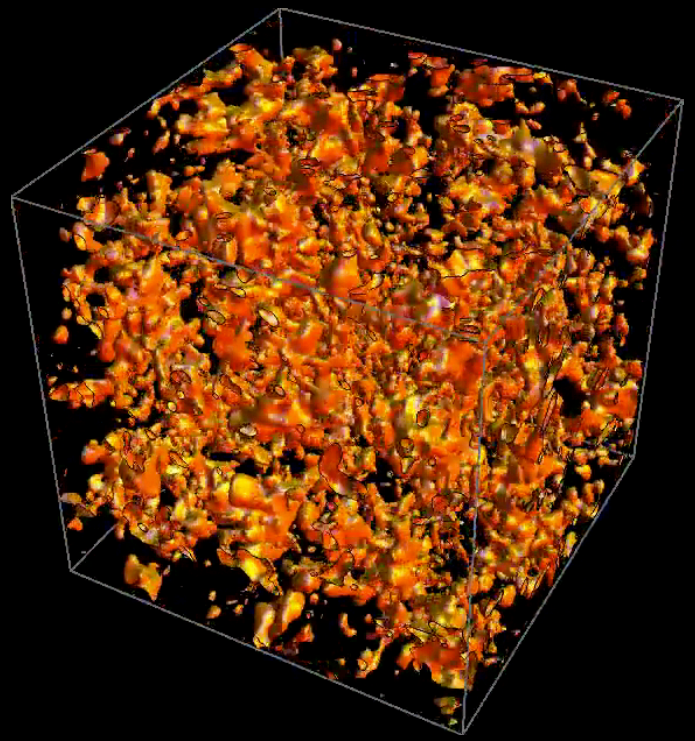
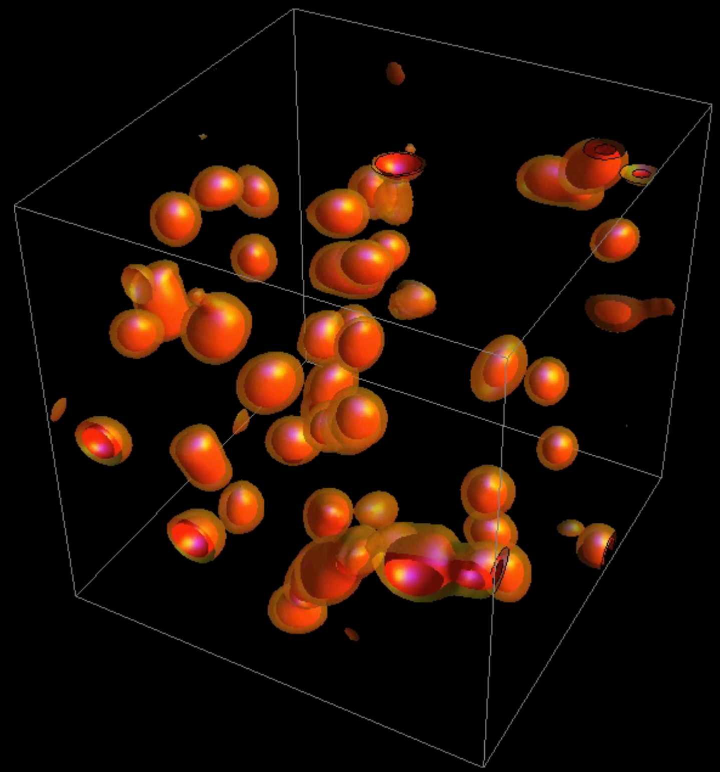
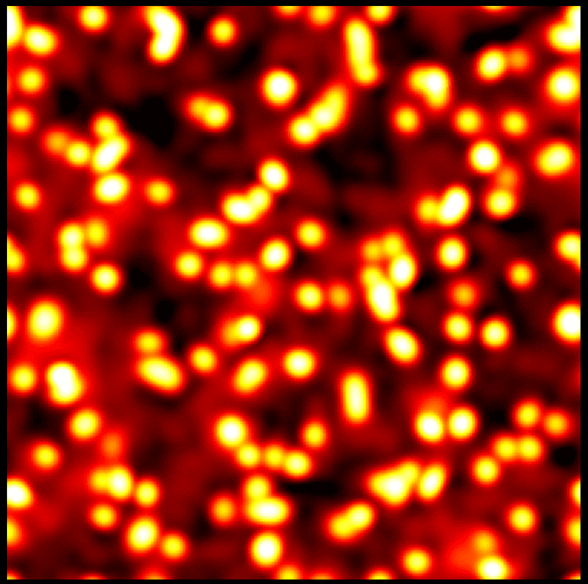
Nonlinear Dynamics after Inflation







