A Spin on Wave Dark Matter

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RICE

with Mudit Jain [2109.04892]

5-minute summary

spin-s field as dark matter

Einstein +

Klein-Gordon ($s = 0$)

Proca ($s = 1$)

Fierz-Pauli ($s = 2$)

non-relativistic limit = multicomponent Schrödinger-Poisson

spin-s fields as light dark matter

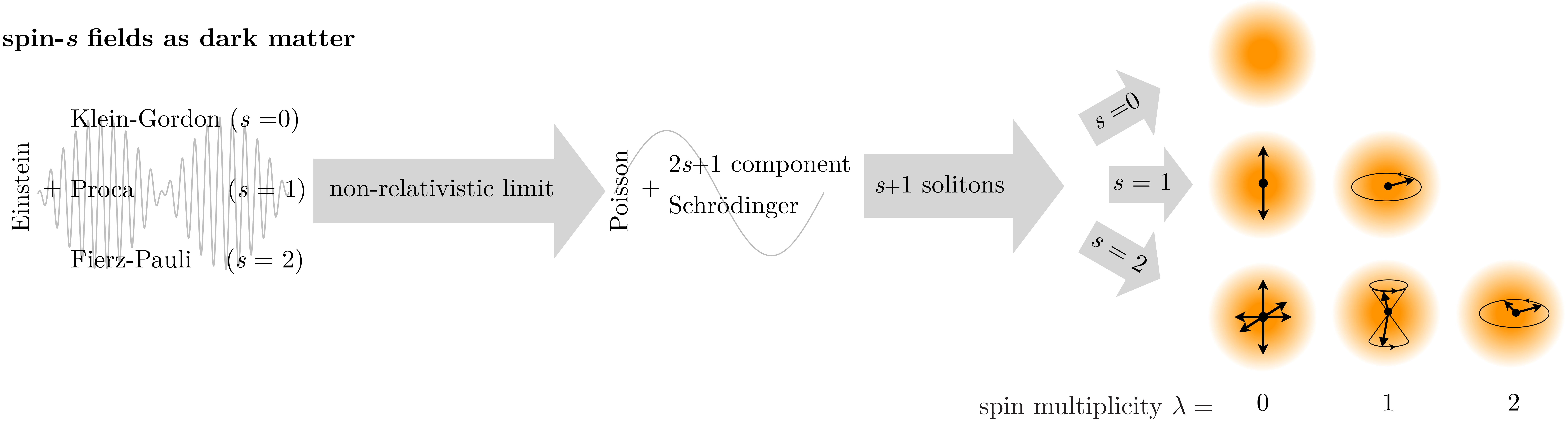
Einstein + Klein-Gordon ($s = 0$) / Proca ($s = 1$) / Fierz-Pauli ($s = 2$)

$\xrightarrow{\text{non-relativistic limit}}$

Poisson + $2s$+1 component Schrödinger

p-solitons

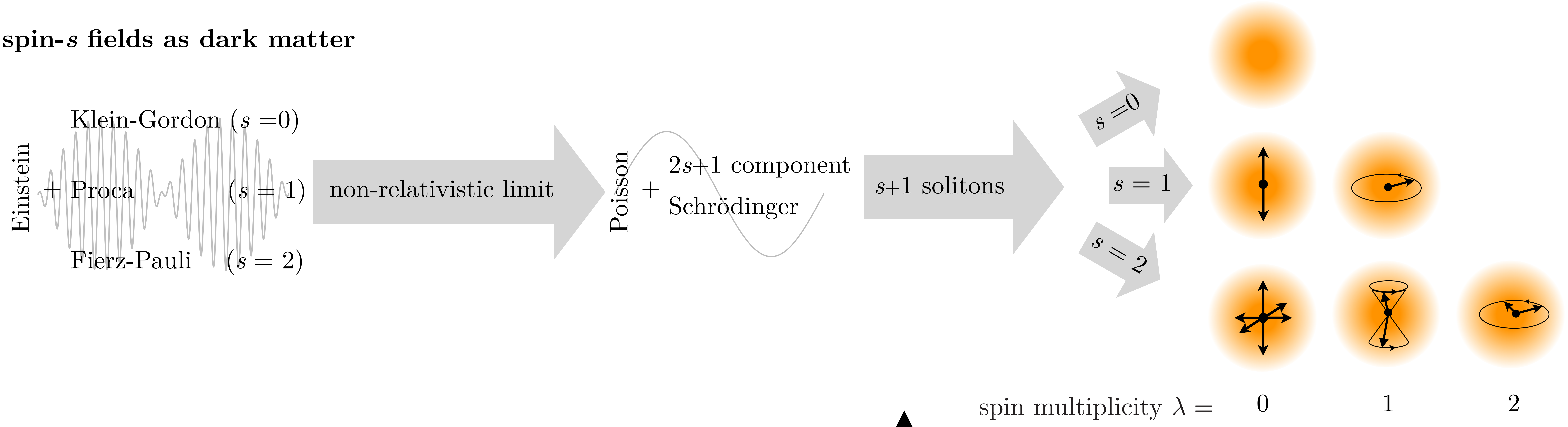
spin-s fields as dark matter

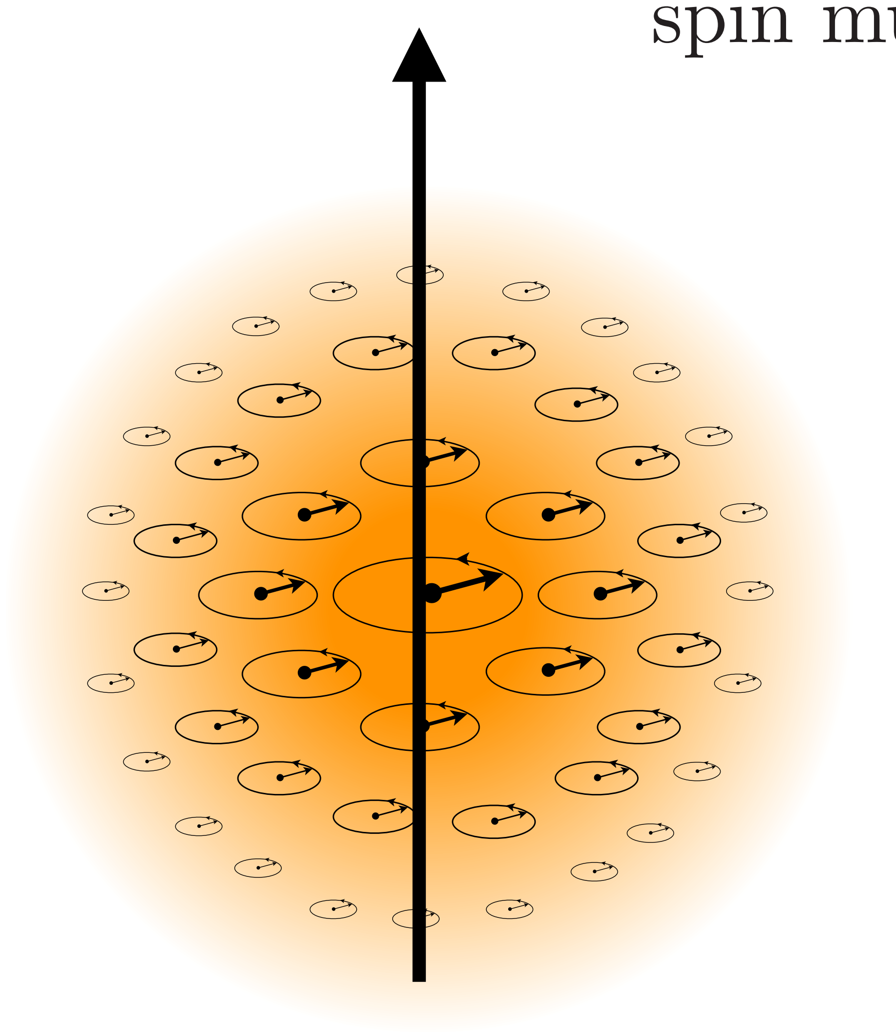


- and their linear superpositions

macroscopic intrinsic spin!

spin-s fields as dark matter

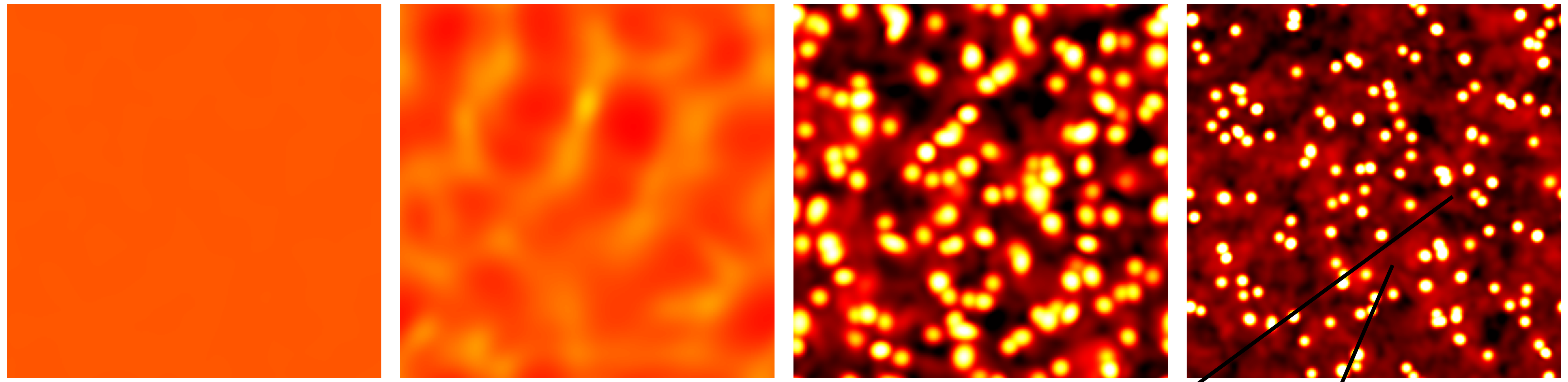




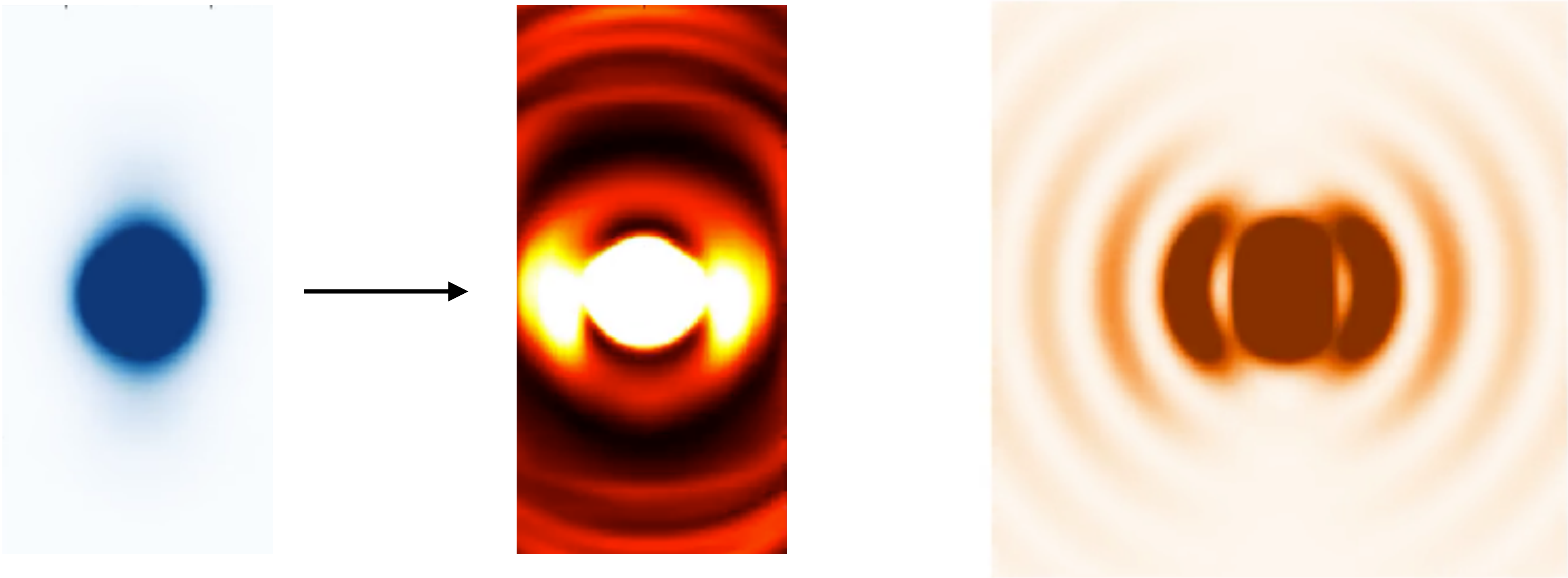
macroscopic spin $\boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

N = # of particles in soliton

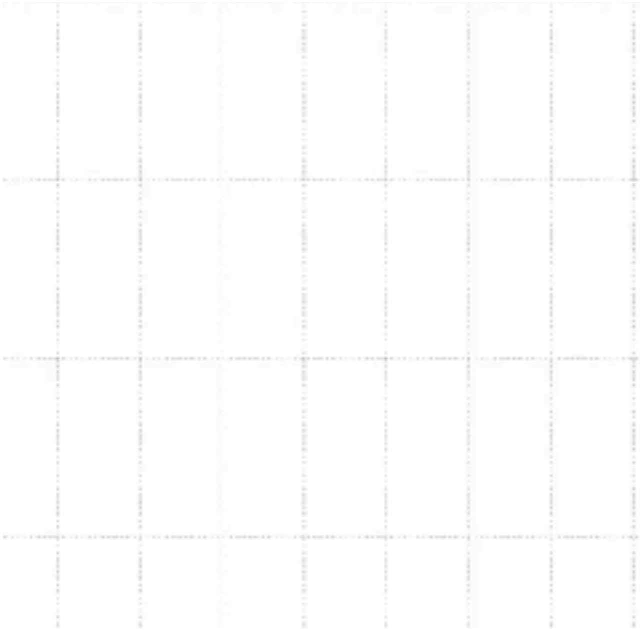
formation mechanisms + astrophysical implications

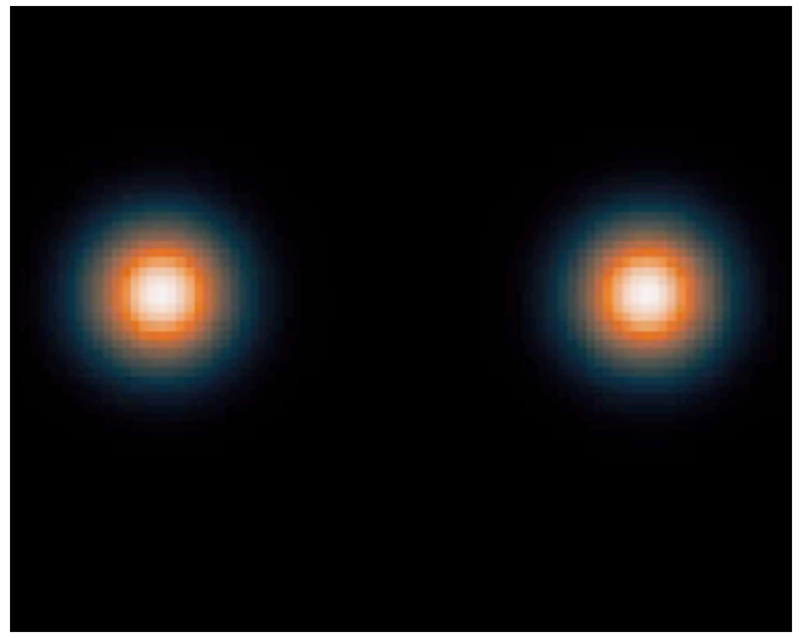


formation mechanisms & interactions



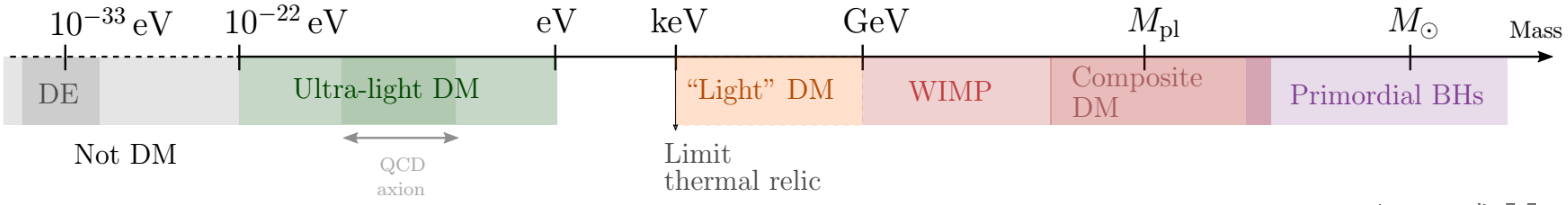
observational implications





motivation

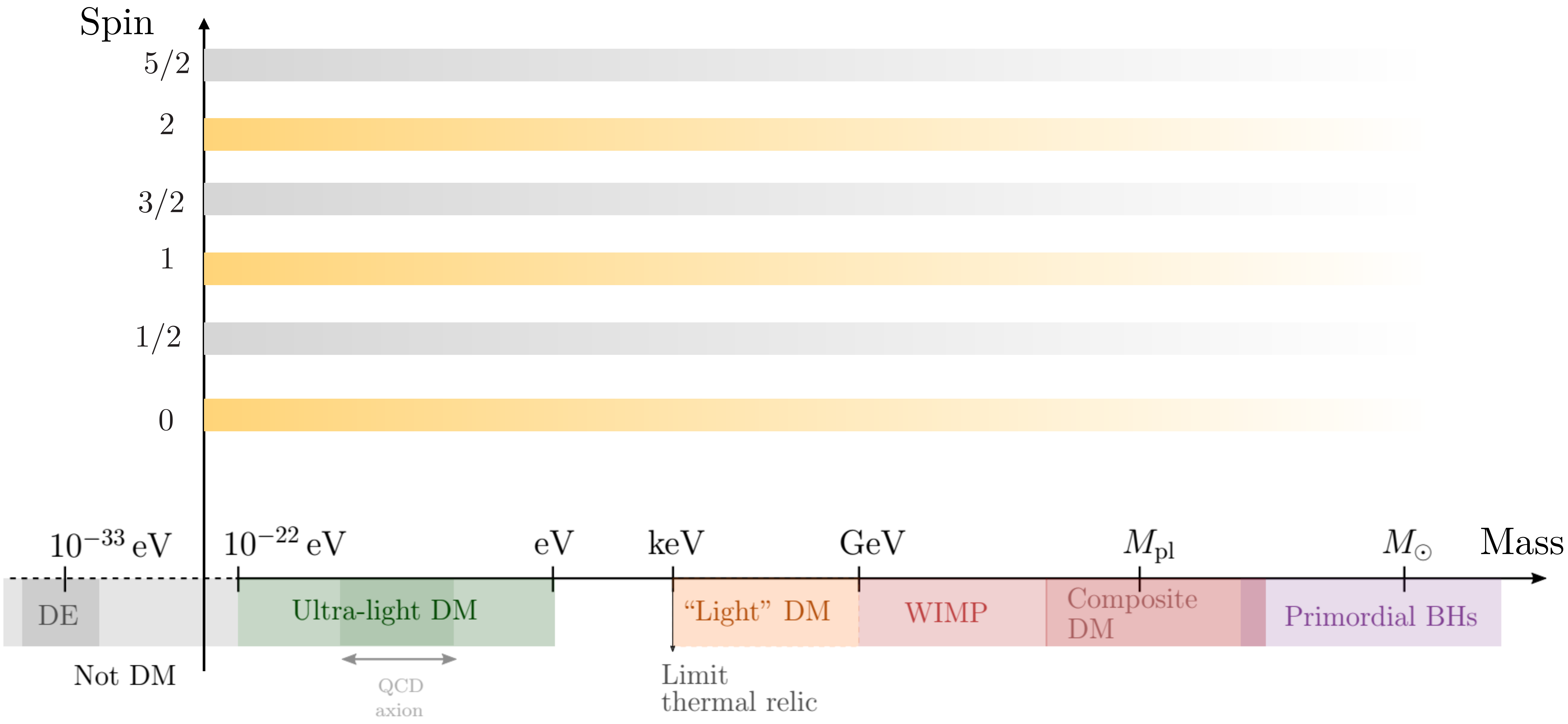
dark matter mass ?



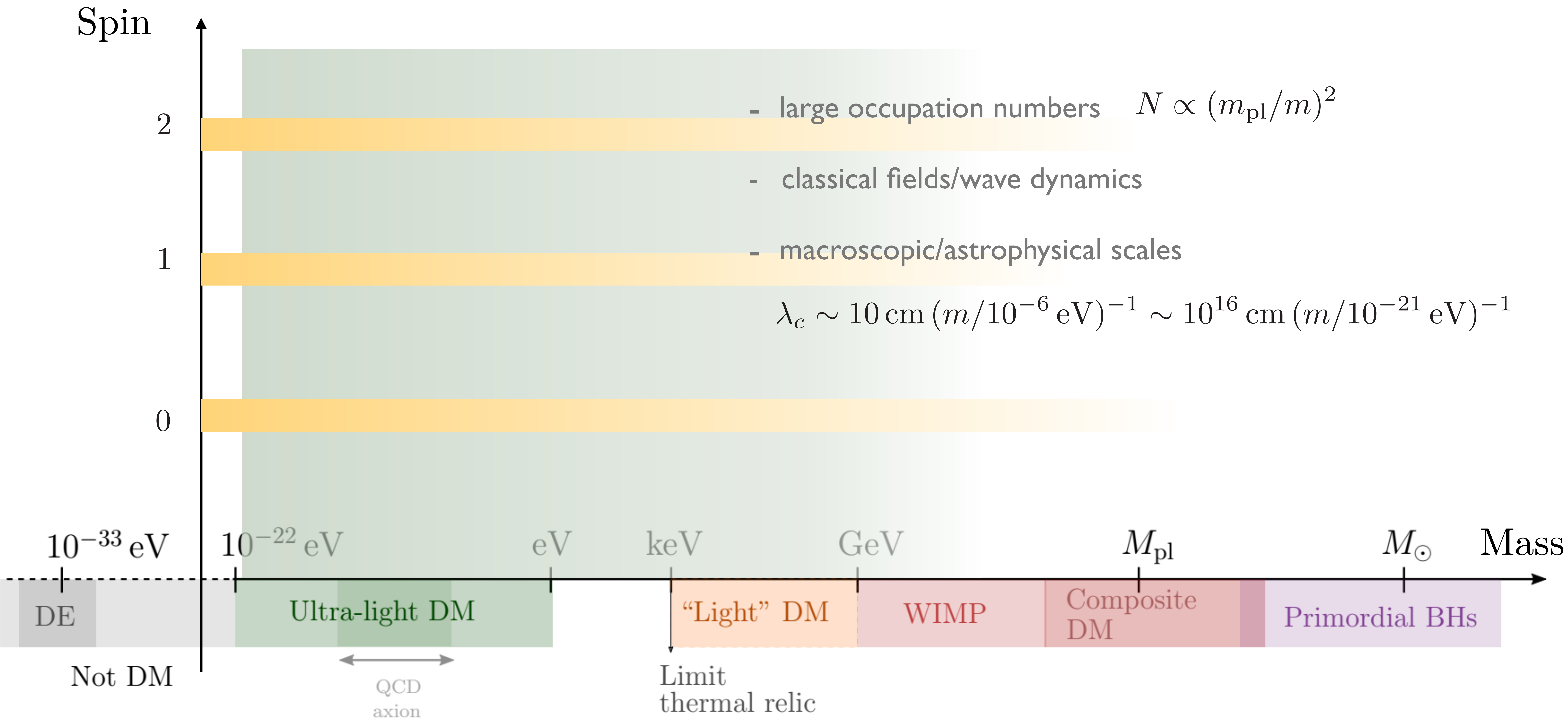
ultra-light — see Dan Grin's talk

image credit: E. Ferreira

dark matter spin ?



