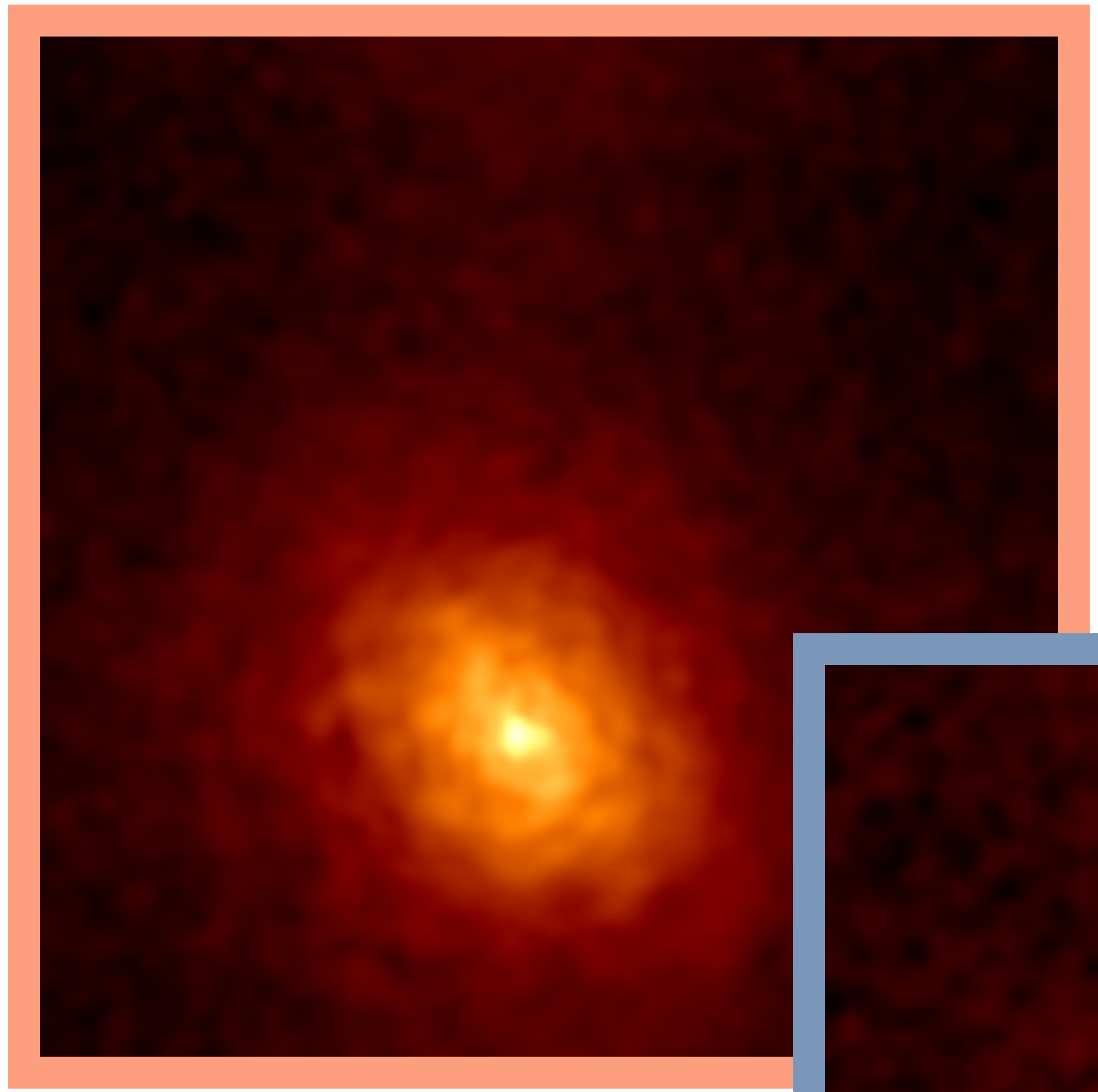
same $\lambda_{\rm dB}$ in the outer regions.

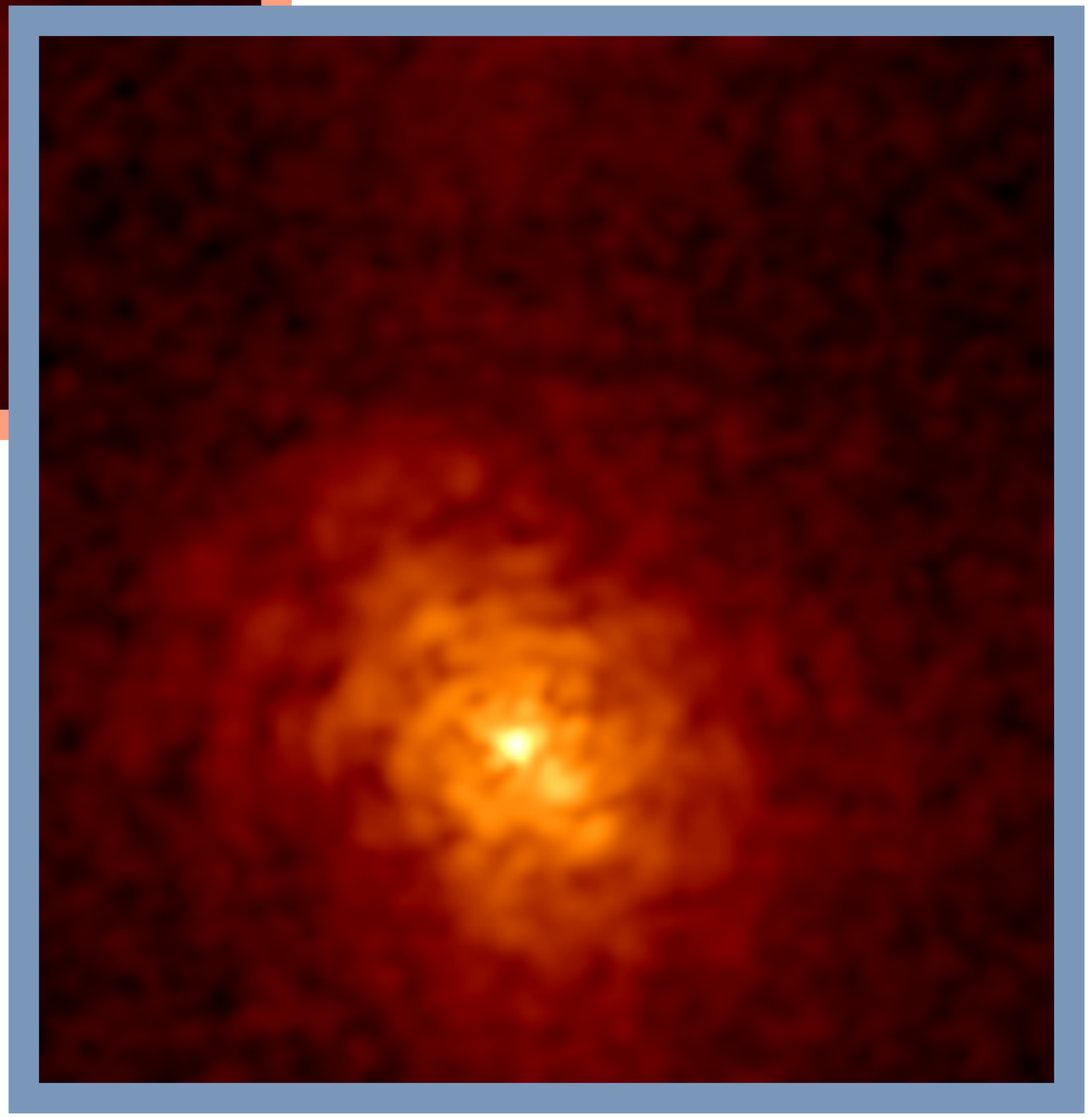




gravitational implications (examples)

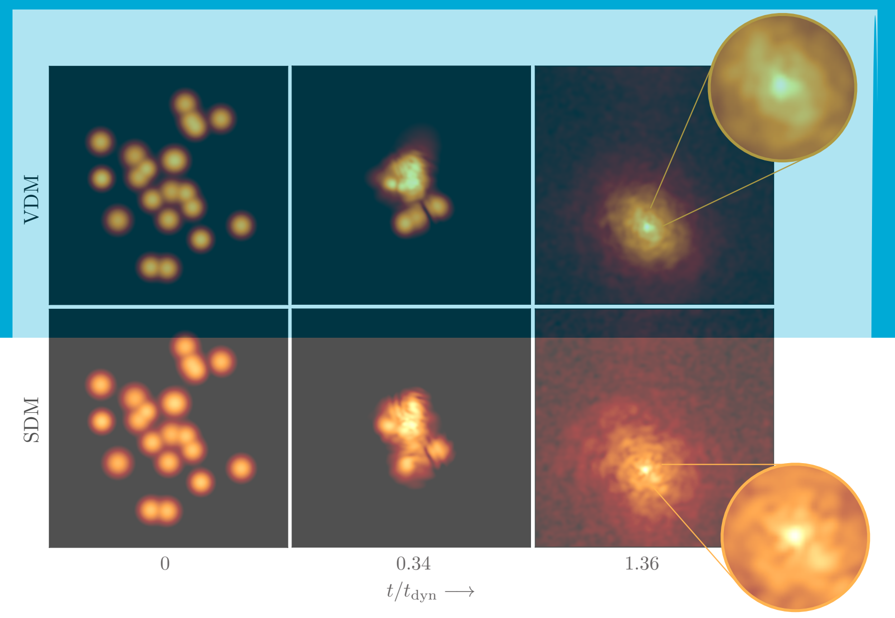
- dynamical heating of stars
  Church et. al (2021), Dalal & Kravtsov (2022)
- cusp-core, diversity
- lensing

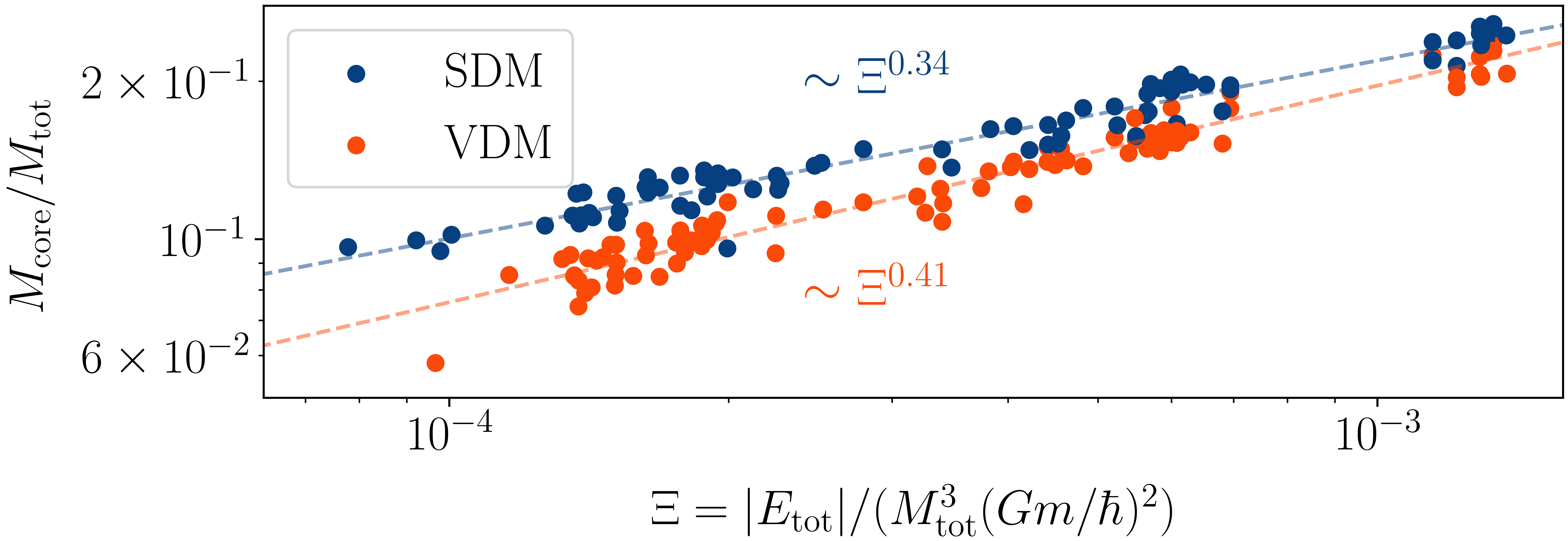




$$\frac{\sqrt{\langle \mathrm{int}^2_{(1)} \rangle}}{\sqrt{\langle \mathrm{int}^2_{(0)} \rangle}} = \frac{1}{\sqrt{3}}$$

