Light from Dark Solitons

Mustafa A. Amin (Rice University)

based on

[arXiv: 2103.12082] with Andrew J. Long, Zong-Gang Mou, Paul Saffin

and [arXiv: 2009.11337] with Zong-Gang Mou

soliton ?

Image Credit: Heriot-Watt University

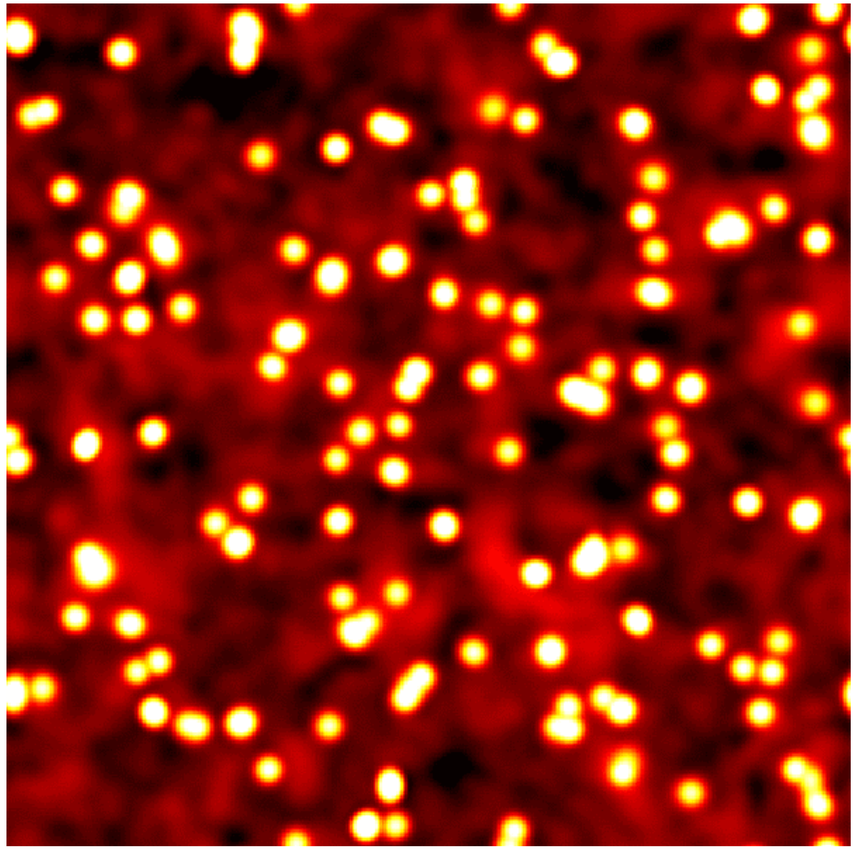
very-long lived, dynamical excitation in a field, with nonlinearities balancing dispersion

- discovered in nonlinear waves in water in canals (John Scott Russell, 1834)
- optics, hydrodynamics, BECs, high energy physics, and cosmology

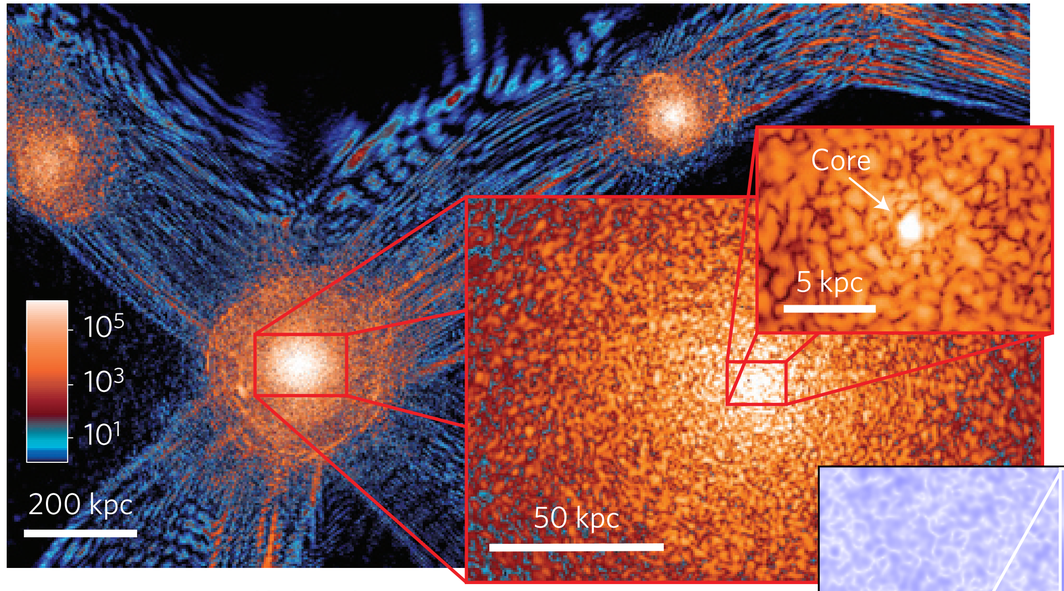
solitons in axion-like fields

"solitons" in axion-like fields can form naturally in the inflaton and in dark matter

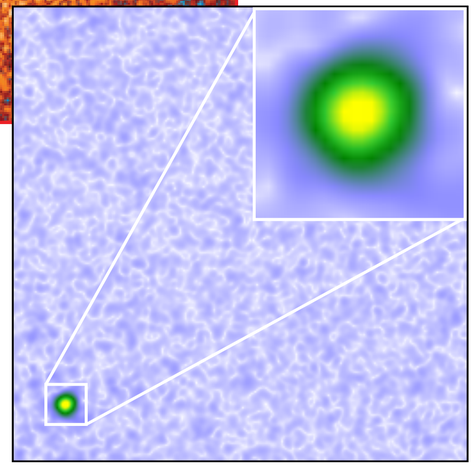
- can have gravitational and non-gravitational effects



MA & Mocz (2019)



Schive et. al (2015)

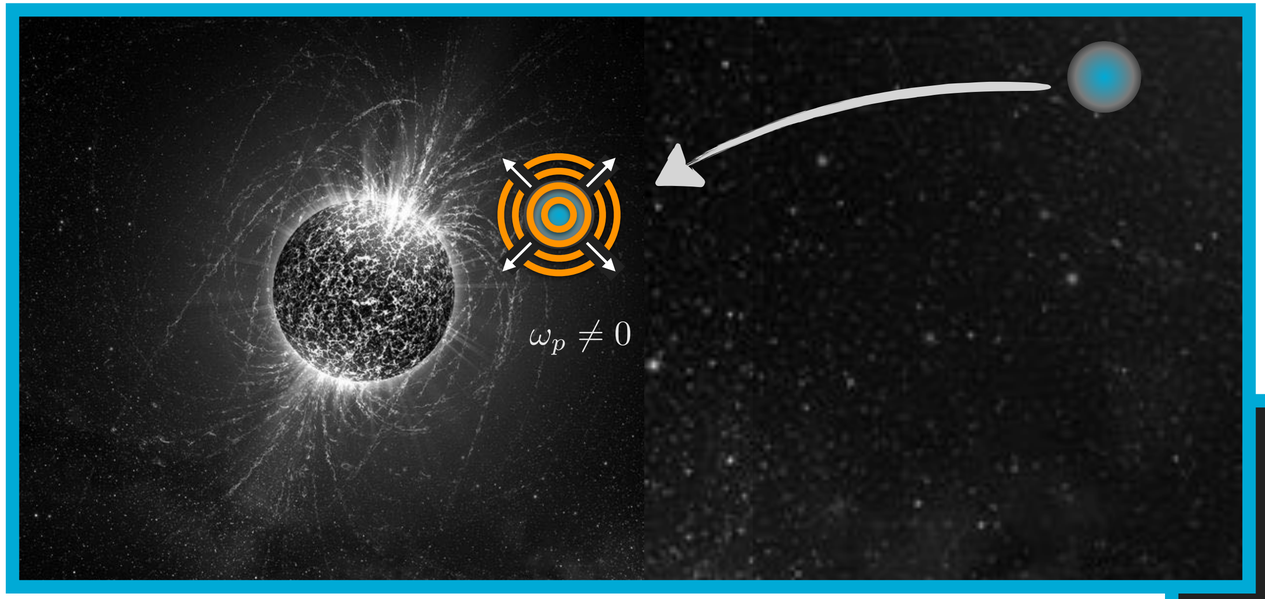


Levkov et. al (2018)

also phase transitions, nucleation around BHs, etc.

main takeaways

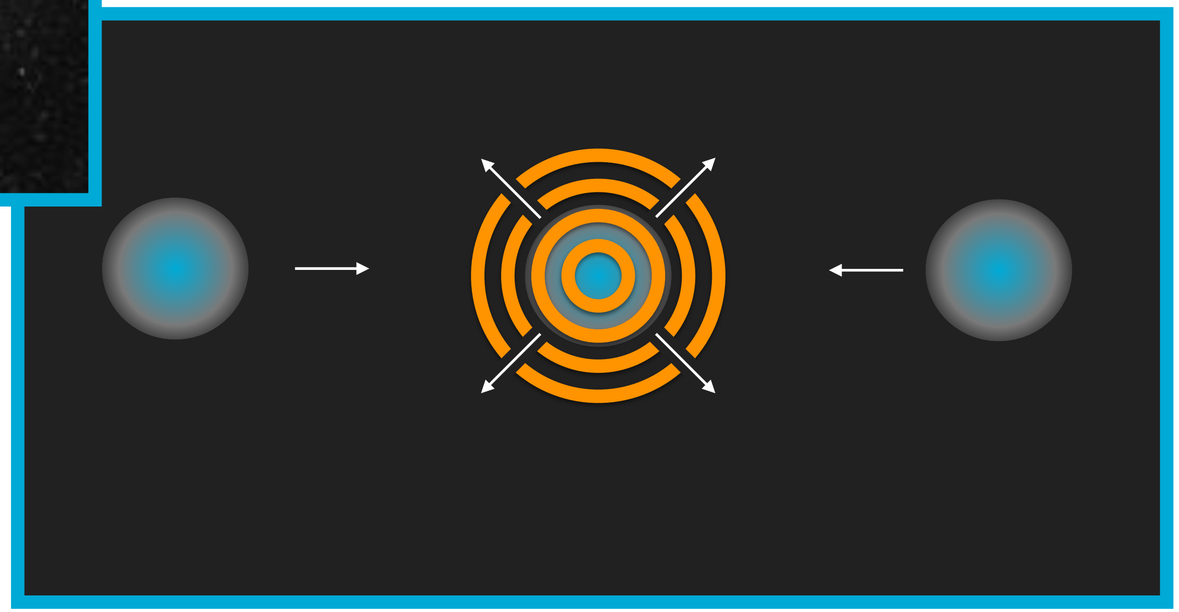
axion stars/oscillons/solitons can radiate energy in electromagnetic fields $g_{a\gamma}\phi\mathbf{E}\cdot\mathbf{B}$



radiation from solitons in external electromagnetic fields
(with or without plasma)

+ early universe effects ?

radiation from collisions of solitons
*gravitational and electromagnetic possible



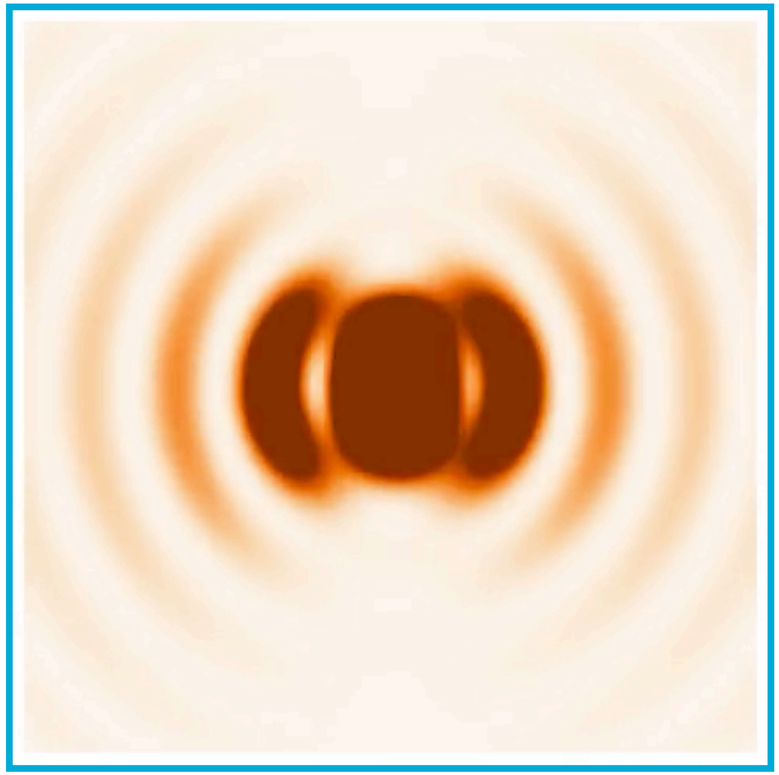
+ Fast Radio Bursts (maybe)

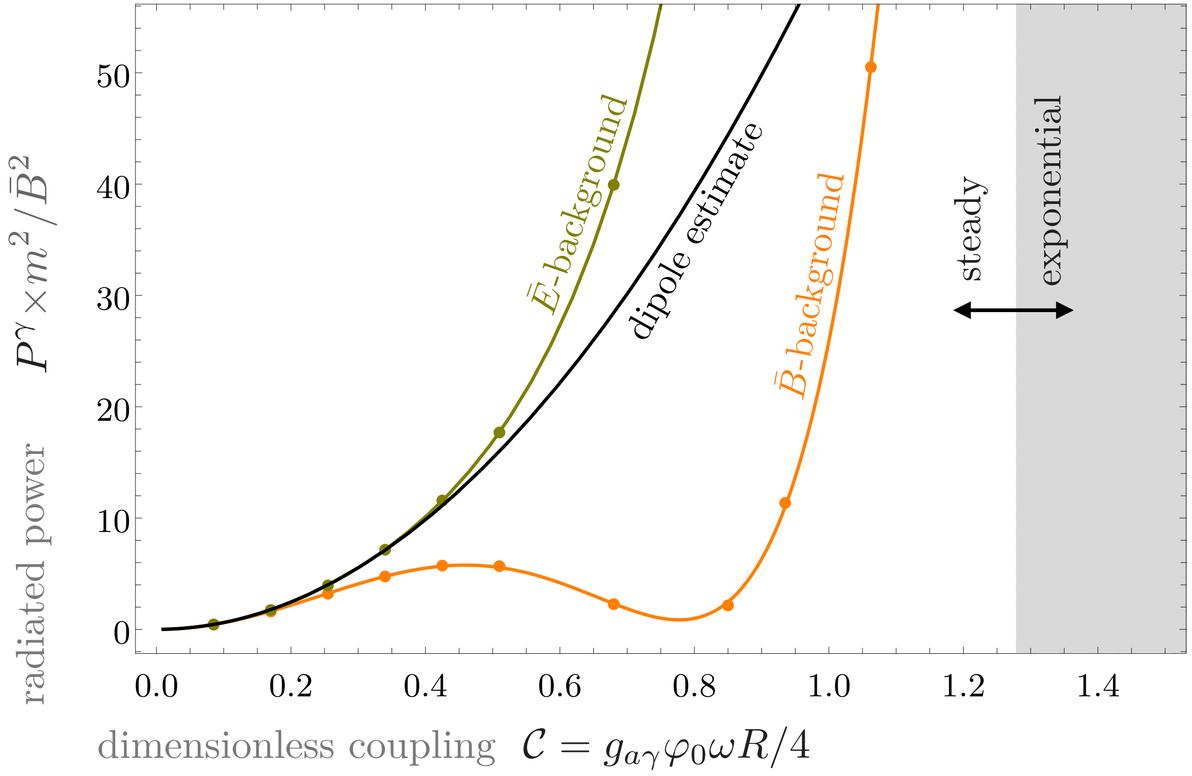
main takeaways

Mou

axion stars/oscillons/solitons can radiate energy in electromagnetic fields $g_{a\gamma}\phi\mathbf{E}\cdot\mathbf{B}$

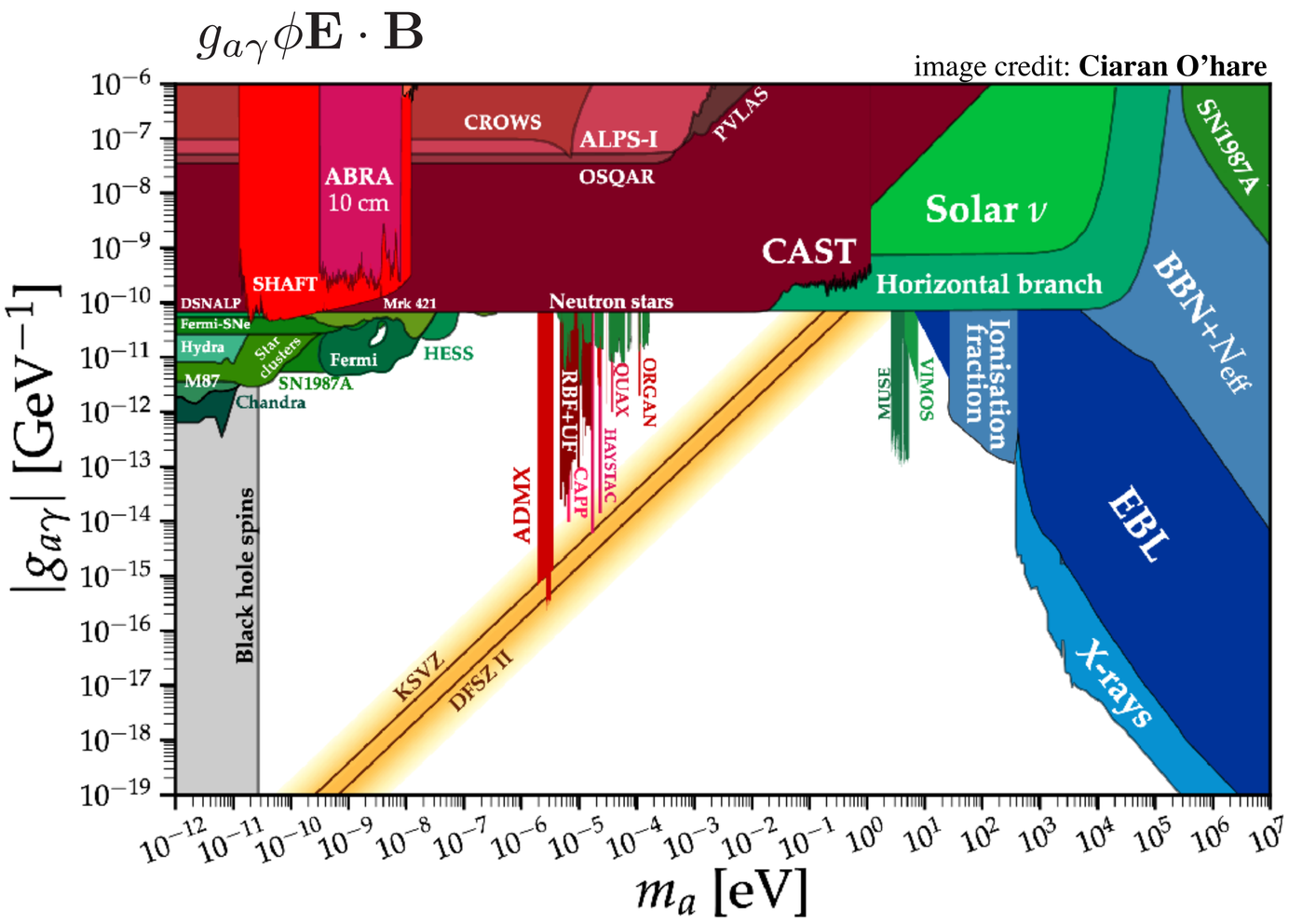
radiated power depends on axion-photon coupling and characteristics of soliton configuration





MA & Mou (2020), MA, Long, Mou & Saffin (2021)

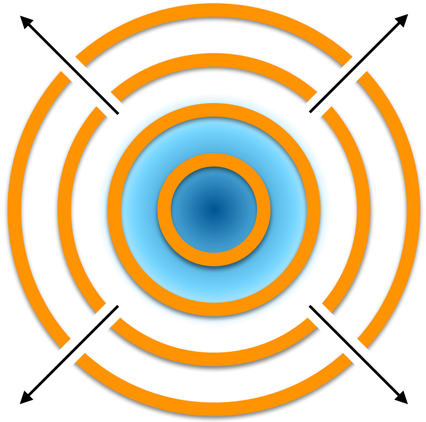
lots of excitement: constrain axion photon coupling



why EM radiation from solitons ?

$g_{a\gamma}\phi\mathbf{E}\cdot\mathbf{B}$

solitons with coherence & large (non-redshifting) central amplitudes — can significantly change expectations of axion-photon conversion

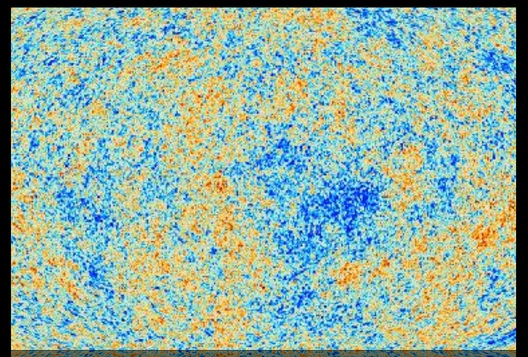


*global picture in terms of the soliton properties, coupling strengths etc.