light, bosonic wave dark matter



models

massive spin-s bosonic field with weak-field gravity

quadratic action for gravitational and massive spin-s fields + leading gravitational interactions

$$S_{\mathrm{EH}} + S_{\mathrm{dark}} = \int d^4x \left[m_{\mathrm{pl}}^2 \mathcal{L}_{\mathrm{GR}}^{(2)}(h) + \mathcal{L}_{m,s}^{(2)} - \frac{1}{2} h^{\mu\nu} T^{s}_{\mu\nu} \right]$$

spin-s bosonic wave dark matter

$$S_{\rm EH} + S_{\rm dark} = \int d^4x \left[m_{\rm pl}^2 \mathcal{L}^{(2)}_{\rm GR}(h) + \mathcal{L}^{(2)}_{m,s} - \frac{1}{2} h^{\mu\nu} T^s_{\mu\nu} \right]$$

$$\mathcal{L}^{(2)}_{m,0}(\phi) = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\,\partial_\mu\phi - \frac{1}{2}m^2\phi^2$$

spin - 0 Klein-Gordon

$$\mathcal{L}^{(2)}_{m,1}(W) = -\frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}\,G_{\mu\nu}G_{\alpha\beta} + \frac{1}{2}m^2\,\eta^{\mu\nu}W_\mu W_\nu$$

$$G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

spin - 1 Proca

$$\begin{aligned}\mathcal{L}^{(2)}_{m,2}(H) = &\frac{1}{2}\eta^{\alpha\beta}\eta^{\lambda\sigma}\eta^{\kappa\rho}\partial_\beta H_{\sigma\rho}\partial_\alpha H_{\lambda\kappa} - \frac{1}{2}\eta^{\alpha\beta}\partial_\beta H\partial_\alpha H \\ &- \eta^{\kappa\rho}\eta^{\sigma\beta}\eta^{\lambda\alpha}\partial_\beta H_{\sigma\rho}\partial_\alpha H_{\lambda\kappa} + \eta^{\lambda\beta}\eta^{\kappa\alpha}\partial_\beta H\partial_\alpha H_{\lambda\kappa} \\ &+ \frac{1}{2}m^2\left[\eta^{\mu\nu}\eta^{\alpha\beta}H_{\mu\nu}H_{\alpha\beta} - \eta^{\sigma\lambda}\eta^{\rho\kappa}H_{\sigma\rho}H_{\lambda\kappa}\right];\end{aligned}$$

spin - 2 Fierz-Pauli

non-relativistic limit

non-relativistic limit, physical d.o.f

$$S_{\rm EH} + S_{\rm dark} = \int d^4x \left[m_{\rm pl}^2 \mathcal{L}^{(2)}_{\rm GR}(h) + \mathcal{L}^{(2)}_{m,s} - \frac{1}{2} h^{\mu\nu} T^s_{\mu\nu} \right]$$

$$= \int \mathrm{d}^4x \left[m_{\rm pl}^2 \mathcal{L}^{(2)}_{\rm GR}(h) + \mathcal{L}^{(2)}_{m,s}(\boldsymbol{\mathcal{F}}) - \frac{1}{2} h_{\mu\nu} \mathcal{T}^{\mu\nu}(\boldsymbol{\mathcal{F}}) + \ldots \right]$$

only includes physical d.o.f (constraints eliminated)

$\phi = [\boldsymbol{\mathcal{F}}]$	spin-0	1 component
$W_i = [\boldsymbol{\mathcal{F}}]_i$	spin-1	3 components
$H_{ij} = [\boldsymbol{\mathcal{F}}]_{ij}$	spin-2	5 components

$2s{+}1$ components

non-relativistic limit

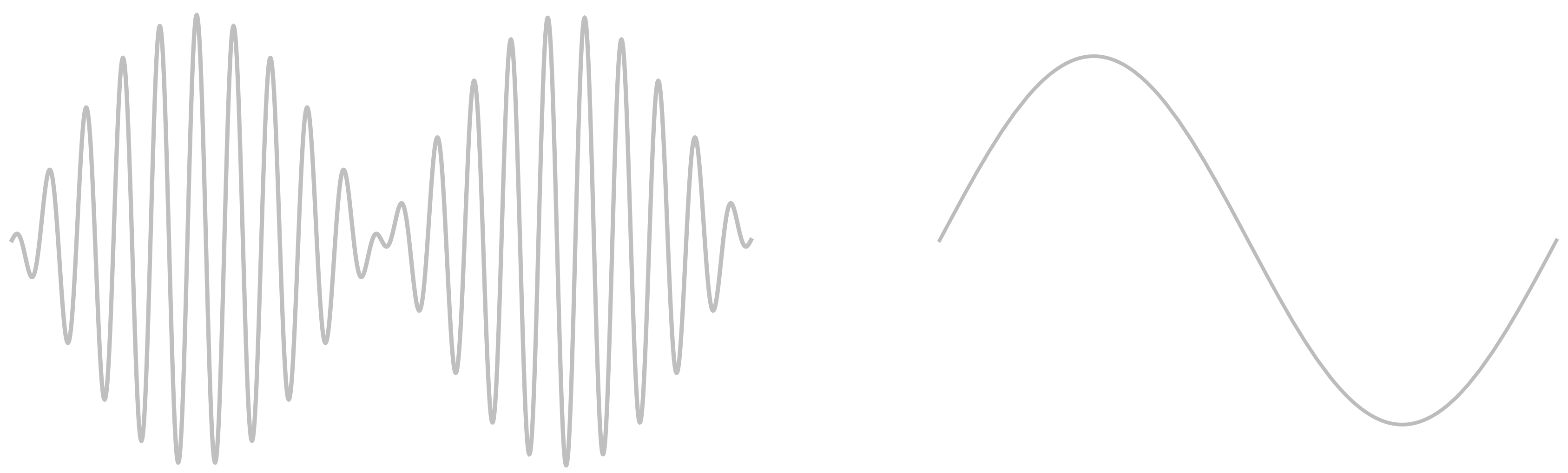
$$S_{\mathrm{EH}} + S_{\mathrm{dark}} = \int d^4x \left[m_{\mathrm{pl}}^2 \mathcal{L}_{\mathrm{GR}}^{(2)}(h) + \mathcal{L}_{m,s}^{(2)} - \frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{s} \right]$$

$$= \int \mathrm{d}^4x \left[m_{\mathrm{pl}}^2 \mathcal{L}_{\mathrm{GR}}^{(2)}(h) + \mathcal{L}_{m,s}^{(2)}(\boldsymbol{\mathcal{F}}) - \frac{1}{2} h_{\mu\nu} \mathcal{T}^{\mu\nu}(\boldsymbol{\mathcal{F}}) + \ldots \right]$$

only includes physical d.o.f (constraints eliminated)

$$\boldsymbol{\mathcal{F}}(\mathbf{x}, t) = \frac{1}{\sqrt{2m}} \left[\underline{e^{-imt}} \, \underline{\tilde{\boldsymbol{\Psi}}(\mathbf{x}, t)} + \mathrm{h.c.} \right]$$

split in “fast” and “slow” parts



non-relativistic limit

$$S_{\rm EH} + S_{\rm dark} = \int d^4x \left[m_{\rm pl}^2 \mathcal{L}^{(2)}_{\rm GR}(h) + \mathcal{L}^{(2)}_{m,s} - \frac{1}{2} h^{\mu\nu} T^s_{\mu\nu} \right]$$

$$= \int \mathrm{d}^4x \left[m_{\rm pl}^2 \mathcal{L}^{(2)}_{\rm GR}(h) + \mathcal{L}^{(2)}_{m,s}(\boldsymbol{\mathcal{F}}) - \frac{1}{2} h_{\mu\nu} \mathcal{T}^{\mu\nu}(\boldsymbol{\mathcal{F}}) + \ldots \right]$$

only includes physical d.o.f (constraints eliminated)

$$\boldsymbol{\mathcal{F}}(\mathbf{x}, t) = \frac{1}{\sqrt{2m}} \left[e^{-imt} \tilde{\boldsymbol{\Psi}}(\mathbf{x}, t) + \mathrm{h.c.} \right]$$

assume $\partial_t \boldsymbol{\Psi} \ll m \boldsymbol{\Psi}, \ \nabla^2 \boldsymbol{\Psi} \ll m^2 \boldsymbol{\Psi}$

$$\mathcal{S}^{\rm eff}_{nr} = \int \mathrm{d}^4x \left[\frac{i}{2} \mathrm{Tr} \left[\boldsymbol{\Psi}^\dagger \dot{\boldsymbol{\Psi}} \right] + \mathrm{c.c.} - \frac{1}{2m} \mathrm{Tr}[\nabla \boldsymbol{\Psi}^\dagger \cdot \nabla \boldsymbol{\Psi}] + m_{\rm pl}^2 \, \Phi \nabla^2 \Phi - m \, \Phi \ \mathrm{Tr}[\boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}] \right]$$

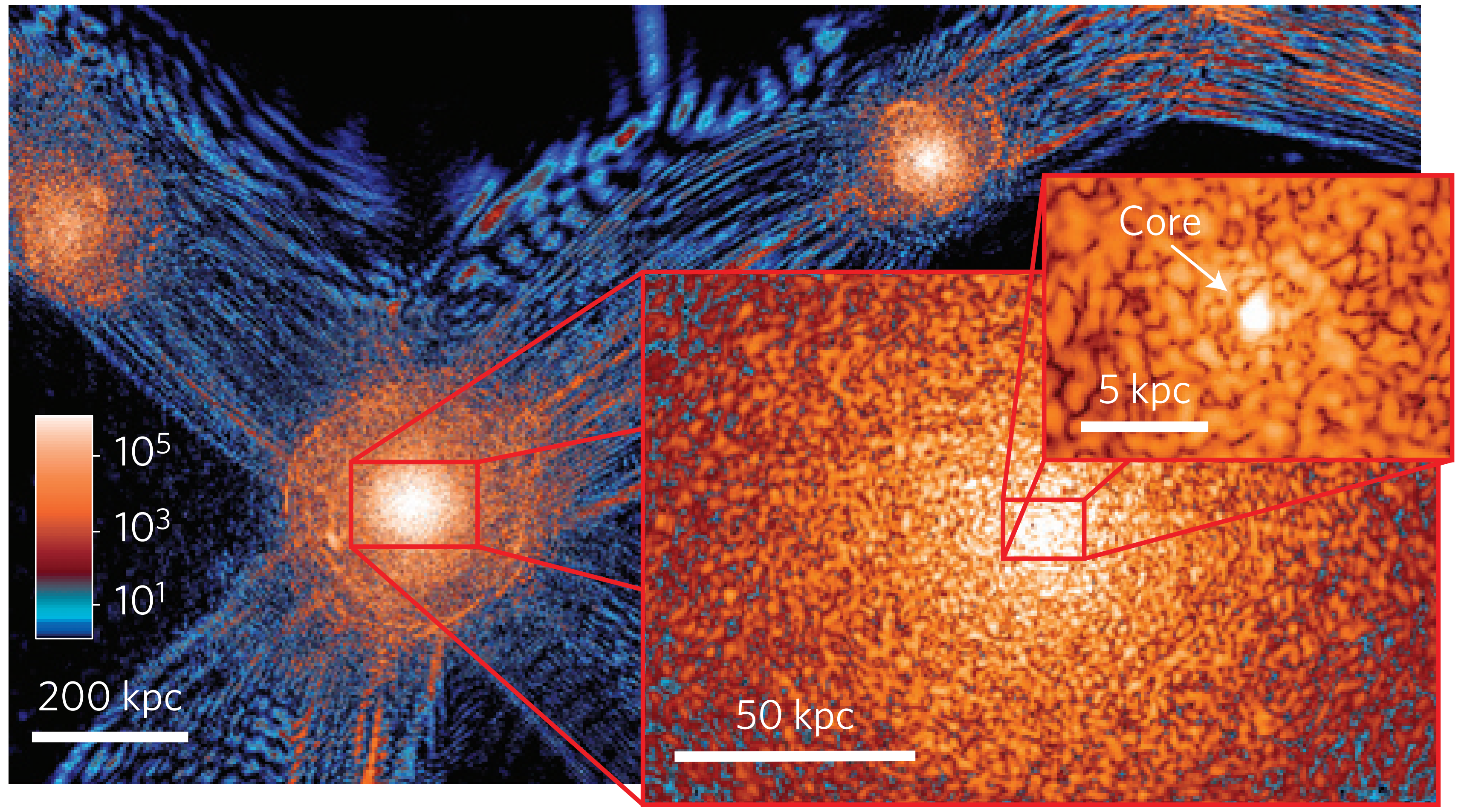
$\Phi = \text{Newtoian gravitational potential}$

$\psi = [\boldsymbol{\Psi}]$	spin-0
$\psi_i = [\boldsymbol{\Psi}]_i$	spin-1
$\psi_{ij} = [\boldsymbol{\Psi}]_{ij}$	spin-2

multi-component Schrodinger-Poisson system!

$$i\frac{\partial}{\partial t}\boldsymbol{\Psi} = -\frac{1}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi}$$
$$\nabla^2\Phi = \frac{m}{2m_{\rm pl}^2}\,\mathrm{Tr}[\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}].$$

extension to FRW: $\partial_t \to \partial_t + 3H/2, \nabla \to \nabla/a$



Schive et. al (2015)

$\psi = [\boldsymbol{\Psi}]$	spin-0	1 component
$\psi_i = [\boldsymbol{\Psi}]_i$	spin-1	3 components
$\psi_{ij} = [\boldsymbol{\Psi}]_{ij}$	spin-2	5 components

much easier to simulate than the relativistic equations

since fast time-scales are integrated out.

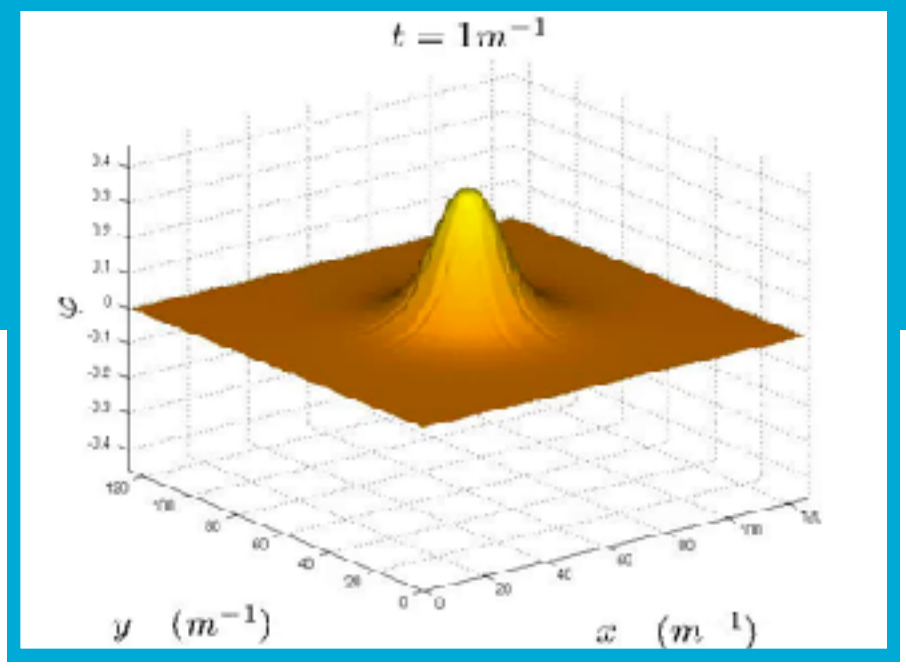
Recent work on non-relativistic case :

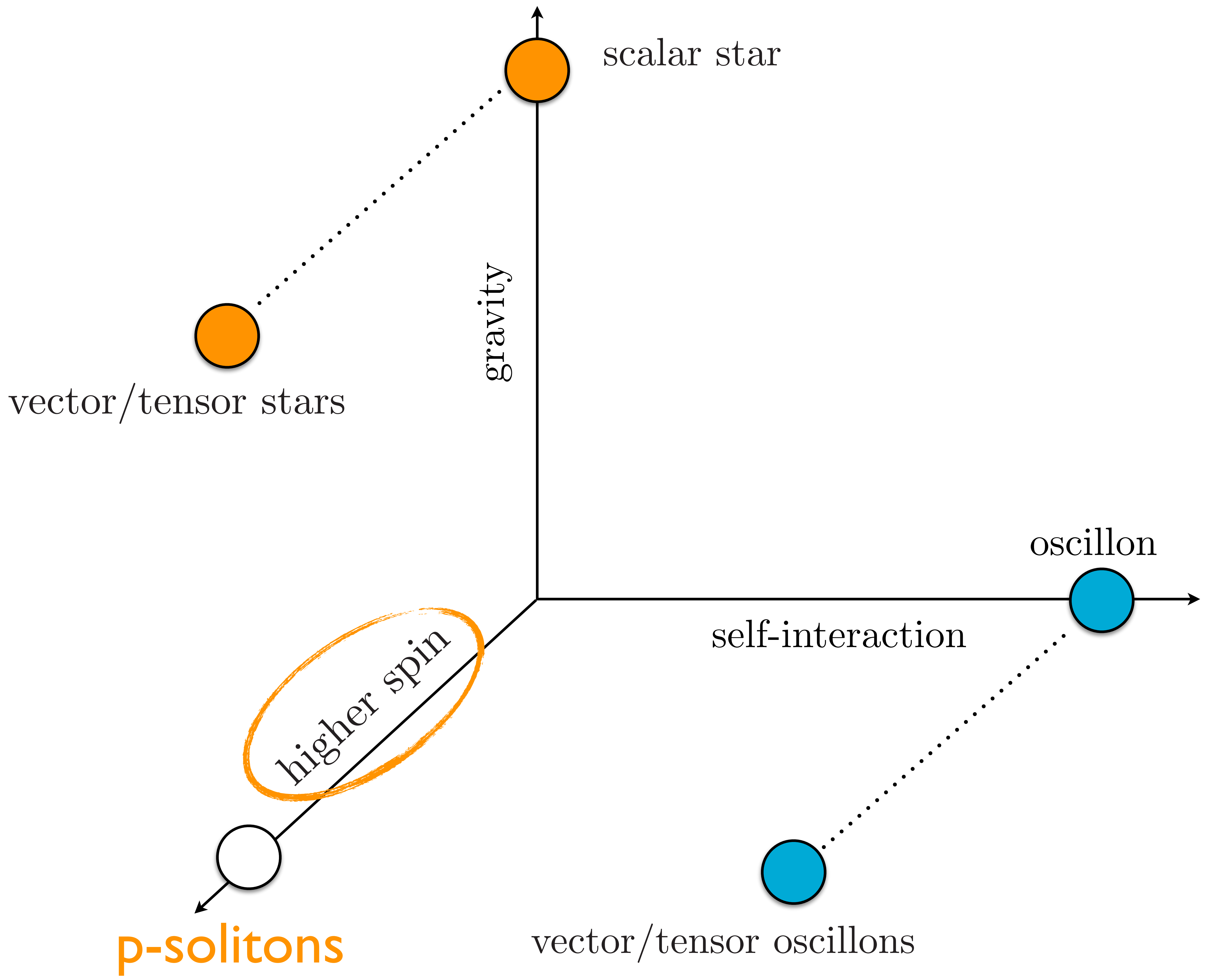
for scalar, see for example example: Guth, Kaiser, Namjoo (2017), Salehian et. al (2021), for vector case Adshead & Lozanov (2021)

polarized solitons

non-topological solitons

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

exceptionally long-lived

soliton solutions: scalar case

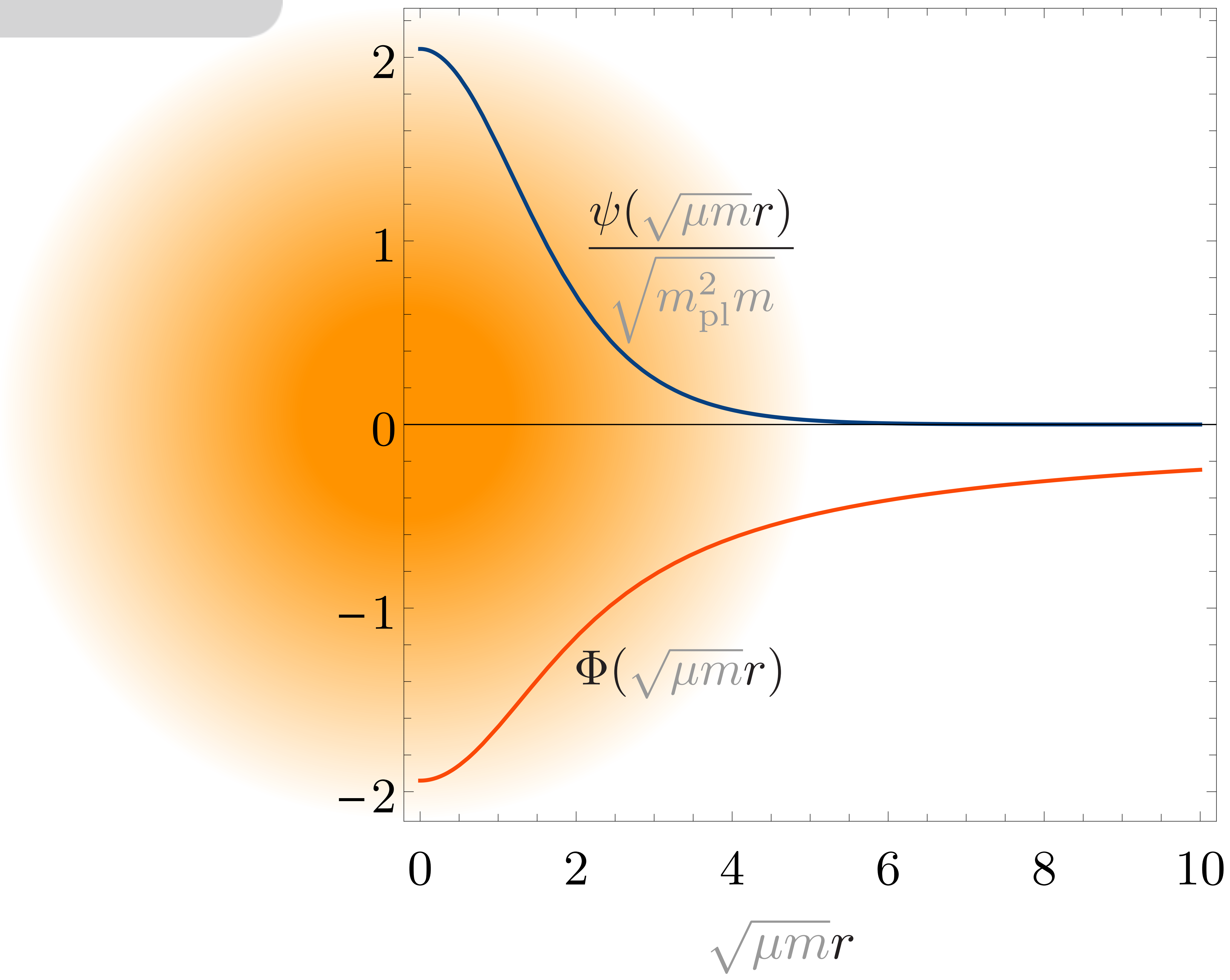
ground state solutions at fixed particle number

$$\Psi(t,\mathbf{x}) = \psi(\mathbf{x})e^{i\mu t}$$

$\mu =$ chemical potential

$$-\mu\,\psi = -\frac{1}{2m}\nabla^2\psi + m\,\Phi\,\psi$$
$$\nabla^2\Phi = \frac{m}{2m_{\rm pl}^2}\,|\psi(\mathbf{x})|^2,$$

$$\phi = \sqrt{\frac{2}{m}}\Re[\Psi e^{-imt}]$$



same "universal" profile

soliton solutions: scalar case

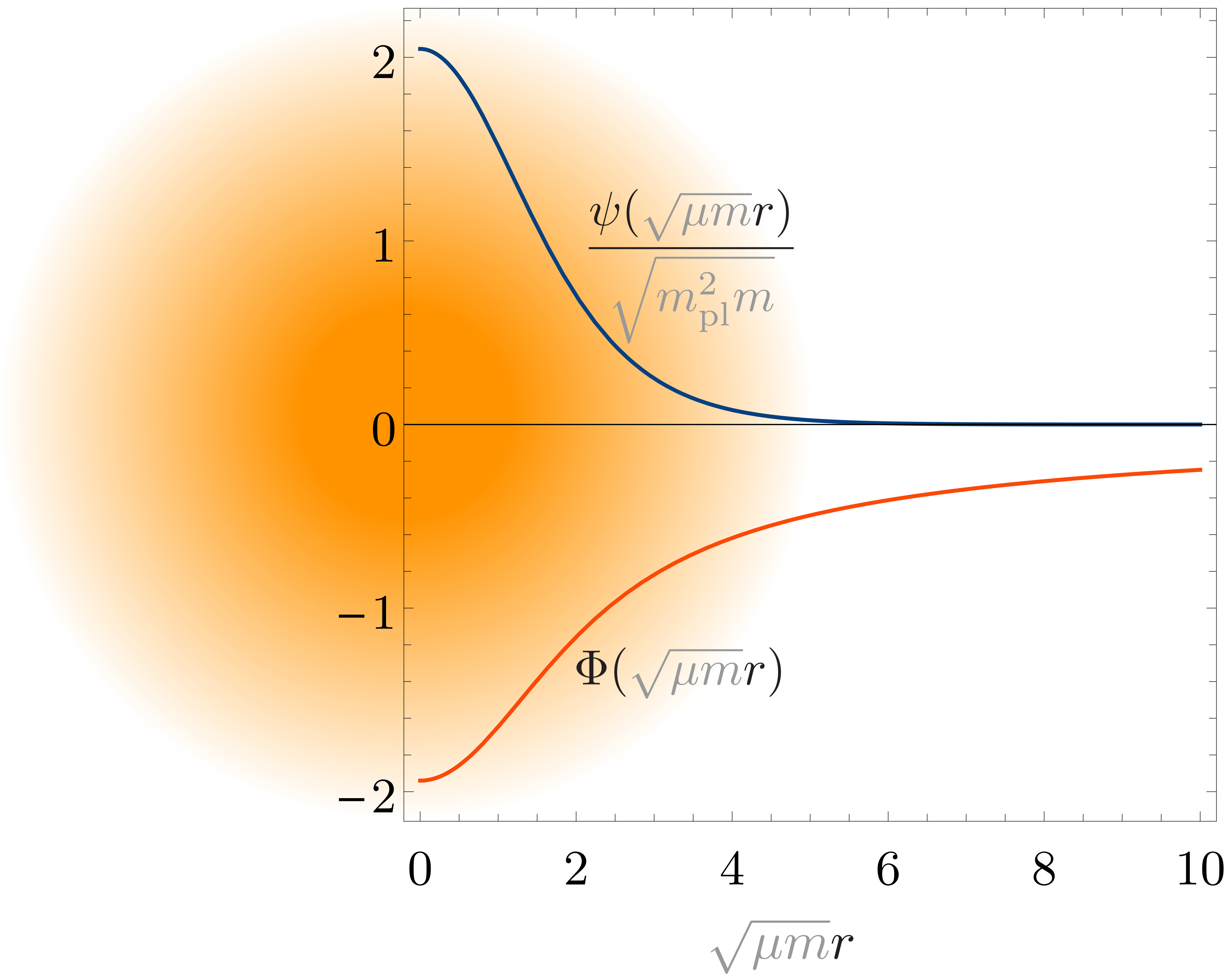
size, particle number, energy and spin

$$L \sim (m/\mu)^{1/2} m^{-1} \qquad \gg m^{-1}$$

$$N = \frac{M}{m} \approx 60.7 \frac{m_{\rm pl}^2}{m^2}\left(\frac{\mu}{m}\right)^{1/2} \qquad \gg 1$$

$$E \approx -19.2 \frac{m_{\rm pl}^2}{m}\left(\frac{\mu}{m}\right)^{3/2}, \qquad \gg m$$

$$\boldsymbol{S}^{\rm tot} = 0$$



$\mu/m \ll 1$ non-relativistic

solitons in a spin-s field

spin-s field, $2s+1$ multiplicity states

$$\mathbf{\Psi}_s^{(\lambda)}(\mathbf{x},t) = \psi(\mathbf{x})e^{i\mu t}\underline{\boldsymbol{\epsilon}_{s,\hat{n}}^{(\lambda)}} \qquad \lambda \in \{-s, \dots, s\}$$

polarization tensors

$s = 1$

$$\boldsymbol{\epsilon}_{1,\hat{z}}^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{\epsilon}_{1,\hat{z}}^{(\pm 1)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