core-halo mass relation



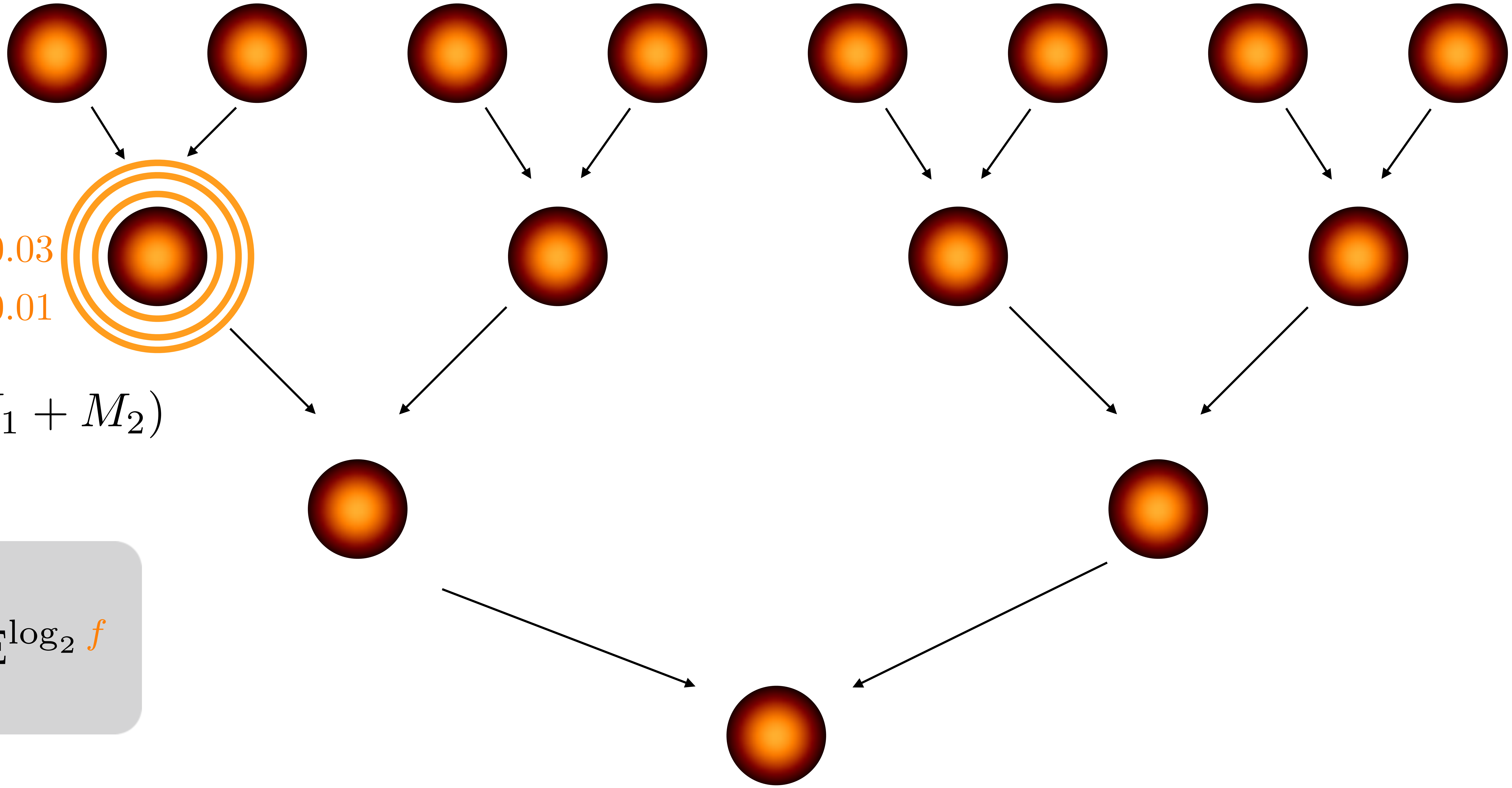


merger tree

$f_\mathrm{v} = 0.56 \pm 0.03$

$f_\mathrm{s} = 0.61 \pm 0.01$

$M_\mathrm{f} = f(M_1 + M_2)$

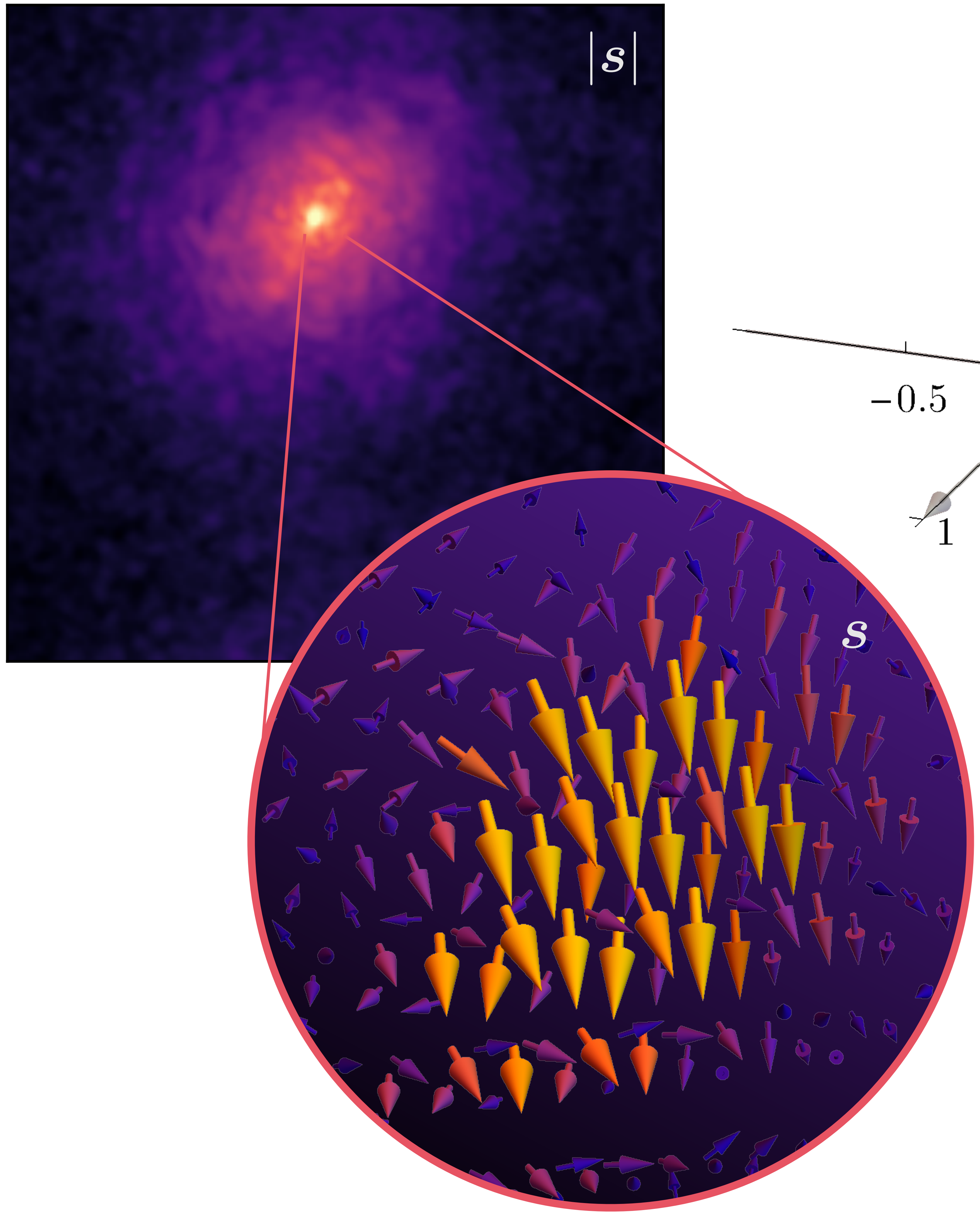


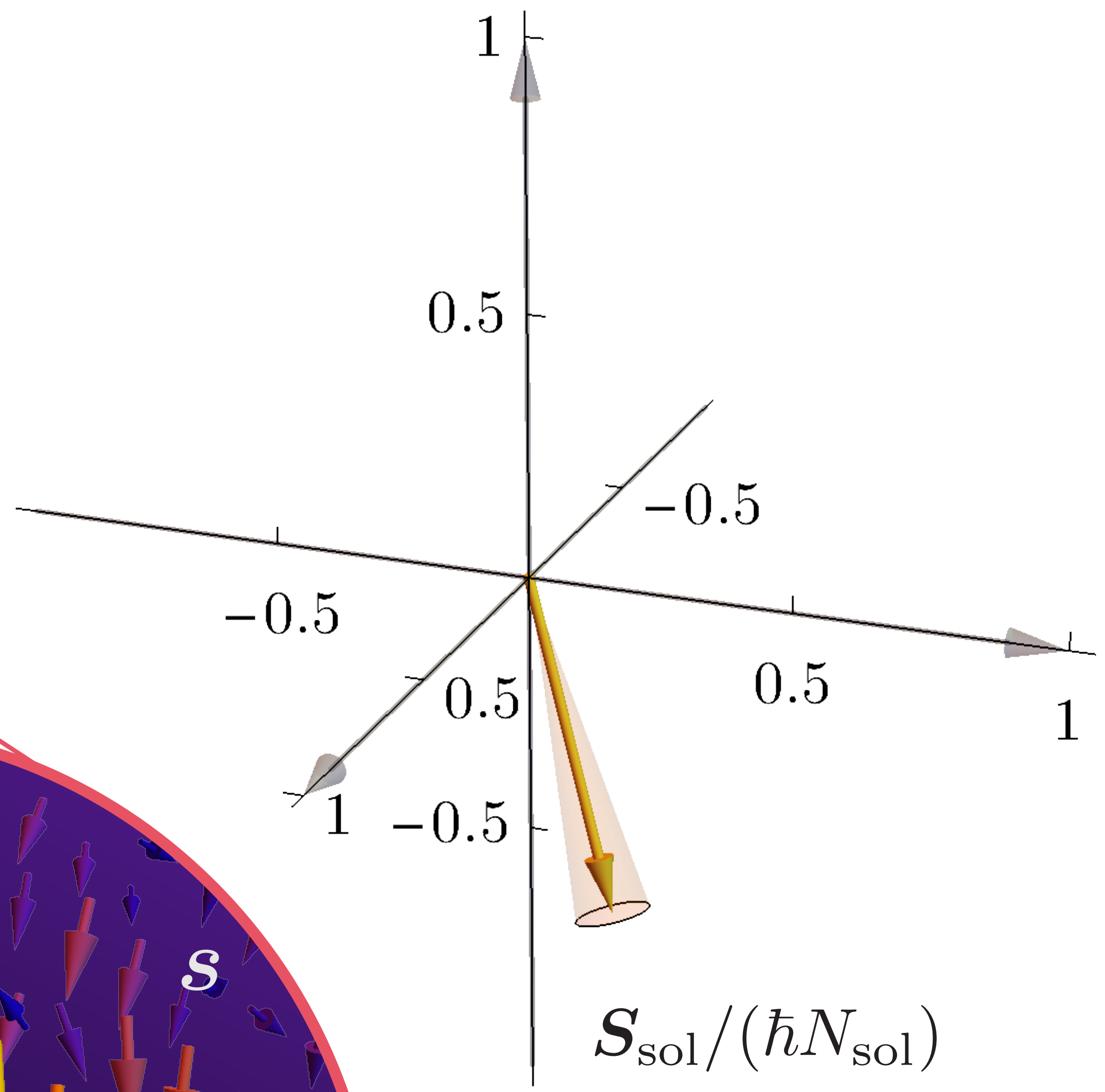
$$\frac{M_\mathrm{core}}{M_\mathrm{tot}} \propto \Xi^{\log_2 f}$$