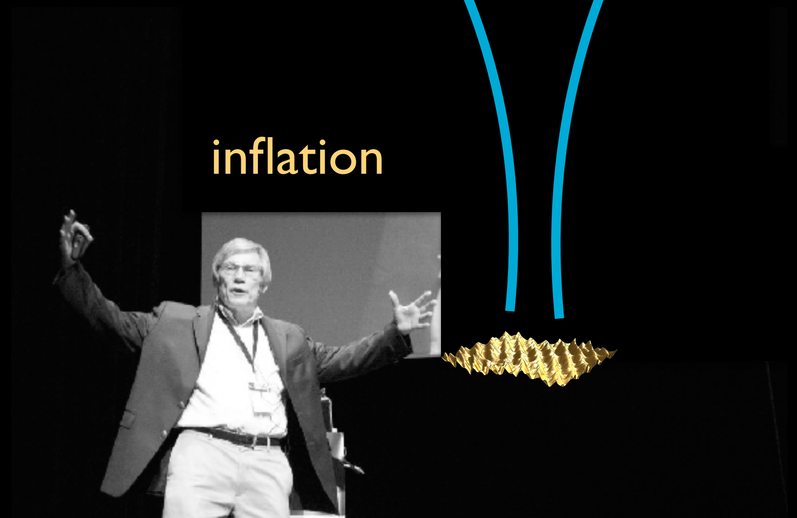
synopsis of talk

- orientation
- *physics of soliton formation, stability and clustering (later)
- light (and gravitational waves) from dark solitons

our cosmic story

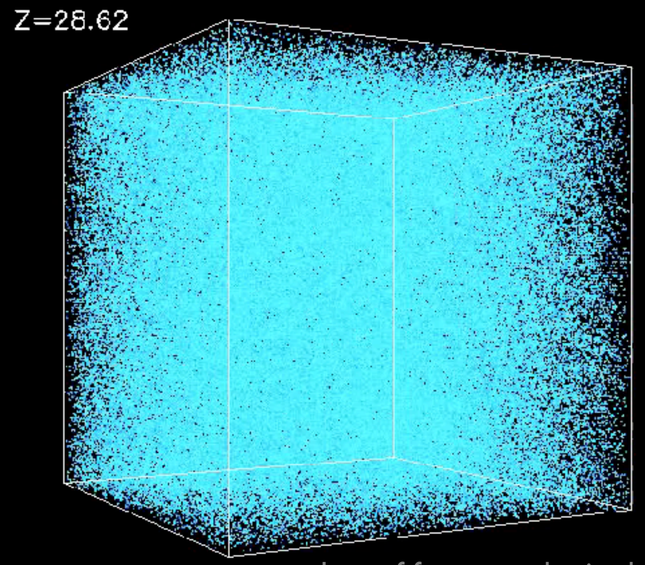


lots of fun high energy physics
+ phase transitions here

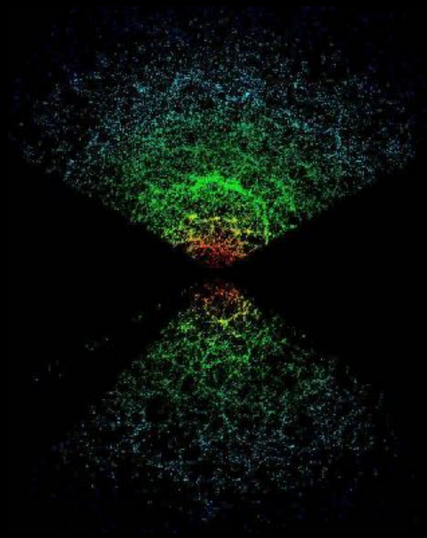


inflation

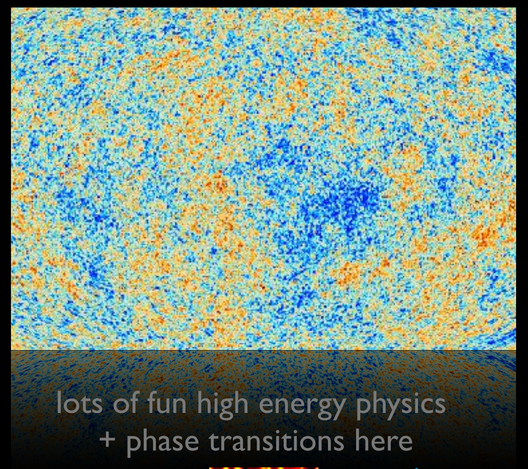
dark matter



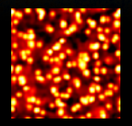
lots of fun astrophysics here

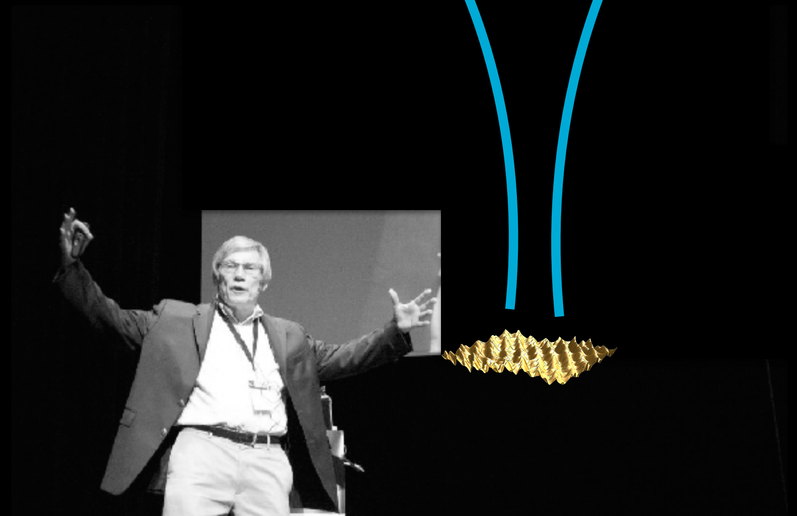


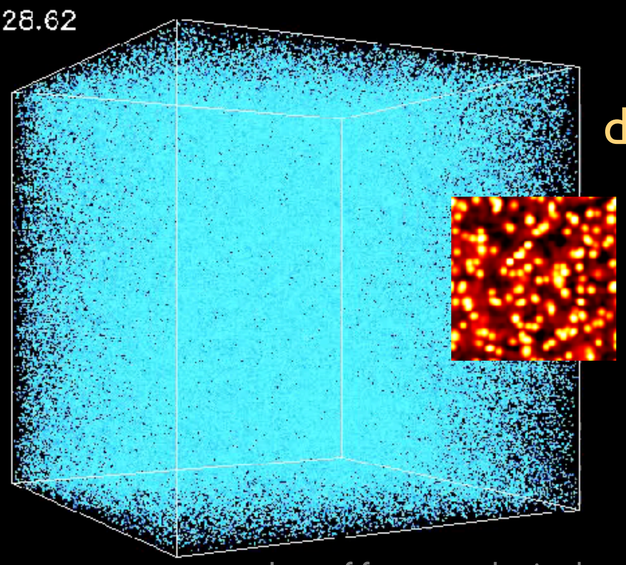
our cosmic story



after inflation







dark matter

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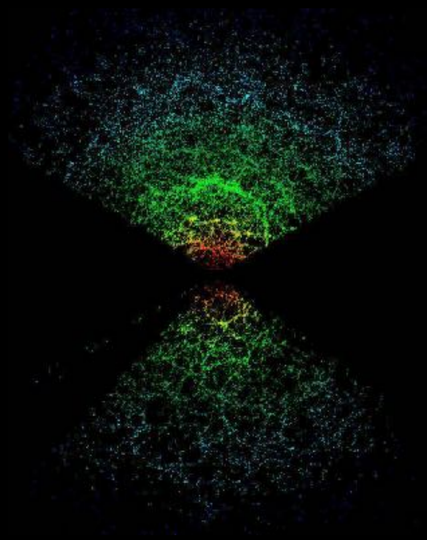
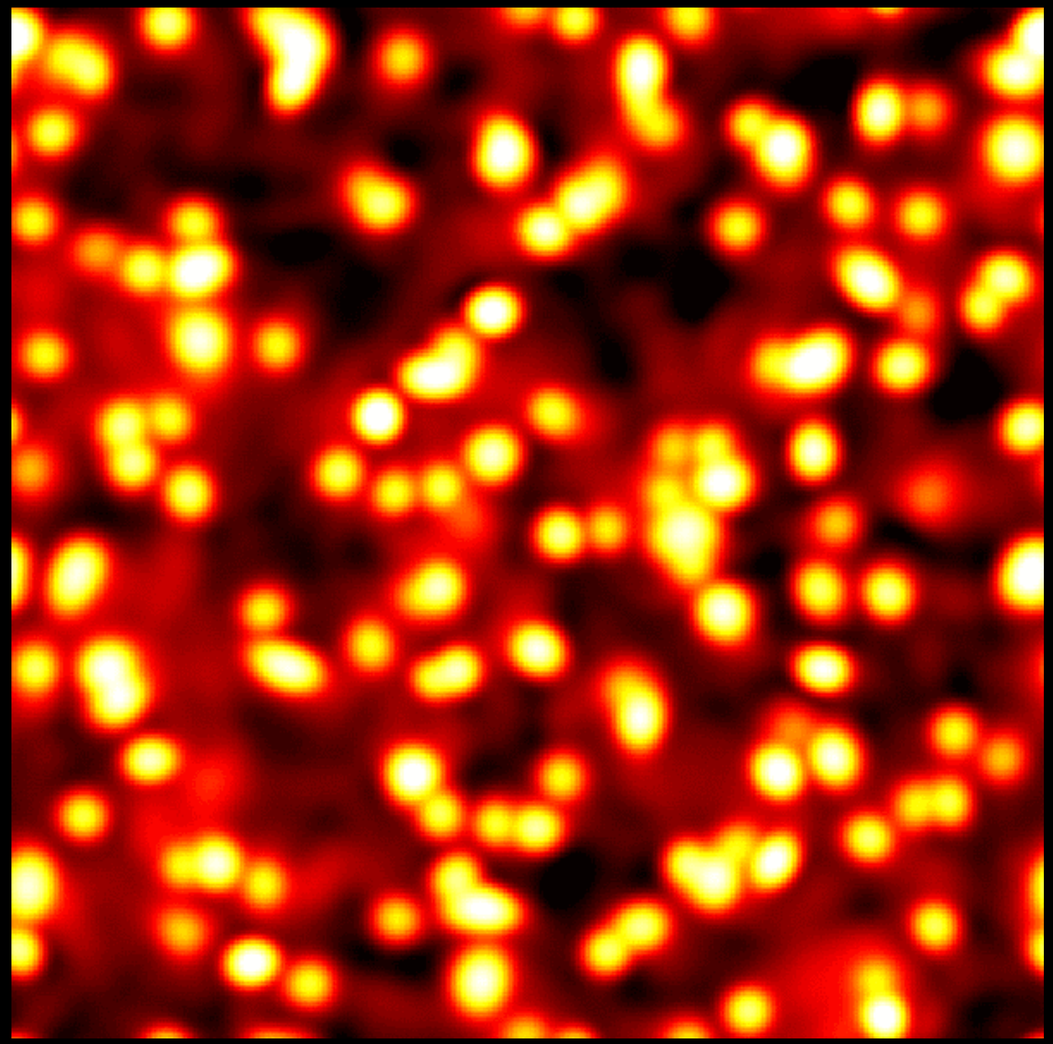


length scales ?



microscopic length scales

end of inflation

sizes scales with inverse mass of the scalar field

macroscopic length scales

but (usually) smaller than galactic scales

dark matter

cosmological scalar fields + gravity + photons

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

relevant for soliton formation, clustering and gravitational wave production

relevant for photon production

* scalar field is real valued

cosmological scalar fields + gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

relevant for soliton properties, formation, clustering and gravitational wave production

relevant for photon production

*if coupling to photons is very large, it can significantly influence formation as well, particularly relevant for (p)reheating applications

cosmological scalar fields + gravity

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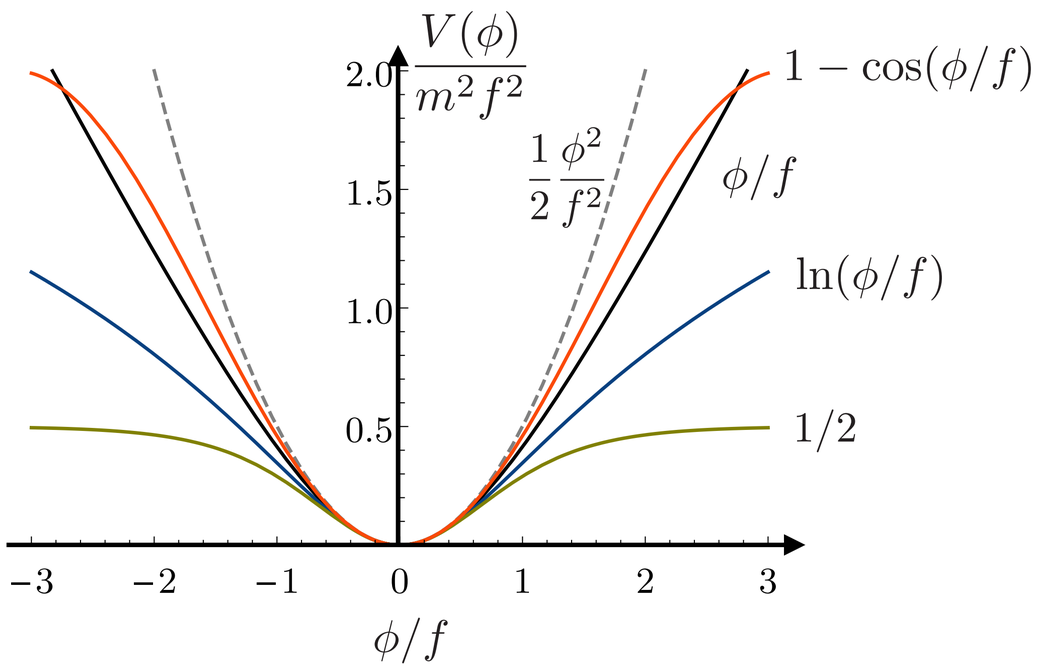


"opening up" of the potential means there is an attractive interaction in addition to gravity

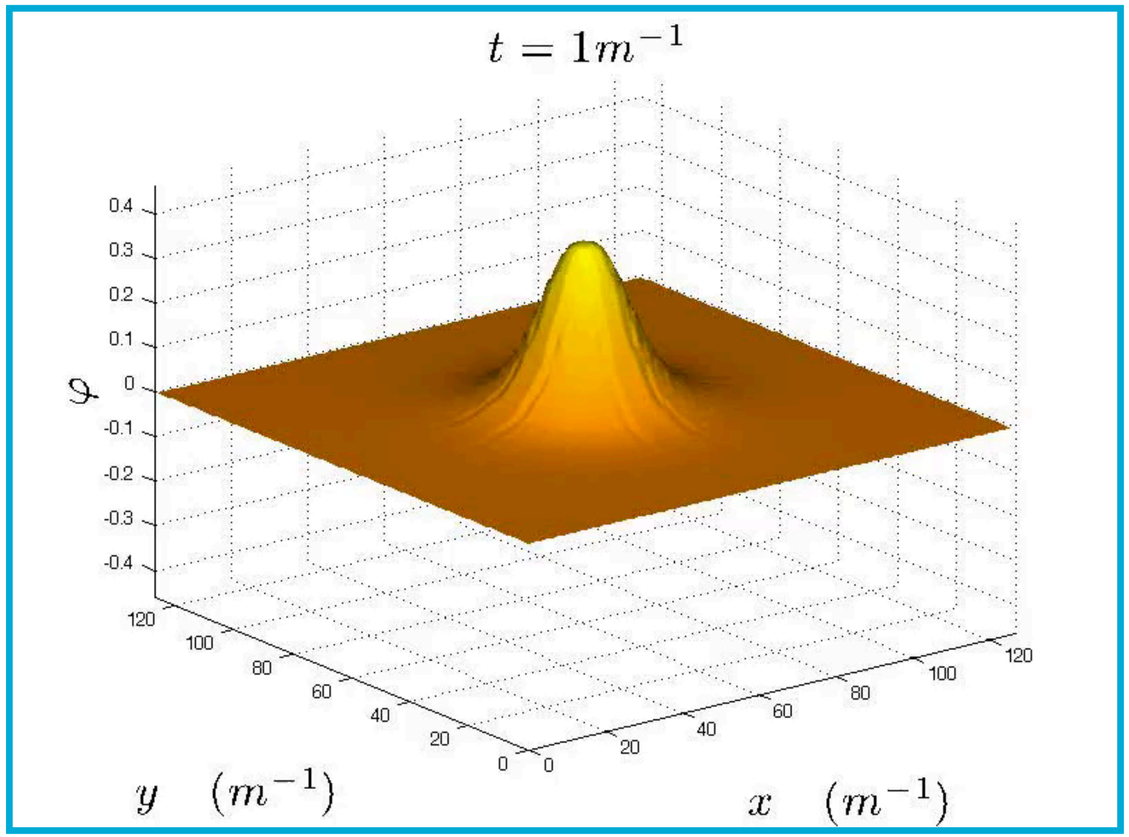
opening up potentials common for inflation and axion-like fields

solitons : oscillons, axion stars …

spatially localized

coherently oscillating

exceptionally long-lived



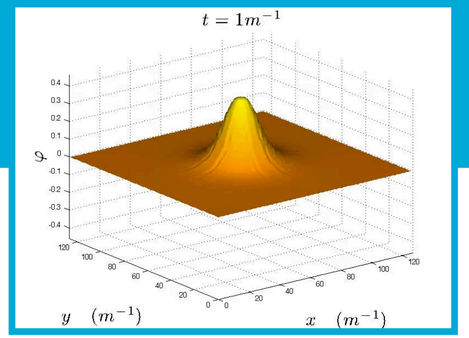
For example:

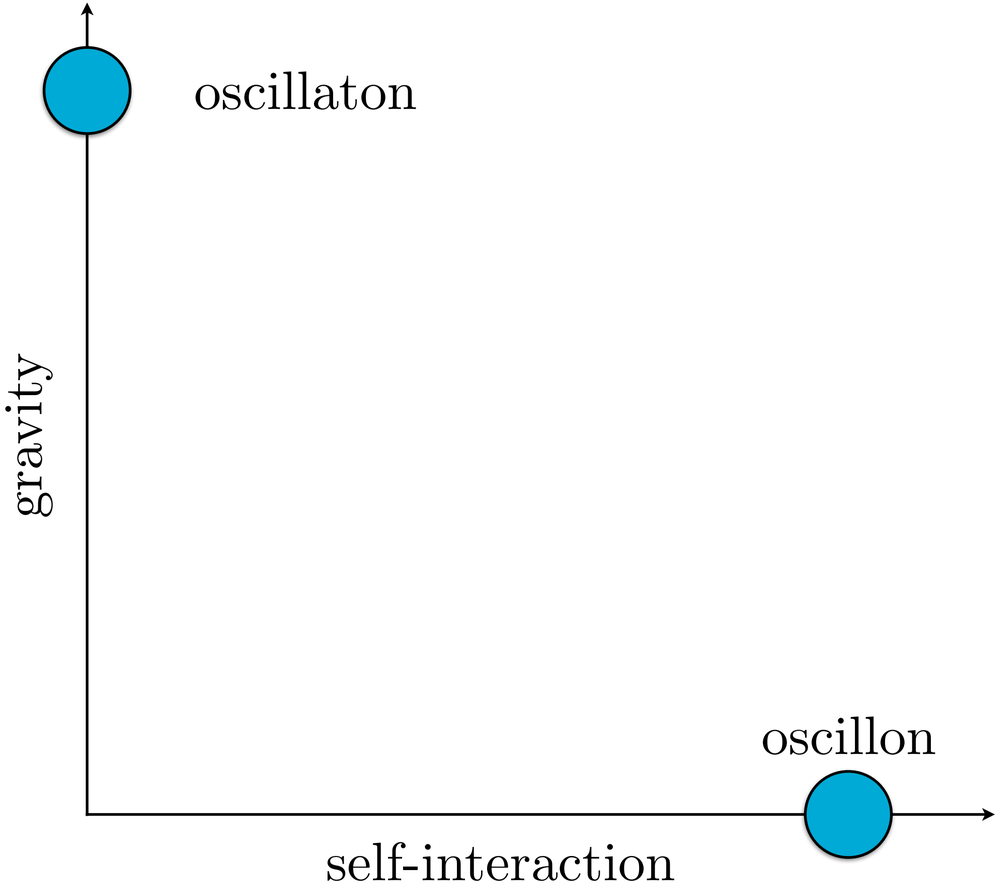
Bogolubsky & Makhankov (1976)
Segur & Kruskal (1987)
Seidel and Sun (1990)
Gleiser (1994)
Copeland et al. (1995)
MA & Shirokoff (2010)
Hertzberg (2011)
MA (2013)
Mukaida et. al (2016)
Zhang, MA, et. al (2020)

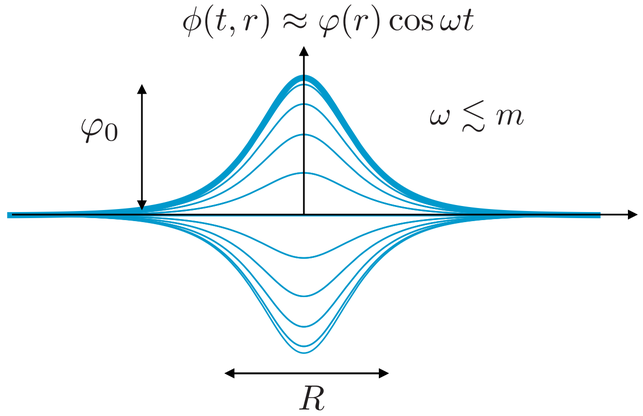
*ask me about lifetimes, see Zhang et. al (2020a, b)