$s = 2$

$$\boldsymbol{\epsilon}_{2,\hat{z}}^{(0)} = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \boldsymbol{\epsilon}_{2,\hat{z}}^{(\pm 1)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & \pm i \\ 1 & \pm i & 0 \end{pmatrix}$$

$$\boldsymbol{\epsilon}_{2,\hat{z}}^{(\pm 2)} = \frac{1}{2}\begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

solitons in a spin-s field

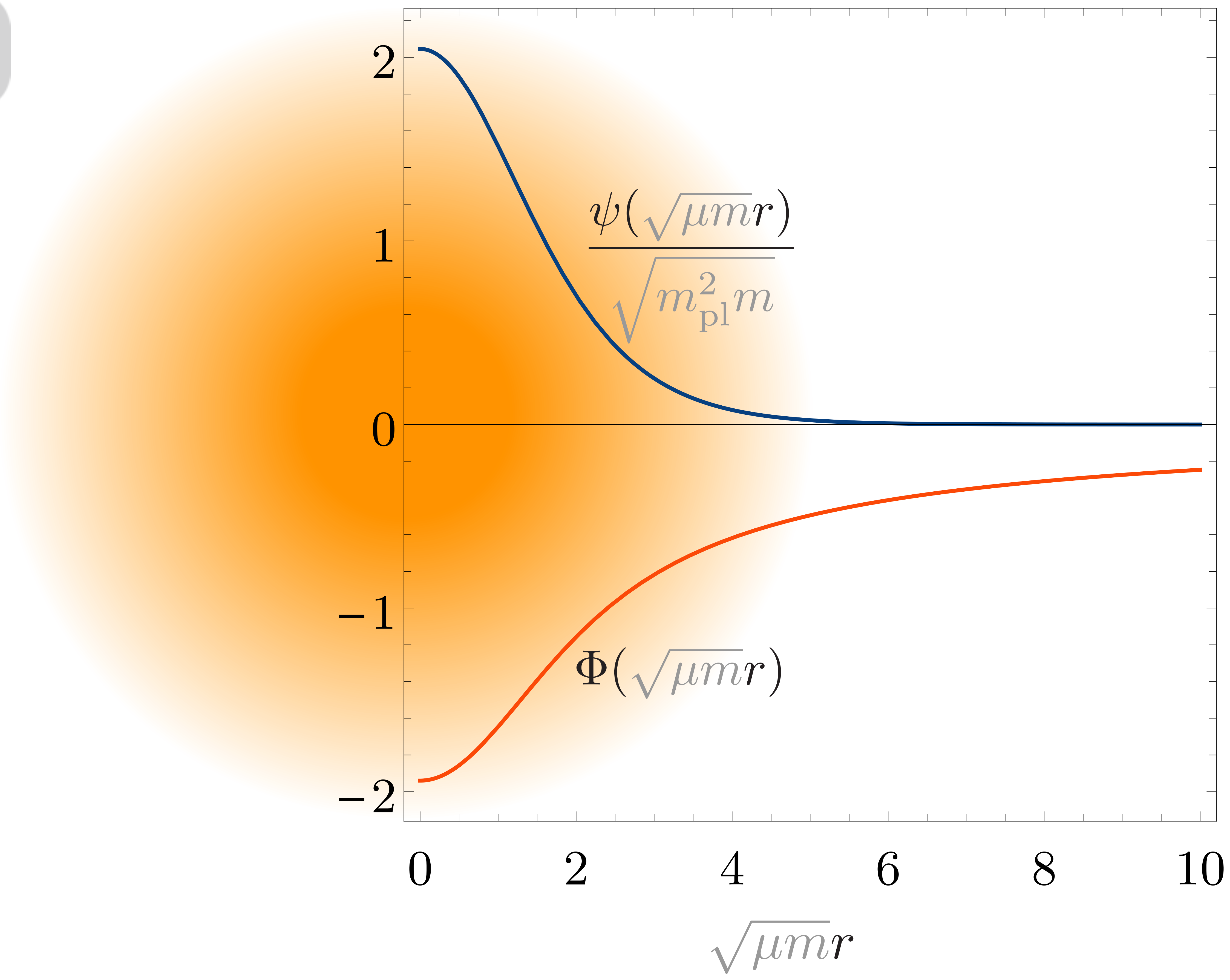
size, particle number, energy and spin

$$L \sim (m/\mu)^{1/2} m^{-1} \qquad \gg m^{-1}$$

$$N = \frac{M}{m} \approx 60.7 \frac{m_{\rm pl}^2}{m^2} \left(\frac{\mu}{m}\right)^{1/2} \qquad \gg 1$$

$$E \approx -19.2 \frac{m_{\rm pl}^2}{m} \left(\frac{\mu}{m}\right)^{3/2}, \qquad \gg m$$

$$\boldsymbol{S}^{\rm tot} = ?$$



$\mu/m \ll 1$ non-relativistic

extremely polarized solitons

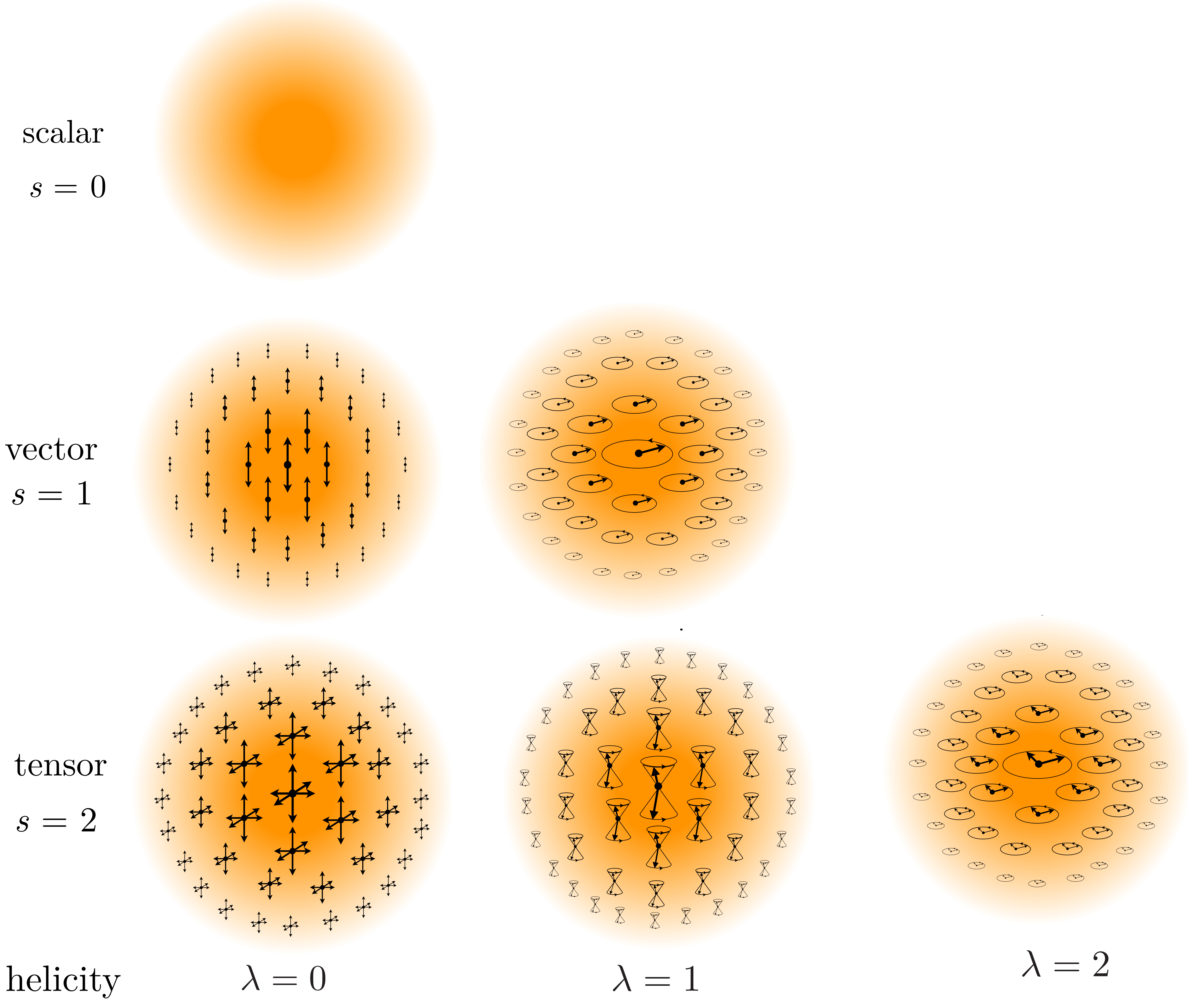
$$\boldsymbol{\Psi}_s^{(\lambda)}(\mathbf{x},t) = \psi(\mathbf{x})e^{i\mu t}\boldsymbol{\epsilon}_{s,\hat{n}}^{(\lambda)} \qquad \lambda \in \{-s, ..., s\}$$

Spin

$$\boldsymbol{S}_{\rm tot} \approx \lambda \times 60.7\frac{m_{\rm pl}^2}{m^2}\left(\frac{\mu}{m}\right)^{1/2}\hat{n}$$

$$= \lambda N \hat{n}$$

$$= \lambda \frac{M}{m}\hat{n}$$



orbital angular momentum is zero

extremally polarized solitons

Spin

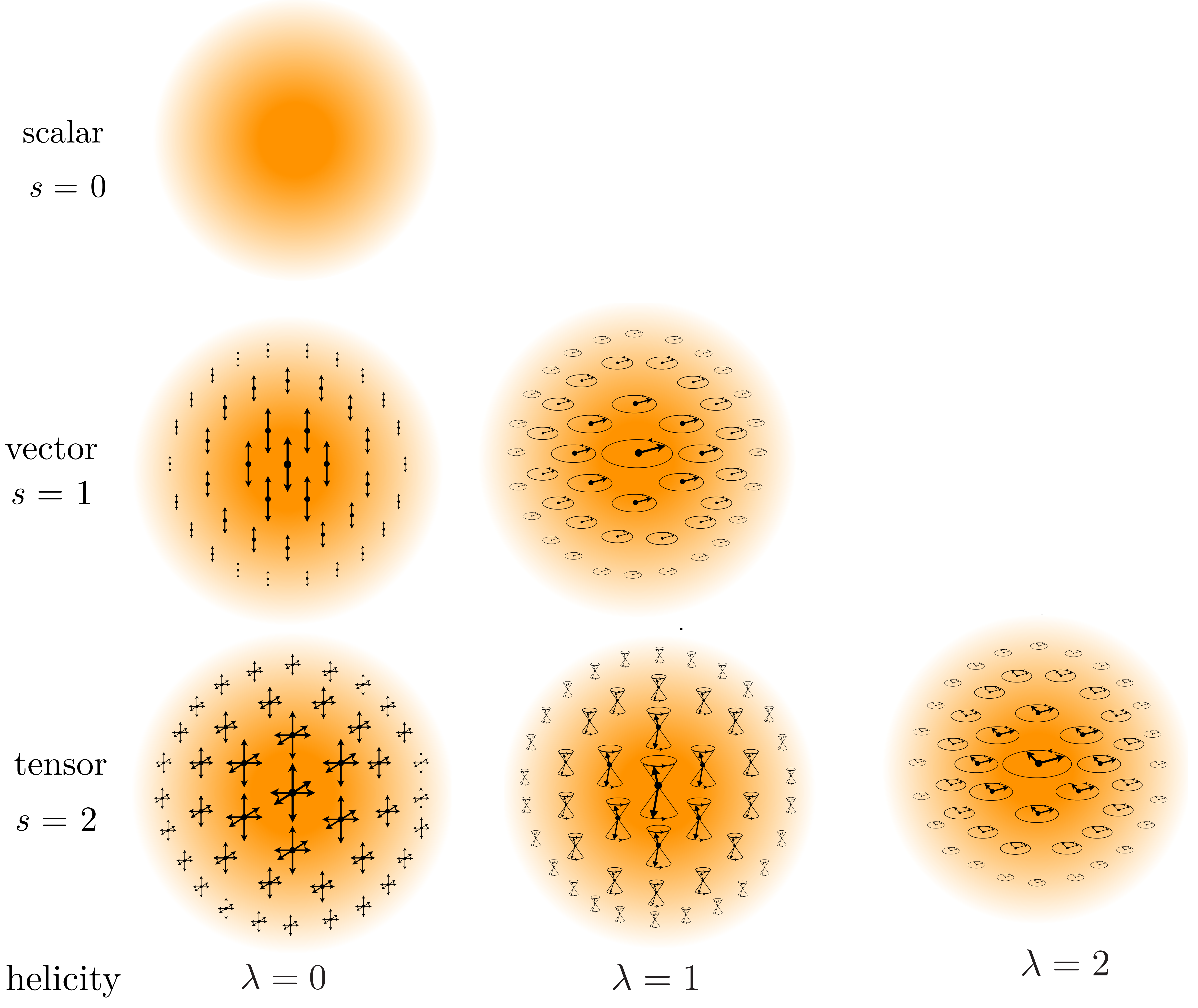
$$\boldsymbol{S}_{\text{tot}} = \lambda \frac{M}{m}\ \hat{n}$$

BHs

$$S_{\text{tot}}/J_{\text{bh}} \ = \ 8\pi\lambda m_{\text{pl}}^2/(aMm)$$

DM halos

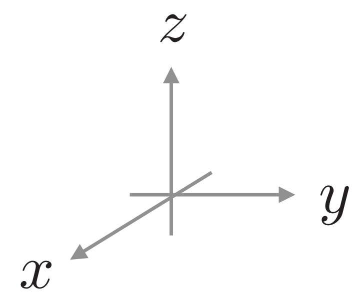
$$\langle\Lambda\rangle_{\text{sol}}/\langle\Lambda\rangle_{\text{DM halo}} \approx 3s > 1$$



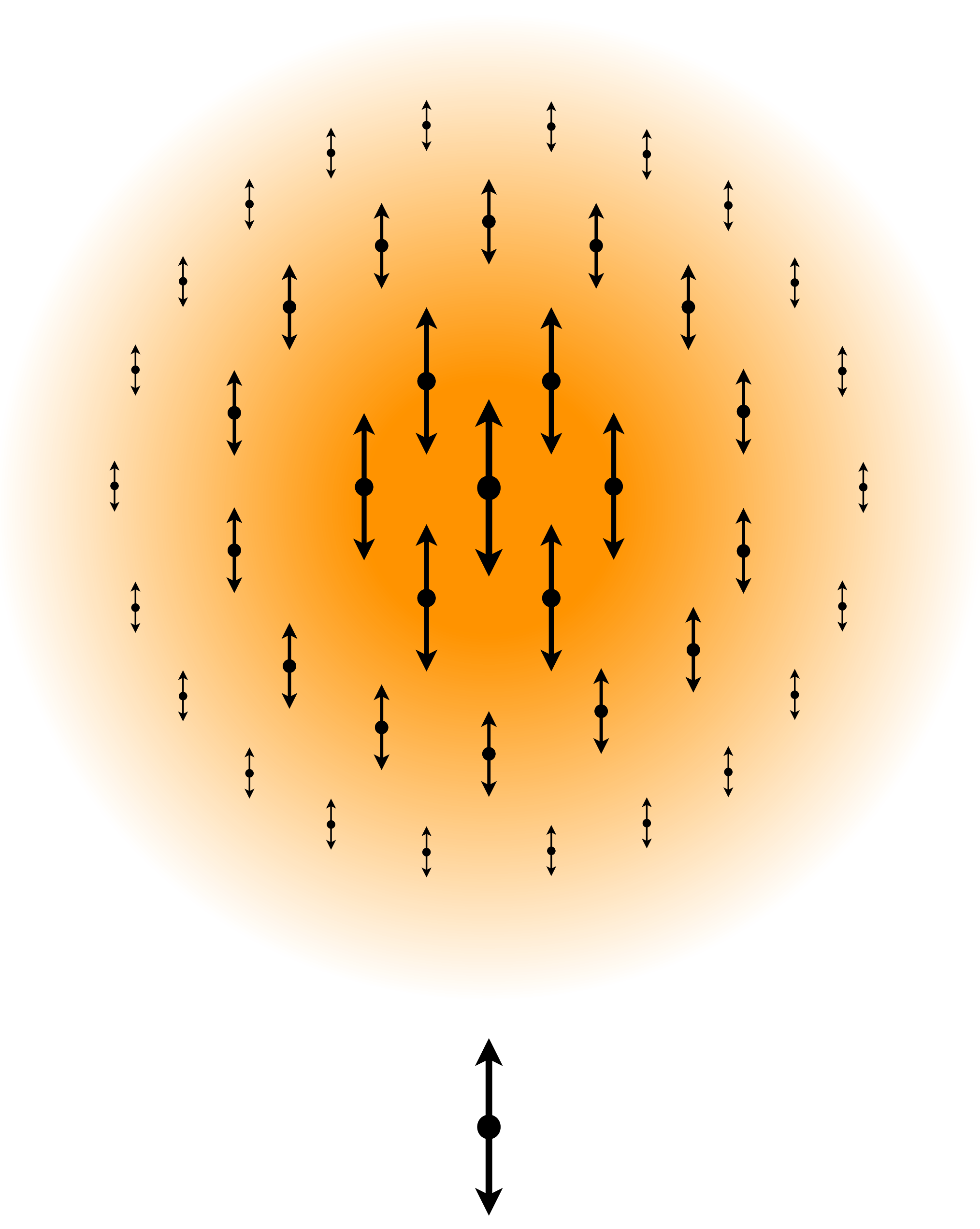
orbital angular momentum is zero

extremal soliton solutions (spin-1)

$$\boldsymbol{S}_{\text{tot}} = \lambda \frac{M}{m}\,\hat{n}$$



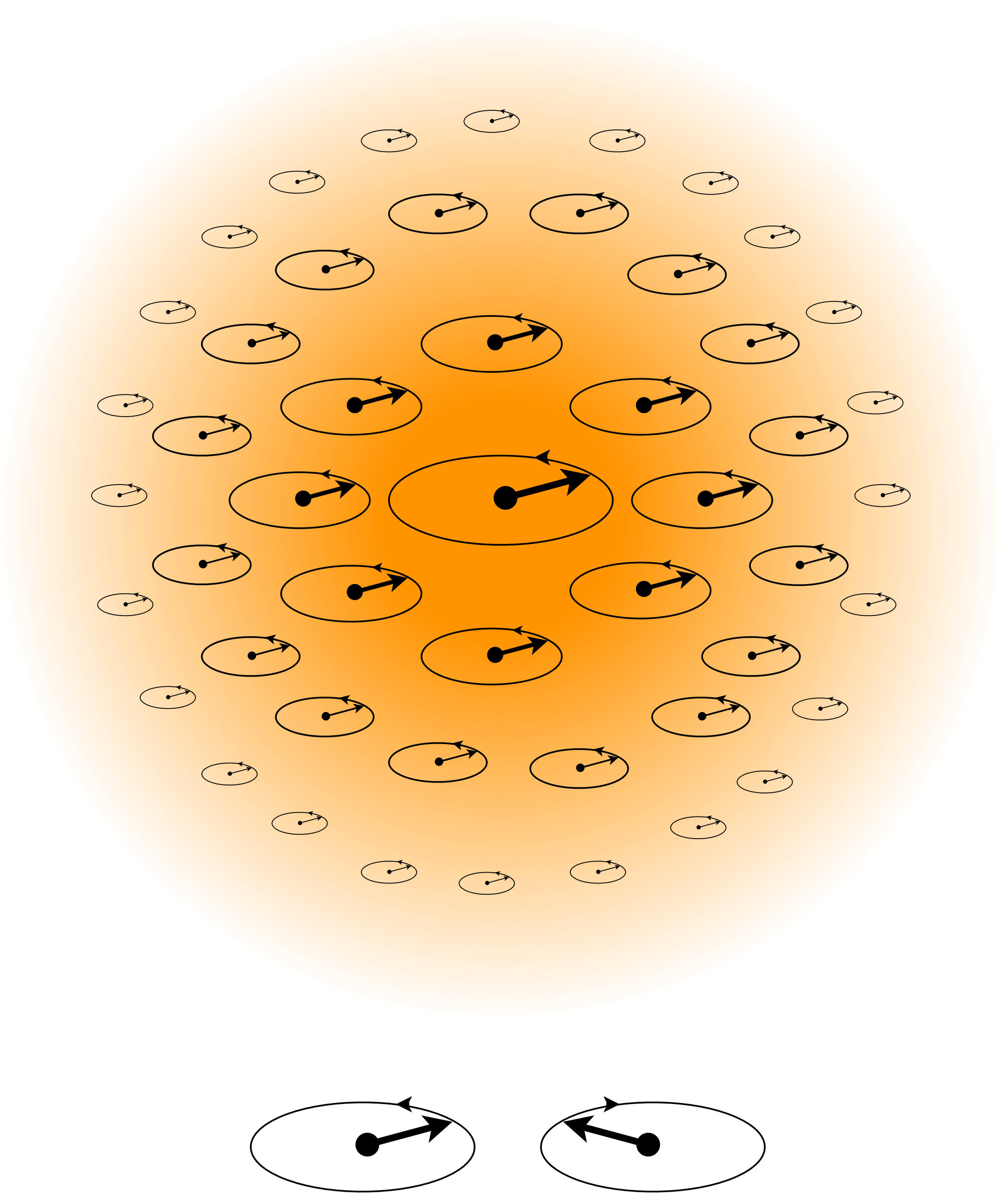
$\boldsymbol{S}_{\text{tot}} = \vec{0}$



$\lambda = 0$

$$\boldsymbol{W}^{(0)}(\mathbf{x},t) = \frac{\sqrt{2}\,\psi(\mathbf{x})}{\sqrt{m}} \begin{pmatrix} 0 \\ 0 \\ \cos\omega t \end{pmatrix}$$

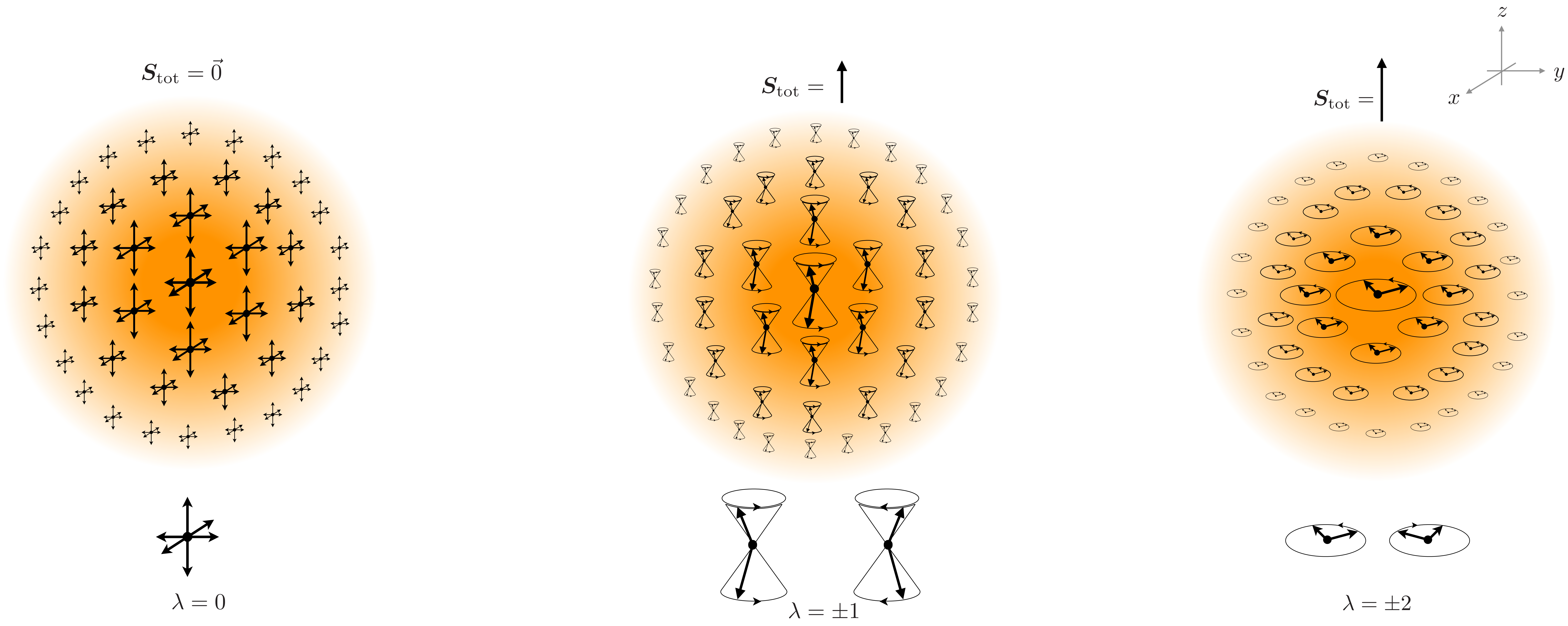
$\boldsymbol{S}_{\text{tot}} = \uparrow$