Du et. al (2017)

intrinsic spin

spin





spin density

$$\boldsymbol{s} = i\hbar \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$$

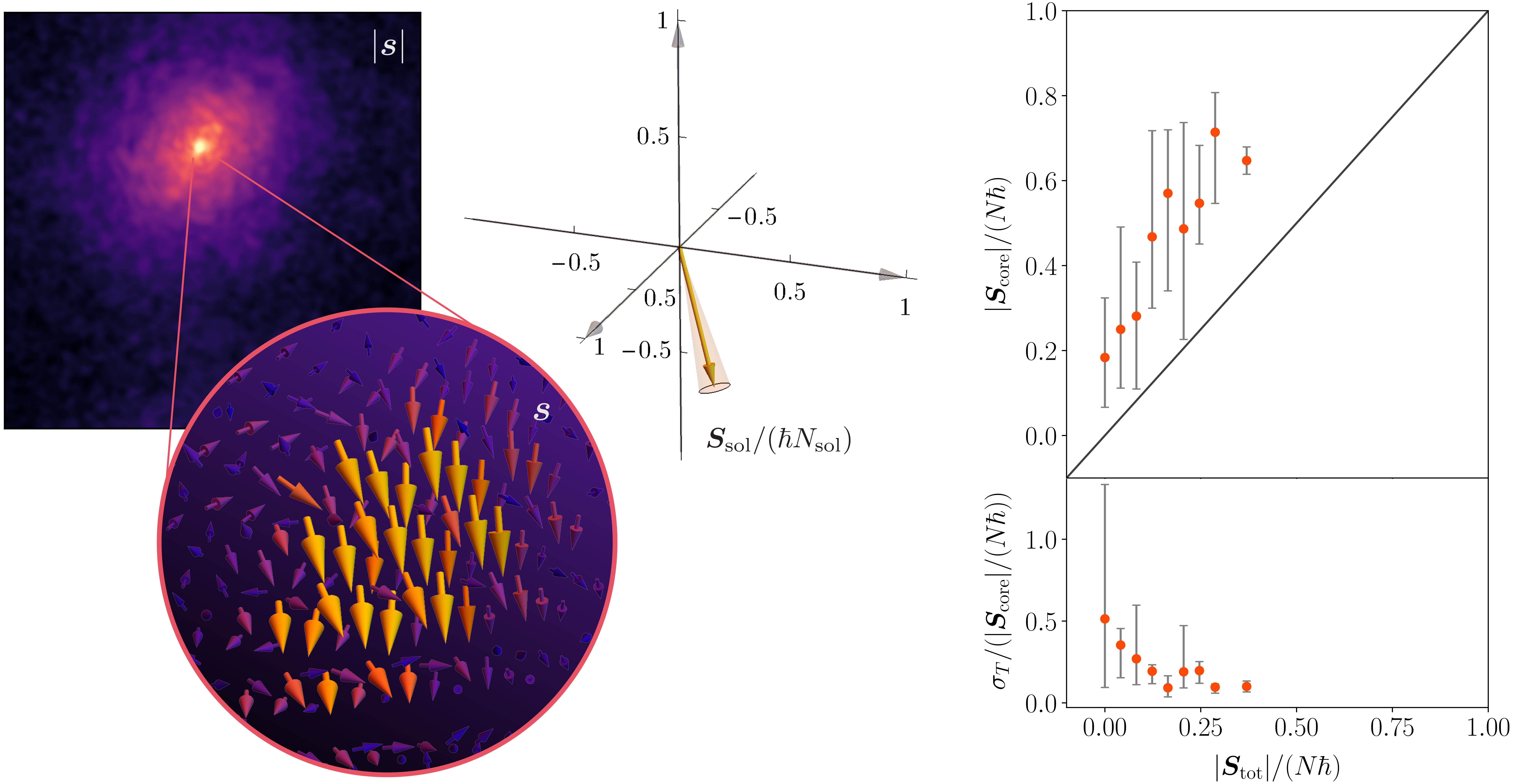
spin

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i\boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$$

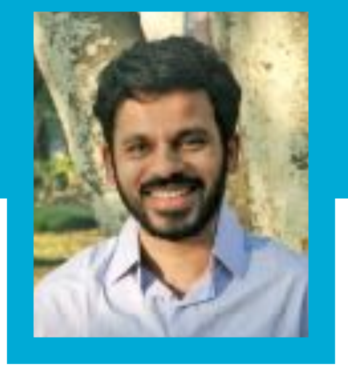
$$\boldsymbol{s} = (2mc/\hbar)\boldsymbol{W} \times (\dot{\boldsymbol{W}} - \nabla W_0)$$

MA, Jain, Karur & Mocz (2022)

generation of spin density



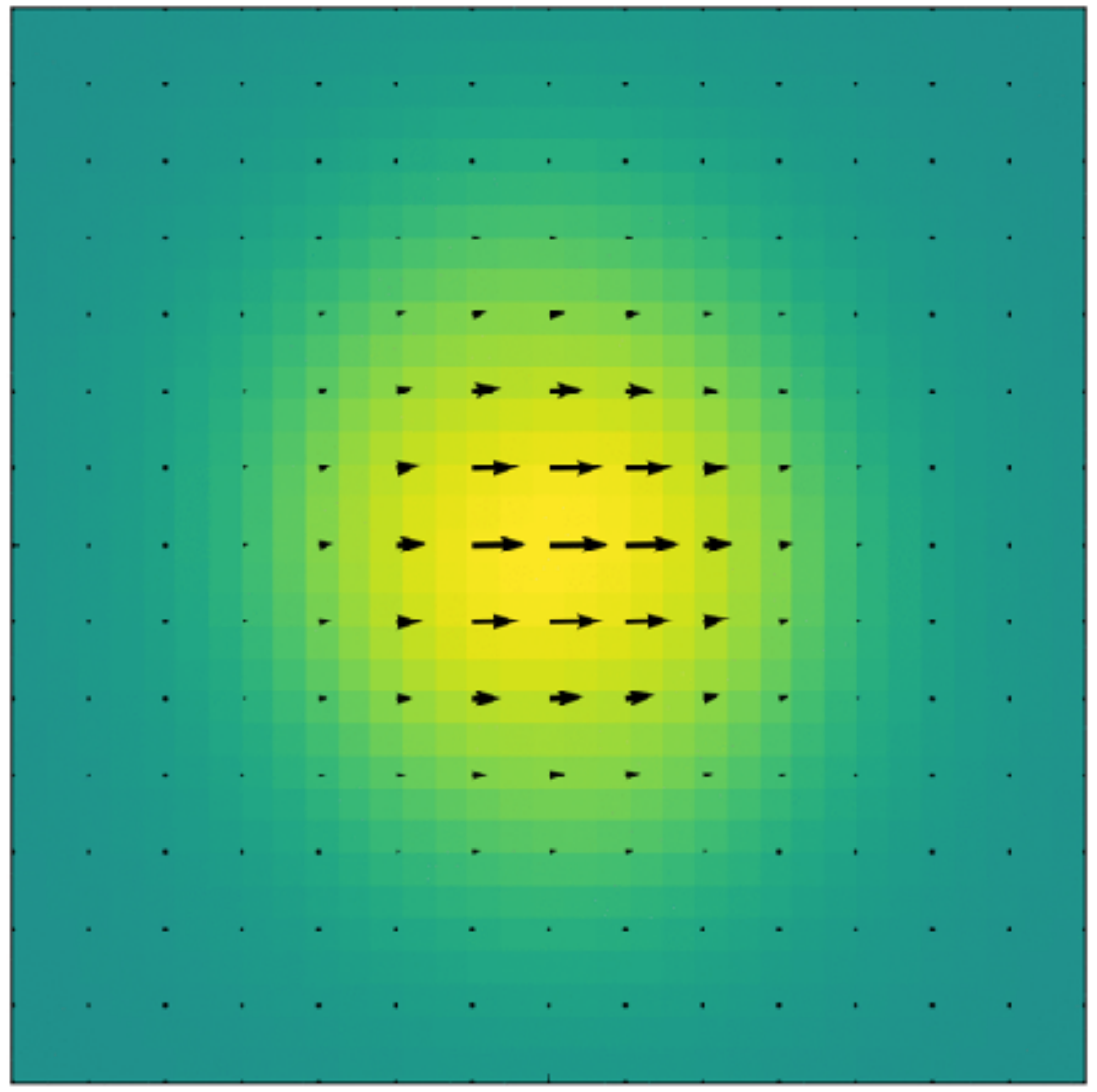
spin with self-interactions: vector oscillons

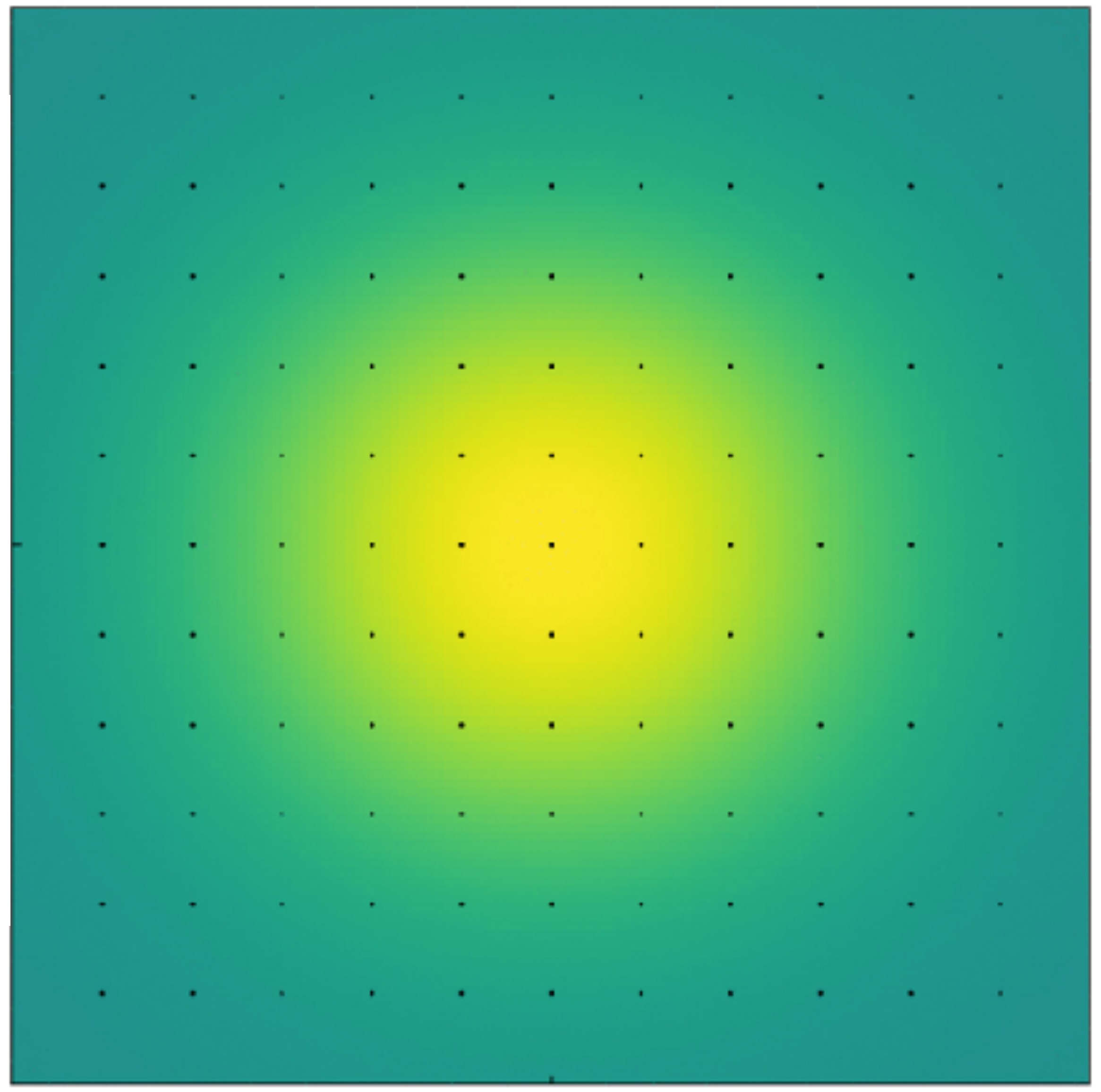
Zhang, Jain & MA (2022)

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} - \frac{m^2}{2}W_\mu W^\mu + \frac{\lambda}{4}(W_\mu W^\mu)^2 - \frac{h}{6}(W_\mu W^\mu)^3$$

$\boldsymbol{S}_{\rm tot} \neq 0$



$\boldsymbol{S}_{\rm tot} = 0$



self-interaction supported

NOT degenerate in energy

spin-spin interaction matters!