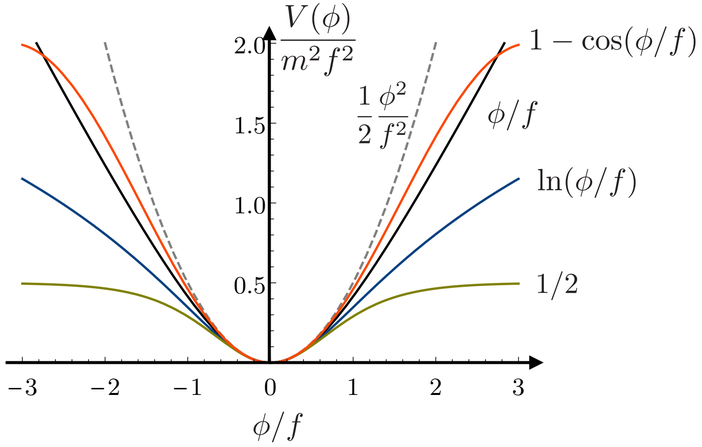
solitons : oscillons, axion stars …

spatially localized, coherently oscillating, long-lived









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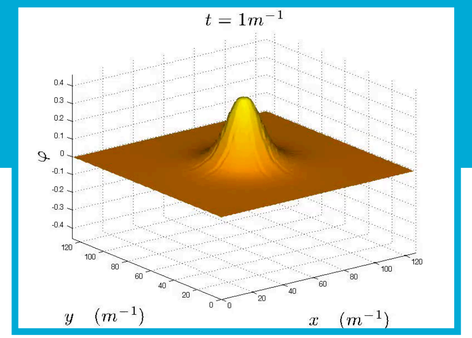
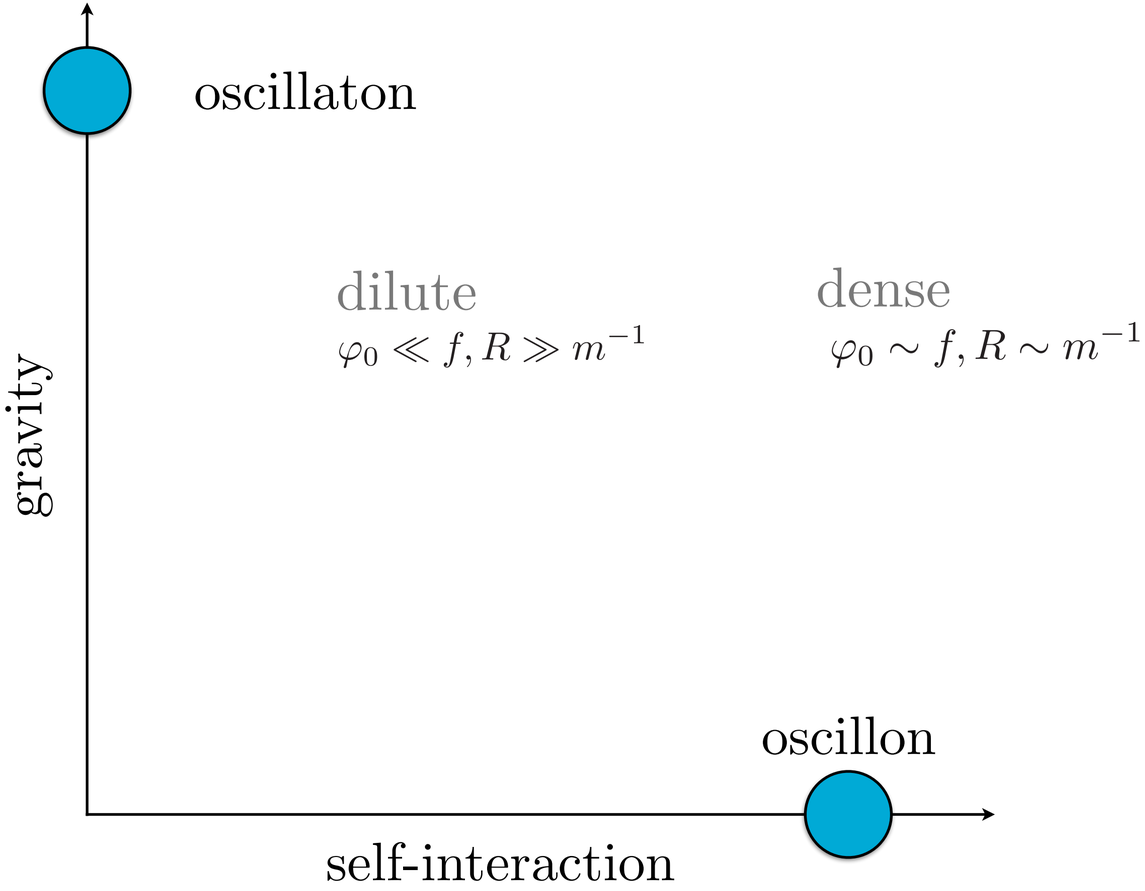
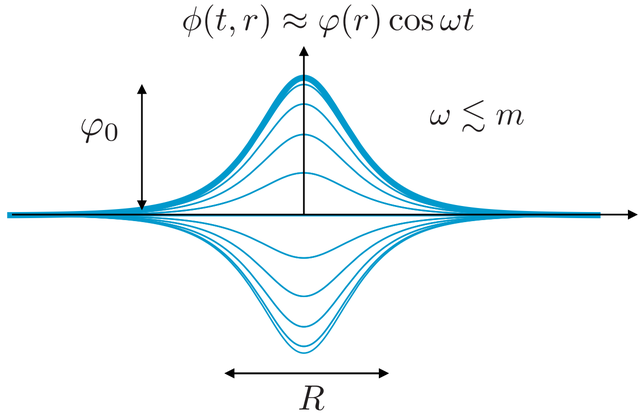
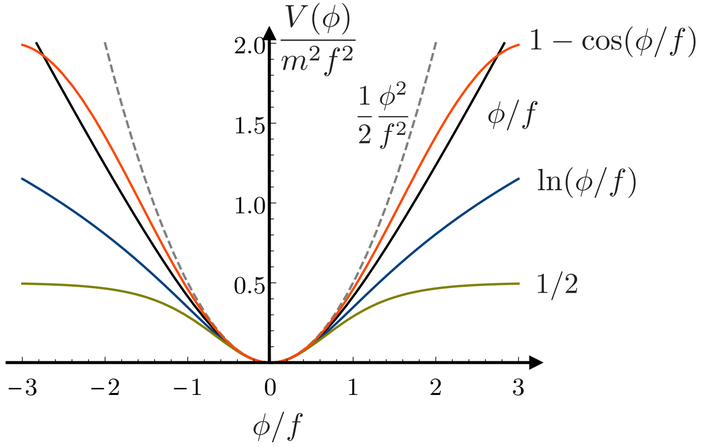




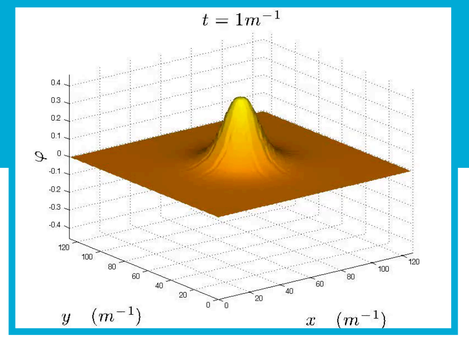


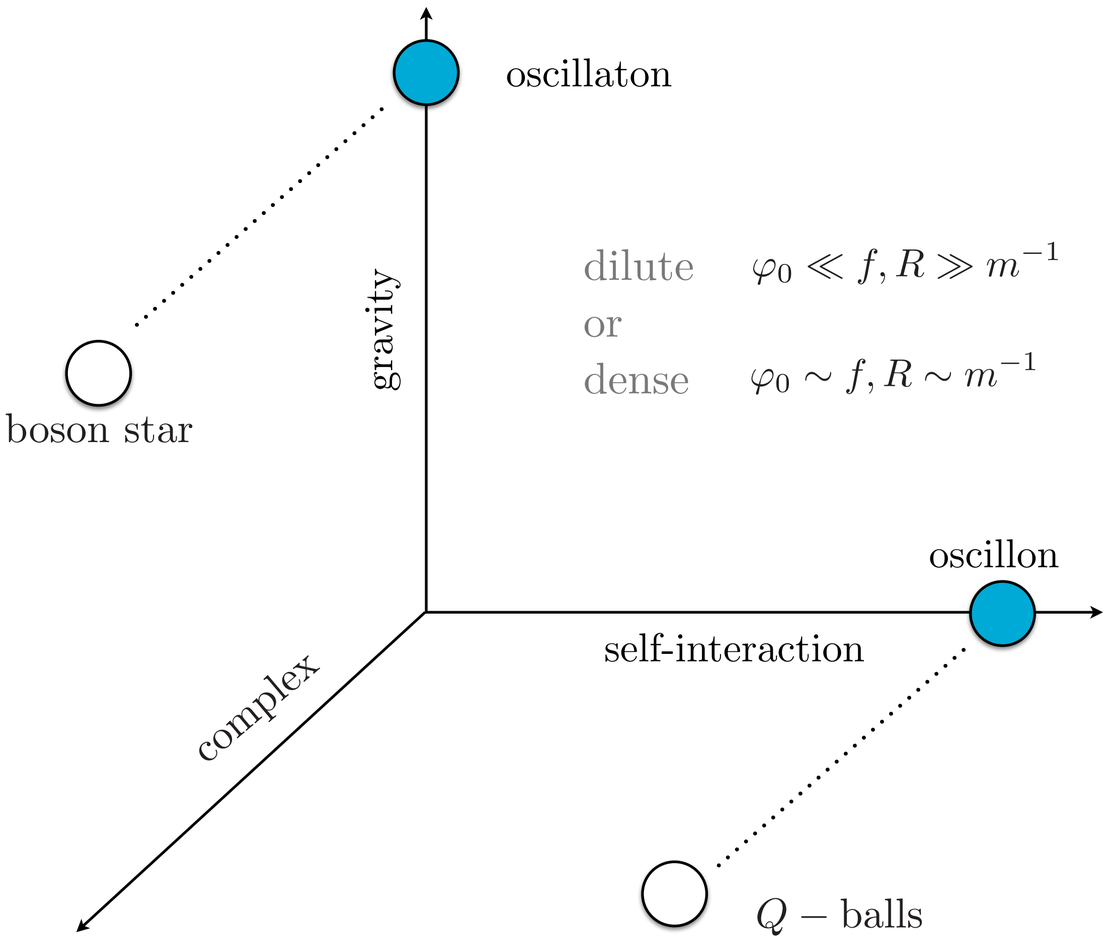


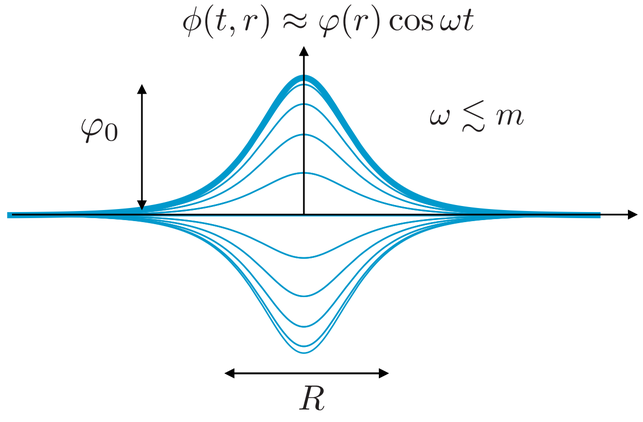
*lifetimes can be much shorter than age of universe for very dense objects, see Zhang et. al (2020a, b)

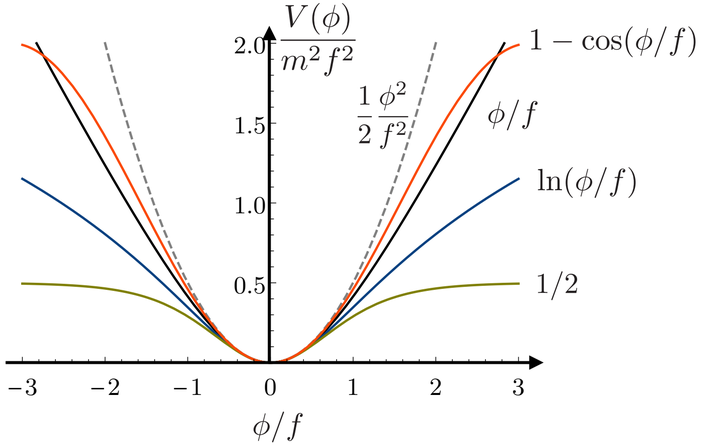
solitons : oscillons, axion stars …

spatially localized, coherently oscillating, long-lived









cosmological scalar fields + gravity + photons

assume “Newtonian" gravity

$$S = \int d^4x\sqrt{-g}\left[\underbrace{\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{relevant for soliton formation, clustering and gravitational wave production}} \underbrace{- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{relevant for photon production}}\right]$$

axion electrodynamics

modified Maxwell's eqns.

$$\dot{\boldsymbol{E}} = \nabla \times \boldsymbol{B} - g_{a\gamma}\left(\dot{\phi}\boldsymbol{B} + \nabla\phi \times \boldsymbol{E}\right)$$

$$\dot{\boldsymbol{B}} = -\nabla \times \boldsymbol{E}\,,$$

$$\nabla \cdot \boldsymbol{E} = -g_{a\gamma}\nabla\phi \cdot \boldsymbol{B}\,,$$

$$\nabla \cdot \boldsymbol{B} = 0\,.$$

effective charge and current densities

$$\rho = -g_{a\gamma}\nabla\phi \cdot \boldsymbol{B}$$

$$\boldsymbol{J} = g_{a\gamma}\left(\dot{\phi}\boldsymbol{B} + \nabla\phi \times \boldsymbol{E}\right)$$

modified Klein-Gordon eq.

$$\ddot{\phi} - \nabla^2\phi + \partial_\phi V = g_{a\gamma}\boldsymbol{E} \cdot \boldsymbol{B}$$

including gravity (non-relativistic)

$$\partial_\phi V \to (1 + 2\Psi)\partial_\phi V$$

$$\nabla^2\Psi = (1/2m_{\rm pl}^2)\rho_\phi$$

electromagnetic radiation from solitons

$g_{a\gamma}\phi\mathbf{E}\cdot\mathbf{B}$

$$\ddot{\boldsymbol{E}} - \boldsymbol{\nabla}^2\boldsymbol{E} = -\boldsymbol{\nabla}\rho - \dot{\boldsymbol{J}}$$

$$\ddot{\boldsymbol{B}} - \boldsymbol{\nabla}^2\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{J}$$

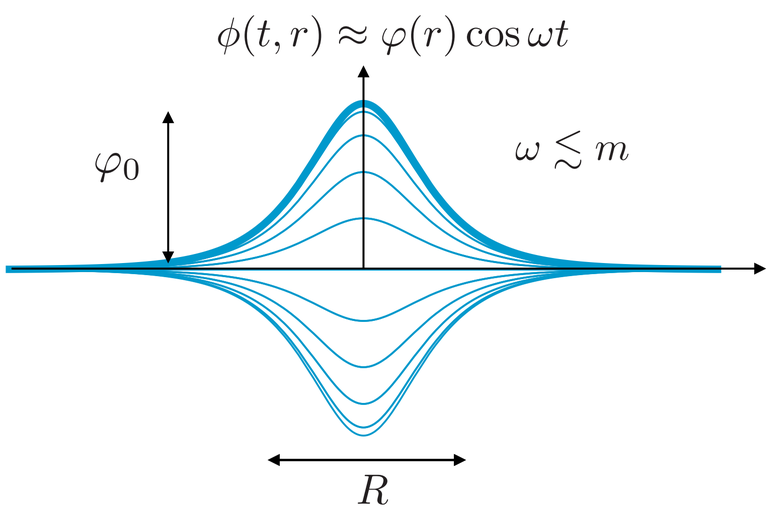
effective charge

$$\rho = -g_{a\gamma}\nabla\phi\cdot\boldsymbol{B}$$

current densities

$$\boldsymbol{J} = g_{a\gamma}\left(\dot{\phi}\boldsymbol{B} + \nabla\phi\times\boldsymbol{E}\right)$$

electromagnetic radiation from solitons



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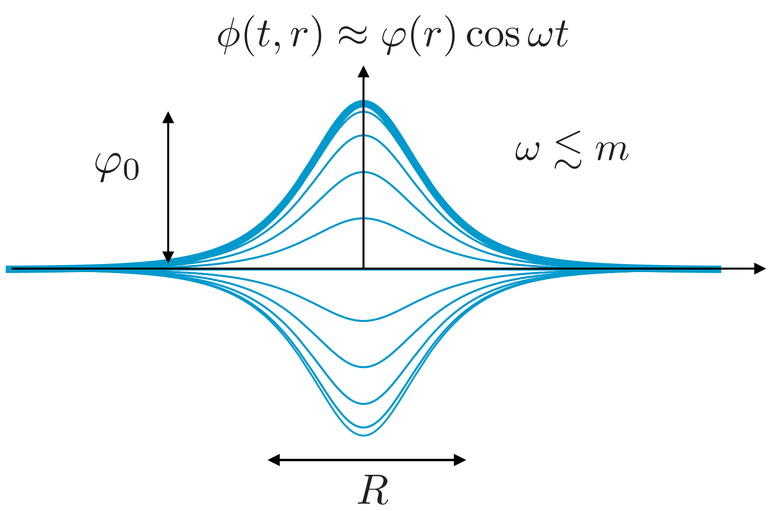
electromagnetic radiation from solitons



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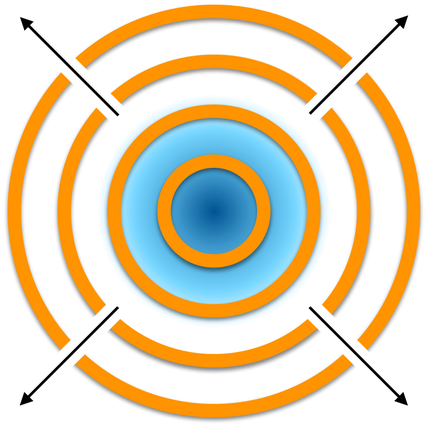

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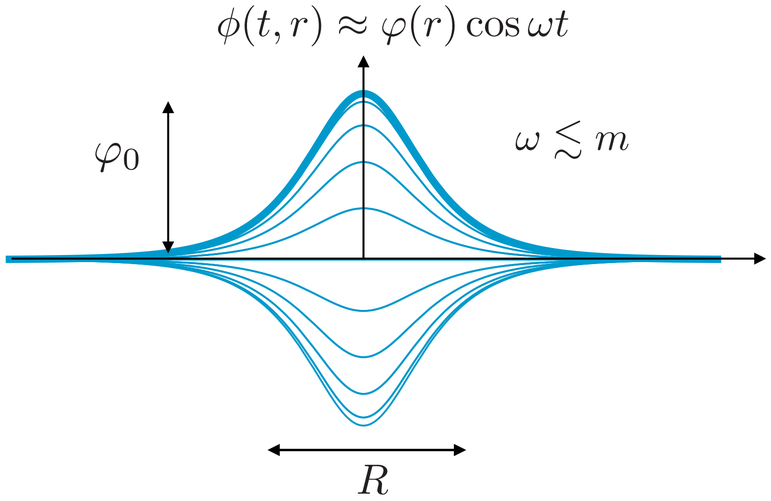

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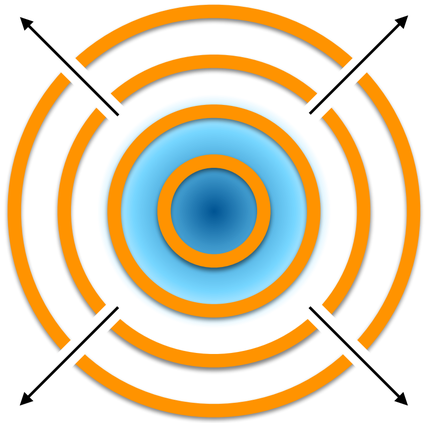
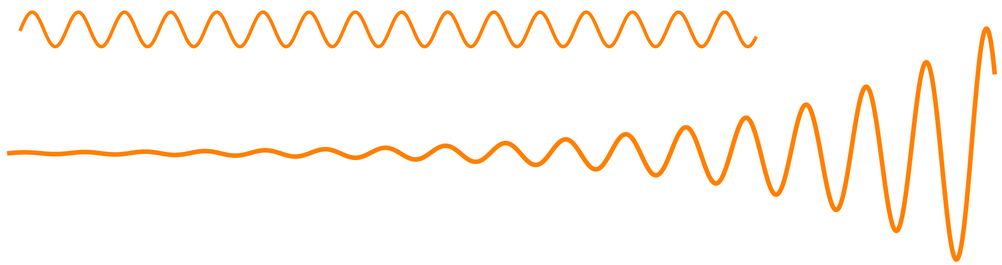


axion field oscillations = periodic coefficients — Floquet theory applies:

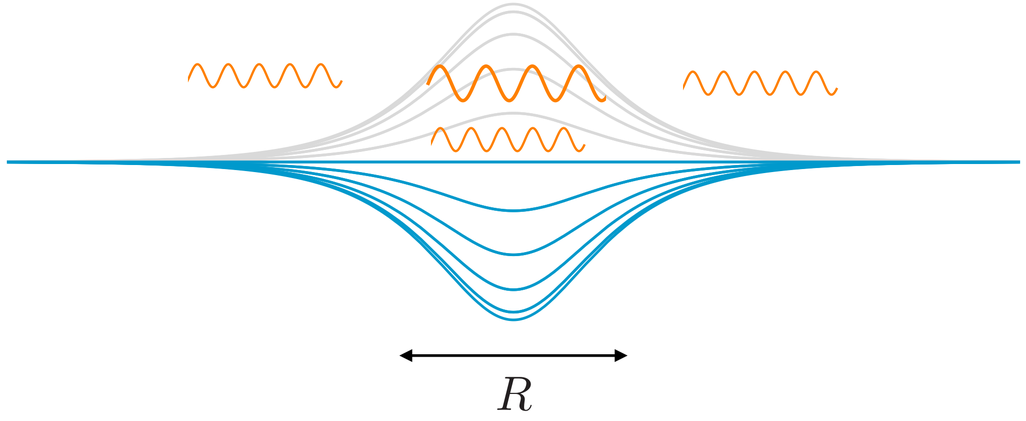
$\boldsymbol{E}, \boldsymbol{B}$

1. steady solutions
2. exponentially growing



EM radiation from solitons: steady vs. explosive

$$\mathcal{C} \equiv \frac{\text{escape time-scale}}{\text{Bose-enhancement time-scale}}$$