$\lambda = \pm 1$

$$\boldsymbol{W}^{(\pm 1)}(\mathbf{x},t) = \frac{\psi(\mathbf{x})}{\sqrt{m}} \begin{pmatrix} \cos\omega t \\ \pm\sin\omega t \\ 0 \end{pmatrix}$$

extremally polarized soliton solutions (spin-2)

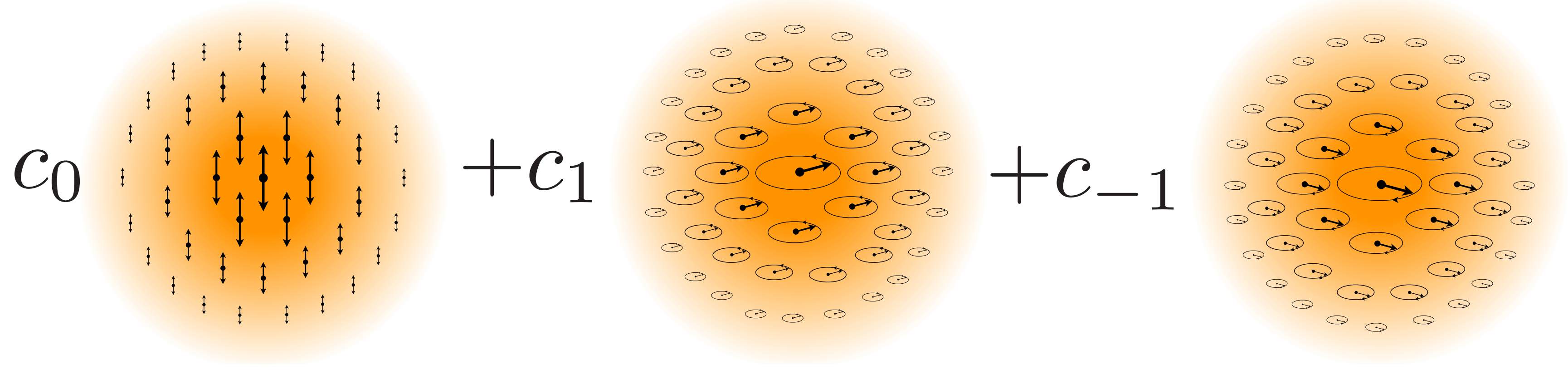


$$\mathbf{H}^{(0)}(\mathbf{x},t) = \frac{\psi(\mathbf{x})}{\sqrt{3m}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cos \omega t$$

$$\mathbf{H}^{(\pm 1)}(\mathbf{x},t) = \frac{\psi(\mathbf{x})}{\sqrt{2m}} \begin{pmatrix} 0 & 0 & \cos \omega t \\ 0 & 0 & \pm \sin \omega t \\ \cos \omega t & \pm \sin \omega t & 0 \end{pmatrix}$$

$$\mathbf{H}^{(\pm 2)}(\mathbf{x},t) = \frac{\psi(\mathbf{x})}{\sqrt{2m}} \begin{pmatrix} \cos \omega t & \pm \sin \omega t & 0 \\ \pm \sin \omega t & -\cos \omega t & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

fractionally polarized solitons



$$\mathbf{\Psi}(\mathbf{x},t) = \psi(\mathbf{x}) \sum_{\lambda} c_\lambda e^{i(\mu t - \varphi_\lambda)} \boldsymbol{\epsilon}^{(\lambda)}_{s,\hat{n}}$$

$$\sum_{\lambda} c_\lambda^2 = 1$$

constructed from a superposition of extremal solitons

same energy and particle number

$$S_k^{\rm tot} = s \int d^3x |\psi(\mathbf{x})|^2 \sum_{\lambda\lambda'} \Re \left[i c_\lambda c_{\lambda'} e^{i(\varphi_\lambda - \varphi_{\lambda'})} \epsilon_{ijk} [\boldsymbol{\epsilon}^{(\lambda')}_{s,\hat{n}} \boldsymbol{\epsilon}^{(\lambda)\dagger}_{s,\hat{n}}]_{ij} \right]$$

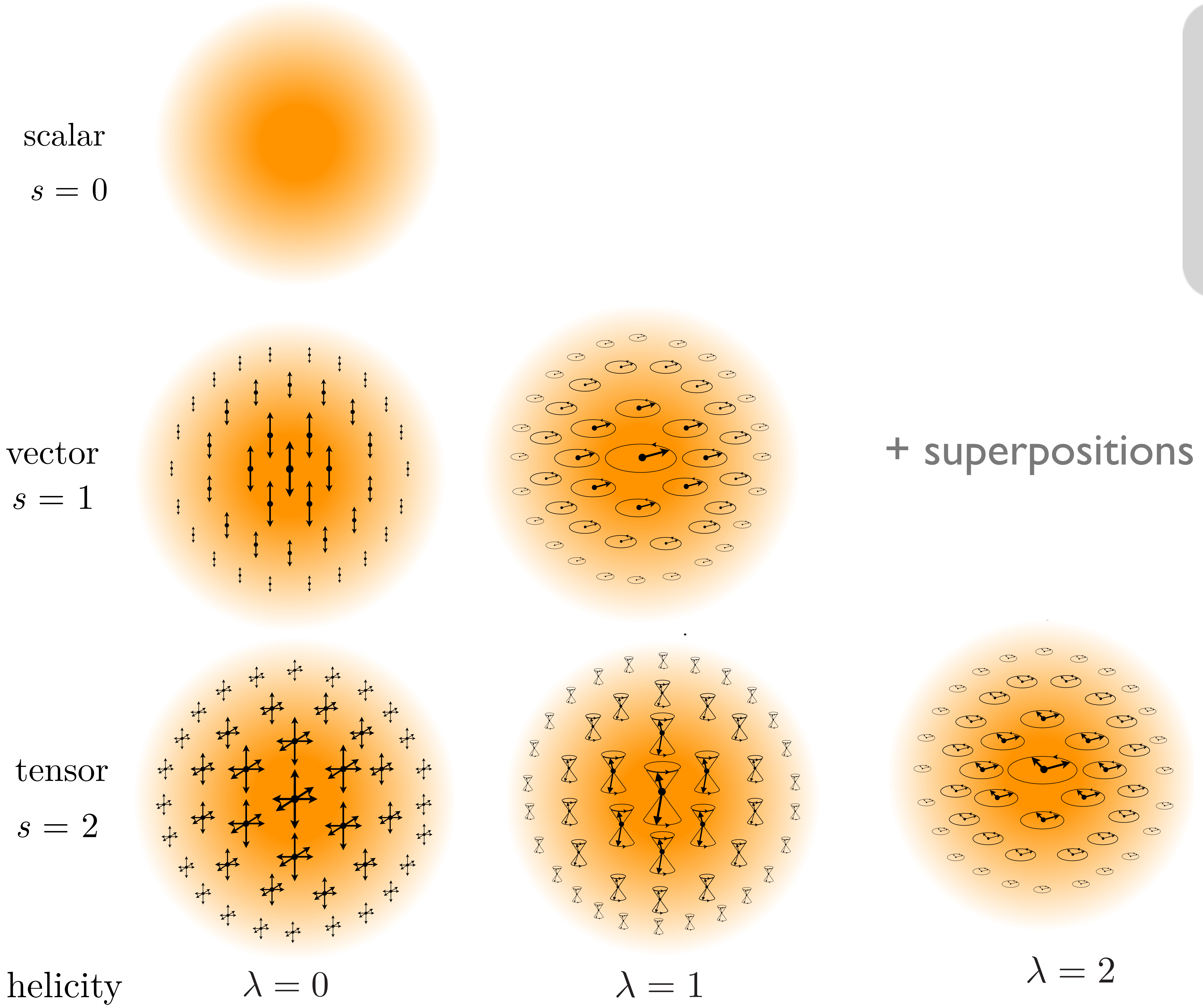
total spin

implications

distinguishability?

same density profile and energy,

and hence gravitational properties (in NR limit)

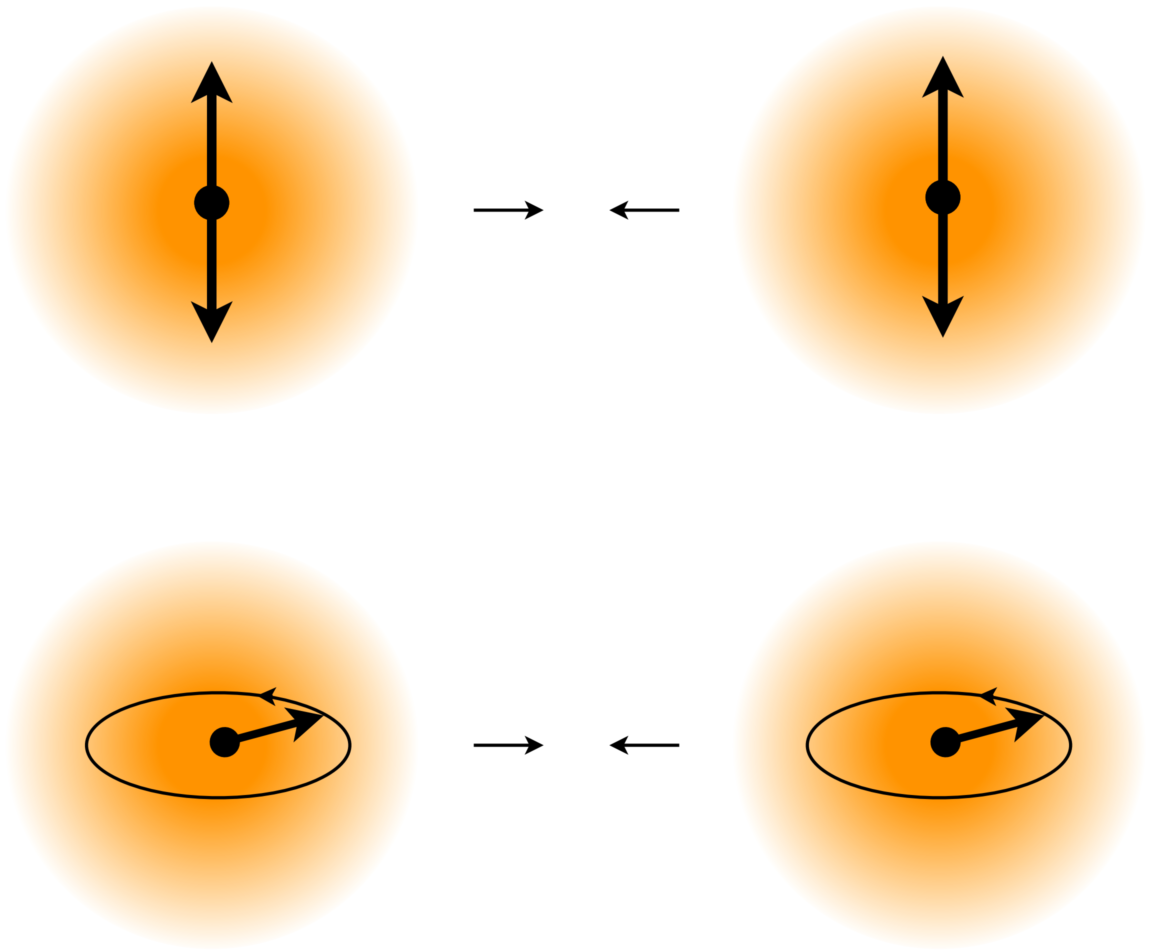


+ superpositions (at fixed particle number)

+ superpositions (at fixed particle number)

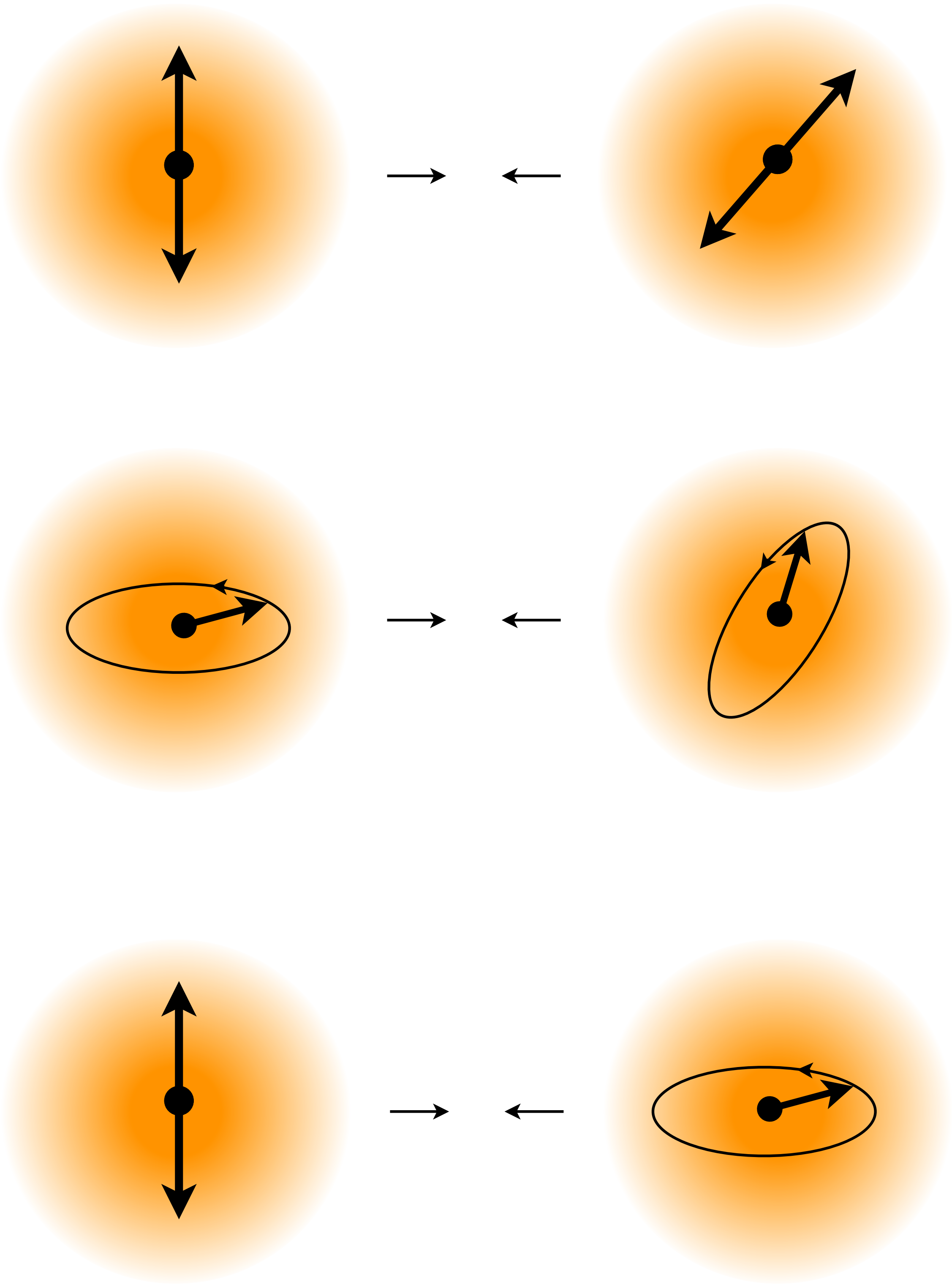
distinguishability?

distinguishable via collisions!



$\boldsymbol{S}_{\mathrm{tot}} \neq 0$

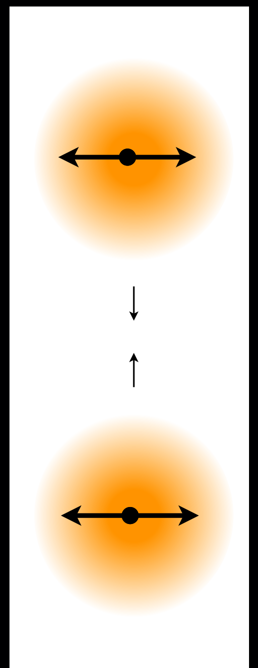
can be replicated by single scalar field

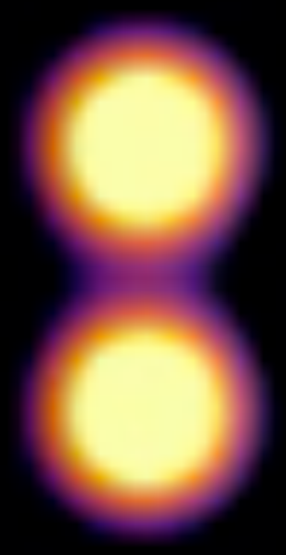


cannot be replicated by single scalar field

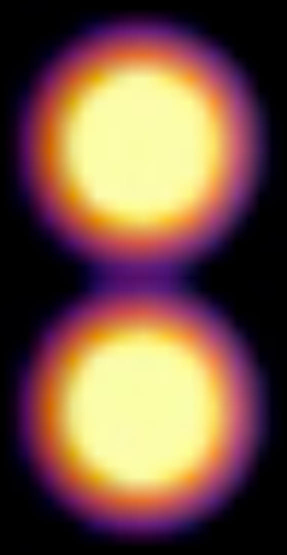
distinguishability via collisions

WORK IN PROGRESS

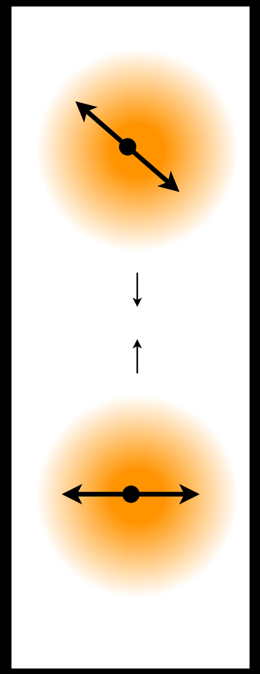




"scalar"



vector



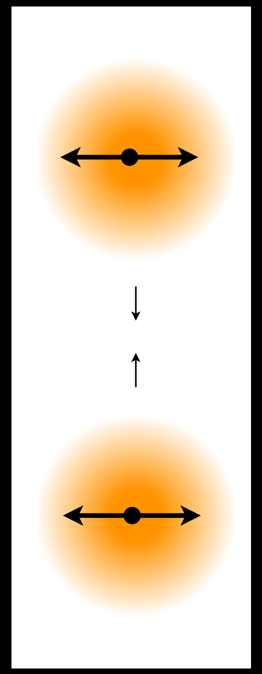
Rohith Karur

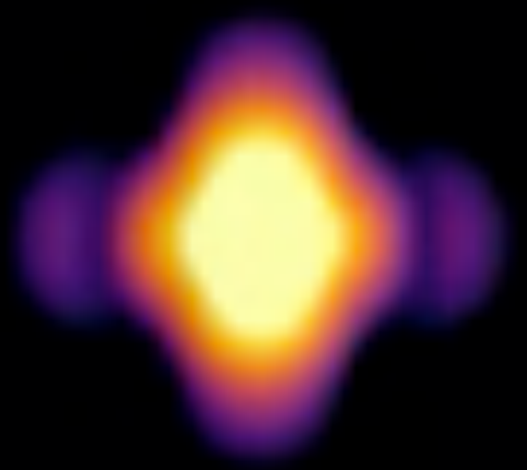
distinguishability via collisions

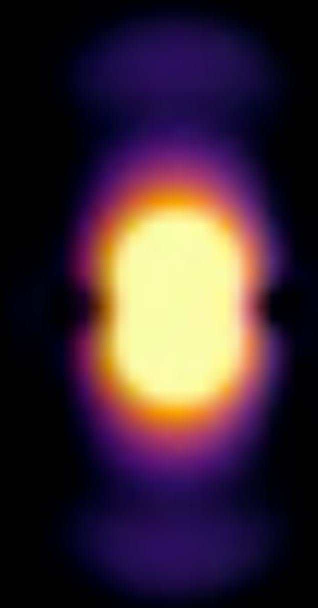
WORK IN PROGRESS

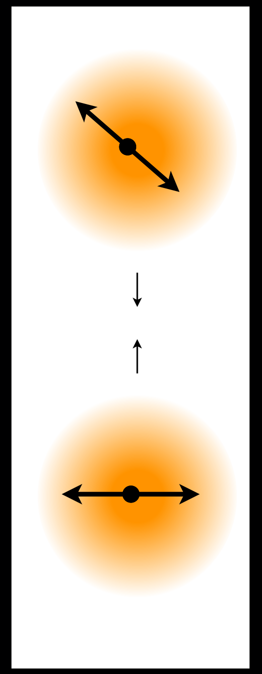
"scalar"