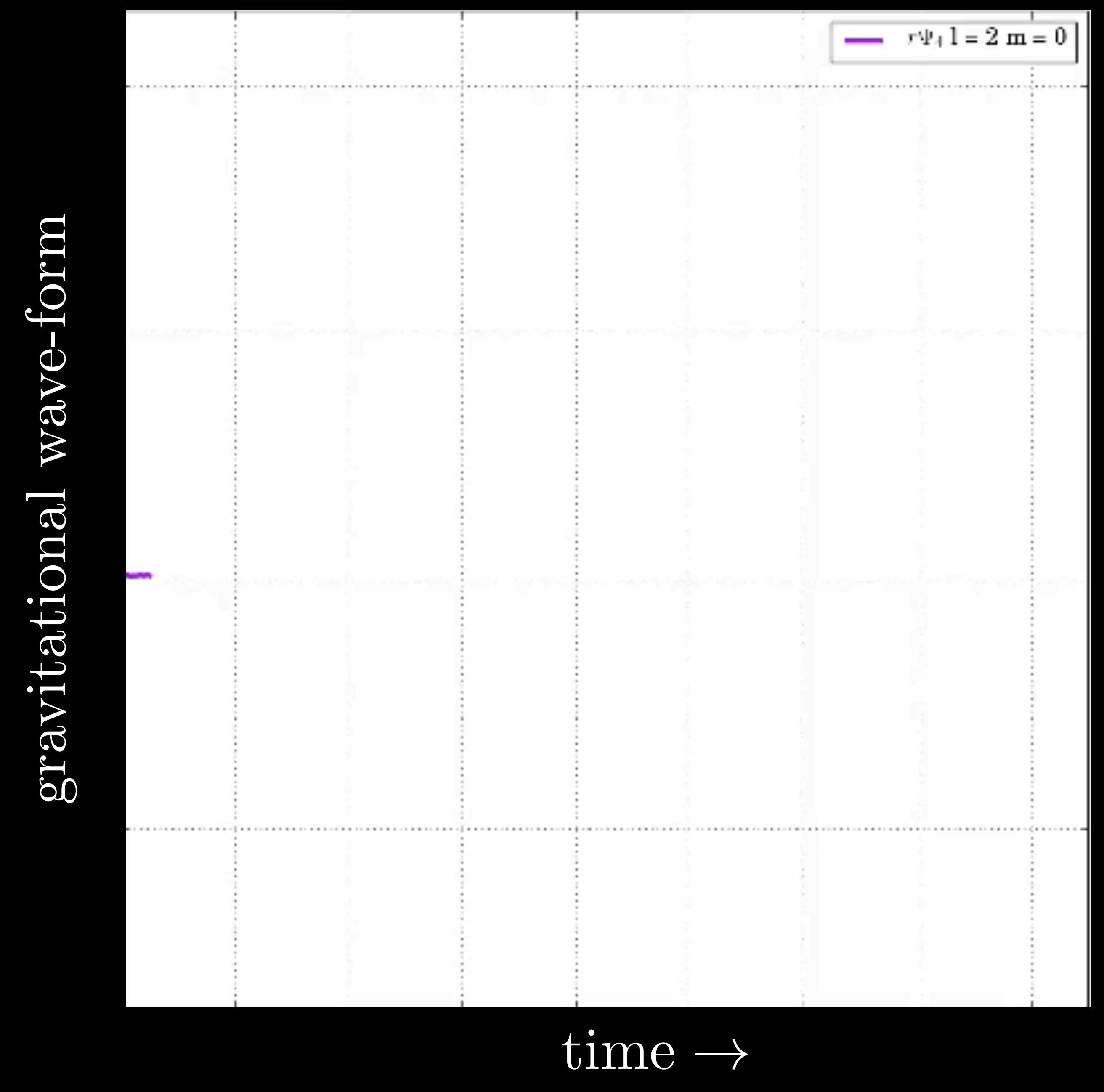
$$\mathcal{L} = \Re[i\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}}] - \frac{1}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} - V_{\rm nl}(\boldsymbol{\Psi}^\dagger, \boldsymbol{\Psi})$$

$$V_{\rm nl}(\boldsymbol{\Psi}^\dagger, \boldsymbol{\Psi}) = -\frac{3\lambda}{8m^2}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})^2 + \frac{5\gamma}{12m^3}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})^3 + \left[\frac{\lambda}{8m^2} - \frac{\gamma}{4m^3}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})\right](\boldsymbol{S}\cdot\boldsymbol{S}),$$

implications of non-zero spin solitons?

gravitational waves

different for higher spin solitons?

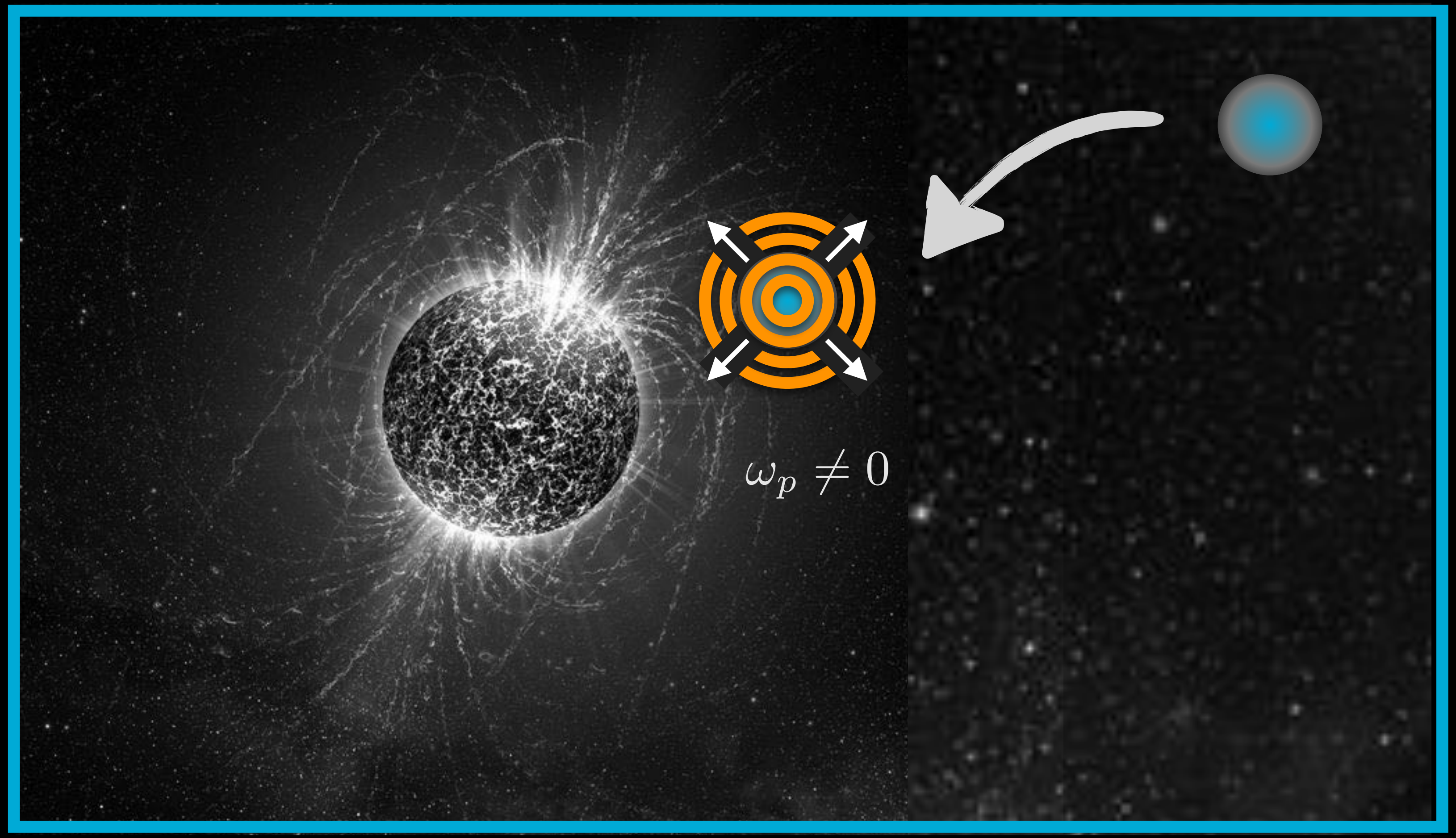


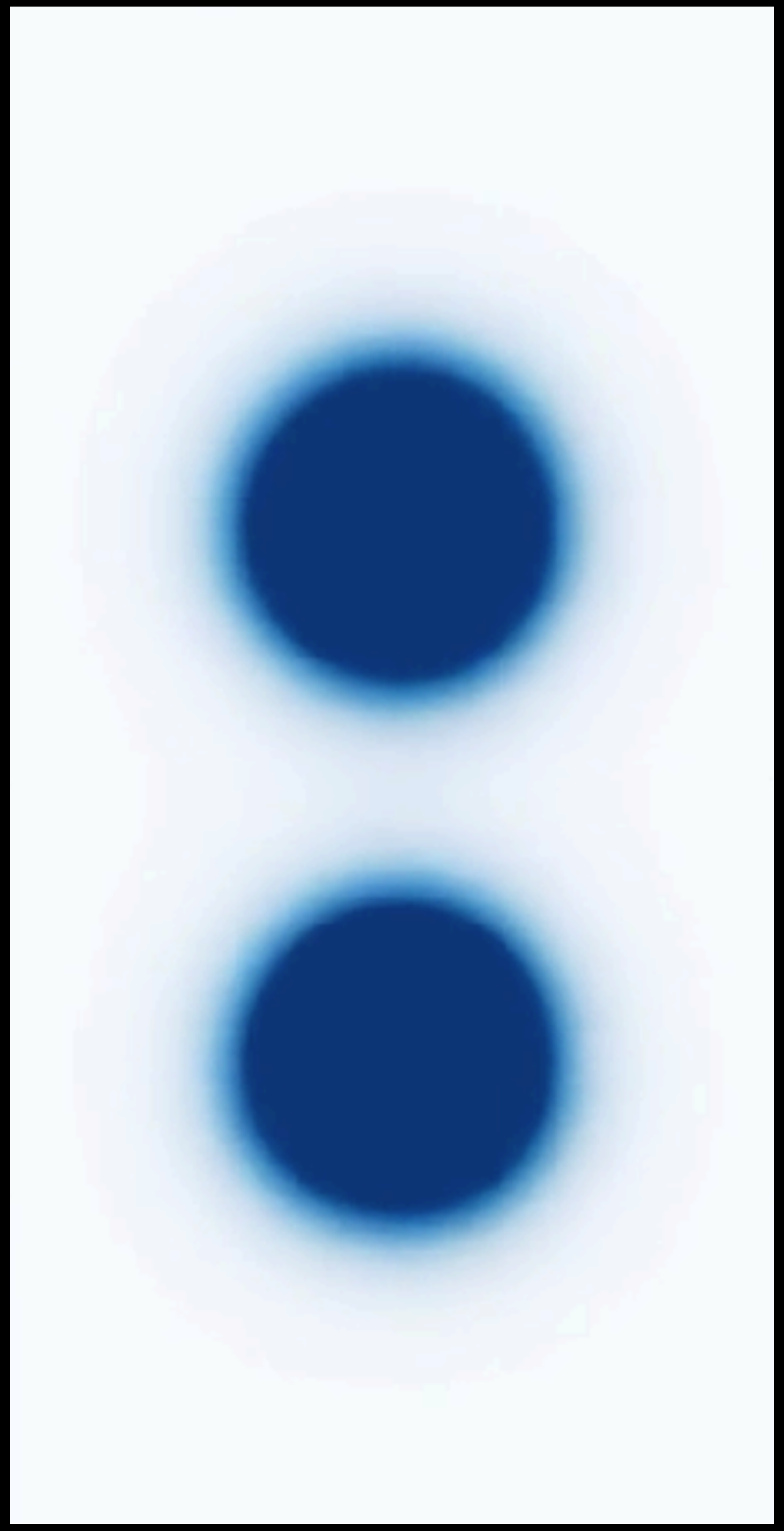
Helfer, Lim, Garcia, MA (2016)

above for compact scalar solitons using full numerical GR

electromagnetic coupling and radiation (axion + photons)