Hertzberg 2010

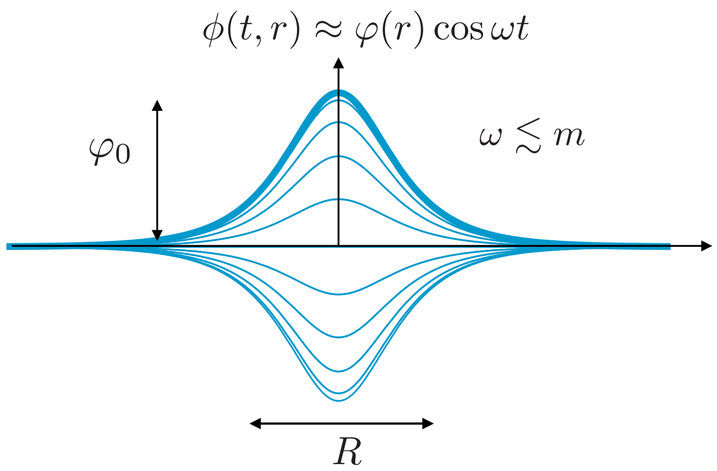
EM radiation from solitons: steady vs. explosive

$$\mathcal{C} \equiv \frac{\text{escape time-scale}}{\text{Bose-enhancement time-scale}}$$

$$\equiv \frac{R}{\mu_{\rm hom}^{-1}} \approx \frac{1}{4} g_{a\gamma} \varphi_0 \omega R$$

numerator and denominator are calculable "by hand"

depends on axion-photon coupling and soliton parameters



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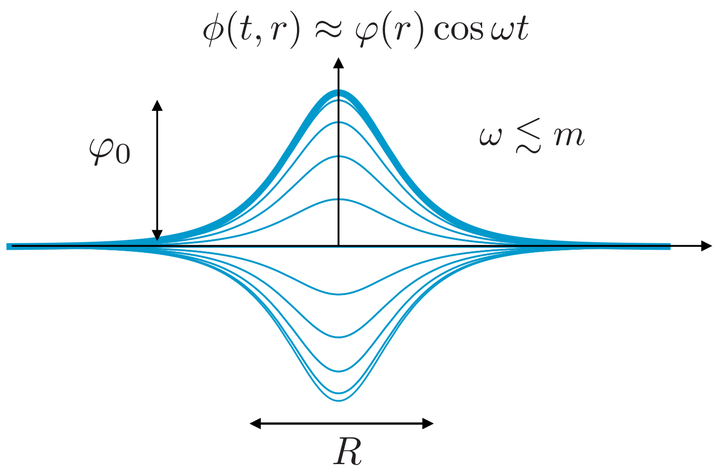
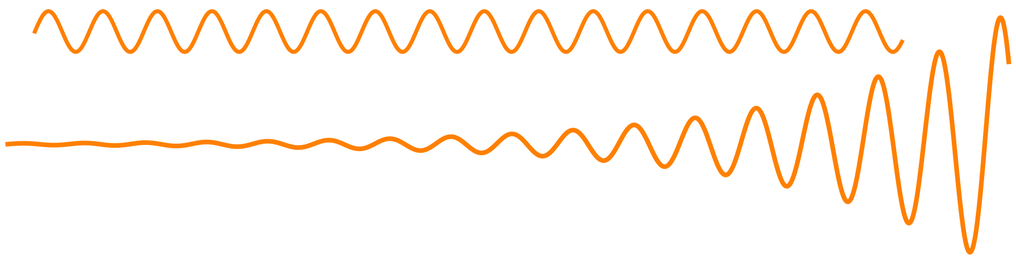
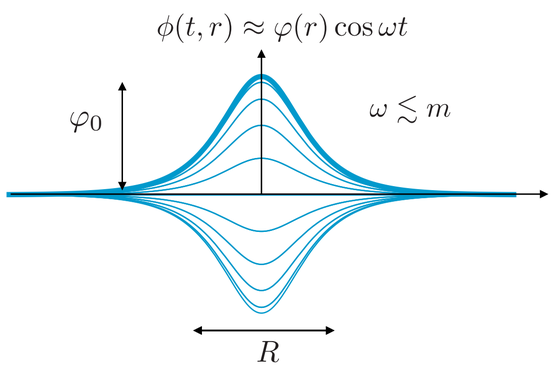


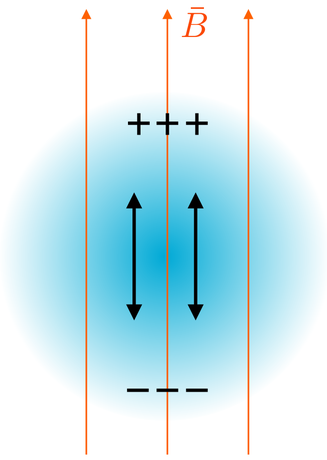
$\mathcal{C} \ll 1 \longrightarrow$

$\mathcal{C} \gtrsim 1 \longrightarrow$

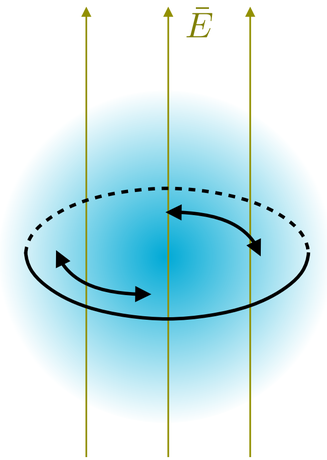


EM radiation from solitons: perturbative calculation





$$\rho = -g_{a\gamma}\nabla\phi\cdot\bar{\boldsymbol{B}}$$



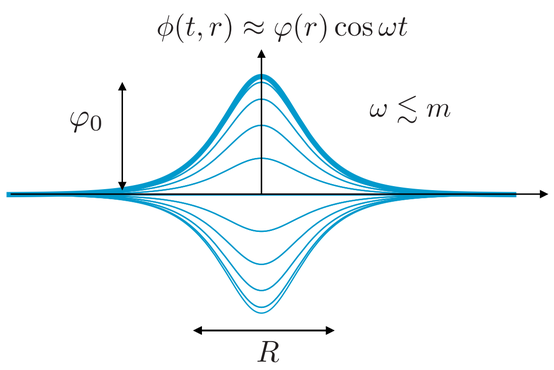
$$\boldsymbol{J} = g_{a\gamma}\left(\dot{\phi}\bar{\boldsymbol{B}} + \nabla\phi\times\bar{\boldsymbol{E}}\right)$$

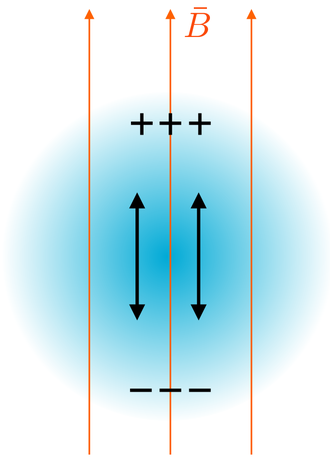
$$\mathcal{C} \sim g_{a\gamma}\varphi_0\omega R \ll 1$$

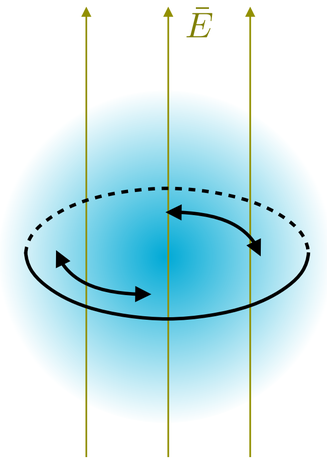
bounded periodic solutions

leading order in $g_{a\gamma}\varphi_0$ only

EM radiation from solitons: dipole radiation







$$\mathcal{C} \sim g_{a\gamma}\varphi_0\omega R \ll 1$$

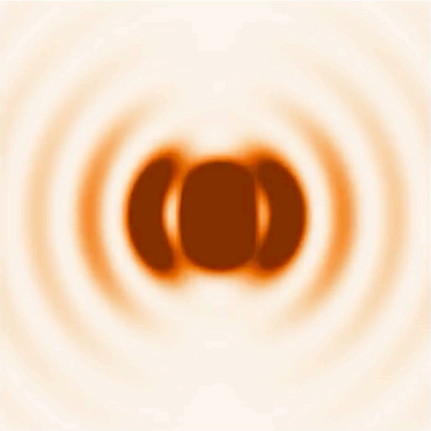
bounded periodic solutions

$$\rho = -g_{a\gamma}\nabla\phi\cdot\bar{\boldsymbol{B}} \qquad \boldsymbol{J} = g_{a\gamma}\left(\dot{\phi}\bar{\boldsymbol{B}} + \nabla\phi\times\bar{\boldsymbol{E}}\right)$$

leading order in $g_{a\gamma}\varphi_0$ only

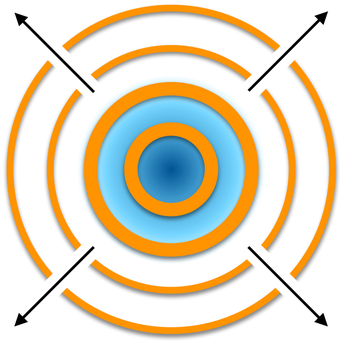
$$\frac{dP^{\gamma}_{(2)}}{d\Omega} = \frac{g_{a\gamma}^2\omega^4\tilde{\varphi}^2(\omega)}{32\pi^2}\left[\left(\hat{\boldsymbol{x}}\times\bar{\boldsymbol{B}}\right)^2 + \left(\hat{\boldsymbol{x}}\times\bar{\boldsymbol{E}}\right)^2 - 2\hat{\boldsymbol{x}}\cdot\left(\bar{\boldsymbol{E}}\times\bar{\boldsymbol{B}}\right)\right]\left(1+\cos\left(2\omega t - 2\omega|\boldsymbol{x}|\right)\right)$$

dipole radiation



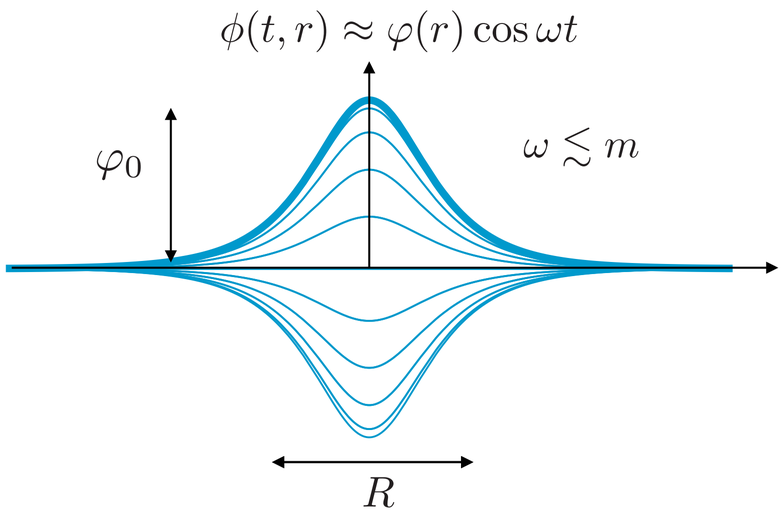
EM radiation & soliton profile

leading order in $g_{a\gamma}\varphi_0$ only



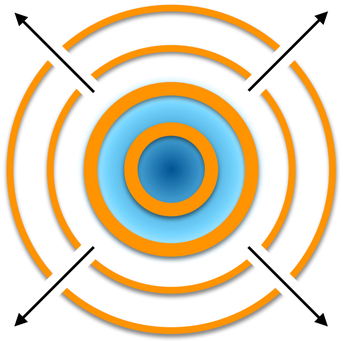
$$\left\langle \frac{dP^{\gamma}_{(2)}}{d\Omega} \right\rangle_t = \frac{g_{a\gamma}^2 \omega^4 \tilde{\varphi}^2(\omega)}{32\pi^2} \bar{\boldsymbol{B}}^2 \sin^2\theta$$

spatial Fourier transform of soliton profile at radiating frequency wavenumber



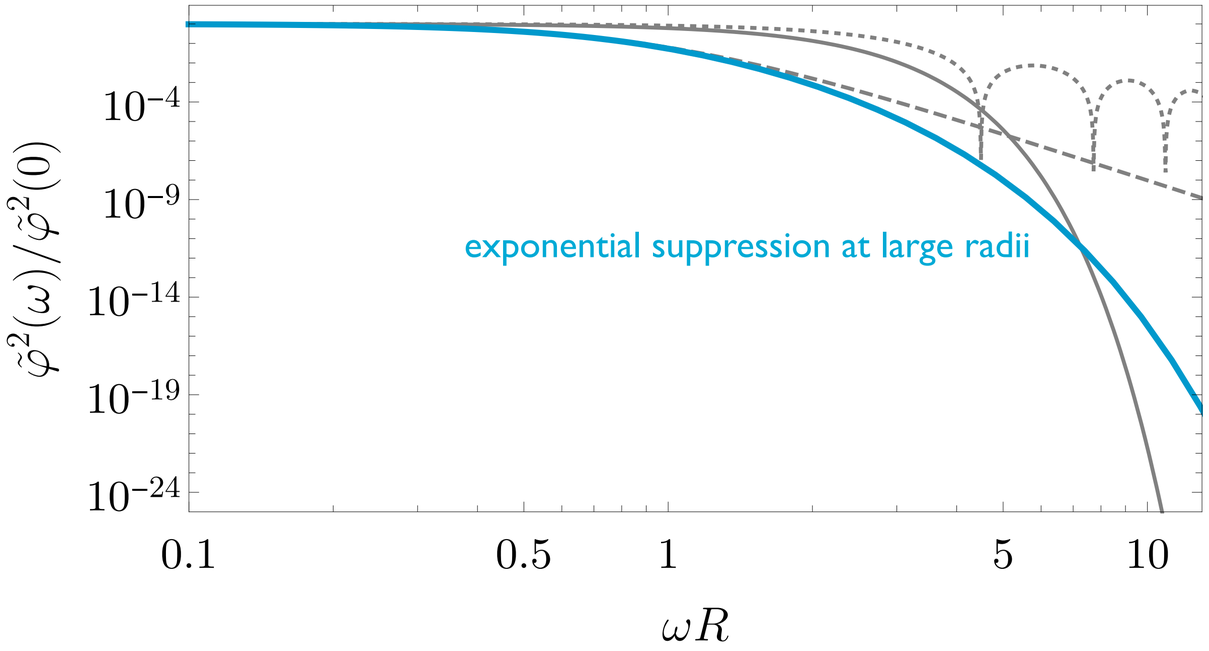
EM radiation & soliton profile

leading order in $g_{a\gamma}\varphi_0$ only



$$\left\langle \frac{dP^{\gamma}_{(2)}}{d\Omega} \right\rangle_t = \frac{g_{a\gamma}^2 \omega^4 \tilde{\varphi}^2(\omega)}{32\pi^2} \bar{\boldsymbol{B}}^2 \sin^2\theta$$

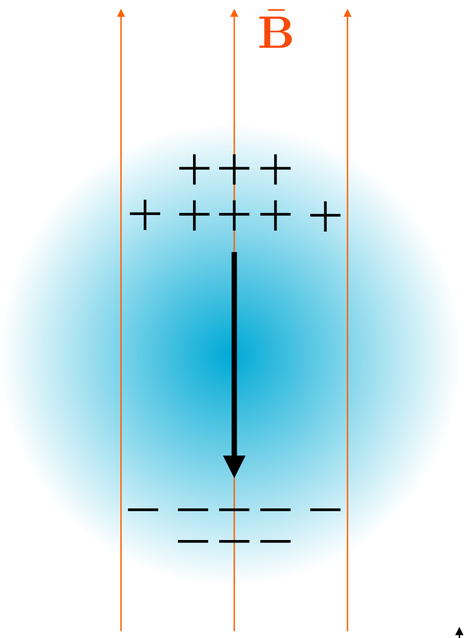
spatial Fourier transform of soliton profile at radiating frequency wavenumber



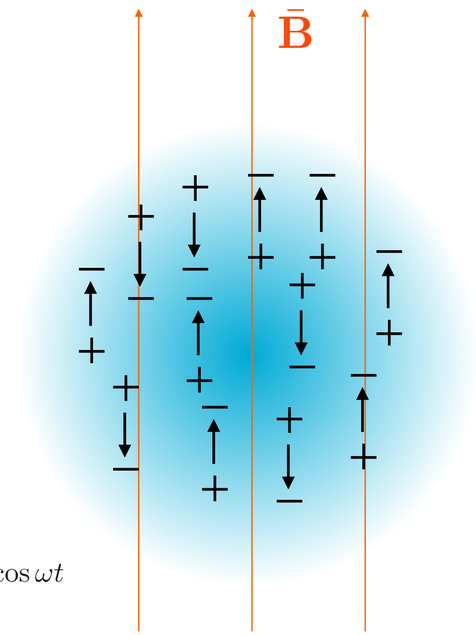
1. use correct physical profile
2. focus on dense, small radius solitons

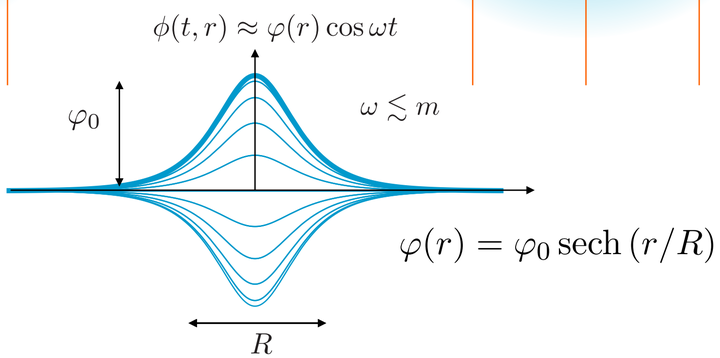
what is the relevance of the soliton? — interference effects

soliton — 1 dipole



N incoherent dipoles





ratio of radiated power

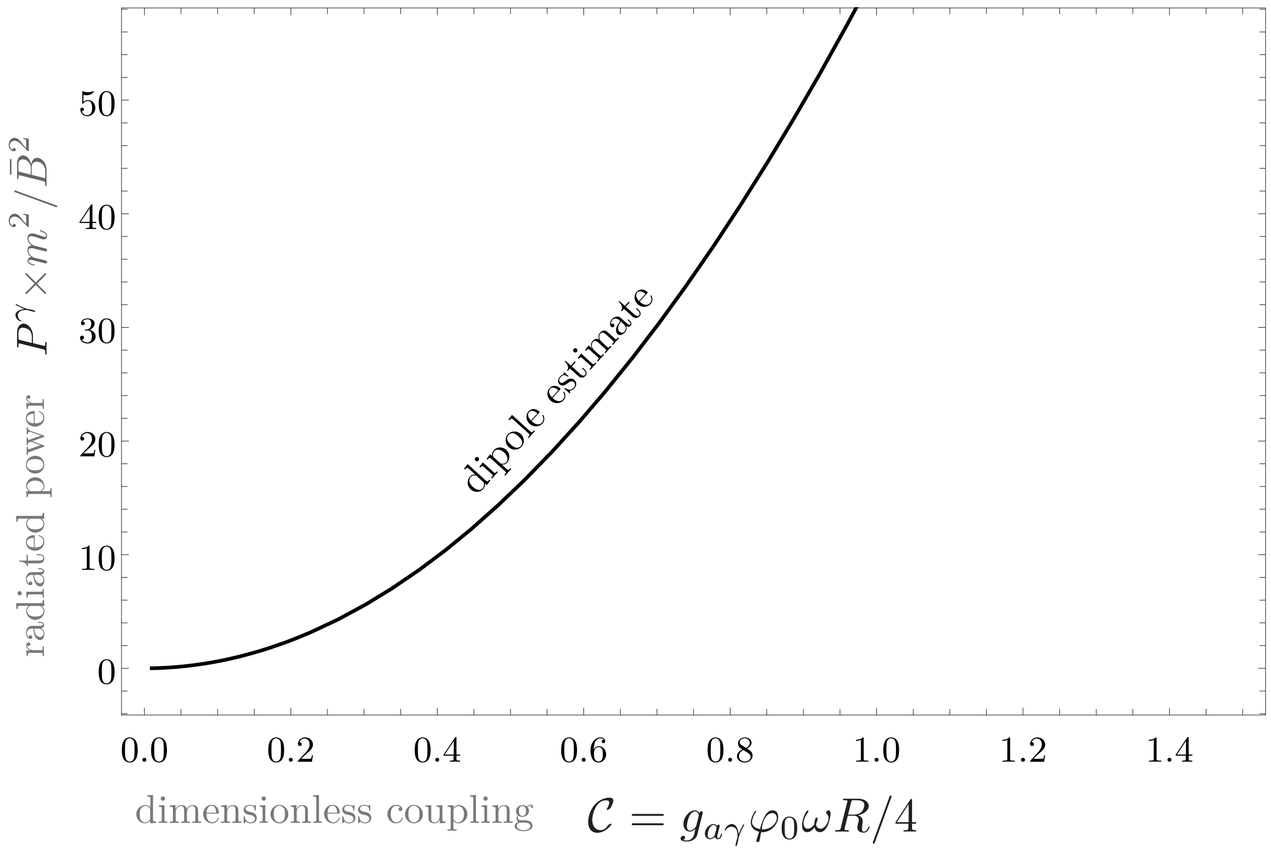
$$\frac{N \text{ incoherent dipoles}}{\text{coherent soliton}} \sim \frac{e^{\pi\omega R}}{N}$$