vector







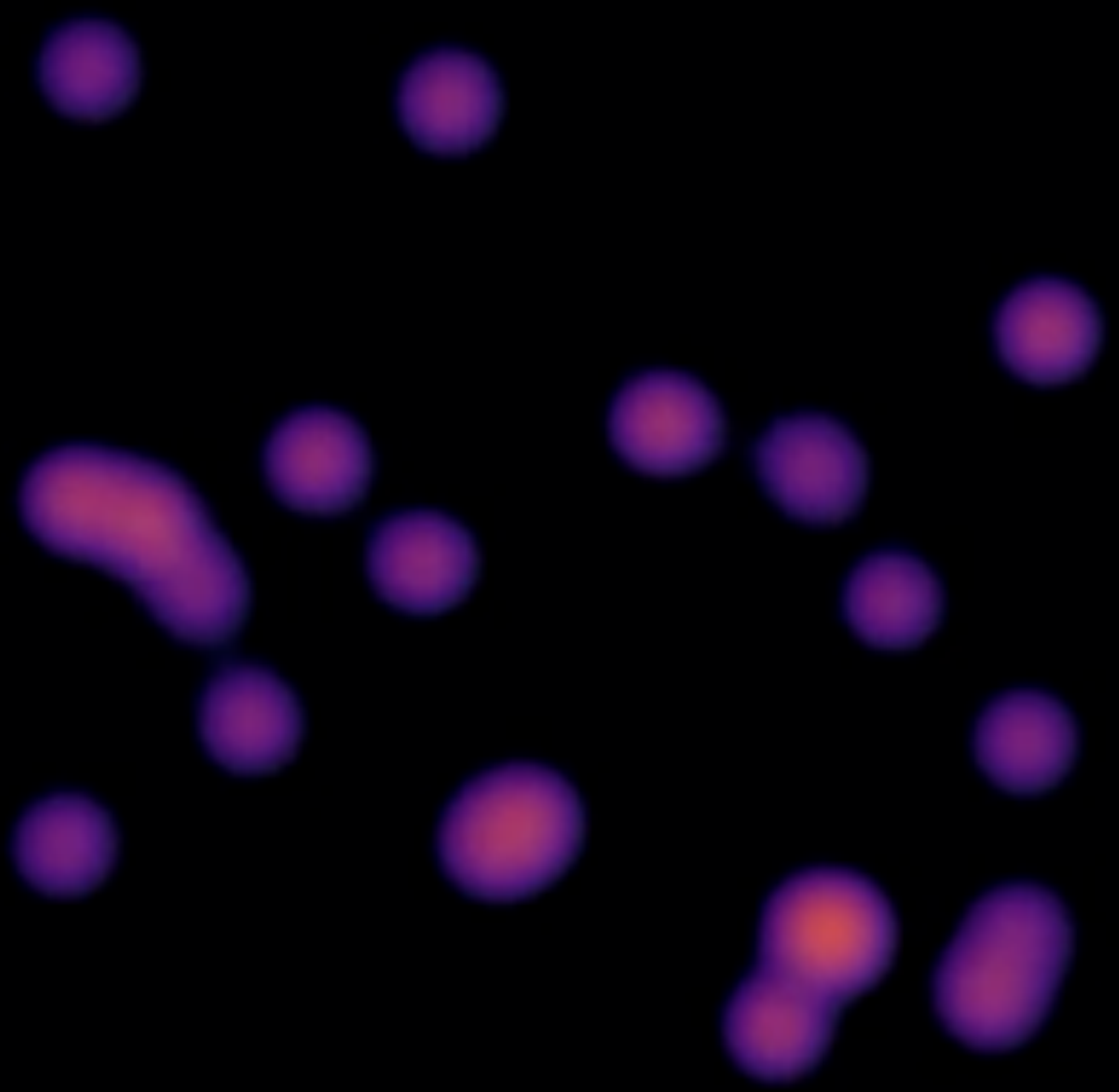


Rohith Karur

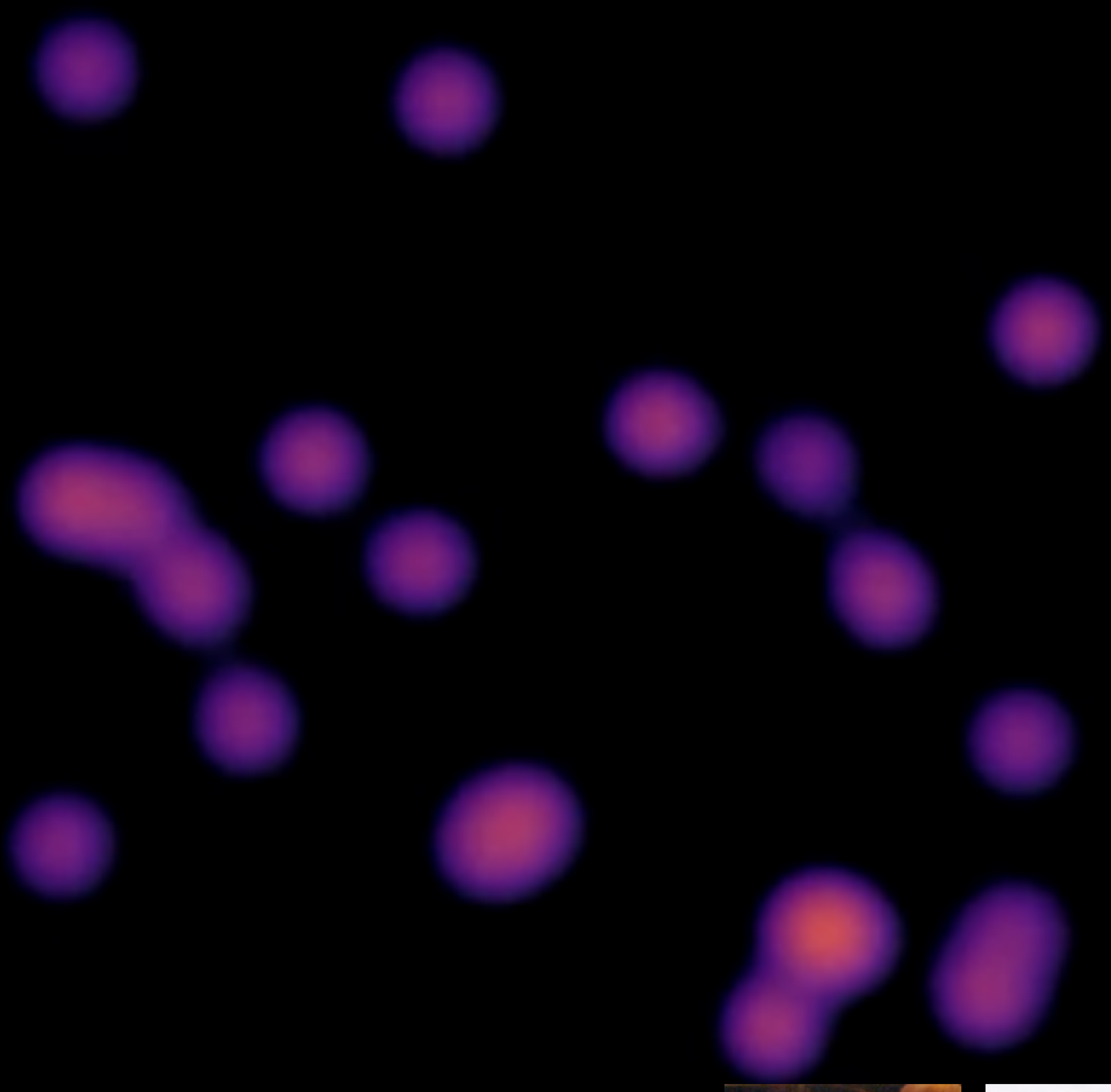
more interference in scalars compared to vector case

WORK IN PROGRESS

scalar

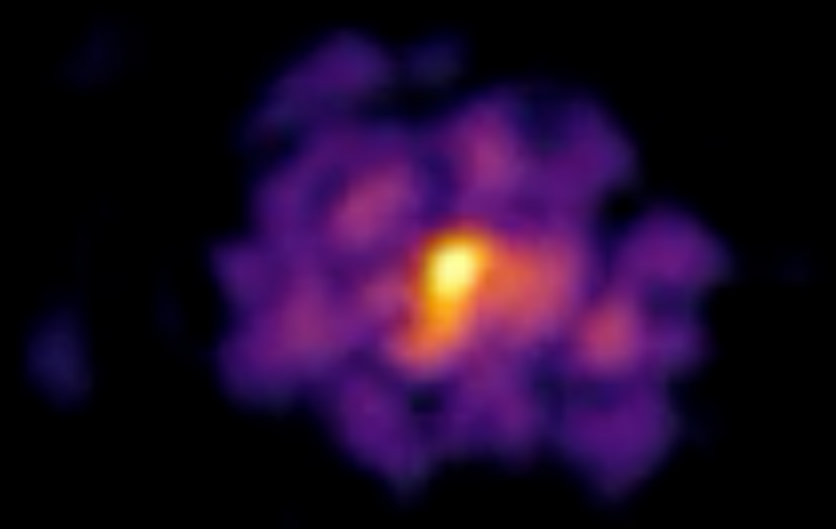


vector



more interference in scalars compared to vector case

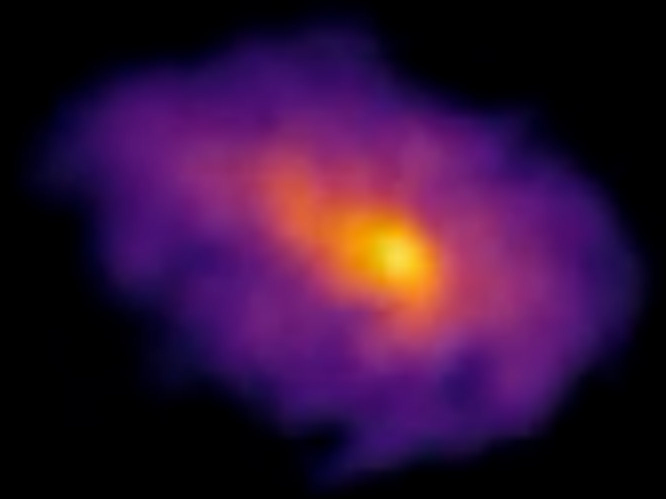
scalar



vector

less interference

central soliton spin: $\boldsymbol{S}^{\rm tot} \sim (0.1 - 0.5) \times N\hbar$

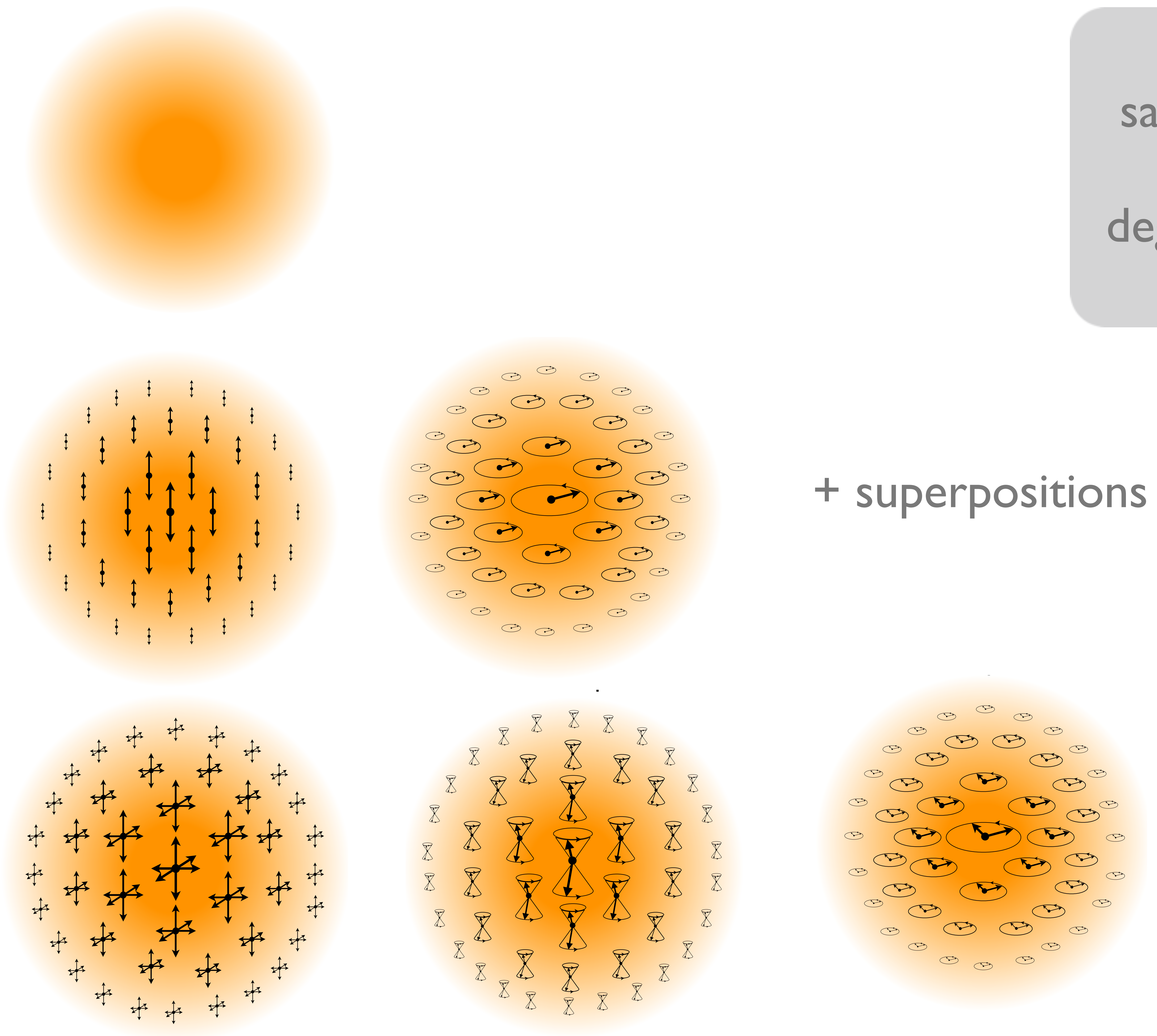


distinguishability?

scalar
$s = 0$

vector
$s = 1$

tensor
$s = 2$



helicity $\lambda = 0$ $\lambda = 1$ $\lambda = 2$

same density profile and energy

degeneracy lifted by self-interaction or relativistic corrections

+ superpositions (at fixed particle number)

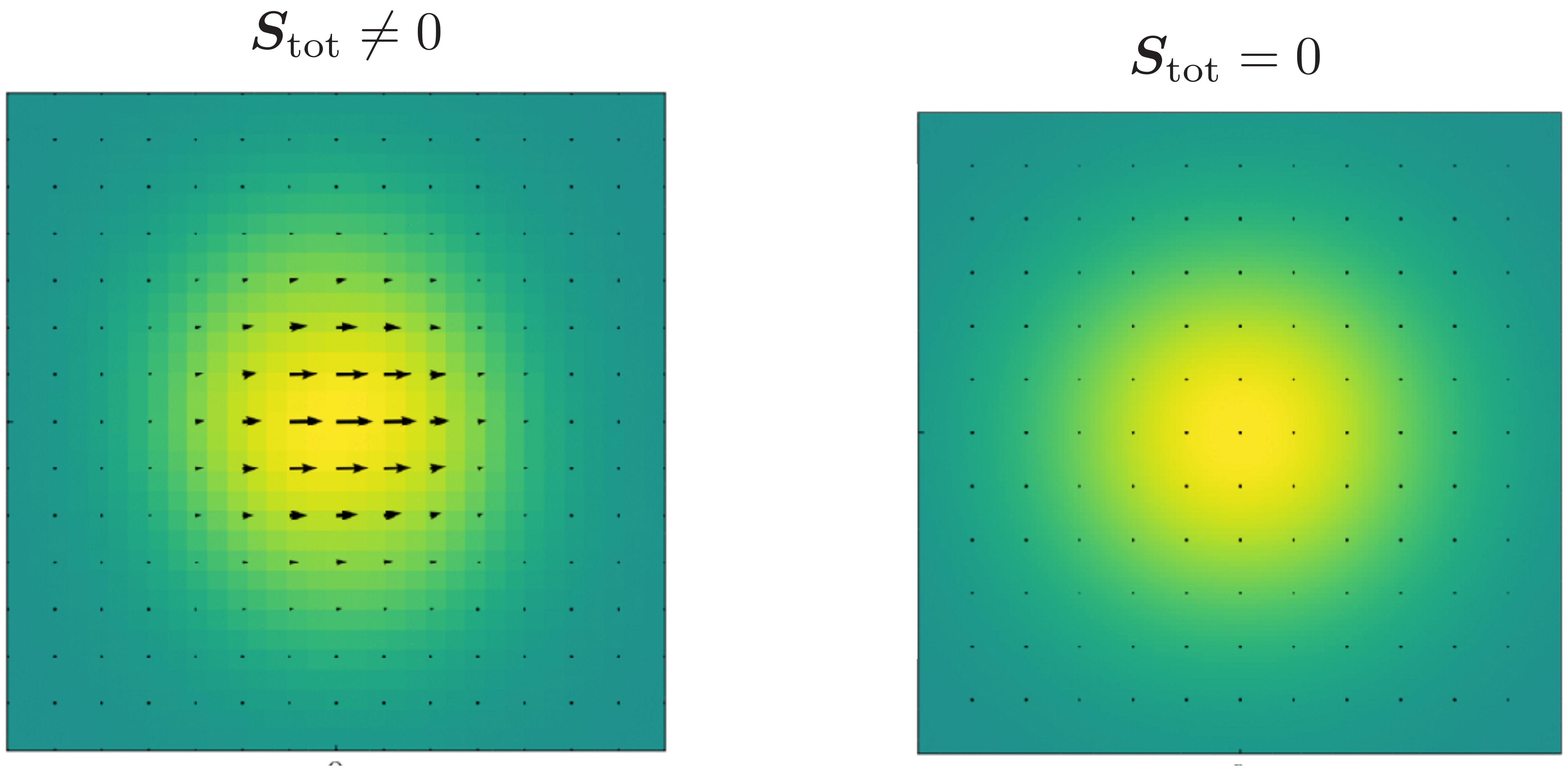
+ superpositions (at fixed particle number)

WORK IN PROGRESS

Vector oscillons

with Zhang and Jain

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}W_\mu W^\mu + \frac{\lambda}{4}(W_\mu W^\mu)^2 - \frac{h}{6}(W_\mu W^\mu)^3$$



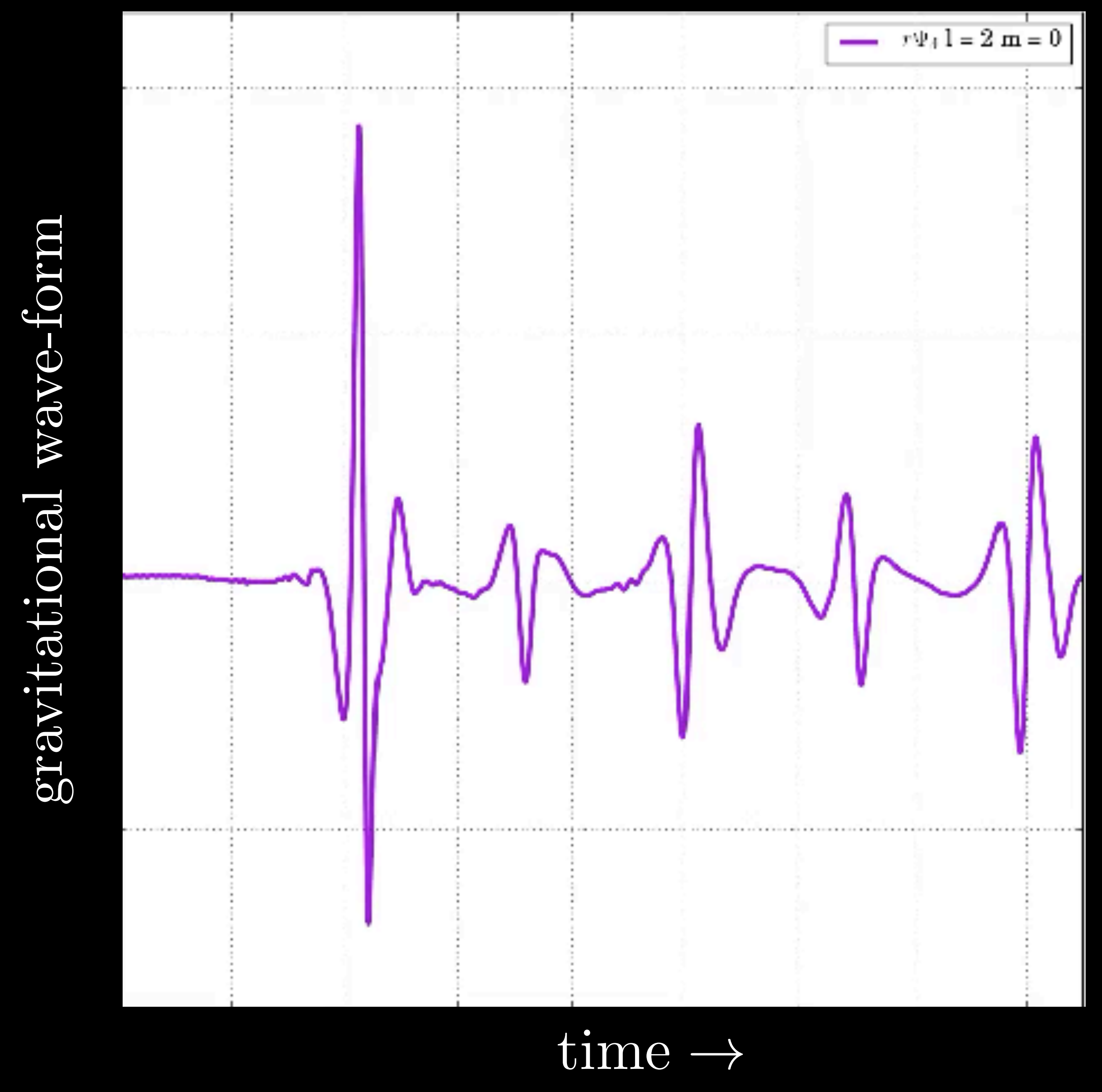
self-interaction supported

NOT degenerate in energy

spin-spin interaction matters!

gravitational waves

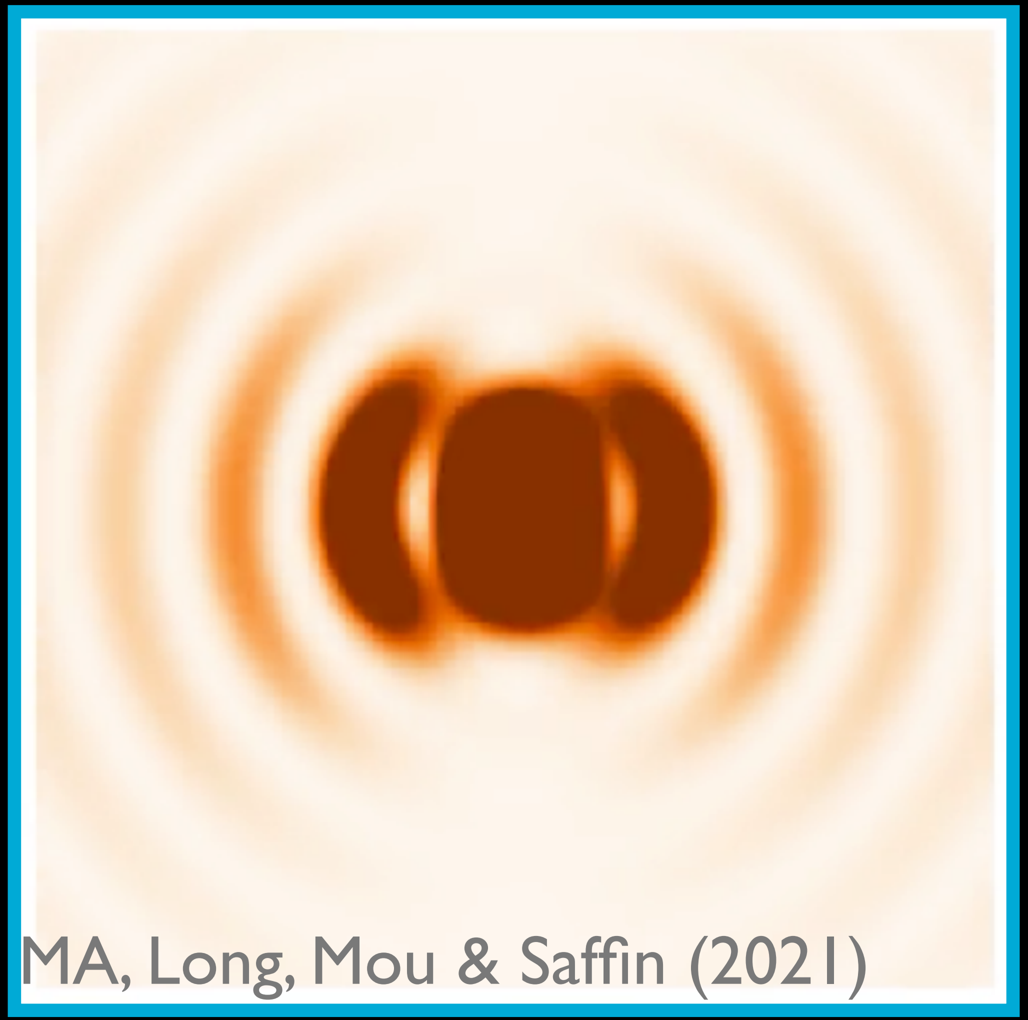
different for higher spin solitons?