MA, Long, Mou & Saffin (2021)



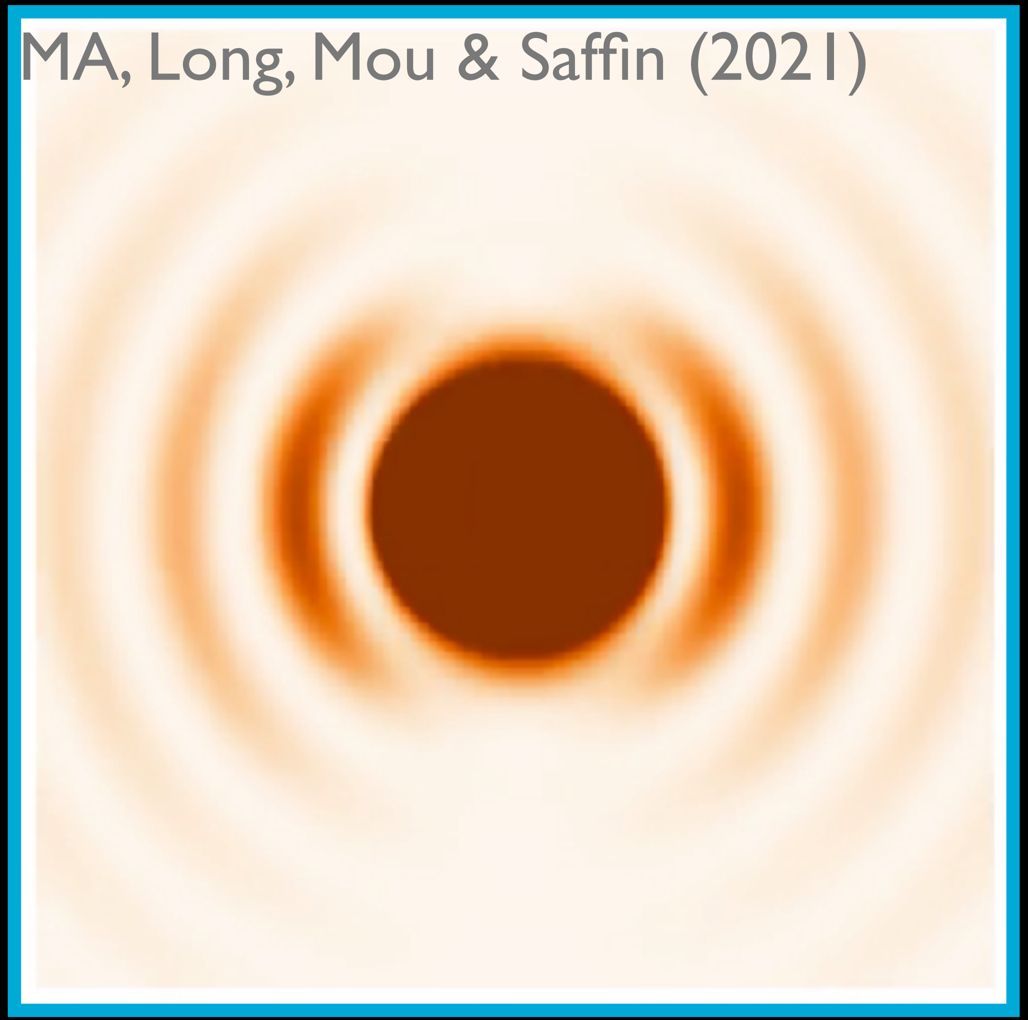


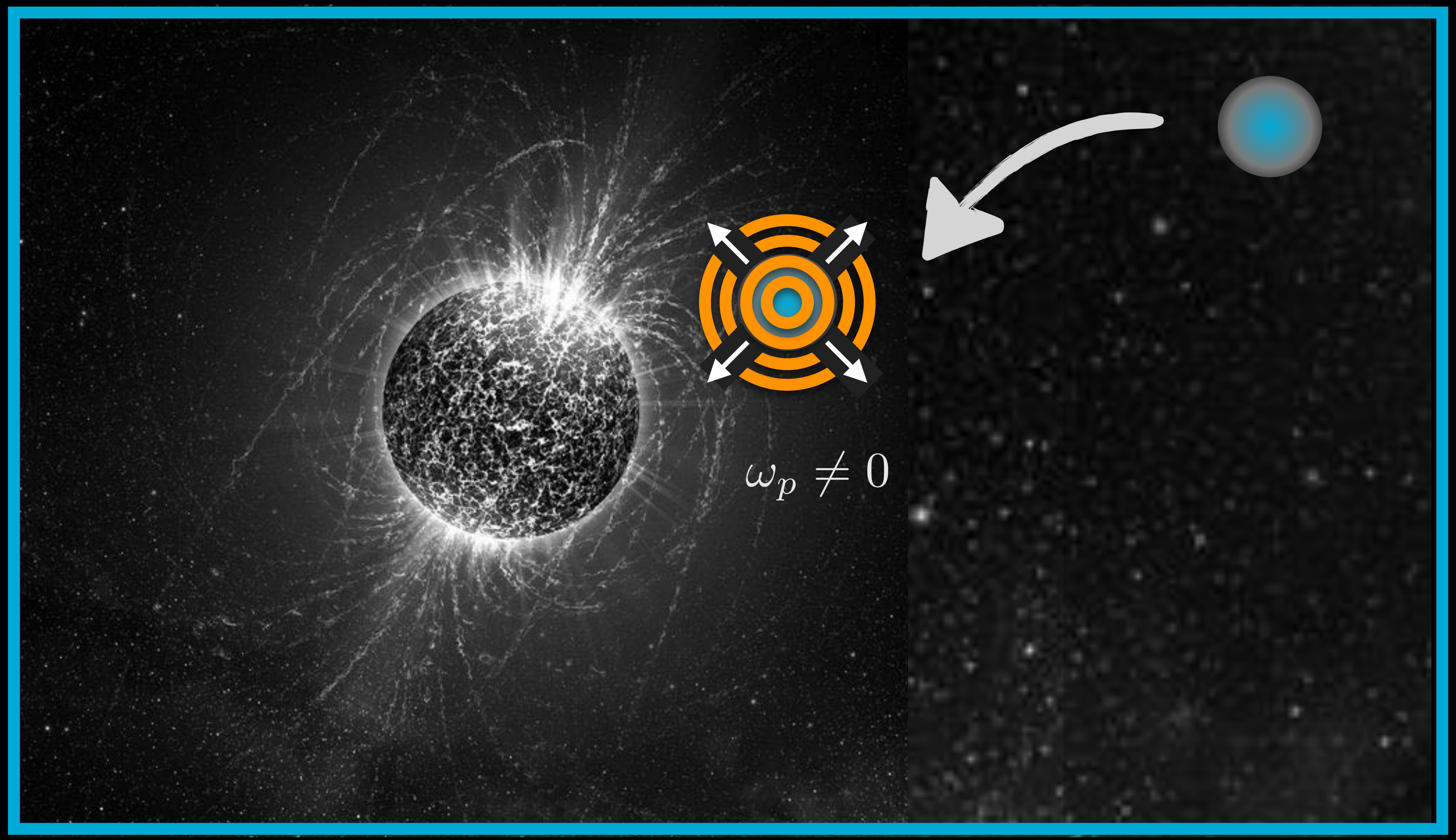
MA & Mou (2020)

$$\mathcal{L}_{int} \sim g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Motivated by earlier work by Hertzberg & Schippacaise (2018)

spin-s + photons: spin of soliton & polarization of photons



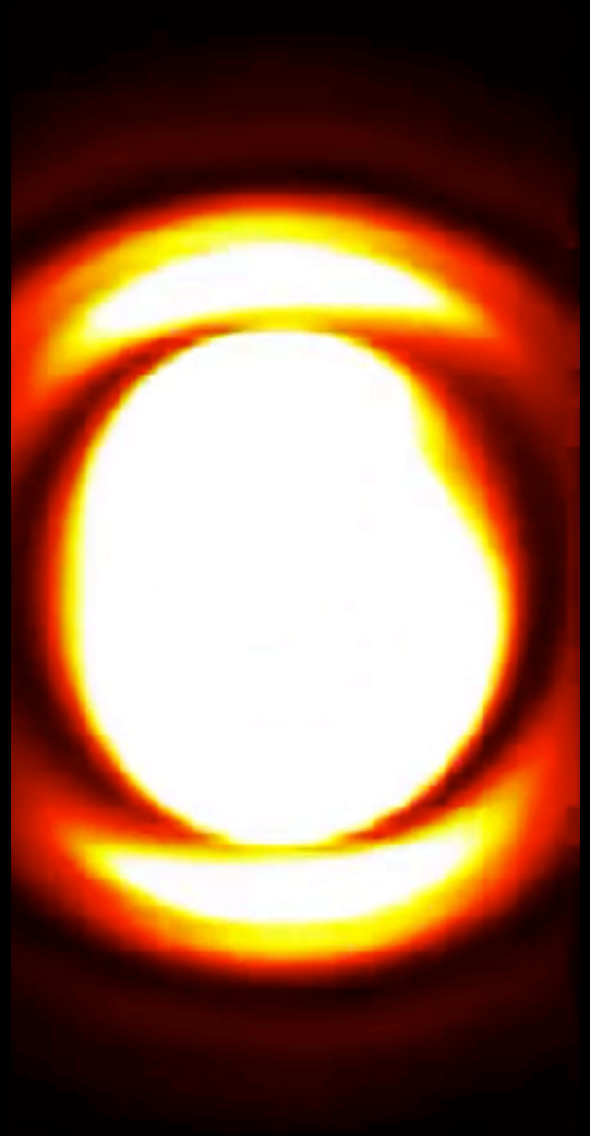


$$\mathcal{L}_{int} \sim \begin{cases} g^2_{W\gamma} W_\mu W^\mu \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-1 \\ g^2_{H\gamma}(H_{\mu\nu}H^{\mu\nu} - H^2) \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-2 \end{cases}$$