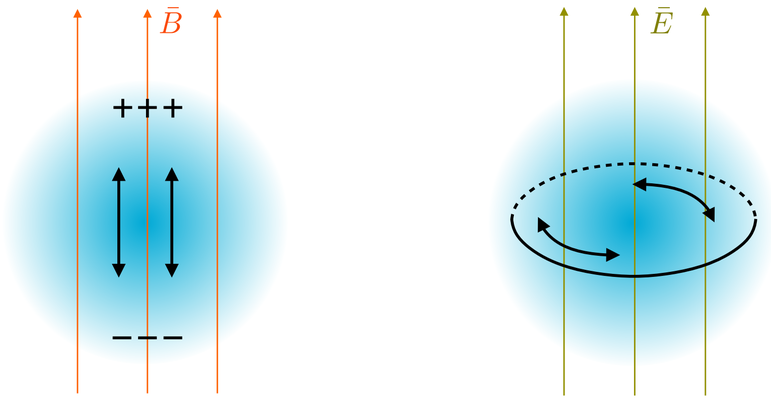
lose power by sub-dividing *too* much

EM radiation at moderate coupling

higher order in $g_{a\gamma}\varphi_0$

$$\mathcal{C} \sim g_{a\gamma}\varphi_0 \omega R \lesssim 1$$



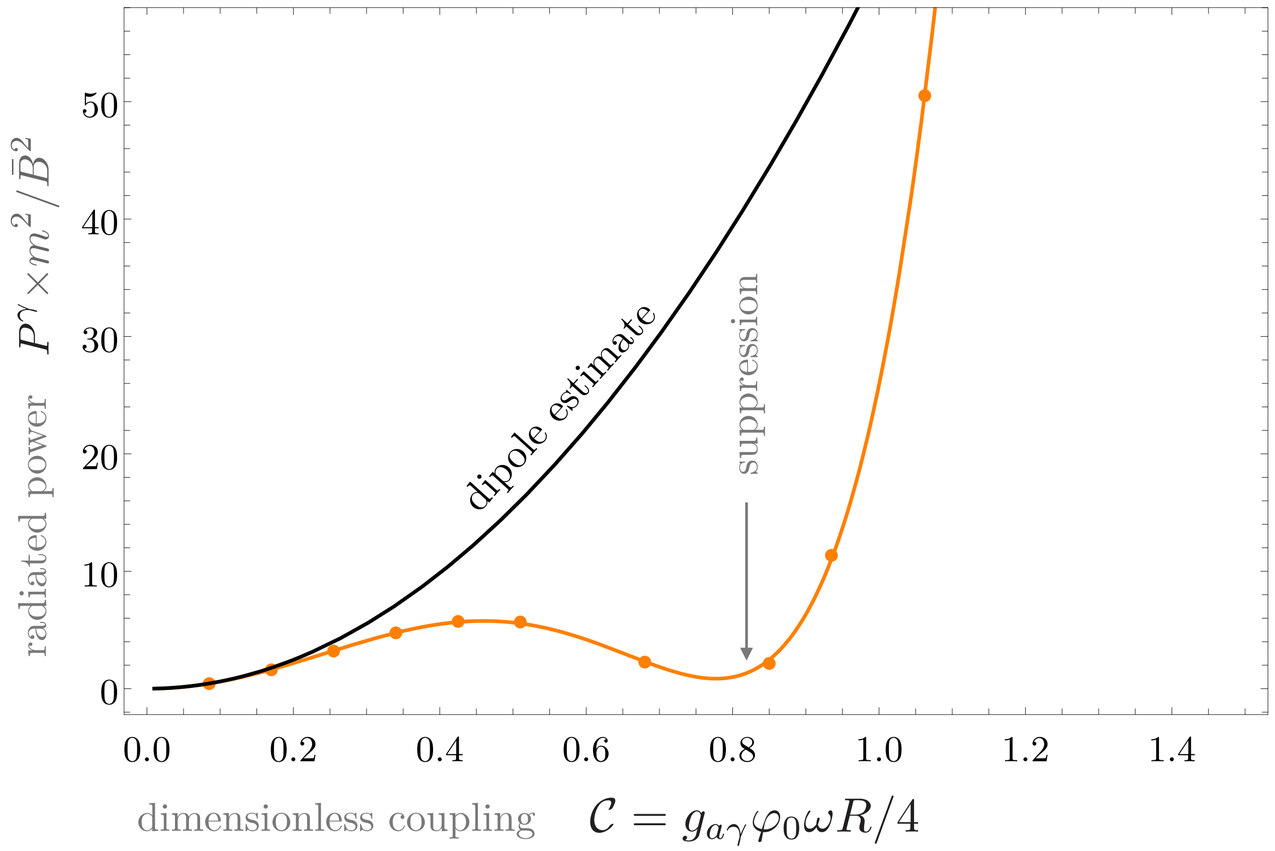


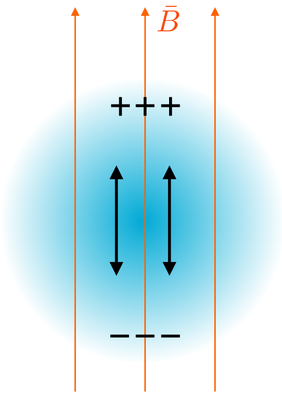
*keep soliton fixed, change axion-photon coupling

EM radiation at moderate coupling

higher order in $g_{a\gamma}\varphi_0$

$$\mathcal{C} \sim g_{a\gamma}\varphi_0 \omega R \lesssim 1$$





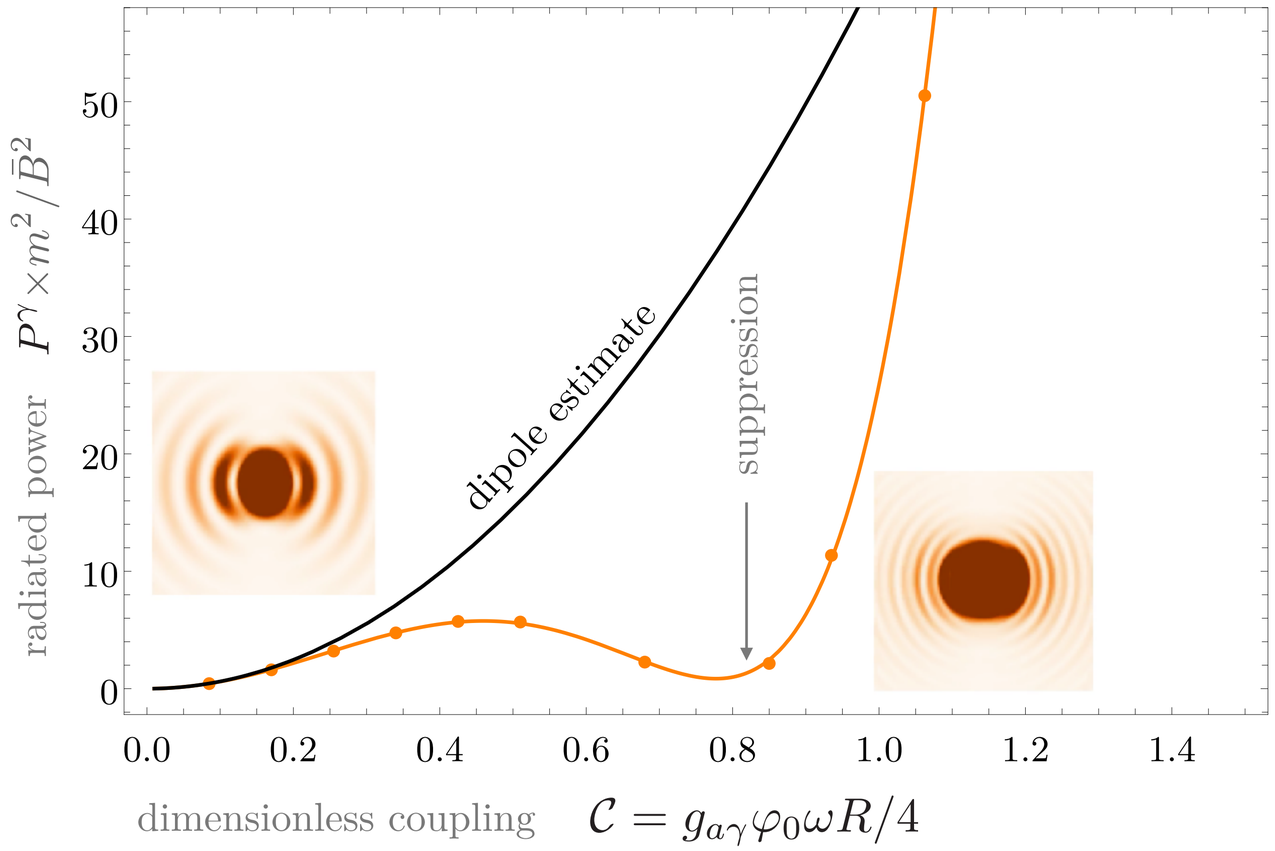
- dipole est. works well for small coupling (as expected)
  - suppression for B background at intermediate coupling !

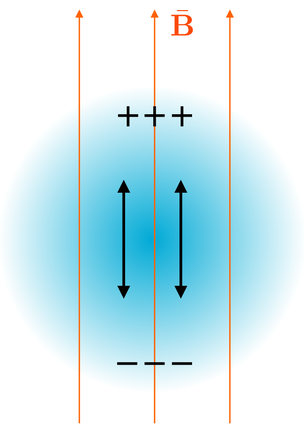
*keep soliton fixed, change axion-photon coupling

EM radiation at moderate coupling

higher order in $g_{a\gamma}\varphi_0$

$$\mathcal{C} \sim g_{a\gamma}\varphi_0 \omega R \lesssim 1$$





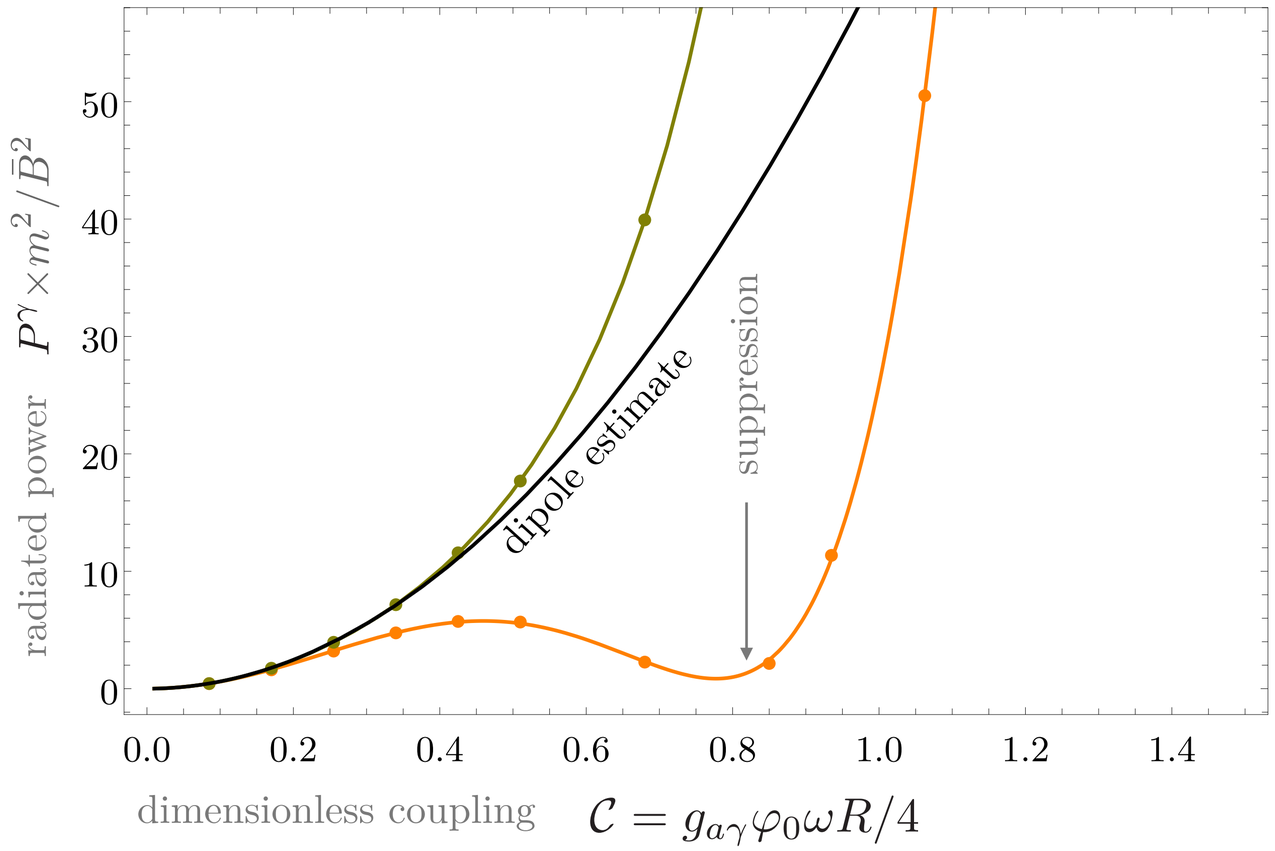
- dipole est. works well for small coupling (as expected)
  - suppression for B background at intermediate coupling !
  - change in dominant frequency of radiation

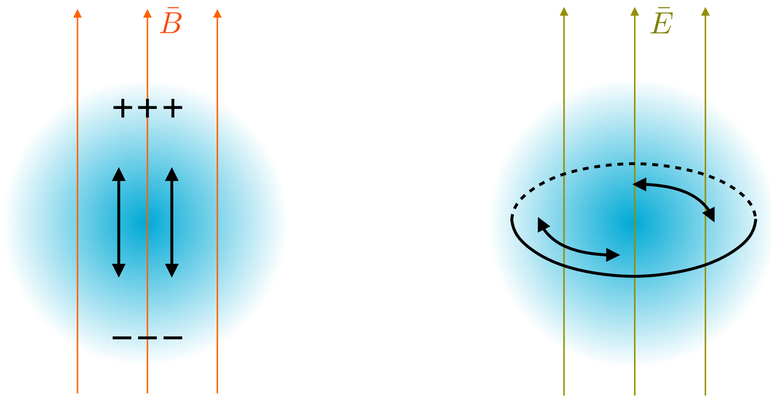
*keep soliton fixed, change axion-photon coupling

steady EM radiation at moderate coupling

higher order in $g_{a\gamma}\varphi_0$

$\mathcal{C} \sim g_{a\gamma}\varphi_0 \omega R \lesssim 1$





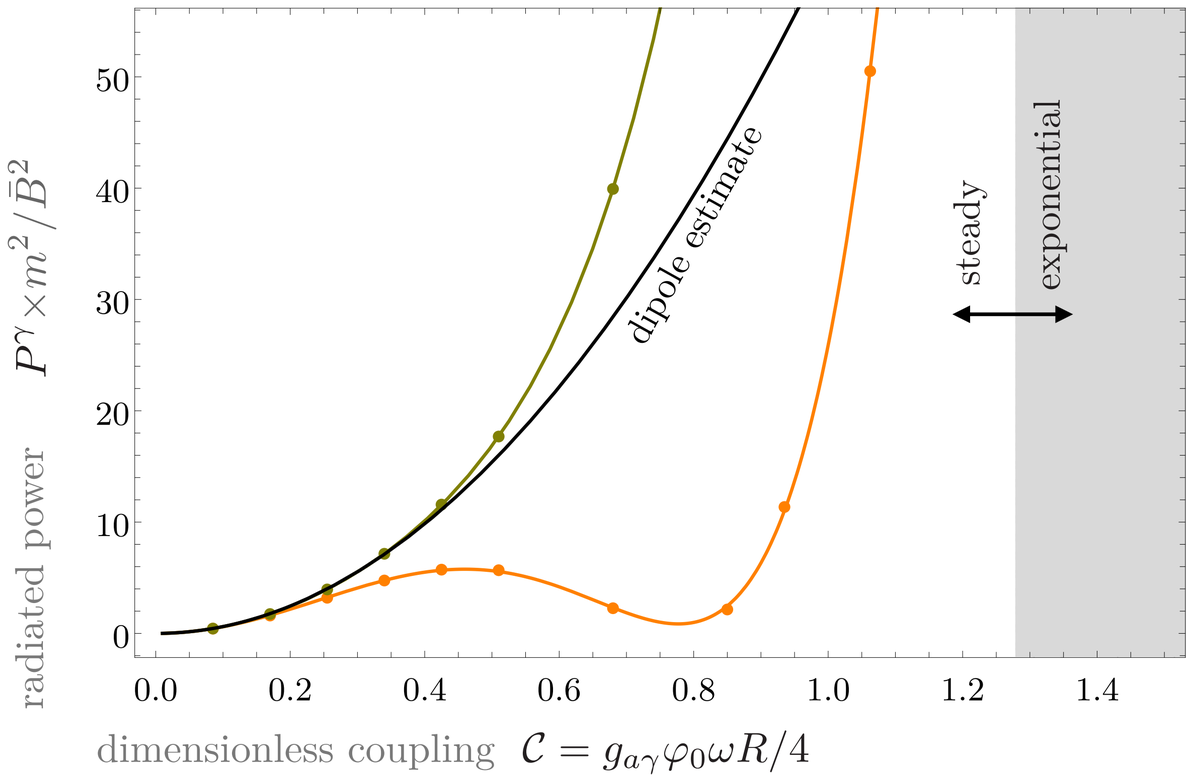
- dipole est. works well for small coupling (as expected)
  - suppression for B background at intermediate coupling !
  - difference between E and B begins to show

*keep soliton fixed, change axion-photon coupling

explosive radiation

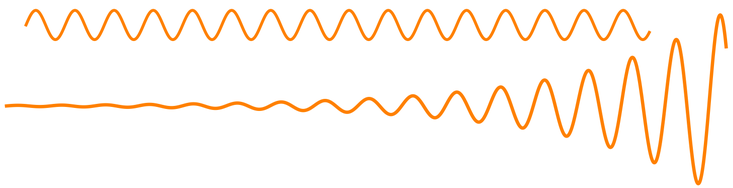
non perturbative $g_{a\gamma}\varphi_0$

$\mathcal{C} \sim g_{a\gamma}\varphi_0\omega R$



$\mathcal{C} \ll 1 \longrightarrow$

$\mathcal{C} \gtrsim 1 \longrightarrow$



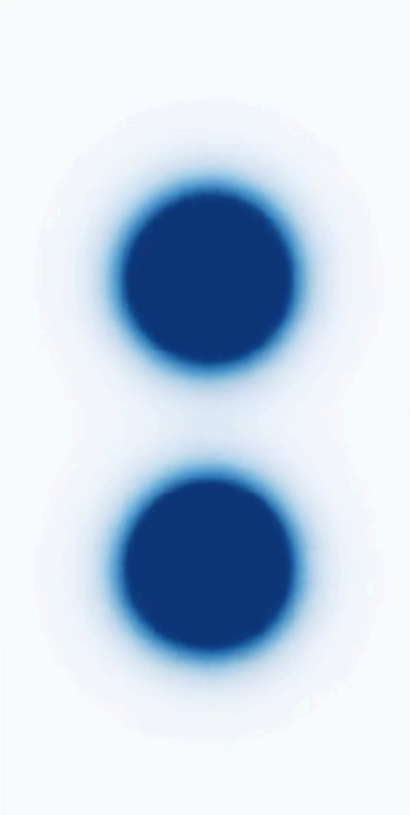
- parametric resonance does not even require background EM fields, quantum fluctuations are enough
- can be efficient for works for dilute and dense cases [see Hertzberg & Schiappacasse (2018), MA & Mou (2020)]

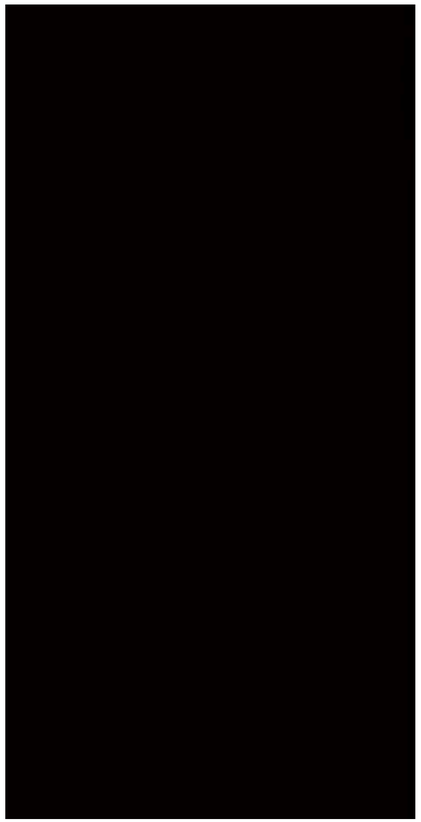
*keep soliton fixed, change axion-photon coupling

explosive production from soliton mergers

MA & Mou (2020)

- no emission before merger $\mathcal{C} \lesssim 1$
- explosive after merger $\mathcal{C} \gtrsim 1$
- a threshold & resonant effect



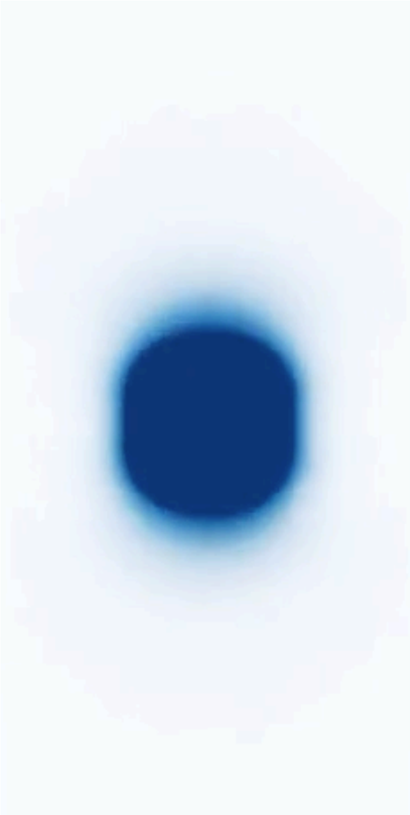


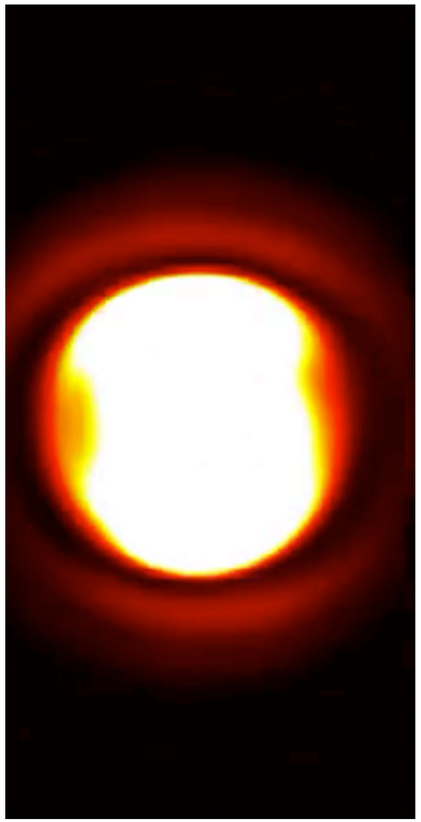
*simulation for dense, self-interaction supported oscillons. For analytics of dilute case see Hertzberg & Schippanaise (2018).

explosive photon production from soliton mergers

~30% of total energy goes into axion waves

~20% of remaining goes into EM radiation





$$E_\gamma \sim 0.1 \times M_{\rm osc} c^2 \sim 10^{35} \left(\frac{f}{10^{10}\,{\rm GeV}}\right)^2 \left(\frac{10^{-5}{\rm eV}}{m}\right) {\rm GeV}$$

$$\nu_\gamma = \frac{\omega}{2\pi} \approx \frac{m}{2\pi} \simeq (2\ {\rm GHz}) \left(\frac{m}{10^{-5}\ {\rm eV}}\right)$$