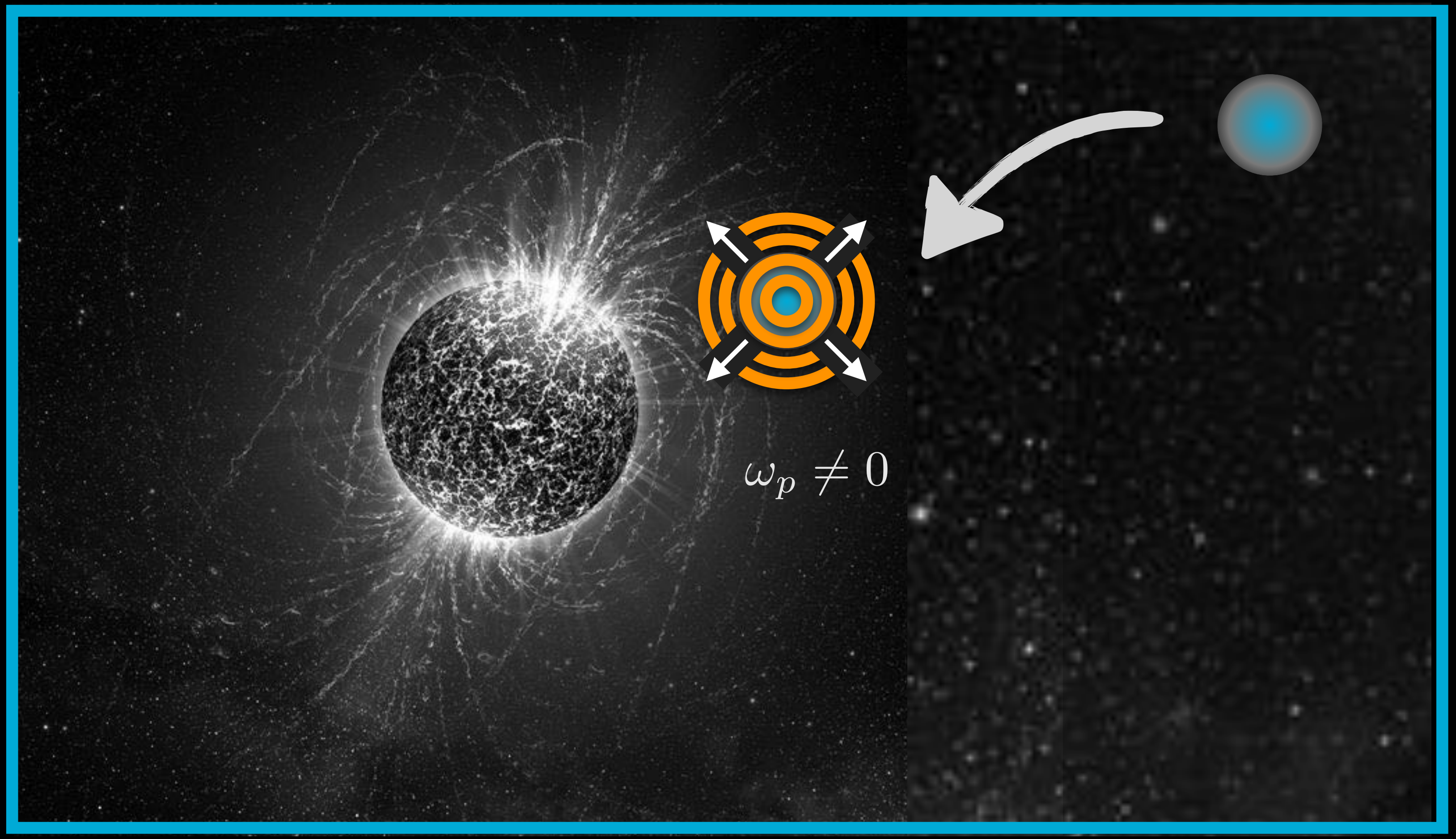
above for compact scalar solitons using full numerical GR

Helfer, Lim, Garcia, MA (2016)

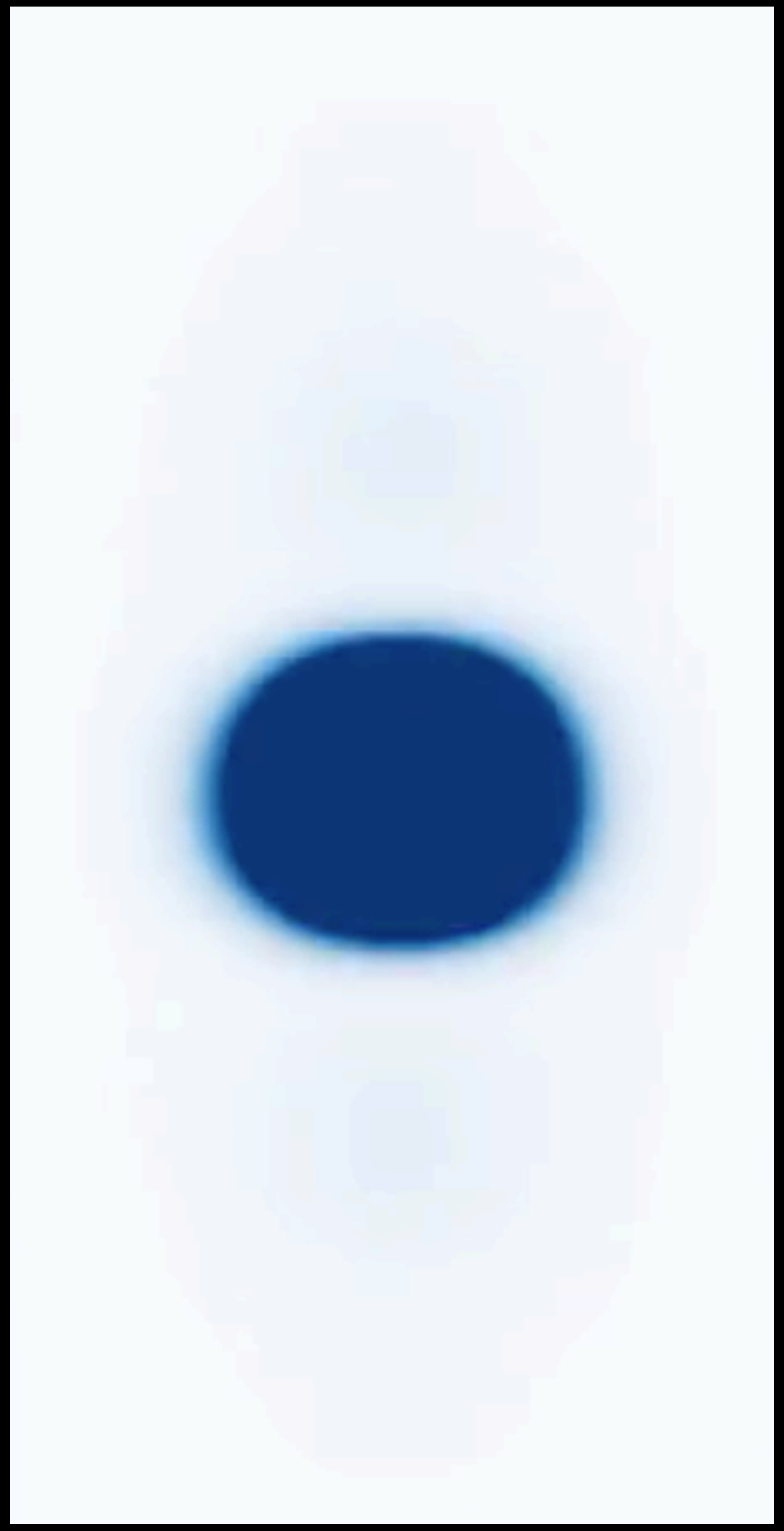
electromagnetic coupling and radiation (axion + photons)



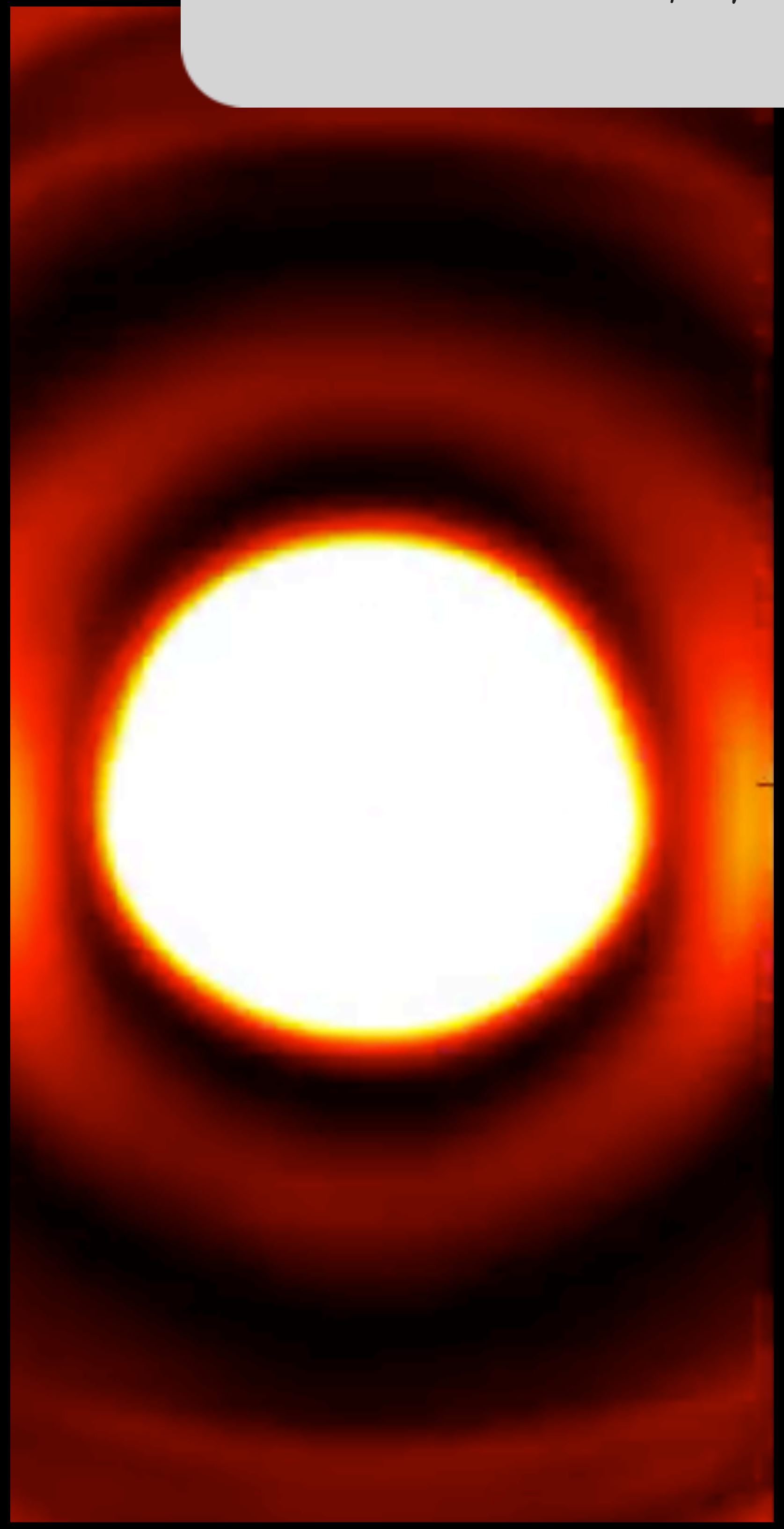
MA, Long, Mou & Saffin (2021)



see for example: Buckley, Dev, Ferrer, Huang (2021)



MA & Mou (2020)



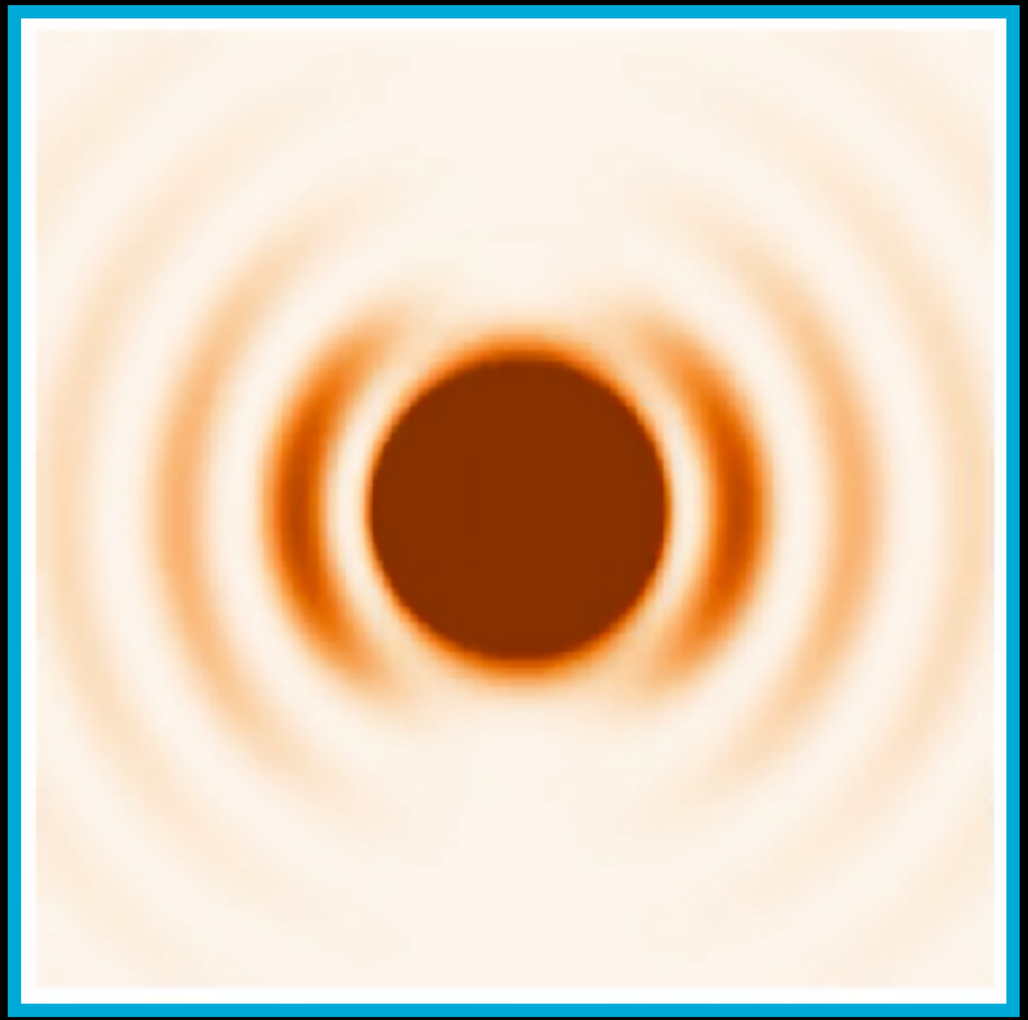
$$\mathcal{L}_{int} \sim g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

spin-s + photons: spin of soliton & polarization of photons

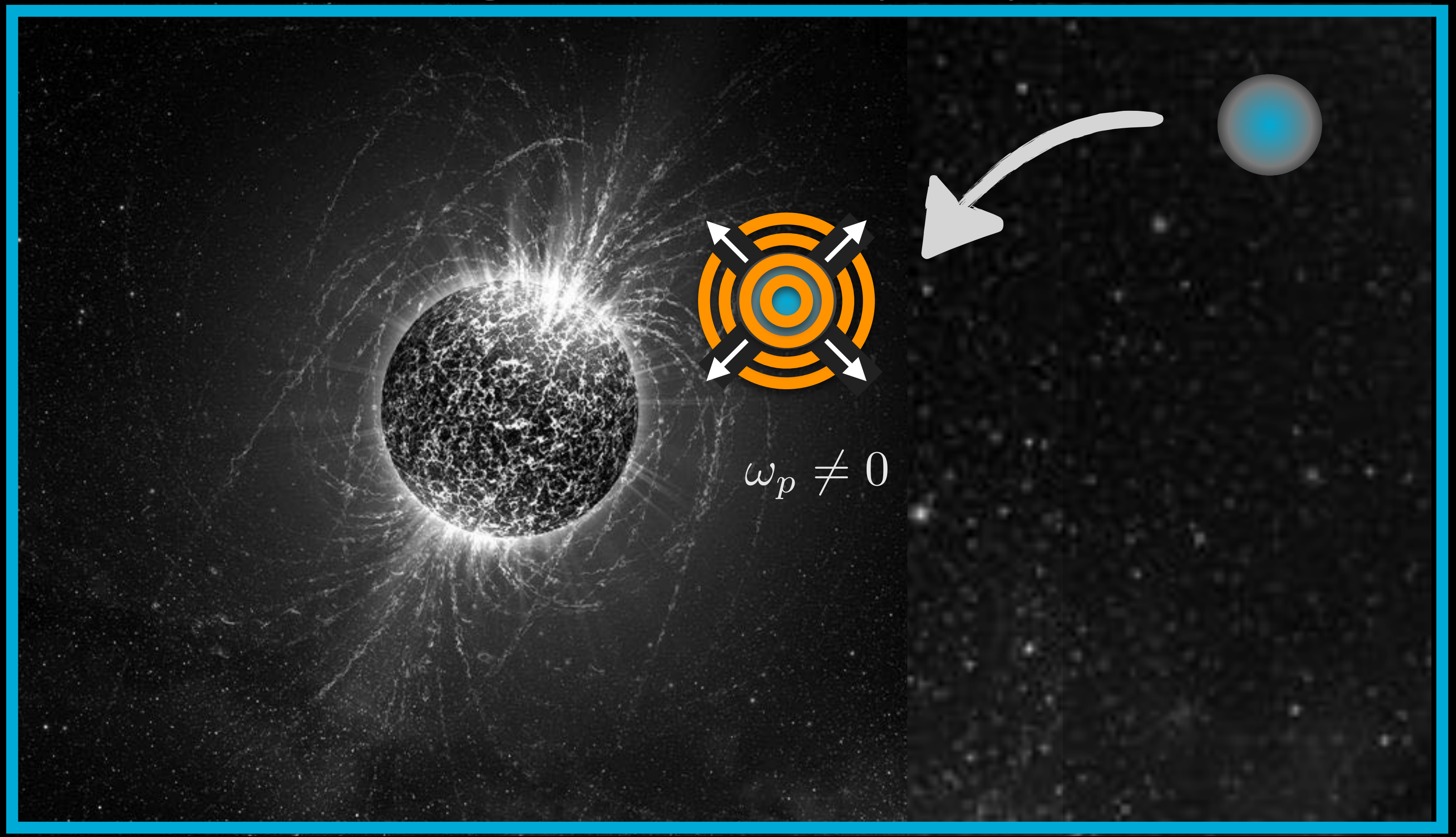
$$\mathcal{L}_{int} \sim \begin{cases} g^2_{W\gamma} W_\mu W^\mu \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-1 \\ g^2_{H\gamma}(H_{\mu\nu}H^{\mu\nu} - H^2) \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-2 \end{cases}$$

$$\sim g^2_{\mathcal{F}\gamma} \, \mathrm{Tr}[\boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{F}}] \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} \qquad \text{NR limit}$$

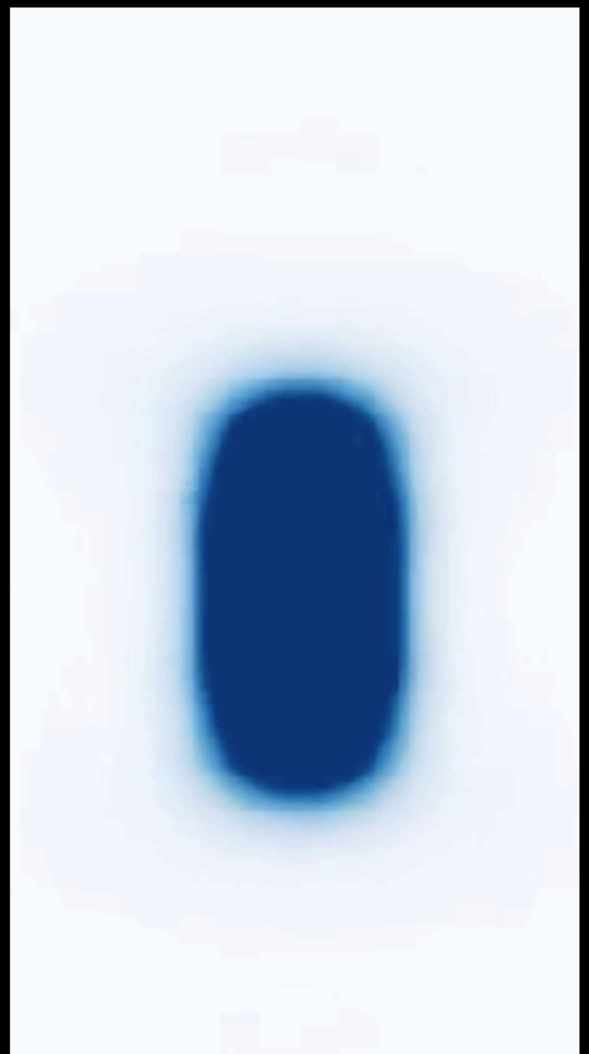
$F_{\mu\nu}G^{\mu\nu}$



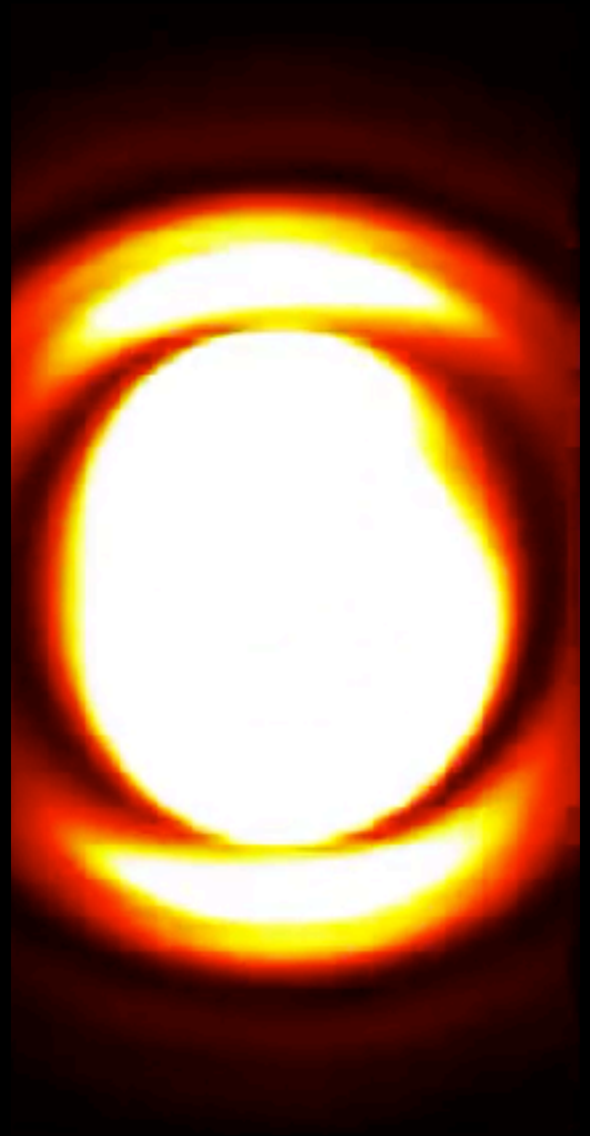
MA, Long, Mou & Saffin (2021)



see for example: Buckley, Dev, Ferrer, Huang (2021)

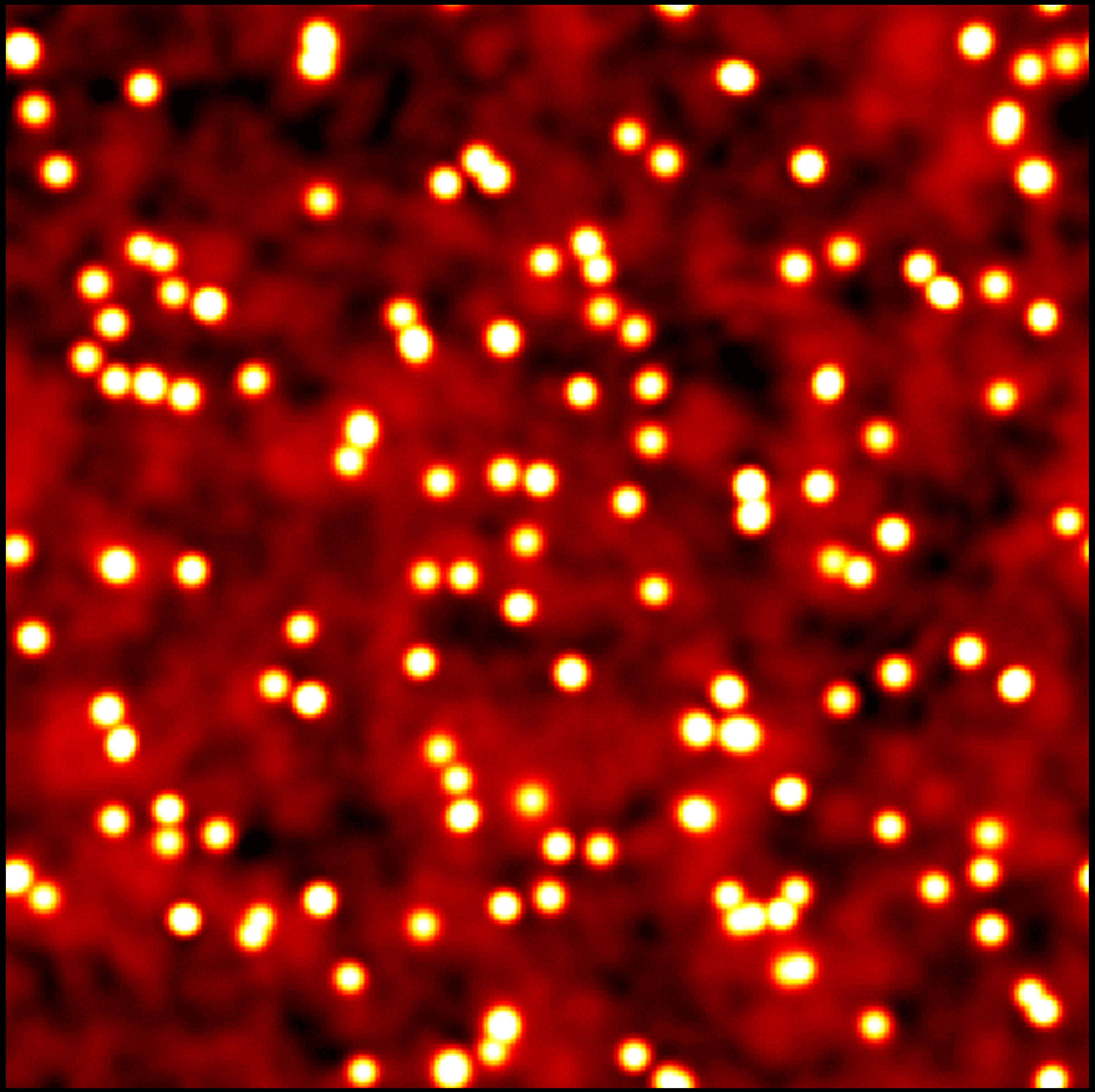


MA & Mou (2020)



formation mechanisms (vector case *in progress*)