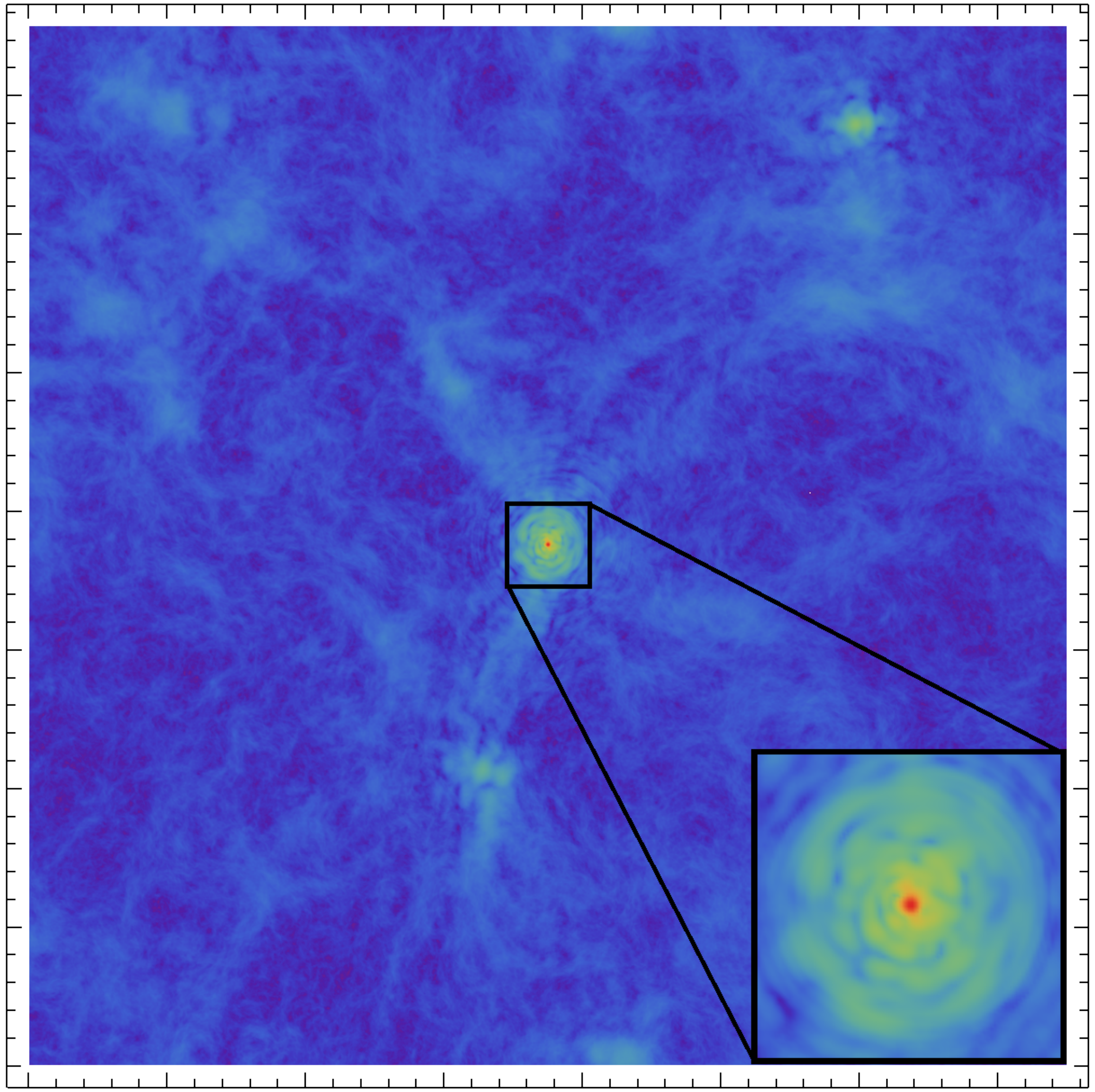
$$\sim g^2_{\mathcal{F}\gamma} \, \mathrm{Tr}[\boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{F}}] \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} \qquad \text{NR limit}$$

$\epsilon^2 F_{\mu\nu}\mathcal{G}^{\mu\nu}$

$F_{\mu\nu}G^{\mu\nu}$

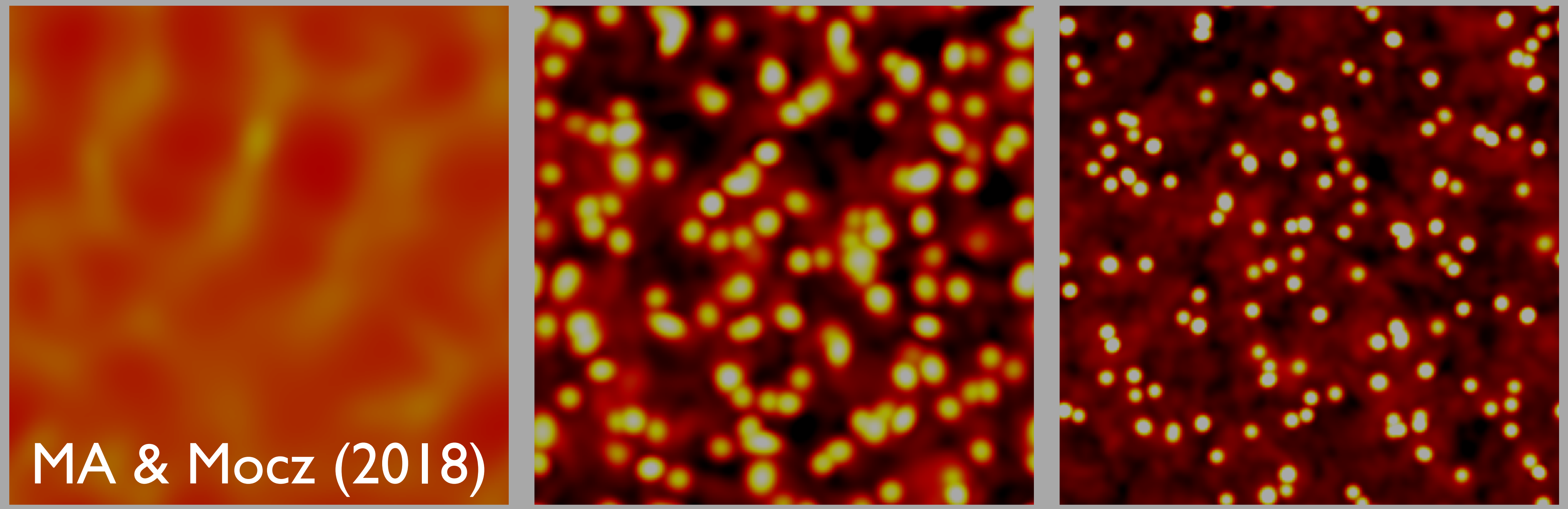


formation mechanisms?



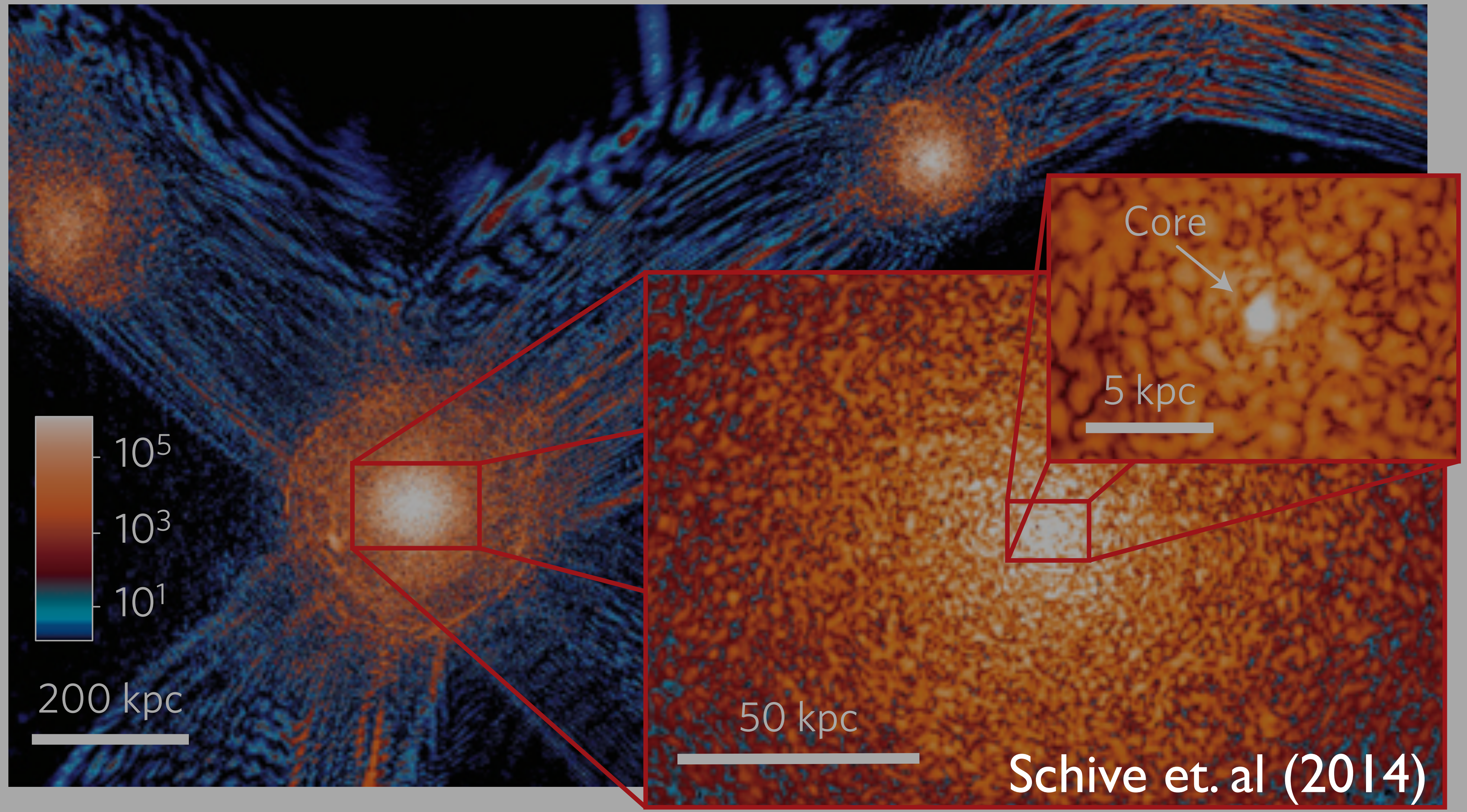
Gorghetto, Hardy, March-Russell, Song & West (2022)

misaligned scalar + self-interactions?



in progress

gravitational in late universe



gravitational particle production to nonlinear structures

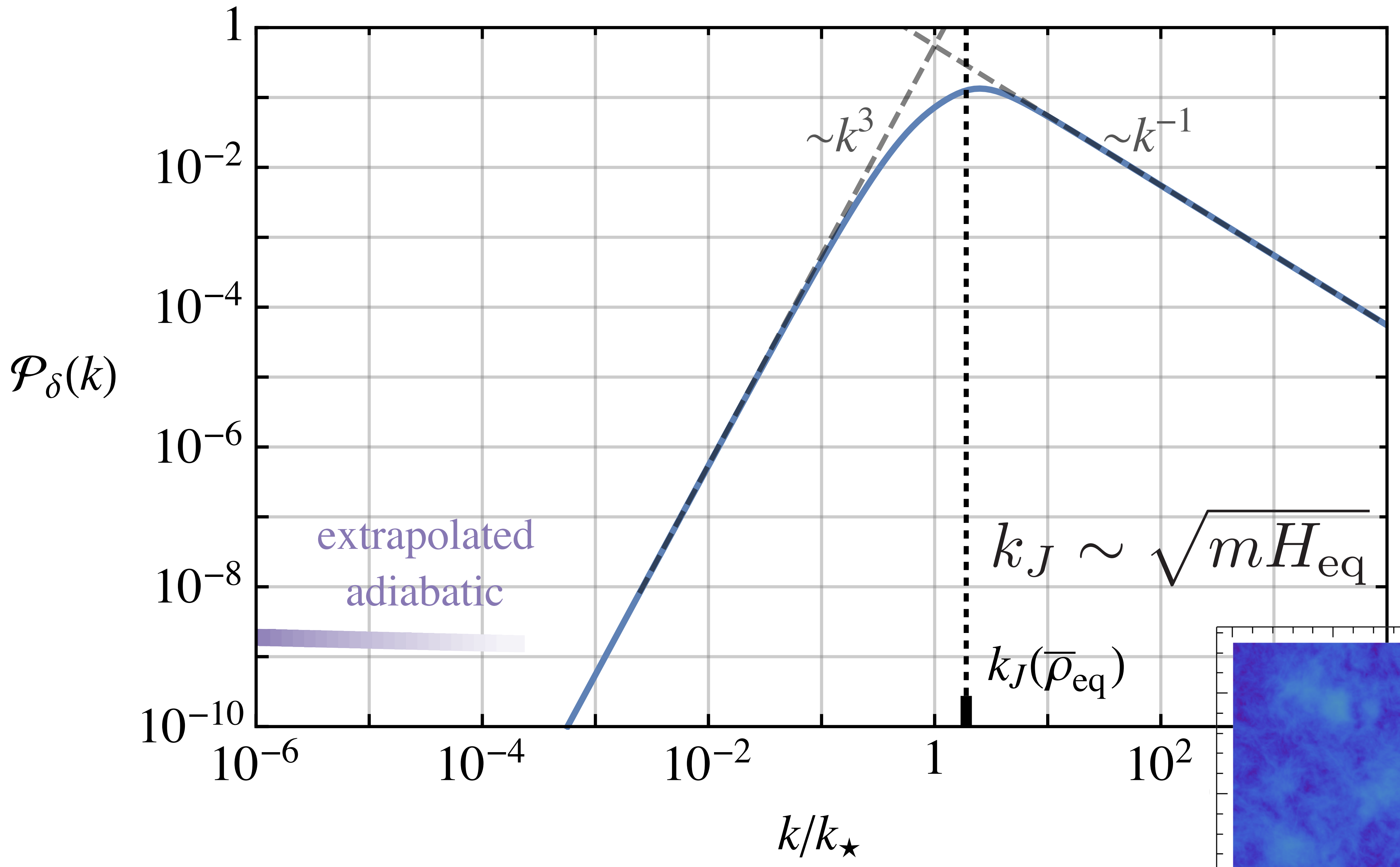
cannot easily do ultralight dark photons