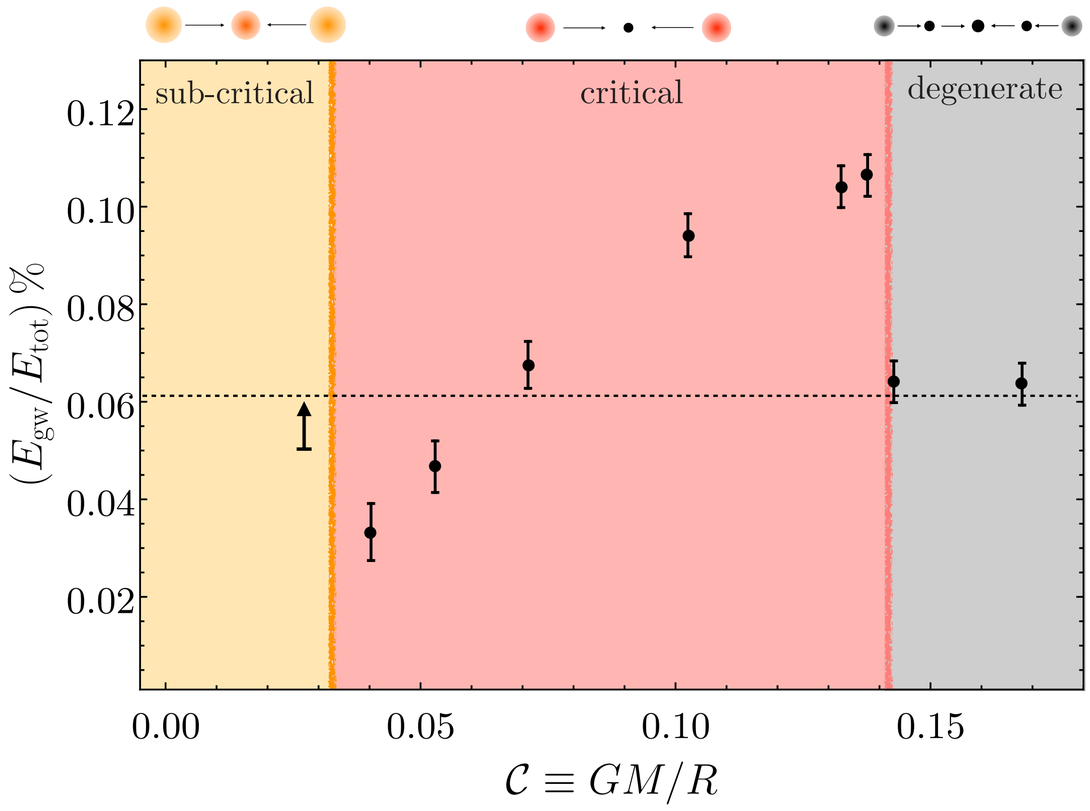
gravitational waves from collisions ? (a small digression)

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \cancel{V_{\rm nl}(\phi)}$$

ignore self-interactions

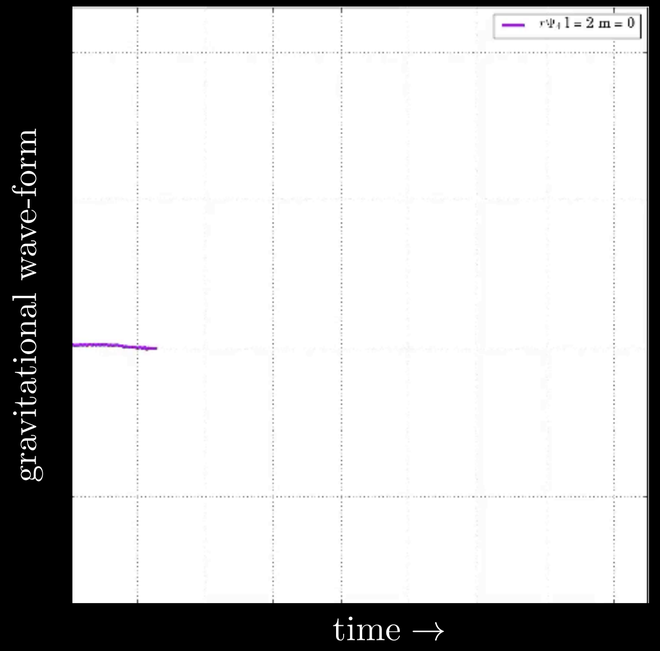
interested in gravitational wave emission from ultra-compact solitons

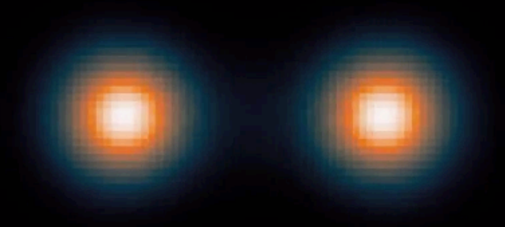



Helfer, Lim, Garcia & MA (2018)

gravitational waves ?

sub-critical collisions (no black hole formation)

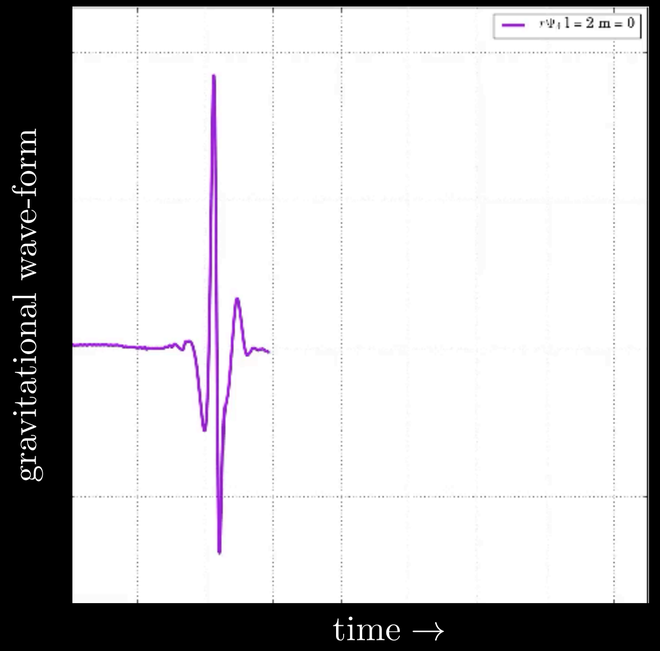




using GRChombo

gravitational waves ?

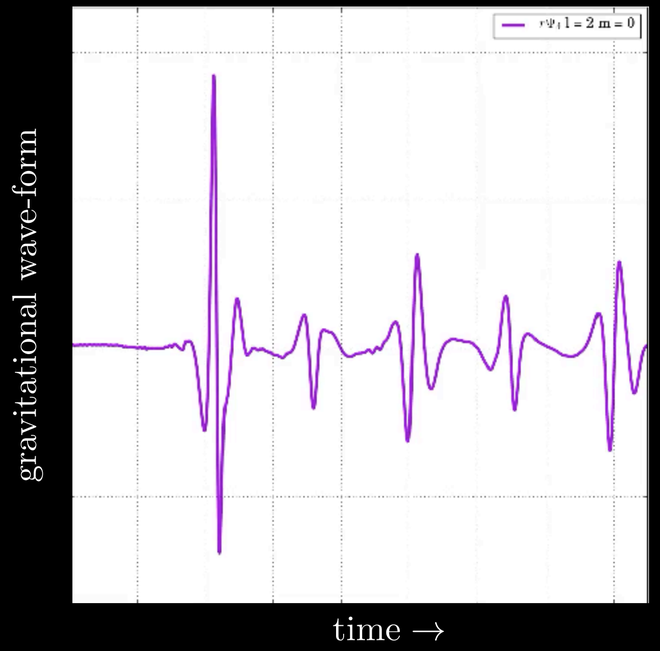
sub-critical collisions (no black hole formation)



using GRChombo

gravitational waves ?

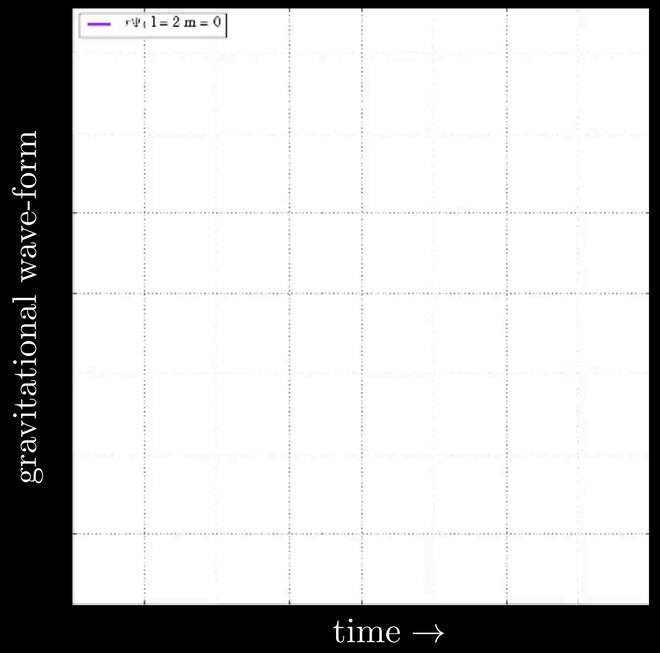
sub-critical collisions (no black hole formation)



using GRChombo

gravitational waves ?

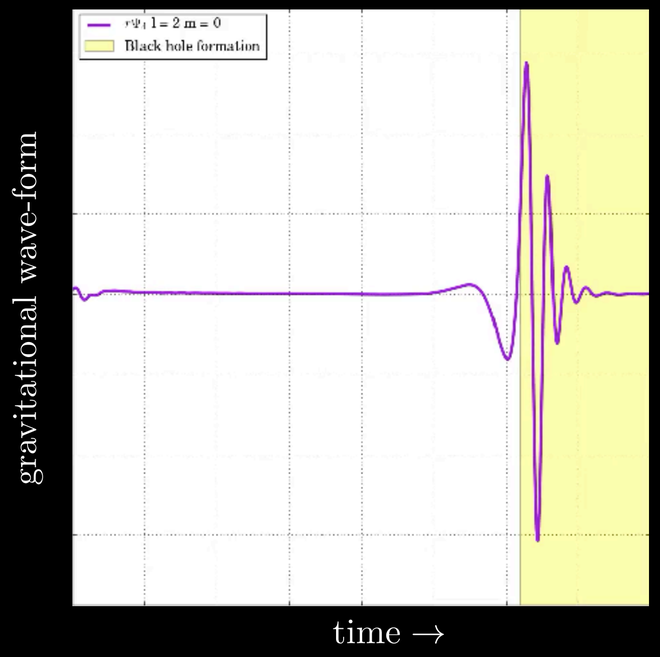
critical collisions (black hole formation)



using GRChombo

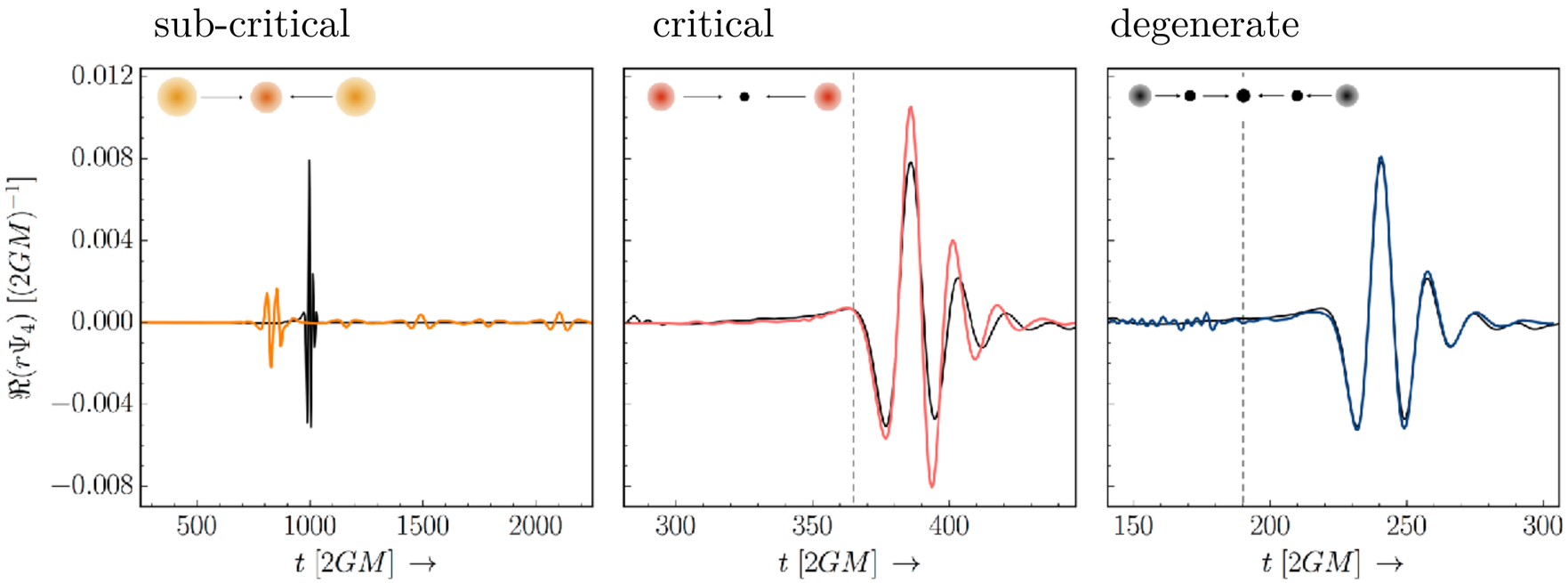
gravitational waves ?

critical collisions (black hole formation)



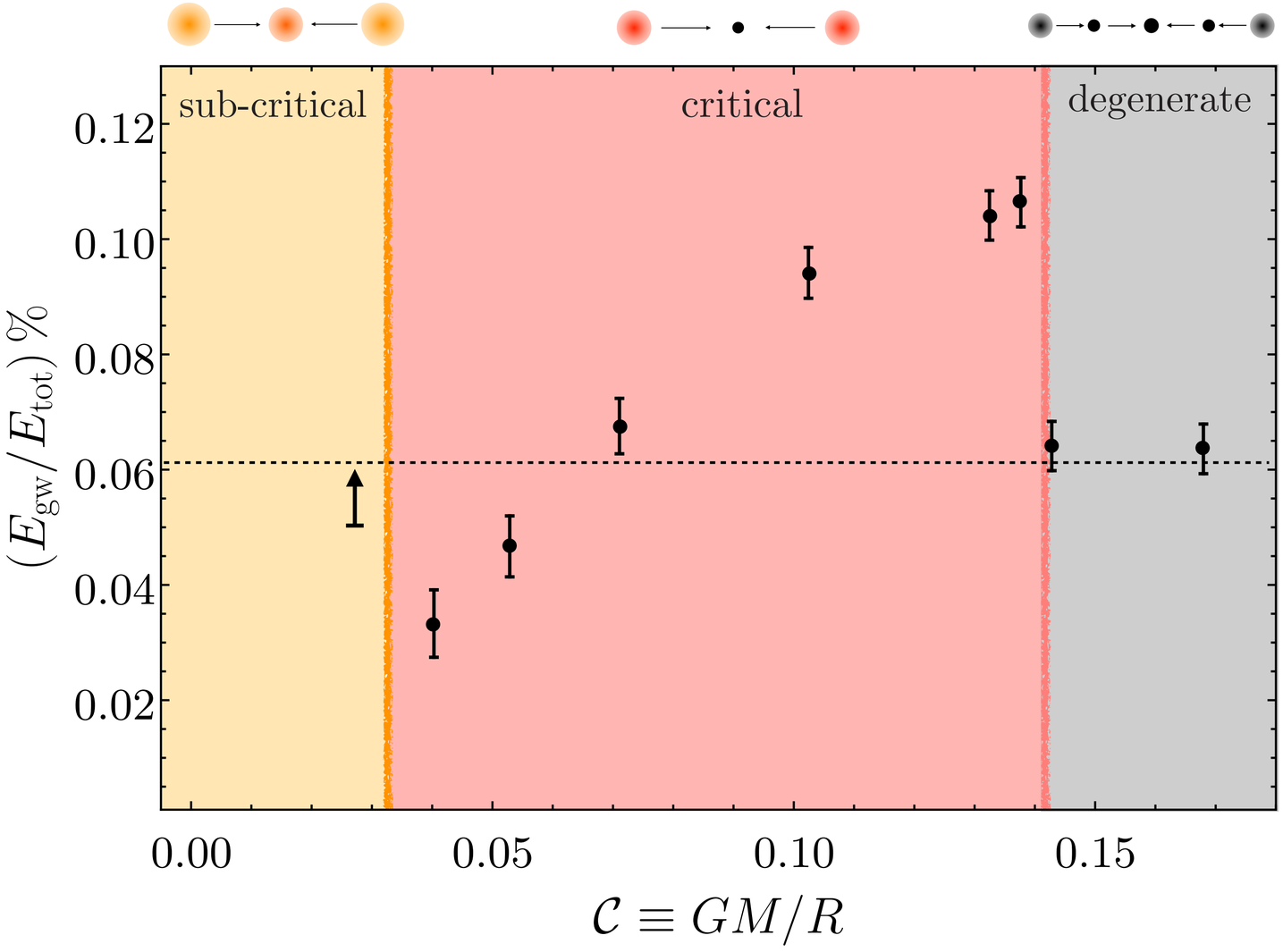
using GRChombo

gravitational waves from collisions ?

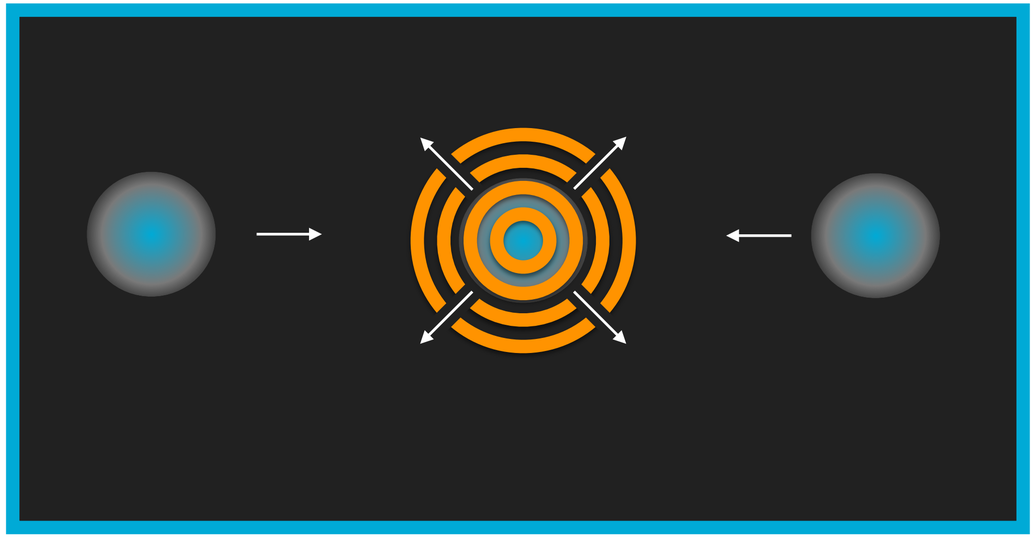


black = corresponding mass black hole g-wave signal

gravitational wave emission and compactness

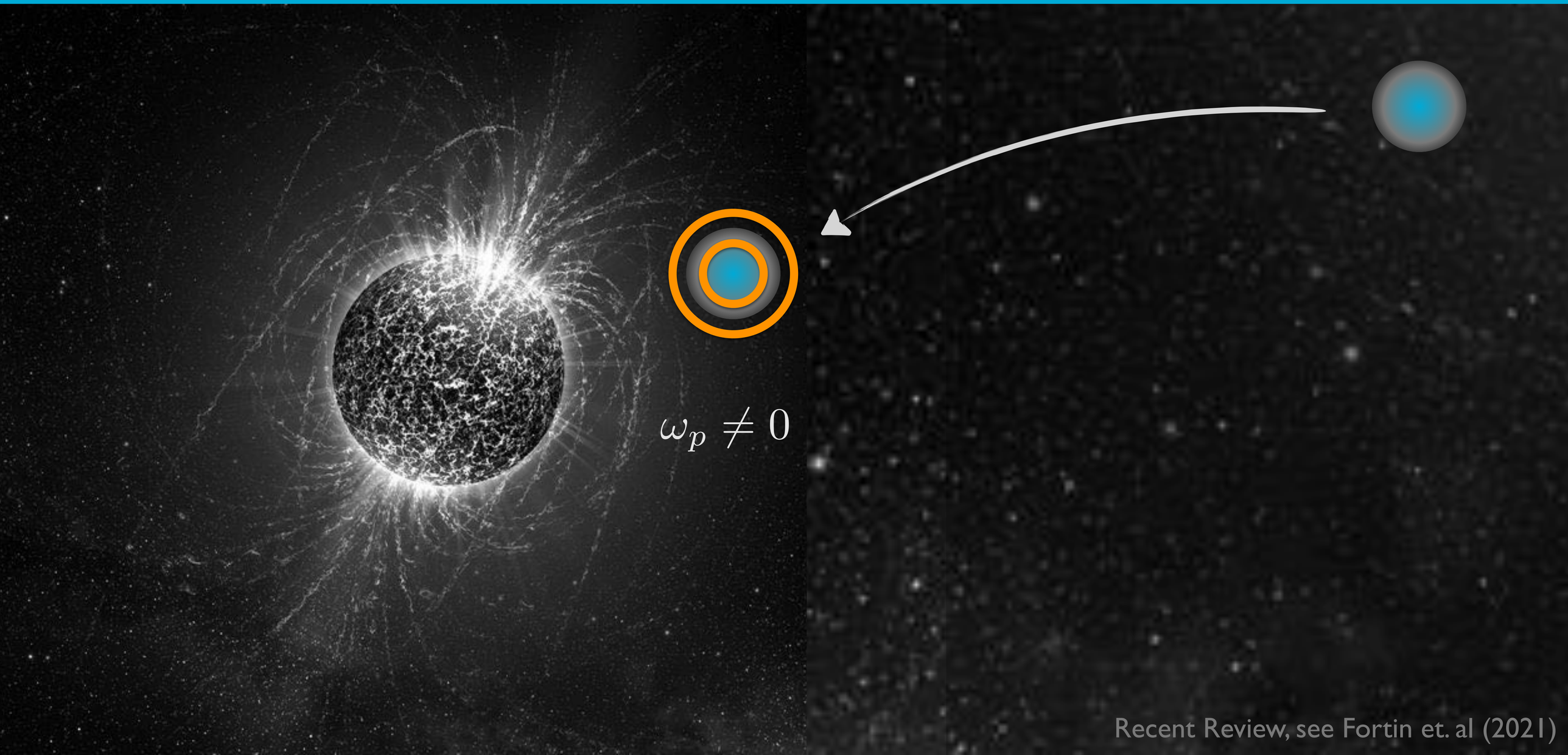


multi-messenger signals ?