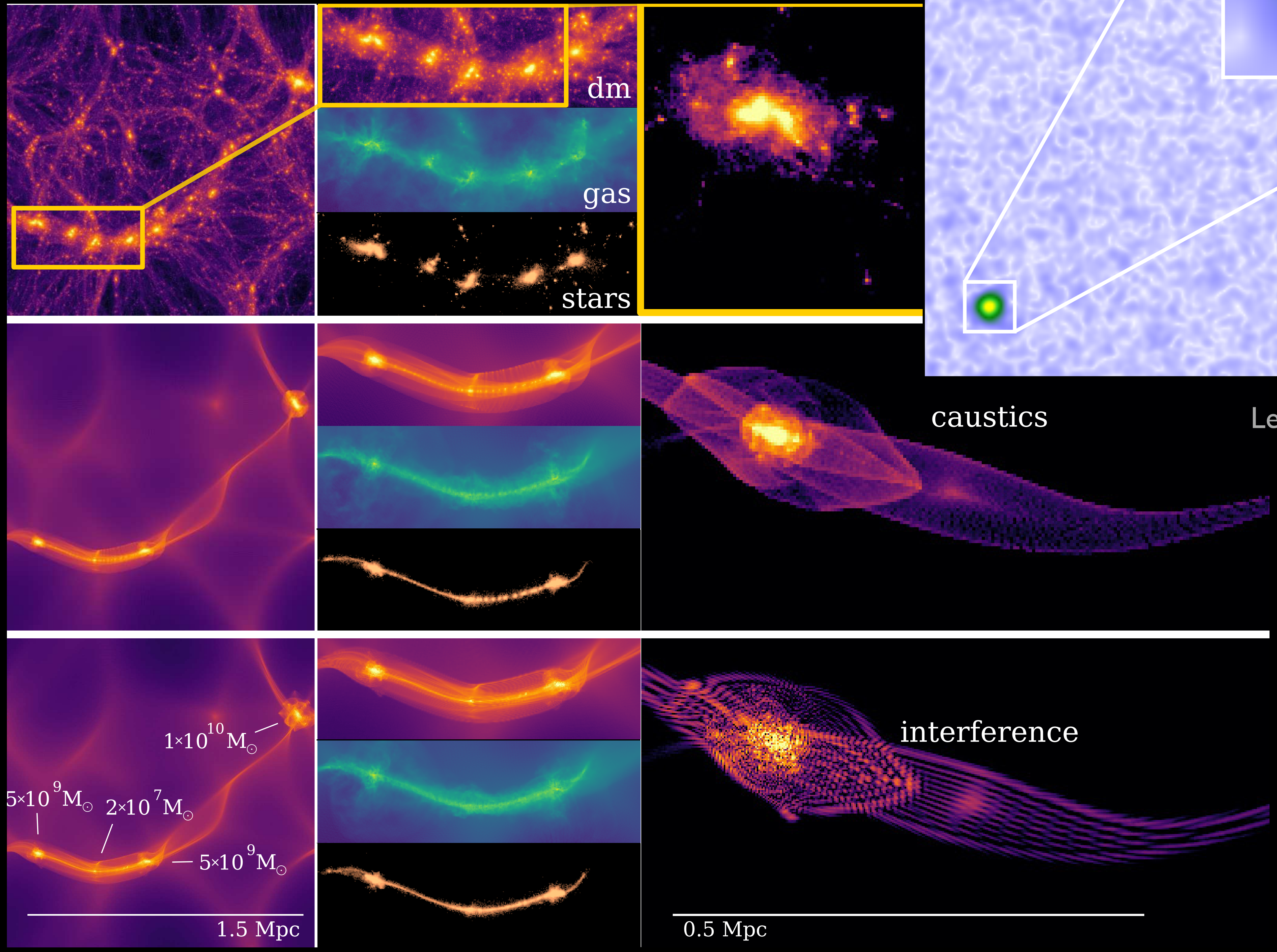
self-interaction driven

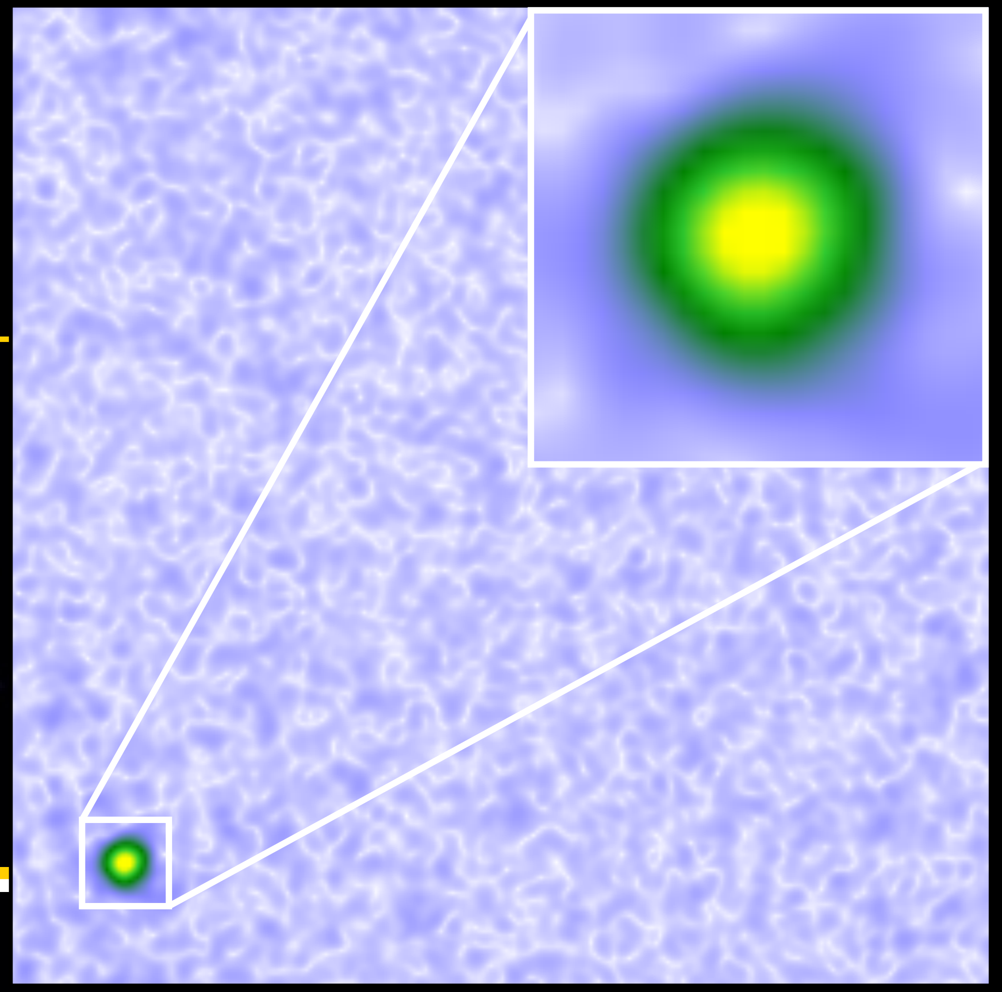


MA & Mocz (2019)

gravitational formation

Mocz et. al (2019)



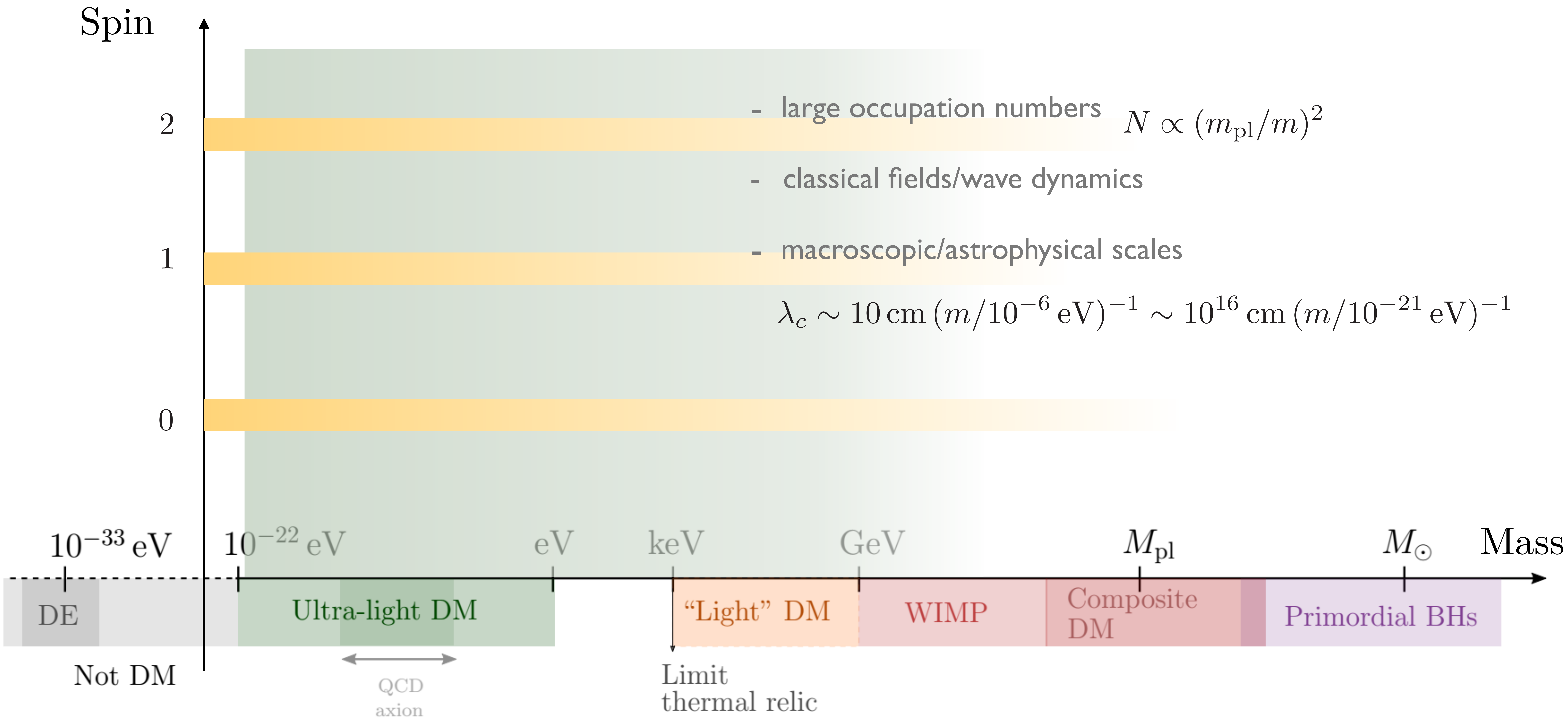


Levkov et. al (2015)

future possibilities …

- formation mechanisms for higher spin dark matter (misalighnment?, gravitational?)
- BH superradiance with higher spin fields
- dynamical friction
- vortices
- pulsar timing measurements
- direct detection
- condensation time scales
- lifetimes of higher spin solitons
- relativistic effects *(for scalar with gravitation, see Salehian, Kaiser, Guth, Namjoo and more recent +Zhang, MA…)*

summary

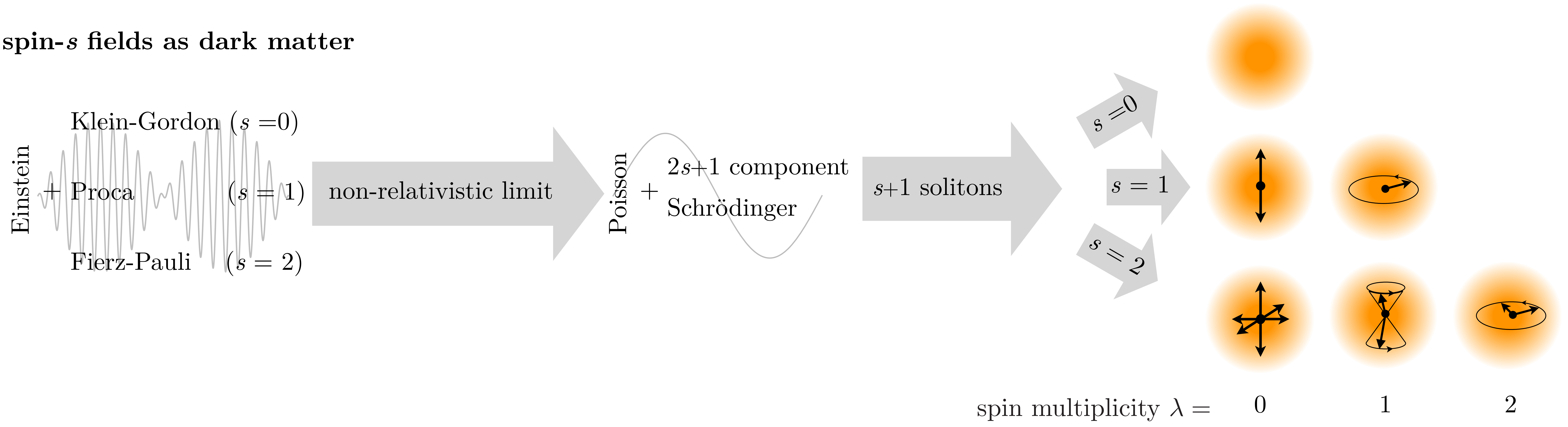


- large occupation numbers $N \propto (m_{\rm pl}/m)^2$
- classical fields/wave dynamics
- macroscopic/astrophysical scales

$$\lambda_c \sim 10\,\mathrm{cm}\,(m/10^{-6}\,\mathrm{eV})^{-1} \sim 10^{16}\,\mathrm{cm}\,(m/10^{-21}\,\mathrm{eV})^{-1}$$

summary

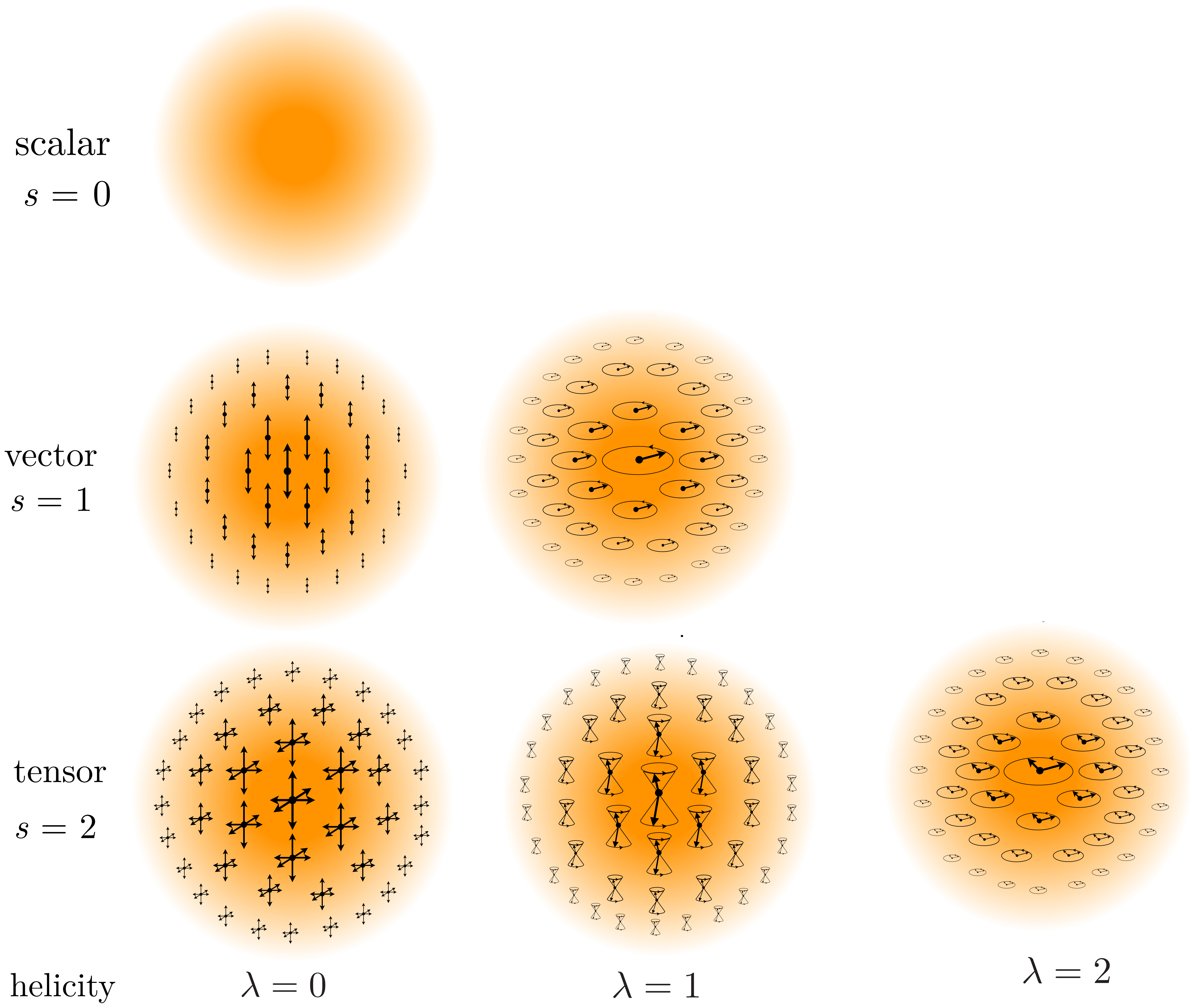
spin-s fields as dark matter



scale separation
- phenomenology/numerical simulations

macroscopic spin $\boldsymbol{S}_{\text{tot}}/\hbar = \lambda N \hat{z}$
N = # of particles in soliton

summary



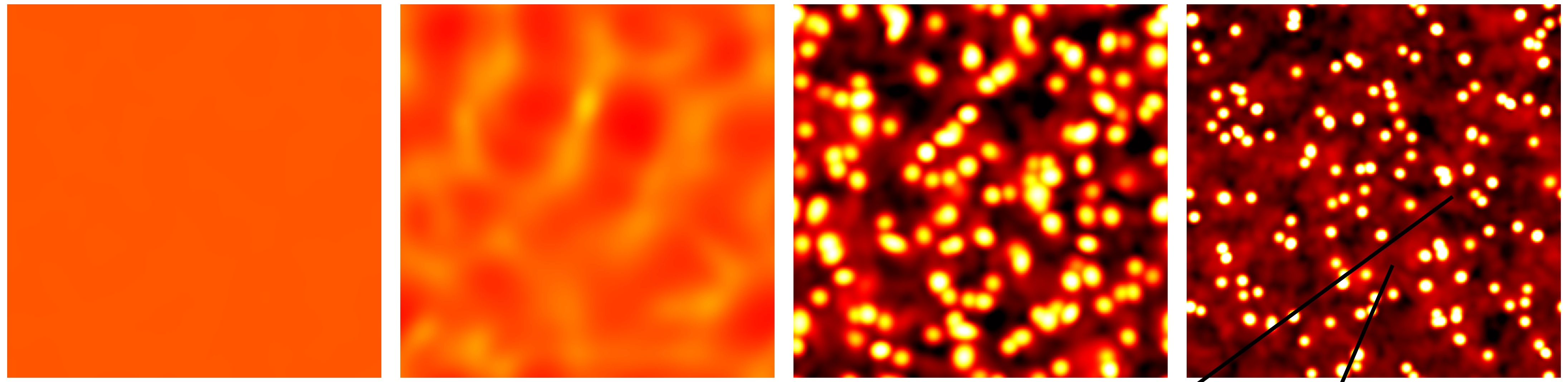
macroscopic spin $\quad \boldsymbol{S}_{\text{tot}}/\hbar = \lambda N \hat{z}$

$N =$ # of particles in soliton

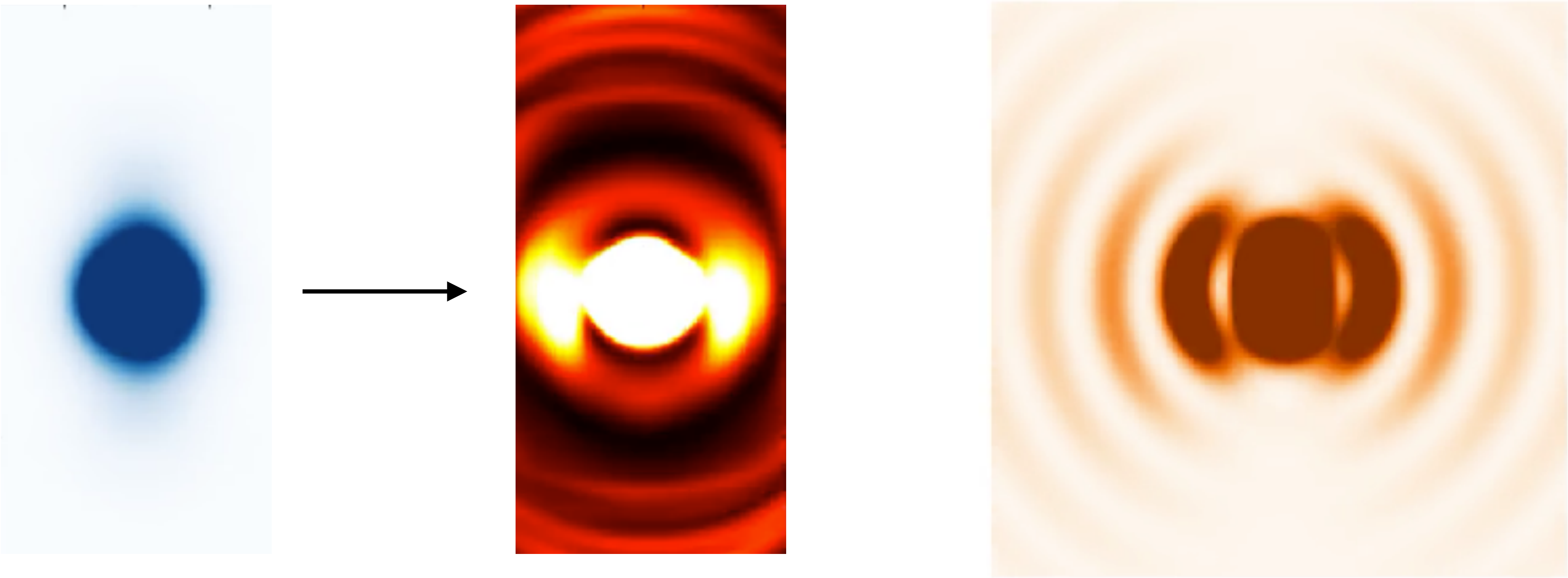
- all lowest energy for fixed N
- bases for partially-polarized solitons

distinguishable via collisions, non-gravitational couplings, g-waves etc.

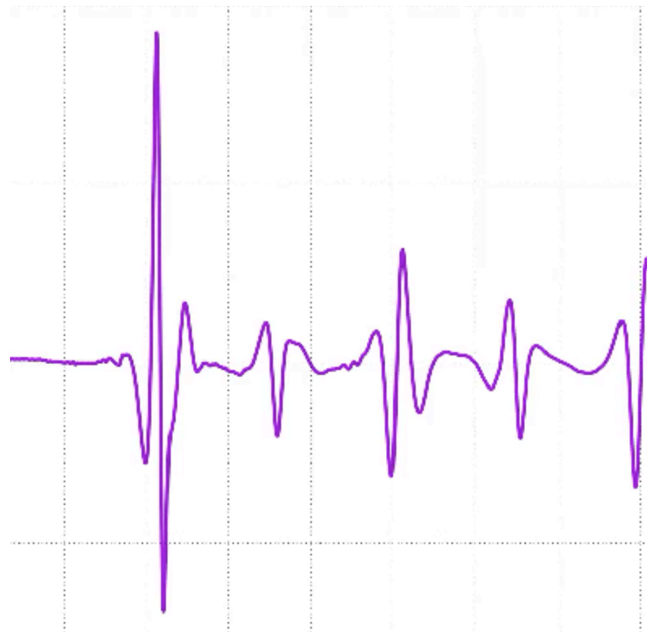
summary

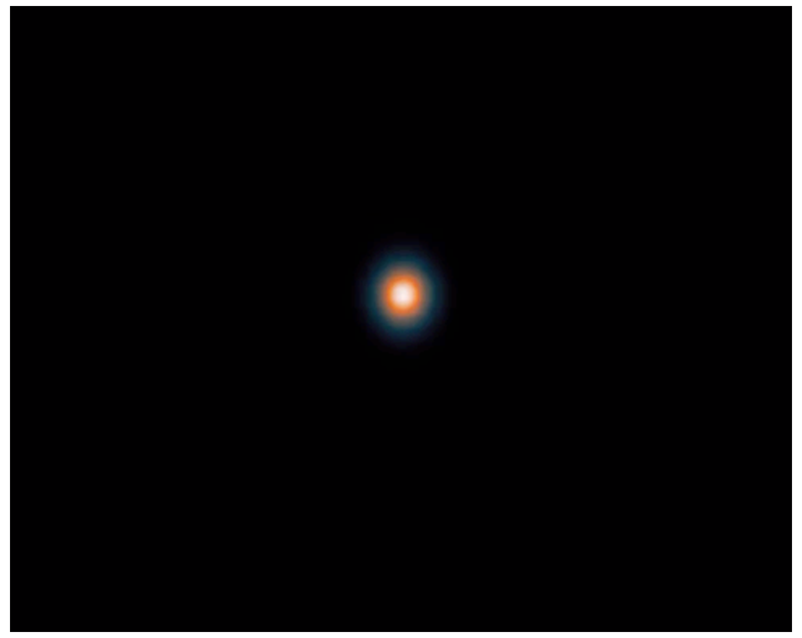


formation mechanisms & interactions



observational implications





extra slides

a different higher energy soliton: the "hedgehogs"

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

$$H_{ij}(\mathbf{x},t) = g(r)\left(3\frac{x^i x^j}{r^2} - \delta_{ij}\right)\cos\omega t$$