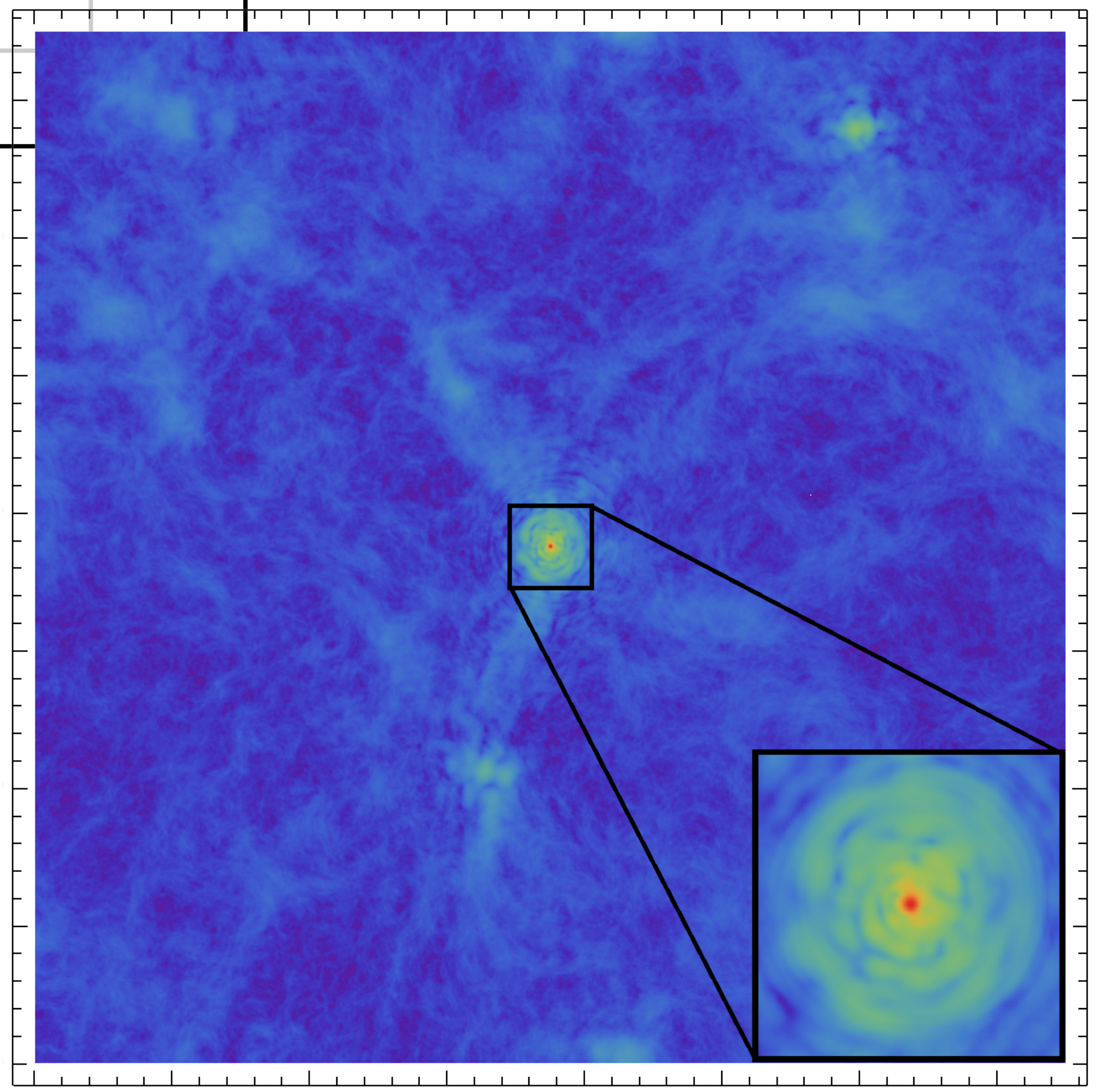
$$\Omega_{\rm vdm} \sim 0.3 \left(\frac{m}{10^{-5}\,{\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^4$$

Graham, Mardon, Rajendran (2016)

Ahmed, Grzadkowski, Socha (2020)

Kolb & Long (2020)





Gorghetto et. al (2022)

$$M_{\rm sol}(a) \sim 10^{-23} M_\odot \left(\frac{a_{\rm eq}}{a}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{3/2}$$

$$R_{\rm sol}(a) \sim 10^4\,{\rm km} \left(\frac{a}{a_{\rm eq}}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{1/2}$$

abundance and detection

$$n = f_s \overline{\rho}(t_0)/M \quad \simeq 10^{20}\text{pc}^{-3} \left(\frac{f_s}{0.05}\right) \left(\frac{\rho_{\text{local}}}{0.5\,\text{GeV}/\text{cm}^3}\right) \left(\frac{0.1 M_J^{\text{eq}}}{M}\right) \left(\frac{m}{\text{eV}}\right)^{3/2}$$

$$\begin{aligned}\Gamma &\simeq n\pi R^2 v_{\text{rel}} \\ &\simeq \frac{0.1}{\text{yr}} \left(\frac{m}{\text{eV}}\right)^{1/2} \left(\frac{0.1 M_J^{\text{eq}}}{M}\right)^3 \left(\frac{v_{\text{rel}}}{10^{-3}}\right) \left(\frac{f_s}{0.05}\right) \left(\frac{\rho_{\text{local}}}{0.5\,\text{GeV}/\text{cm}^3}\right)\end{aligned}$$

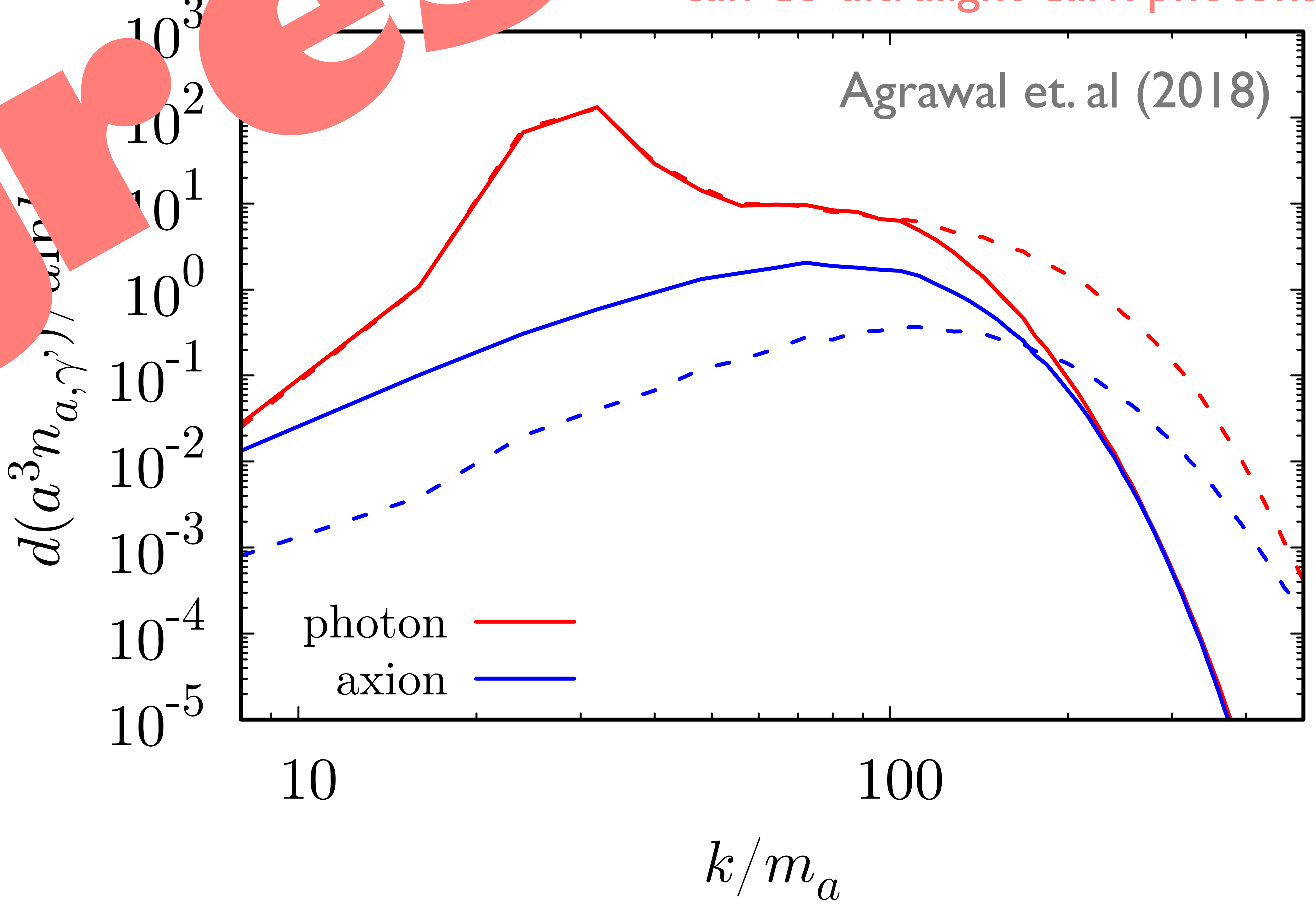
Gorghetto et. al (2022)

misaligned axion to dark photons to nonliear structures

$$\ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$

In Progress

can do ultralight dark photons



Co, Pierce, Zhang, and Zhao (2018)
Dror, Harigaya, and Narayan (2018)
Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018)

generalization to arbitrary spin

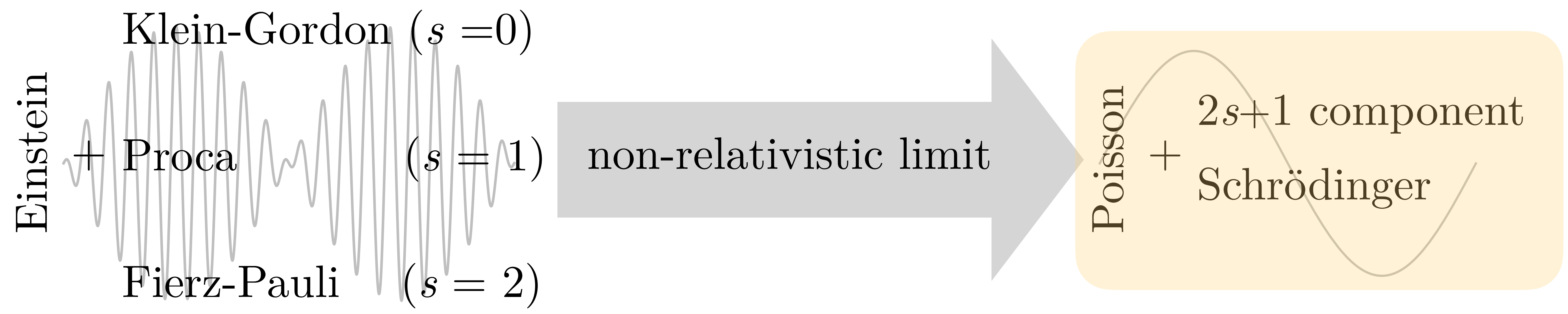
spin-s field as dark matter

Einstein + Klein-Gordon $(s = 0)$
Einstein + Proca $(s = 1)$
Einstein + Fierz-Pauli $(s = 2)$