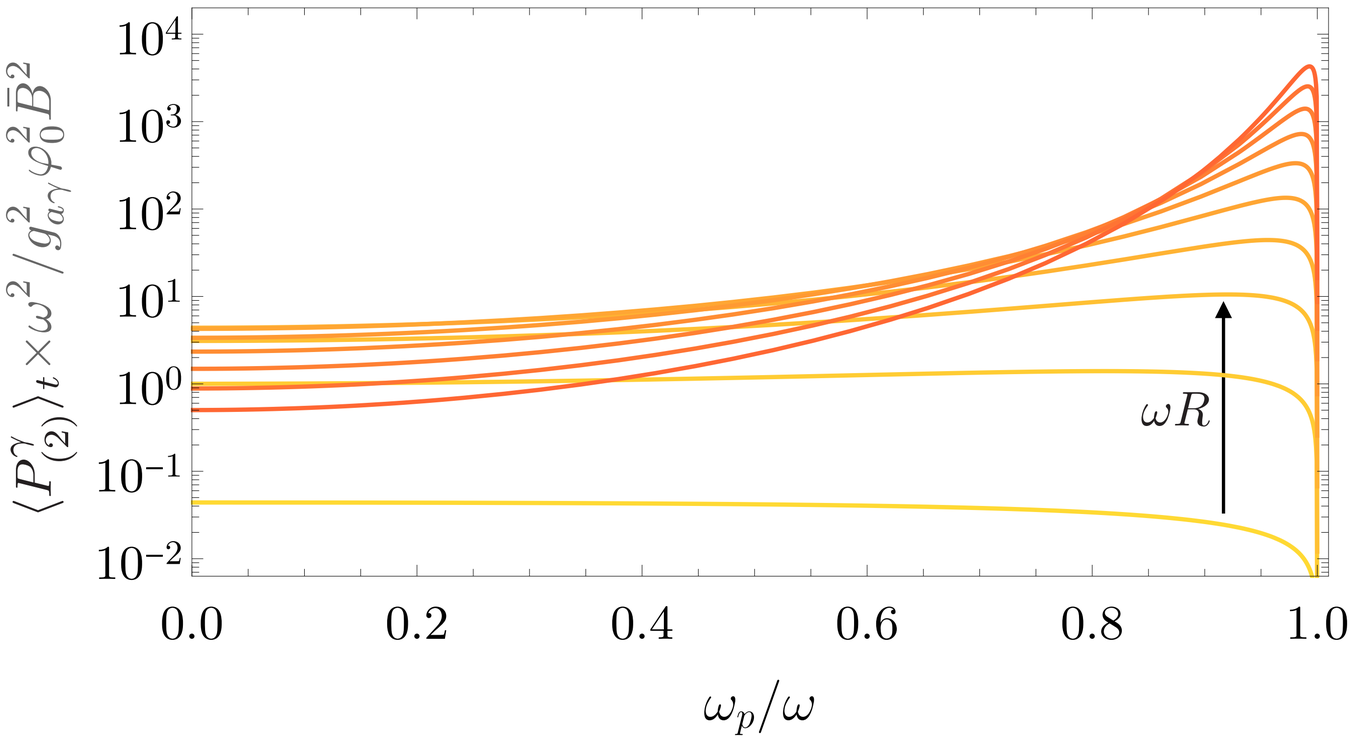
LIGO LabVirgo

solitons falling on to compact stars — plasma effects ?



Recent Review, see Fortin et. al (2021)

solitons falling on to compact stars — resonant conversion!



$$\langle P^\gamma_{(2)}\rangle_t = \frac{g^2_{a\gamma}\omega^4}{12\pi}\frac{\kappa}{\omega}\tilde{\varphi}^2(\kappa)\left(\bar{\boldsymbol{B}}^2 + \bar{\boldsymbol{E}}^2\right)$$

where $\kappa \equiv \sqrt{\omega^2 - \omega_p^2}$

$\omega_p \approx \sqrt{4\pi\alpha n_e/m_e}$ plasma frequency

*This resonance is NOT parametric resonance discussed earlier

some numbers

$$M_{\rm sol} \sim 10^2 f^2/m \simeq (2\times 10^9\ {\rm kg}) \left(\frac{f}{10^{10}\ {\rm GeV}}\right)^2 \left(\frac{m}{10^{-5}\ {\rm eV}}\right)^{-1}$$

$$R_{\rm sol} \sim 2m^{-1} \simeq (4\ {\rm cm}) \left(\frac{m}{10^{-5}\ {\rm eV}}\right)^{-1} .$$

* dense regime only, different for dilute.

*Conservative axion star - compact star interaction rate, there is freedom to change these numbers by orders of magnitude

$$\Gamma \simeq (4\times 10^{-5}\ {\rm hr}^{-1}) \left(\frac{M_\star}{1\,M_\odot}\right) \left(\frac{R_\star}{0.01\,R_\odot}\right) \left(\frac{\rho_{\rm as}}{0.3\ {\rm GeV/cm^3}}\right) \left(\frac{M_{\rm sol}}{10^9\ {\rm kg}}\right)^{-1} \left(\frac{v_{\rm rel}}{10^{-3}}\right)^{-1}$$

some numbers

$$S \simeq (2 \times 10^7\ \mu\text{Jy}) \left(\frac{d_\star}{100\ \text{pc}}\right)^{-2} \left(\frac{m}{10^{-5}\ \text{eV}}\right)^{-3} \left(\frac{g_{a\gamma}}{0.66 \times 10^{-10}\ \text{GeV}^{-1}}\right)^2 \times \left(\frac{f}{10^{10}\ \text{GeV}}\right)^{-2} \left(\frac{\bar{\boldsymbol{B}}}{10^{10}\ \text{G}}\right)^2 \mathcal{F}(\omega R, \omega_p/\omega)\ ,$$

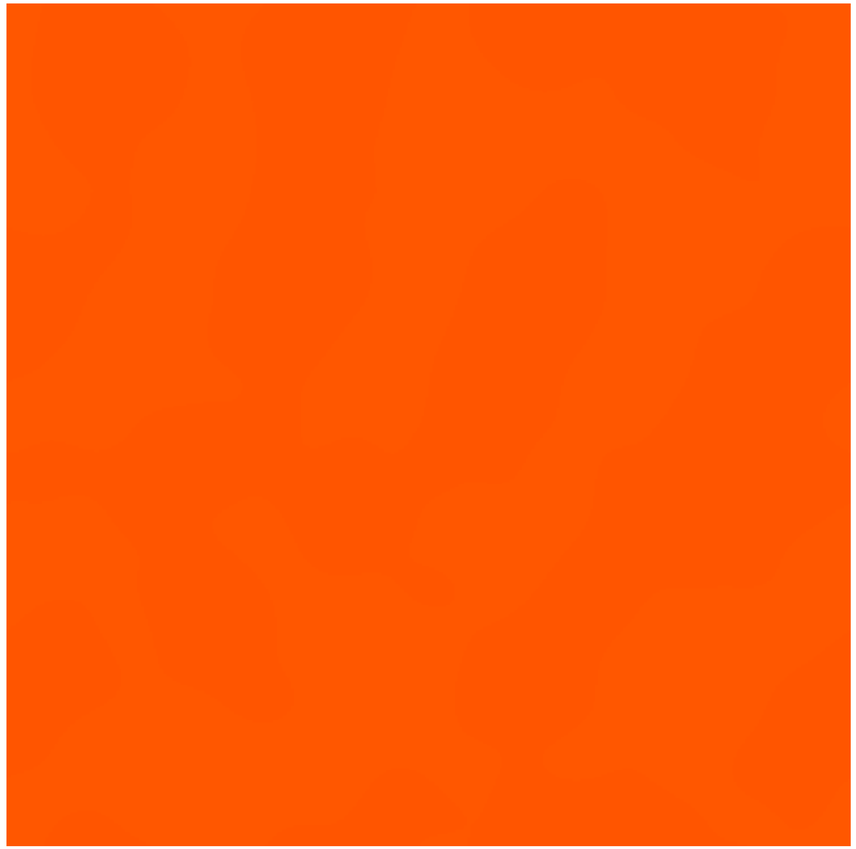
$$\nu_\gamma = \frac{\omega}{2\pi} \approx \frac{m}{2\pi} \simeq (2\ \text{GHz}) \left(\frac{m}{10^{-5}\ \text{eV}}\right)$$

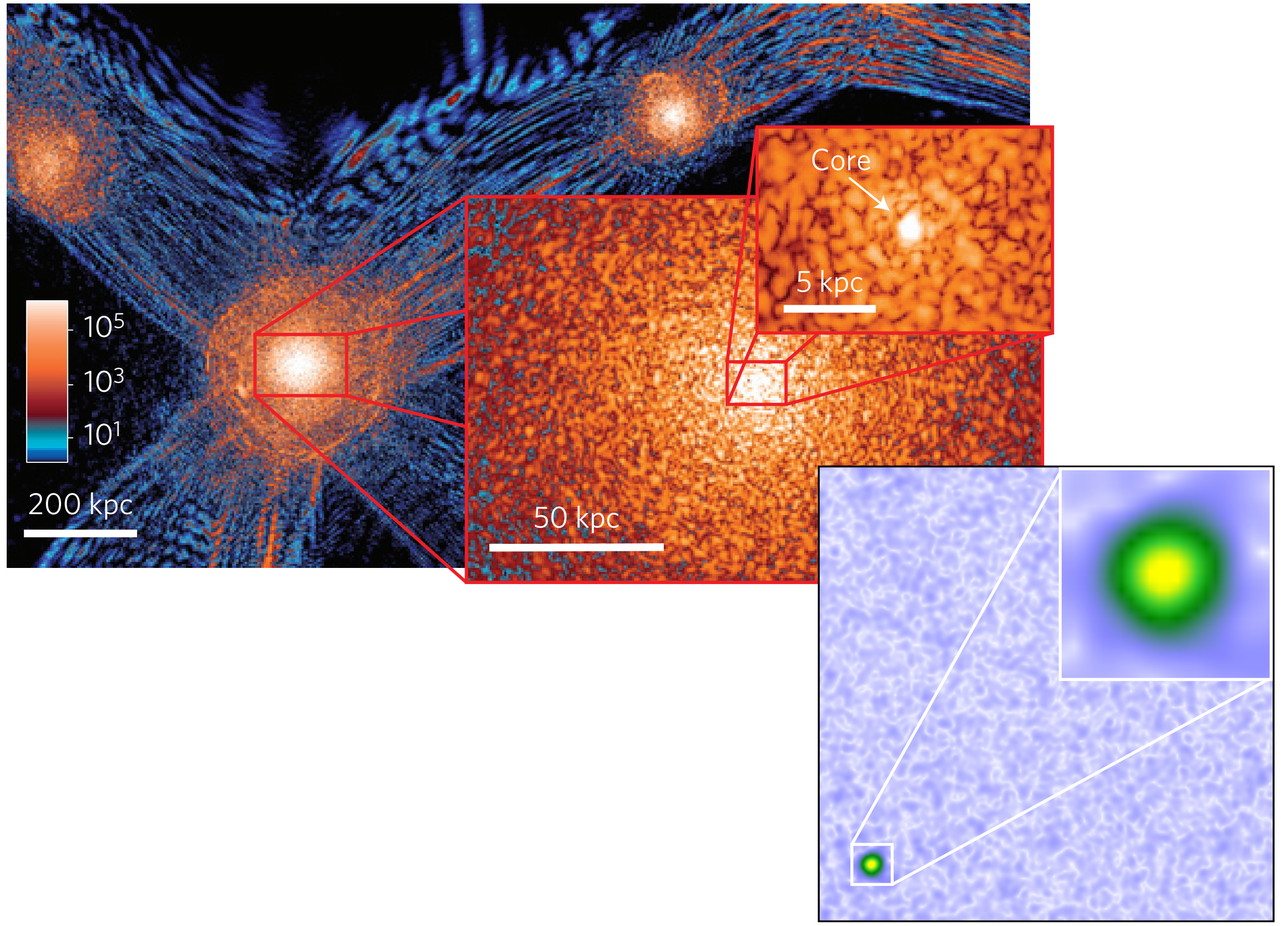
$$\mathcal{F}(\omega R, \omega_p/\omega) \approx \begin{cases} (\pi\omega R)^4 e^{-\pi\omega R}, & \text{for } \omega_p \approx 0 \\ \frac{1}{16}(\pi\omega R)^6 \sqrt{1 - \omega_p^2/\omega^2}, & \text{for } \omega_p \approx \omega \end{cases}$$

* does *not* include large coupling regime.

rates depend on soliton formation mechanisms, lifetimes …

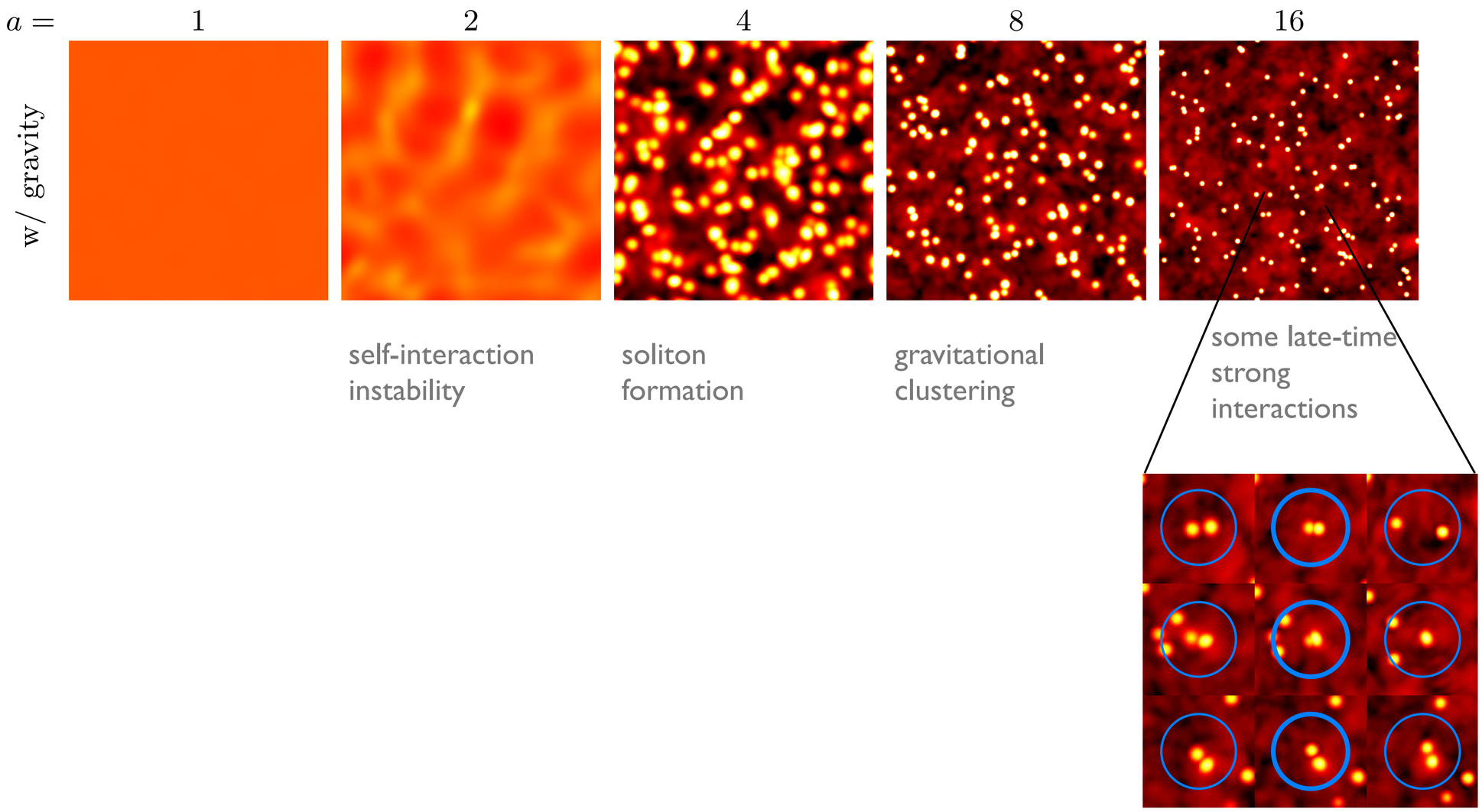
"solitons" in axion-like fields can form naturally in the inflaton and in dark matter





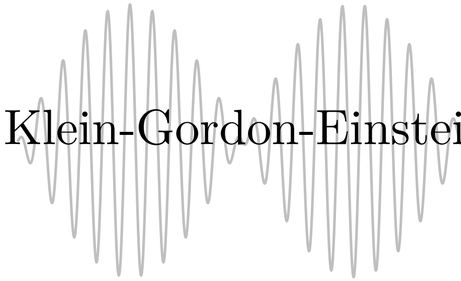
also phase transitions, nucleation around BHs, etc.

example: formation driven by self-interactions



MA & Mocz (2019)

relativistic to non-relativistic effective theory



Klein-Gordon-Einstein

integrate out ‘fast’ modes

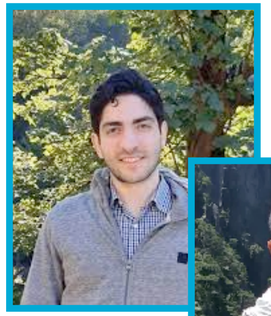
Nonrelativistic EFT for ‘slow’ modes

$=$ Schrödinger-Poisson $+$ corrections

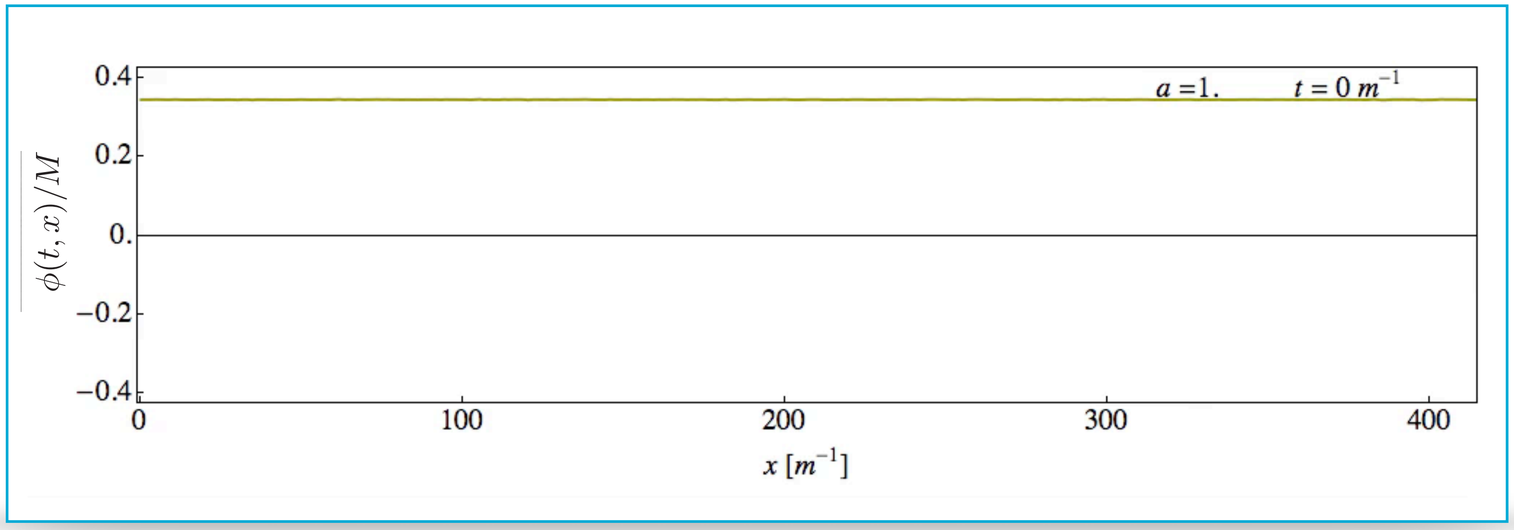
MA, Kaiser, Namjoo, Salehian, Zhang (soon!)