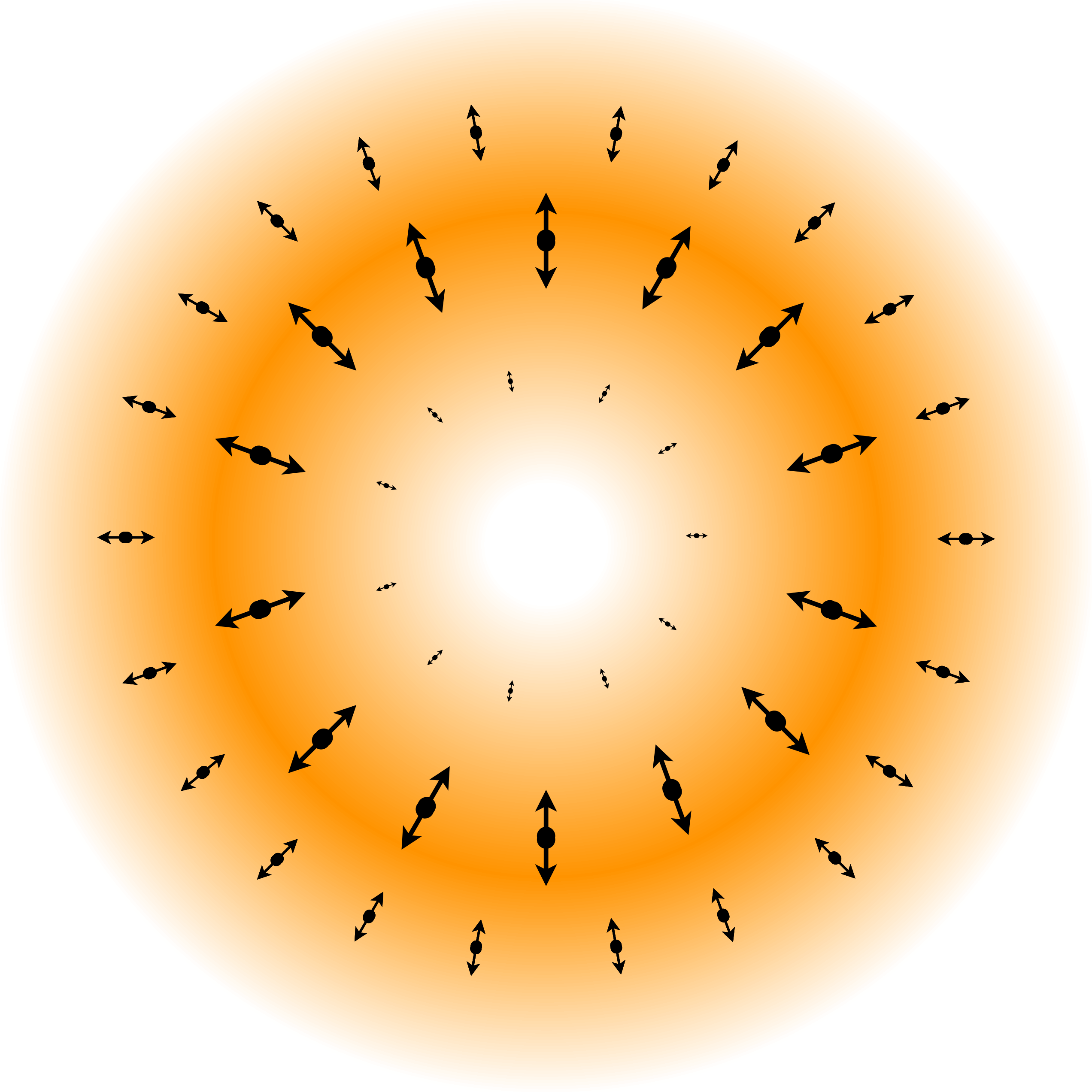
$$E_{\rm hh}^{s=1} \approx 0.33E \quad < 0$$

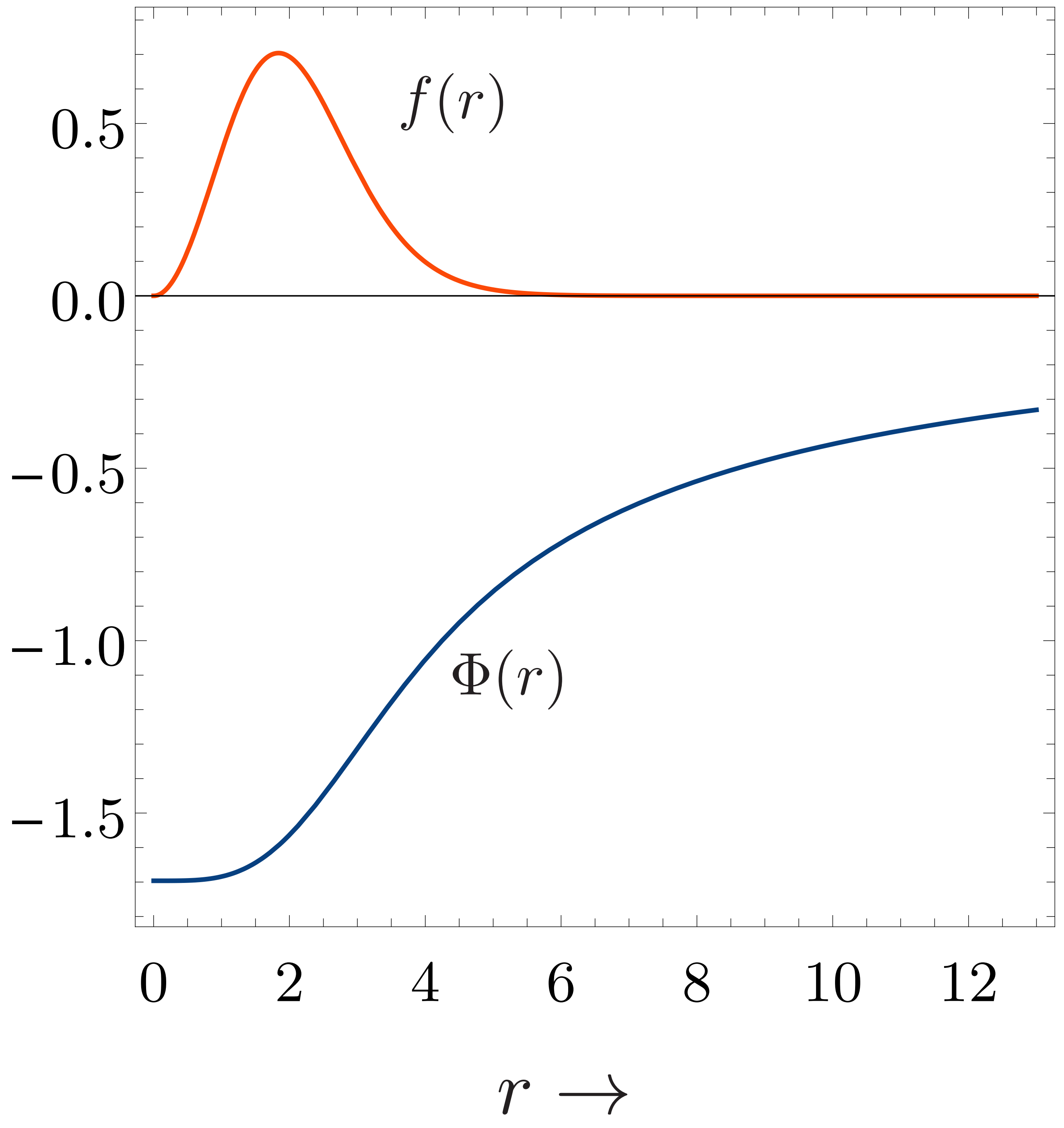
$$E_{\rm hh}^{s=2} \approx 0.17E \quad < 0$$

$$E_{\rm hh}^{s} > E$$

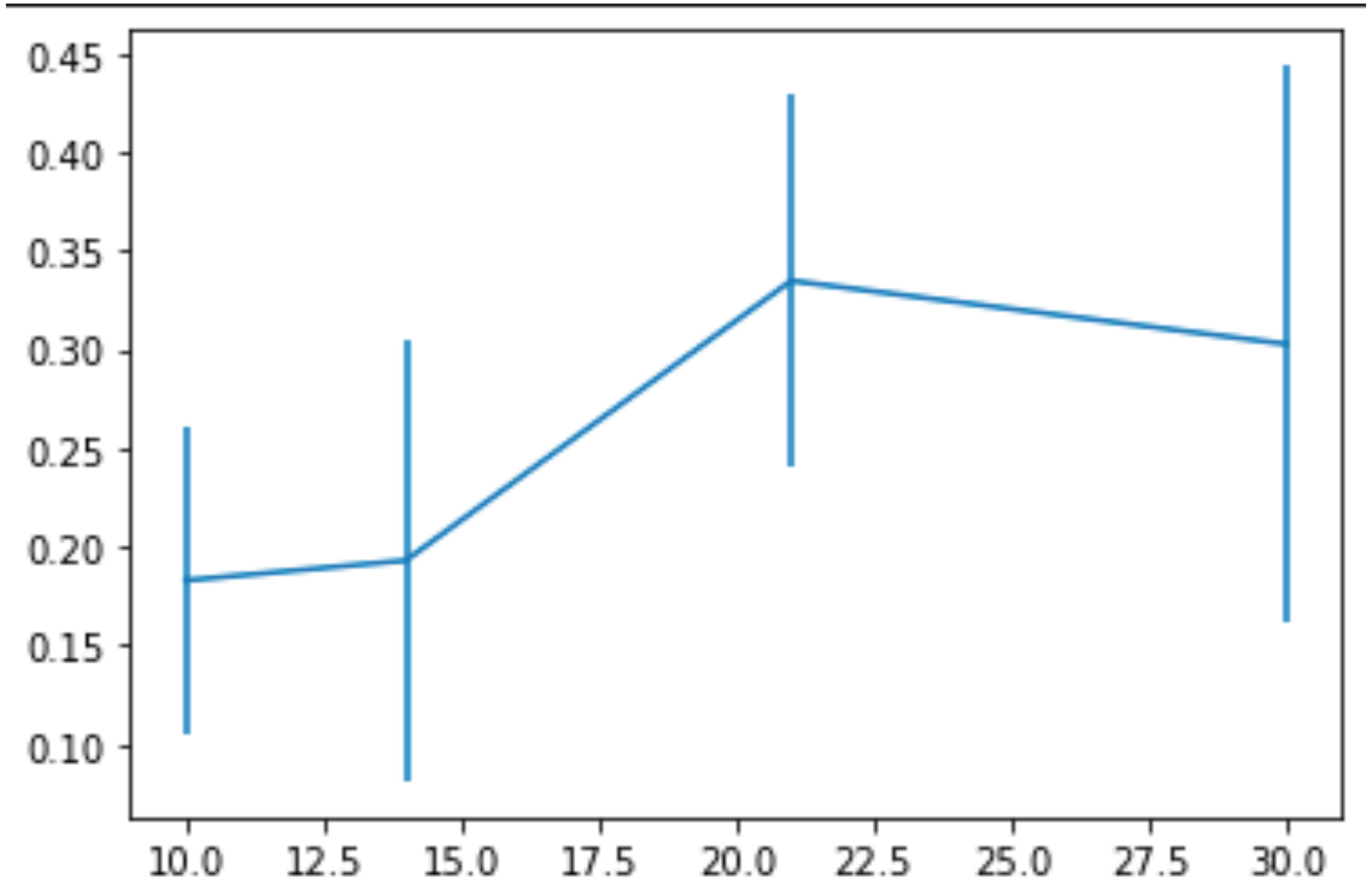
hedgehogs
not ground states

$\boldsymbol{S}_{\rm tot} = \vec{0}$





$\boldsymbol{S}^{\mathrm{tot}}/(\hbar N)$



[redacted]

number of solitons

conserved quantities

$\psi = [\mathbf{\Psi}]$	spin-0
$\psi_i = [\mathbf{\Psi}]_i$	spin-1
$\psi_{ij} = [\mathbf{\Psi}]_{ij}$	spin-2

$$\mathcal{S}_{nr}^{\text{eff}} = \int \mathrm{d}^4x \left[\frac{i}{2}\,\mathrm{Tr}\left[\mathbf{\Psi}^\dagger\dot{\mathbf{\Psi}}\right] + \text{c.c.} - \frac{1}{2m}\,\mathrm{Tr}[\nabla\mathbf{\Psi}^\dagger\cdot\nabla\mathbf{\Psi}] + m_{\text{pl}}^2\,\Phi\nabla^2\Phi - m\,\Phi\,\mathrm{Tr}[\mathbf{\Psi}^\dagger\mathbf{\Psi}]\right]$$

$$N = \int d^3x \mathrm{Tr}[\mathbf{\Psi}^\dagger\mathbf{\Psi}]$$

particle number from “emergent” $\mathrm{U}(1)$

$$E = \int \mathrm{d}^3x\left[\frac{1}{2m}\,\mathrm{Tr}[\nabla\mathbf{\Psi}^\dagger\cdot\nabla\mathbf{\Psi}] + \frac{m^2}{4m_{\text{pl}}^2}\,\mathrm{Tr}[\mathbf{\Psi}^\dagger\mathbf{\Psi}]\int\frac{\mathrm{d}^3y}{4\pi|\mathbf{x}-\mathbf{y}|}\,\mathrm{Tr}[\mathbf{\Psi}^\dagger(\mathbf{y})\mathbf{\Psi}(\mathbf{y})]\right]$$

energy from time translations

$$S_k = s\int d^3x\,\Re\left(i\,\varepsilon_{ijk}[\mathbf{\Psi}\mathbf{\Psi}^\dagger]_{ij}\right),$$

$$L_k = \int d^3x\,\Re\left(i\,\varepsilon_{ijk}\mathrm{Tr}[\mathbf{\Psi}^\dagger\partial_i\mathbf{\Psi}]x^j\right)$$

spin & orbital

from spatial rotations

conserved quantities

$\psi = [\mathbf{\Psi}]$ spin-0
$\psi_i = [\mathbf{\Psi}]_i$ spin-1
$\psi_{ij} = [\mathbf{\Psi}]_{ij}$ spin-2

$$\mathcal{S}_{nr}^{\mathrm{eff}} = \int \mathrm{d}^4 x \left[\frac{i}{2} \mathrm{Tr} \left[\mathbf{\Psi}^\dagger \dot{\mathbf{\Psi}} \right] + \mathrm{c.c.} - \frac{1}{2m} \mathrm{Tr}[\nabla \mathbf{\Psi}^\dagger \cdot \nabla \mathbf{\Psi}] + m_{\mathrm{pl}}^2 \Phi \nabla^2 \Phi - m \, \Phi \, \mathrm{Tr}[\mathbf{\Psi}^\dagger \mathbf{\Psi}] \right]$$

$$S_k = s \int d^3 x \Re \left(i \, \varepsilon_{ijk} [\mathbf{\Psi} \mathbf{\Psi}^\dagger]_{ij} \right),$$

$$L_k = \int d^3 x \Re \left(i \, \varepsilon_{ijk} \mathrm{Tr}[\mathbf{\Psi}^\dagger \partial_i \mathbf{\Psi}] x^j \right)$$

spin & orbital angular momentum

from spatial rotations (independently conserved)

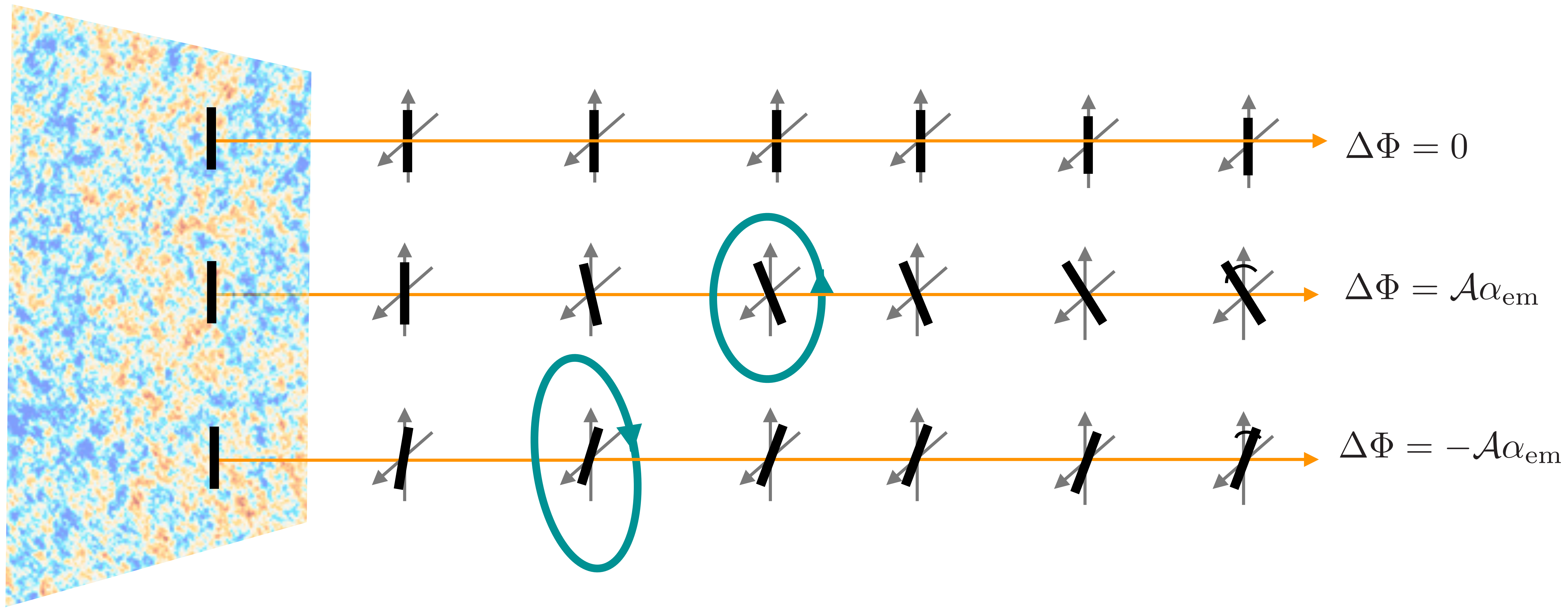
CMB birefringence from ultralight-axion string networks

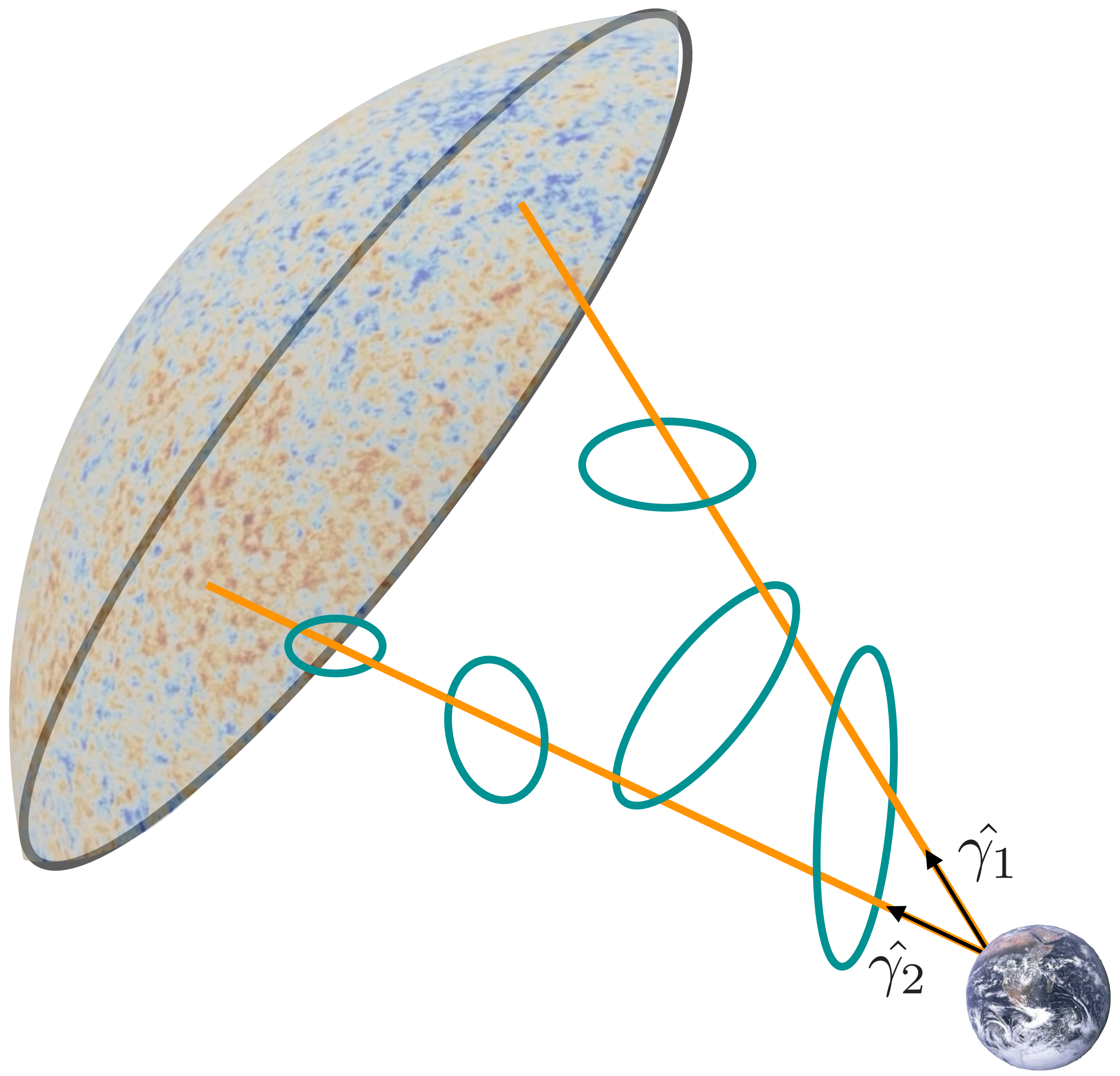
[arXiv:2103.10962]

with Andrew Long and **Mudit Jain**

$$\mathscr{L}_{\mathrm{int}} = -\frac{1}{4}\, g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Delta\Phi = \frac{g_{a\gamma\gamma}}{2}\int_C \mathrm{d}X^\mu\, \partial_\mu a(x) \quad = \pm g_{a\gamma\gamma}\pi f_a = \pm\mathcal{A}\alpha_{\mathrm{em}}$$



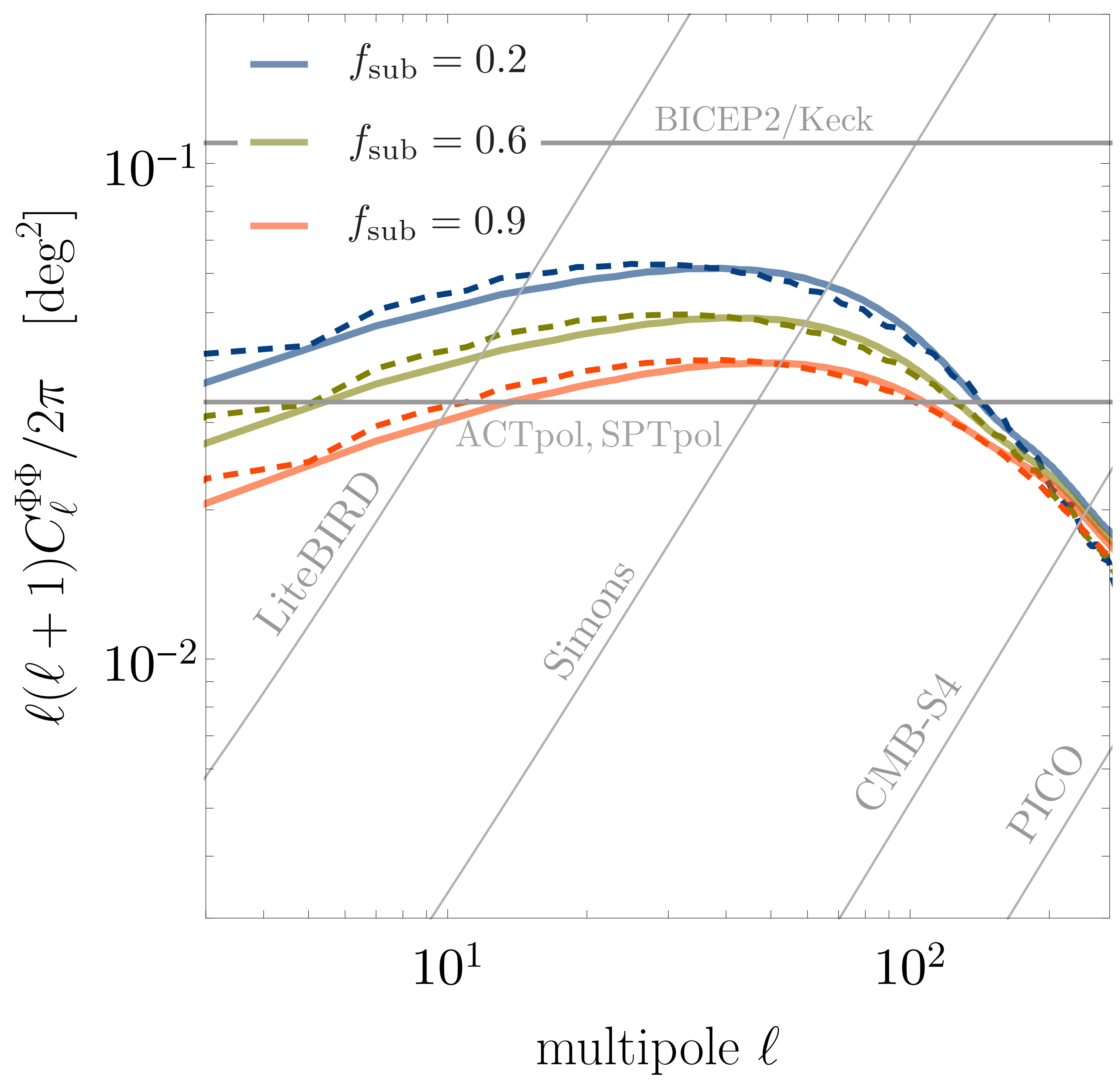


*see Agrawal, Hook & Huang (2019)

[arXiv:2103.10962] with Andrew Long and **Mudit Jain**

CMB birefringence from ultralight-axion string networks

models with sub/super horizon loops



models with decaying networks