Jain & MA (2021)

non-relativistic limit = multicomponent Schrödinger-Poisson

spin-s fields as light dark matter

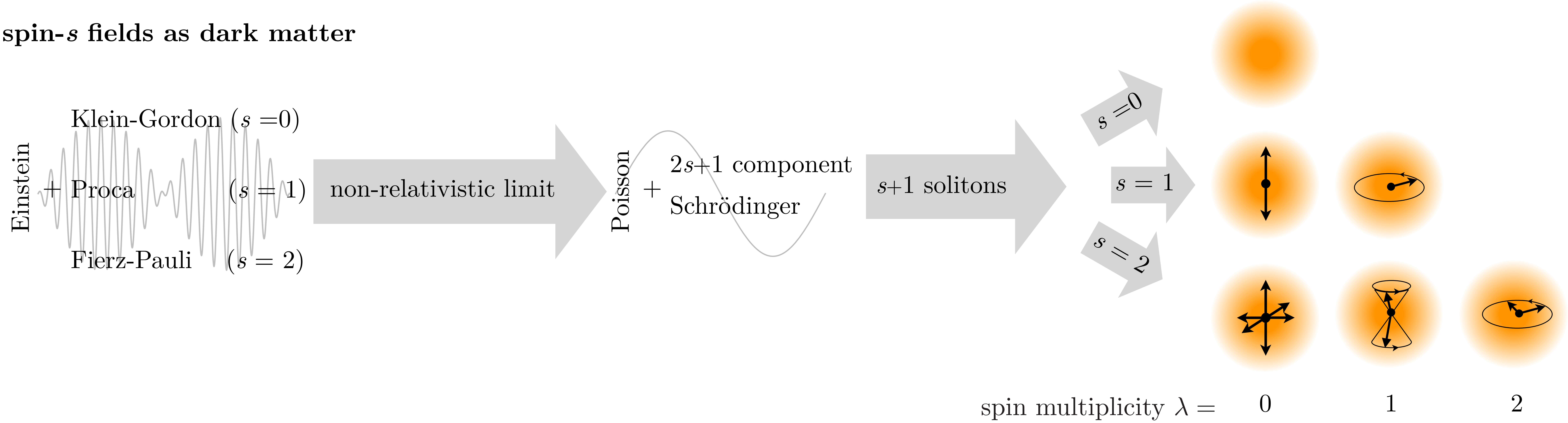


- phenomenology/numerical simulations
- interference ~1/(2s+1)

Jain & MA (2021)

extremally polarized solitons

spin-s fields as dark matter



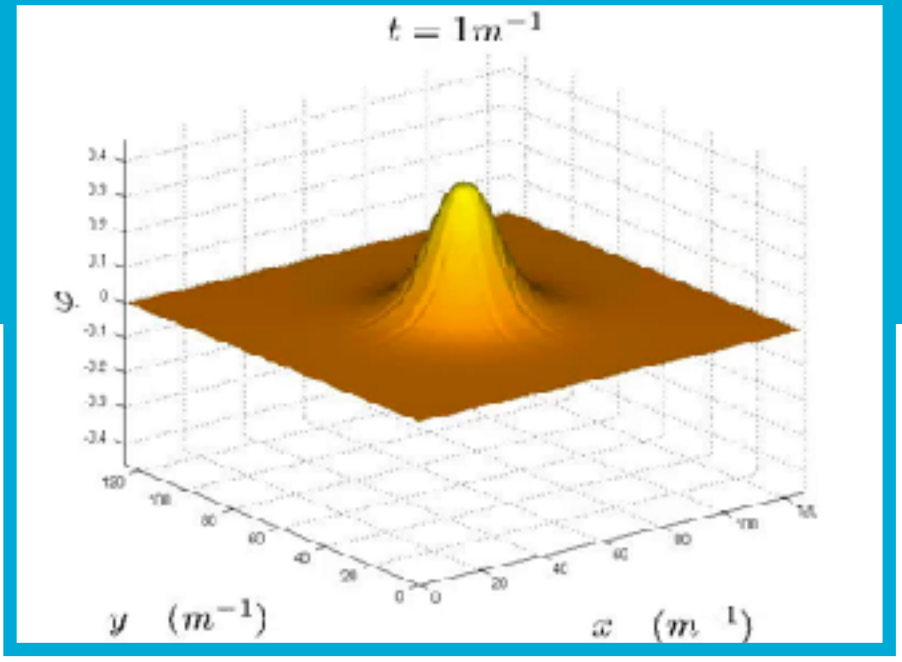
macroscopic spin $\boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

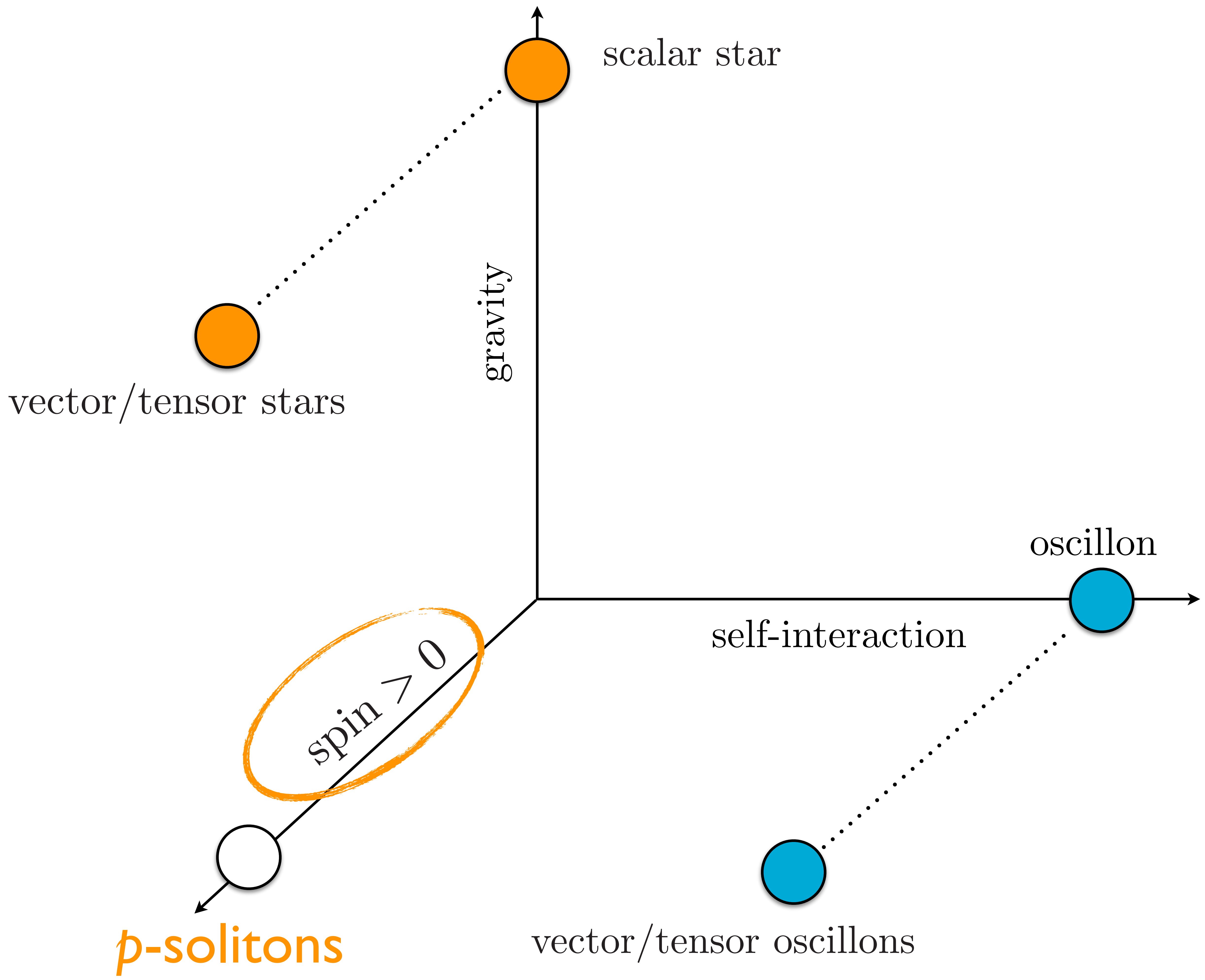
$N =$ # of particles in soliton

Jain & MA (2021)

non-topological solitons

spatially localized, coherently oscillating, long-lived

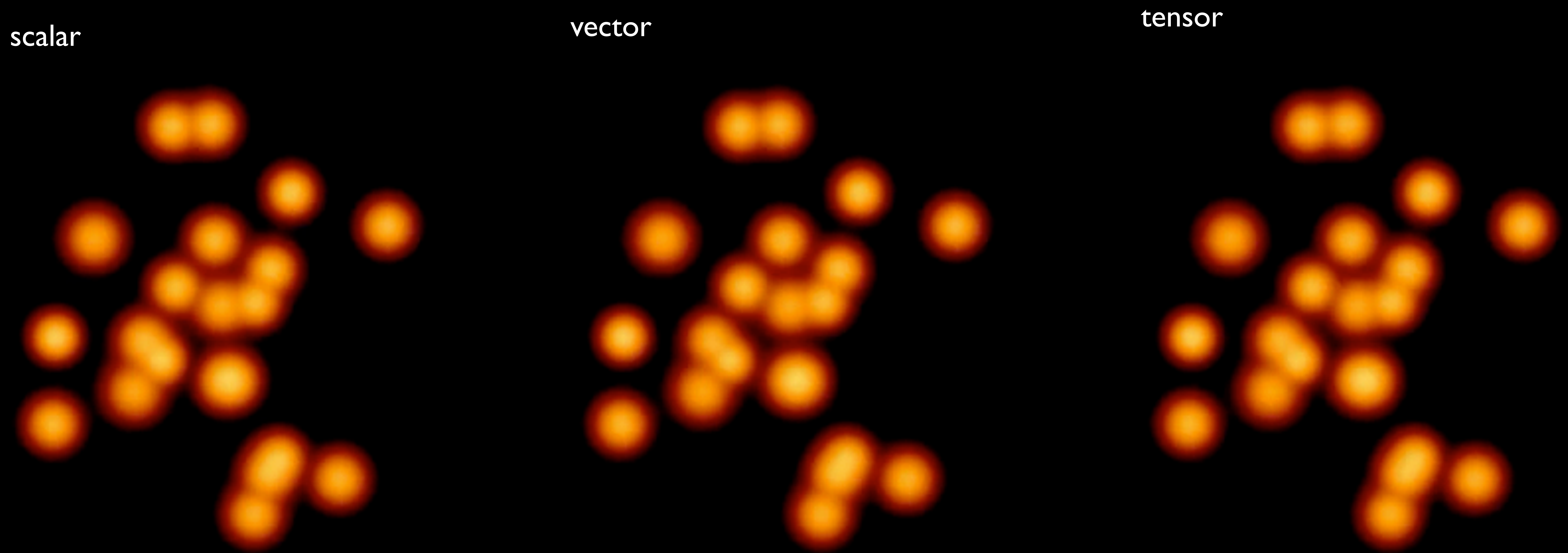




spatially localized

coherently oscillating (components)

exceptionally long-lived


scalar
vector
tensor


scalar
vector
tensor

the near future to do list …

future possibilities …

- formation and survival mechanisms
- BH superradiance with higher spin fields* (already done)
- dynamical friction
- vortices
- direct/indirect detection (interaction with baryons? photons? kinetic mixing)
- condensation time scales
- lifetimes of higher spin solitons
- initial power spectrum of fluctuations in the fields