Namjoo, Guth and Kaiser (2018)

Salehian, Kaiser, Namjoo (2005)

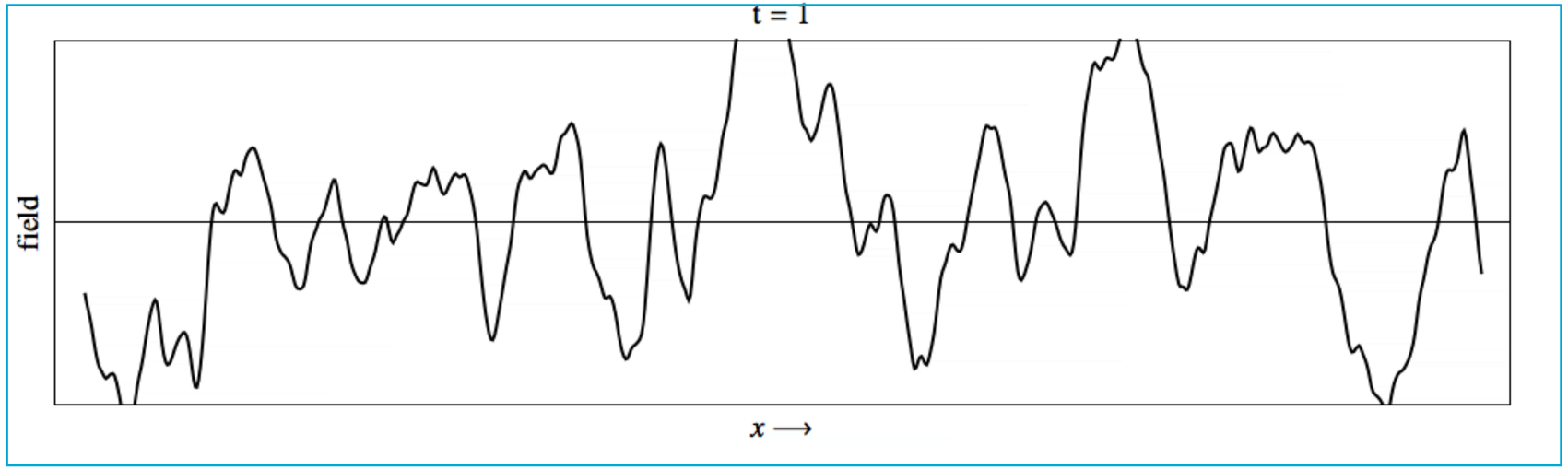


insensitivity to initial conditions

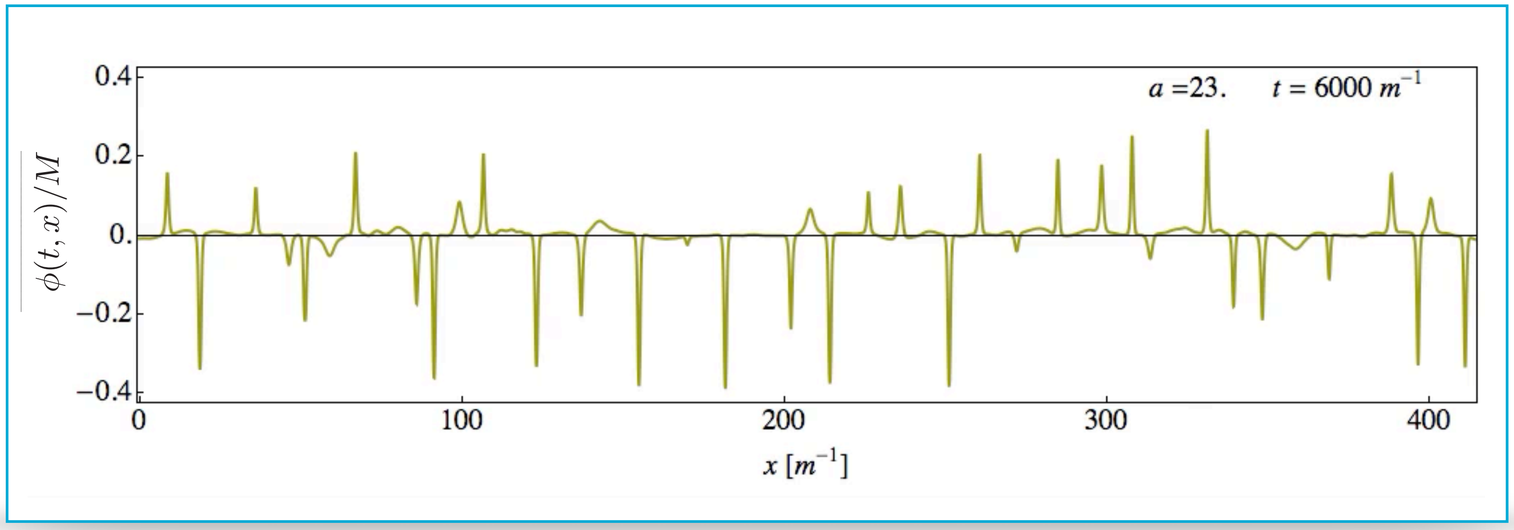


MA (2010)

based on Farhi et. al (2008)

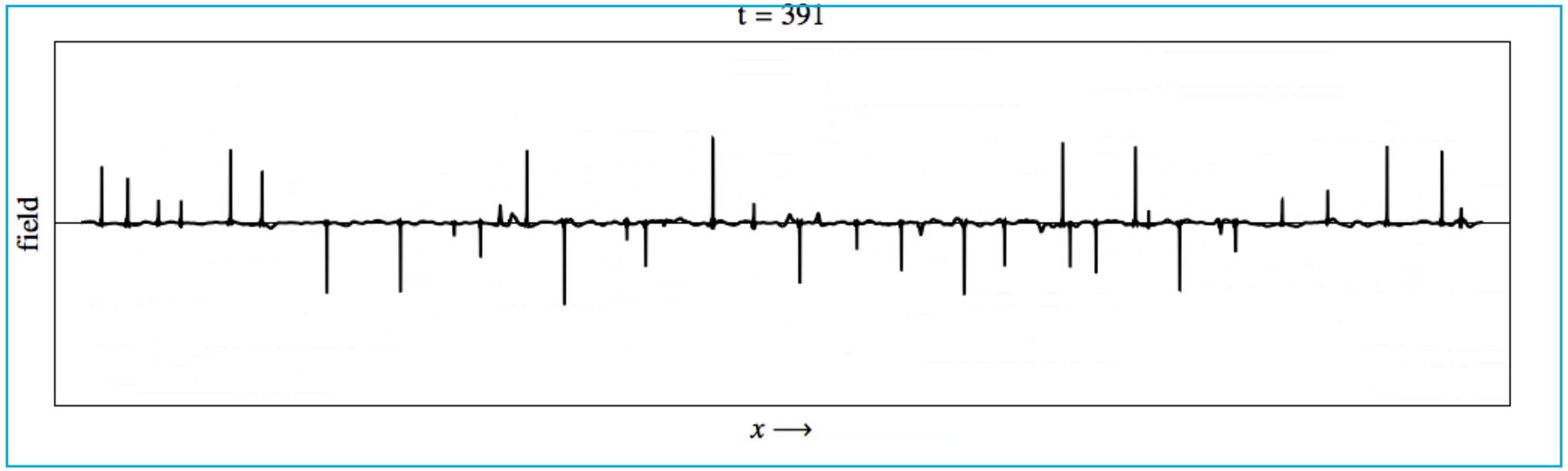


insensitivity to initial conditions

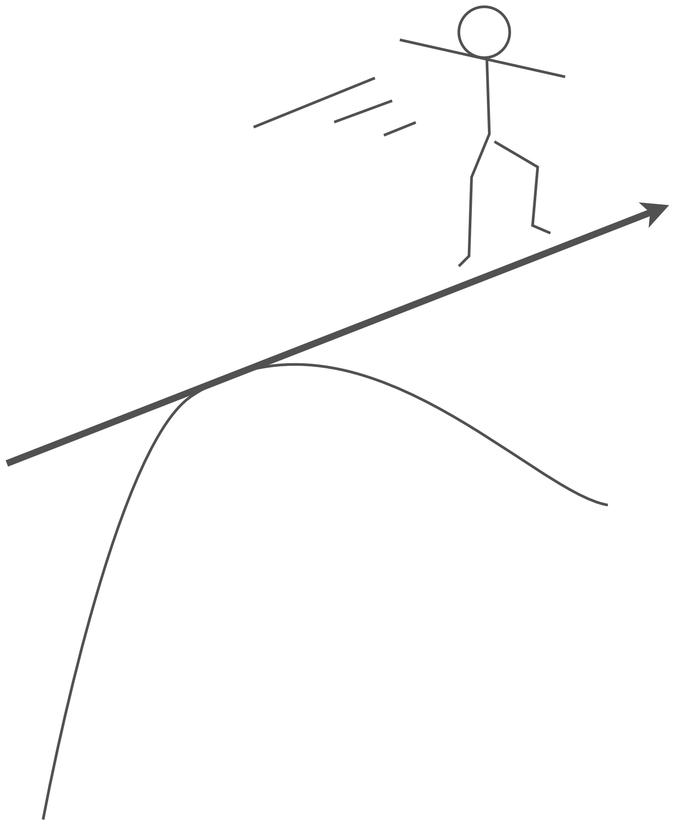


MA (2010)

based on Farhi et. al (2008)



tangential digression (advertisement)



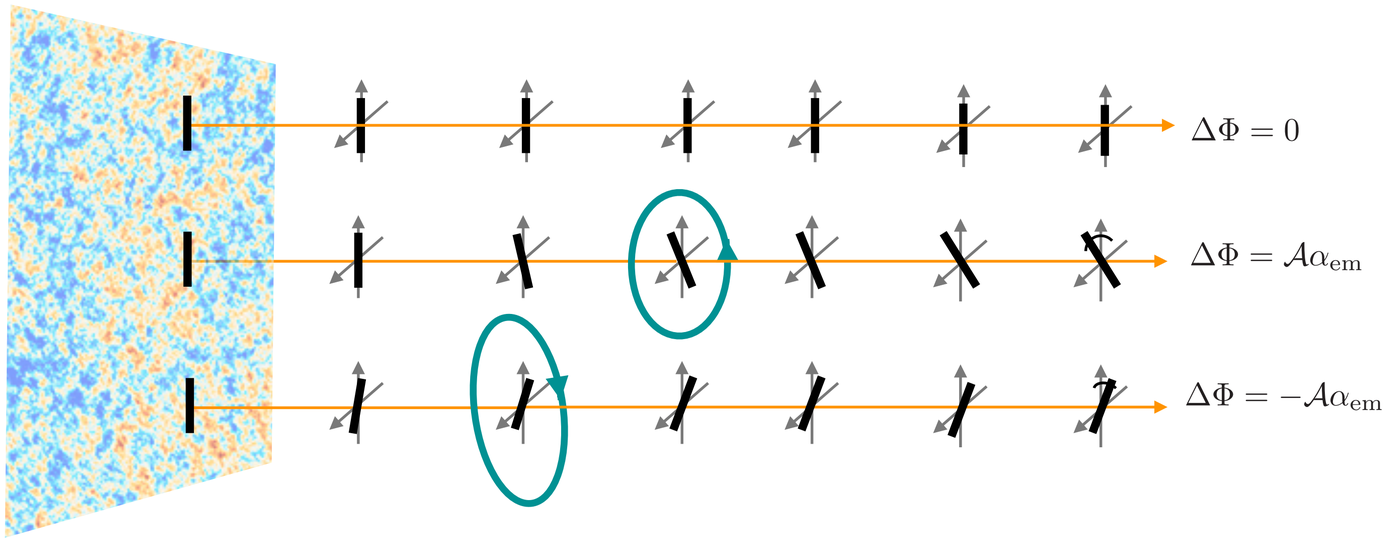
CMB birefringence from ultralight-axion string networks

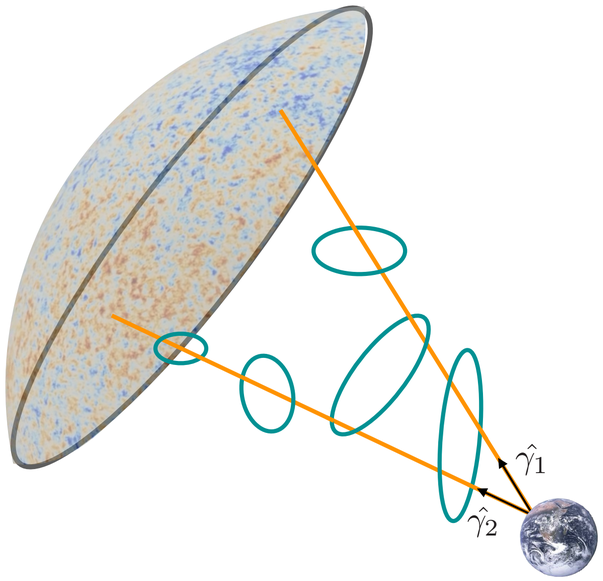
[arXiv:2103.10962]

with Andrew Long and **Mudit Jain**

$$\mathscr{L}_{\rm int} = -\frac{1}{4}\, g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Delta\Phi = \frac{g_{a\gamma\gamma}}{2}\int_C \mathrm{d}X^\mu\, \partial_\mu a(x) \;\; = \pm g_{a\gamma\gamma}\pi f_a = \pm\mathcal{A}\alpha_{\rm em}$$



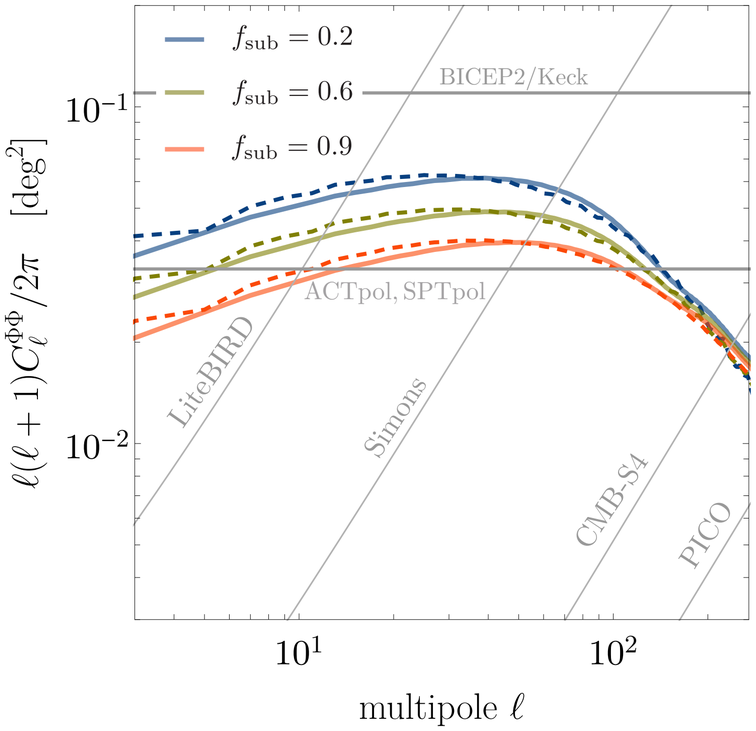


*see Agrawal, Hook & Huang (2019)

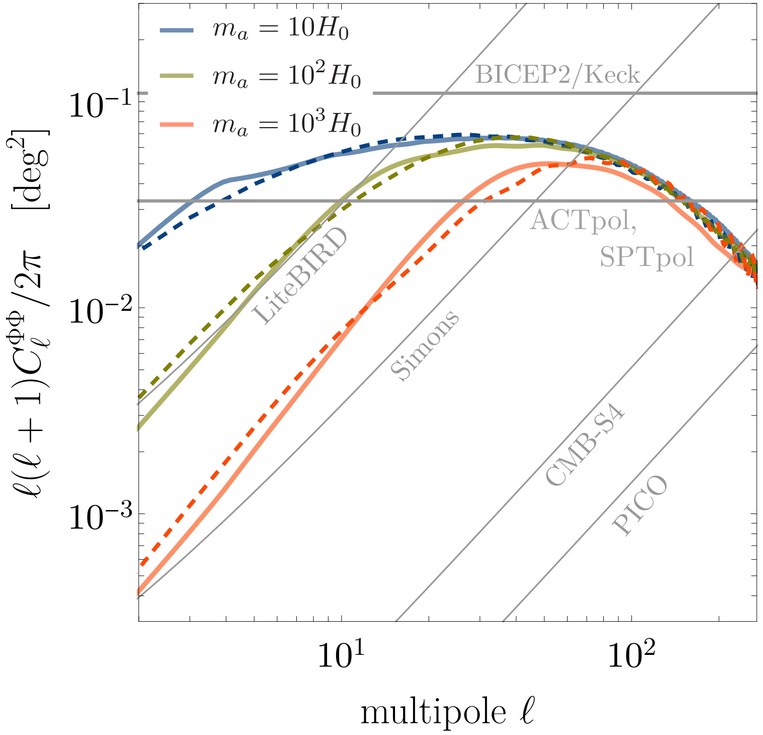
[arXiv:2103.10962] with Andrew Long and **Mudit Jain**

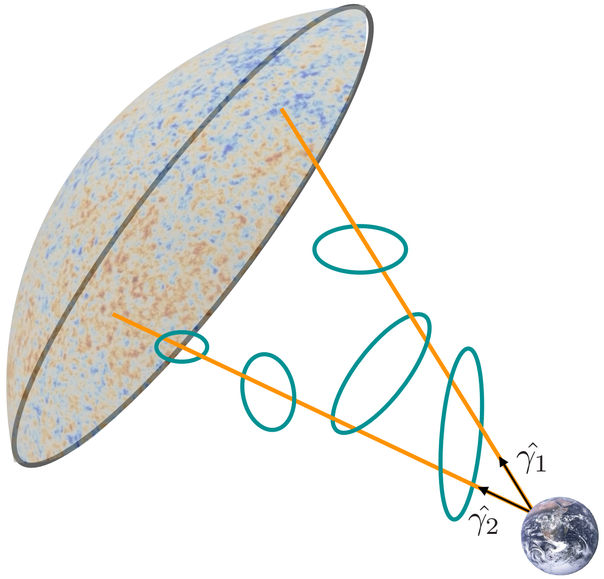
CMB birefringence from ultralight-axion string networks

models with sub/super horizon loops

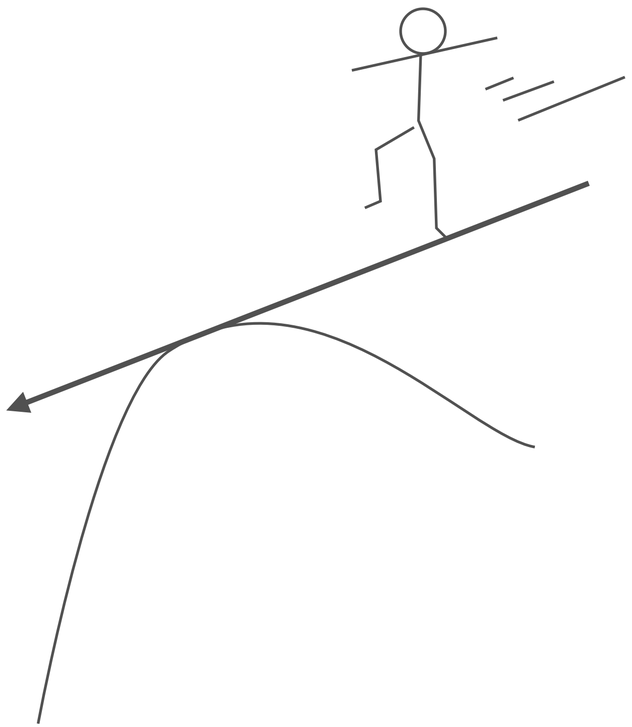


models with decaying networks

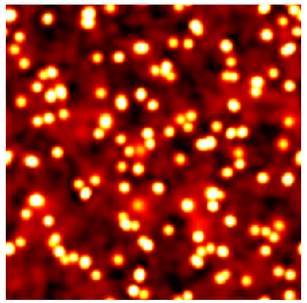


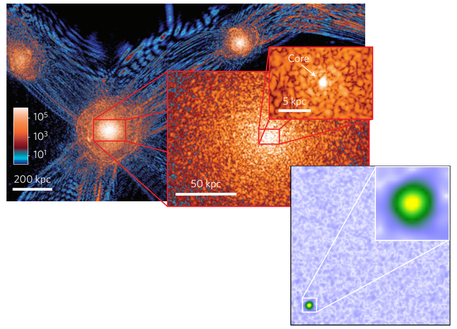


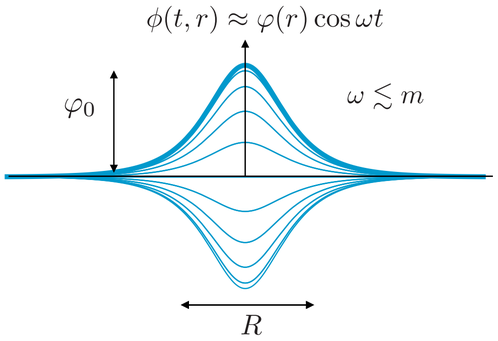
end tangential digression

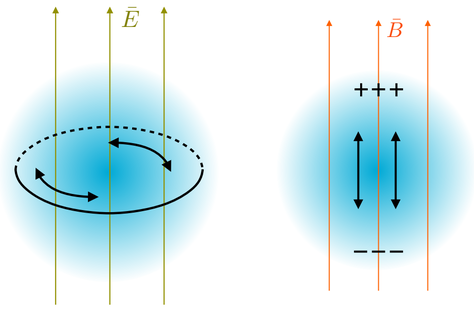


light from dark solitons : summary

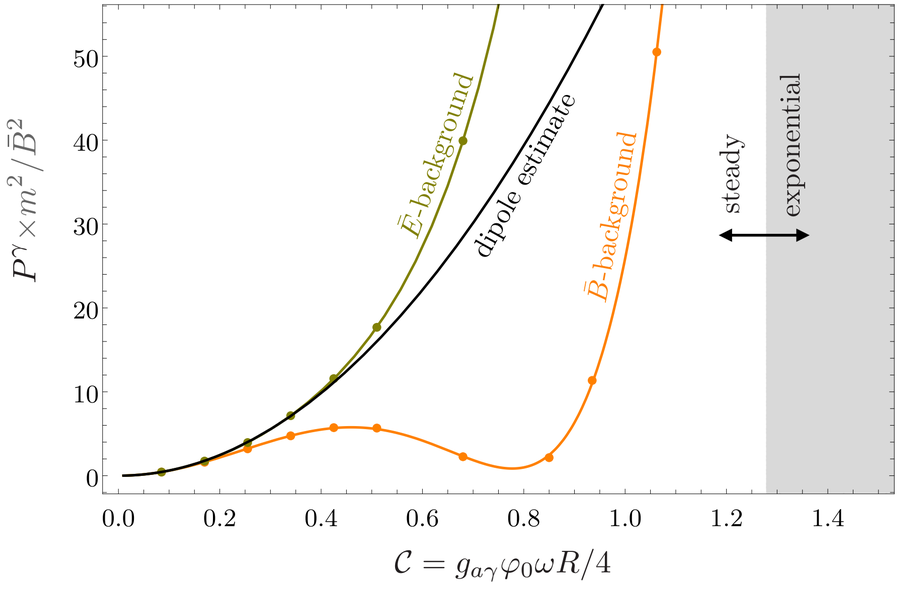


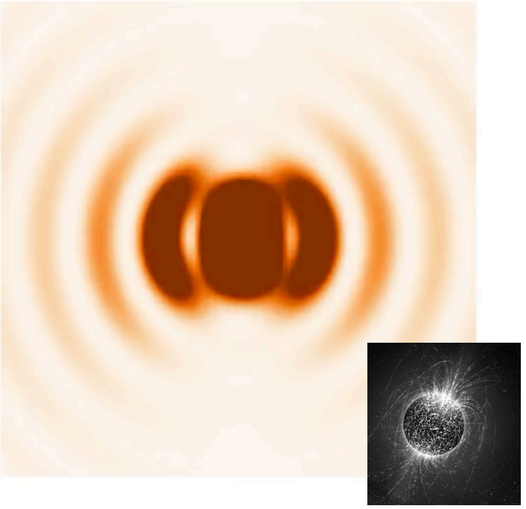


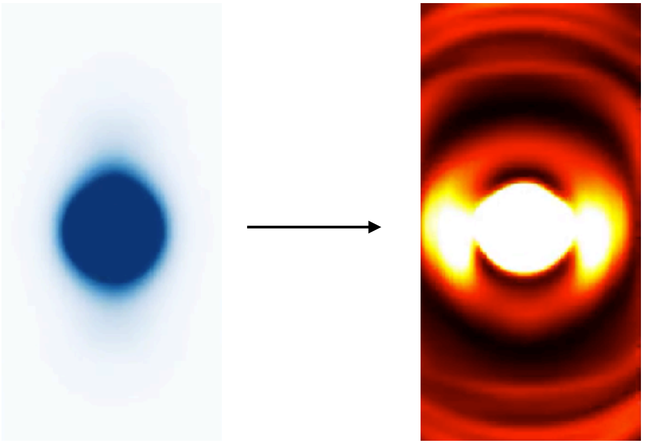




$g_{a\gamma}\phi\mathbf{E}\cdot\mathbf{B}$ perturbative & non-perturbative effects included







[arXiv: 2103.12082]

[arXiv: 2009.11337]