Mustafa A. Amin

Cosmic Controversies

RICE

synopsis

(results more general than end of inflation)

theoretical/numerical results

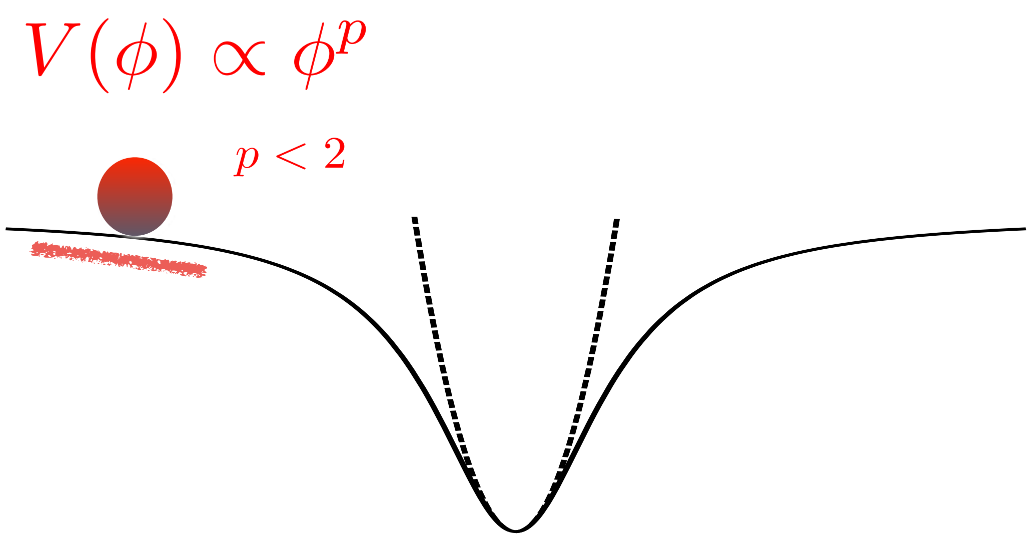
1. instability in oscillating fields
2. formation of solitons (oscillons)
3. eq. of state (with & without solitons)

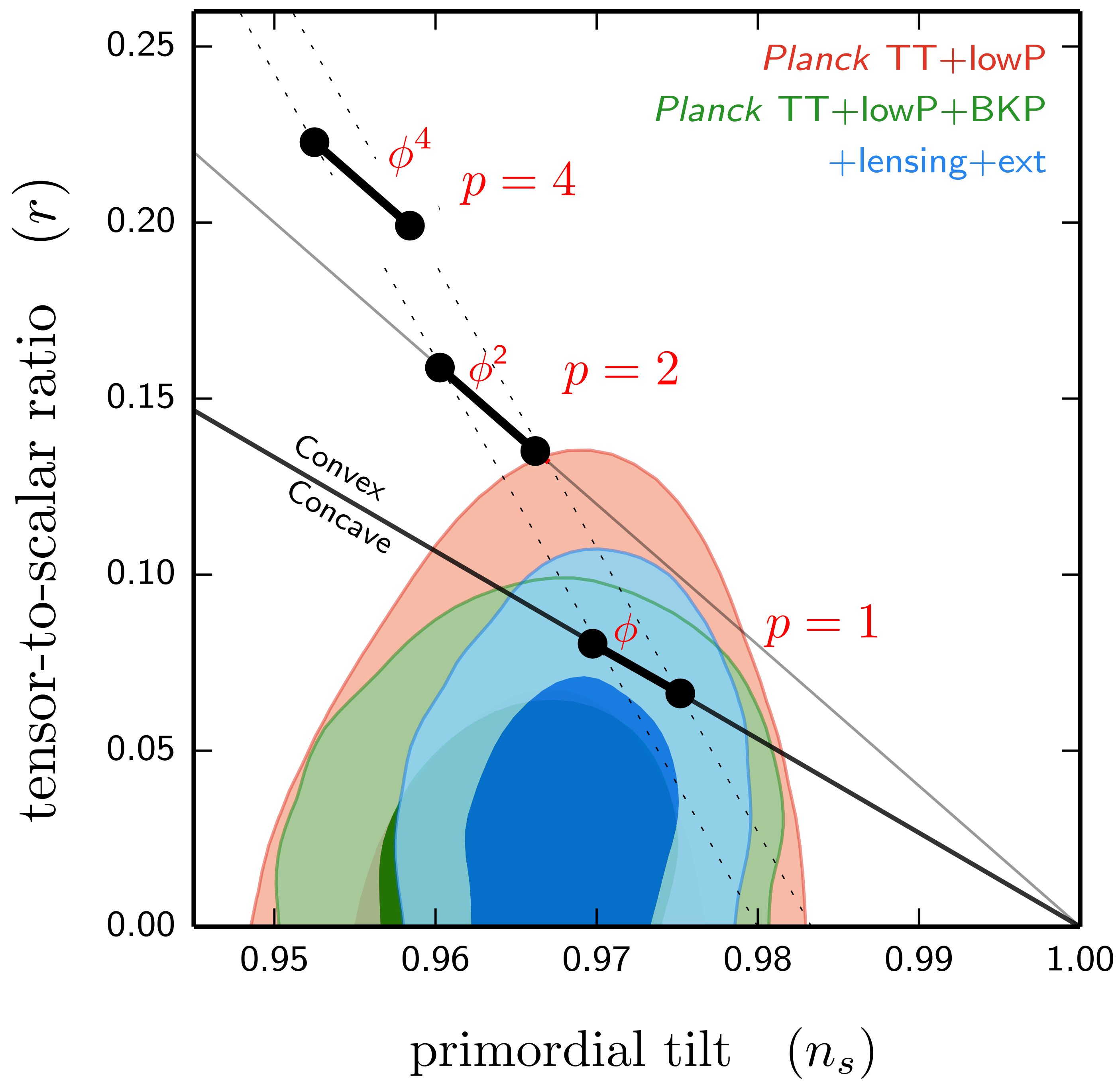
obs. implications

1. expansion history
2. gravitational waves
3. structure formation*

\+ some novel connections to
axions, early dark energy & condensed matter systems

what we “know” about inflation



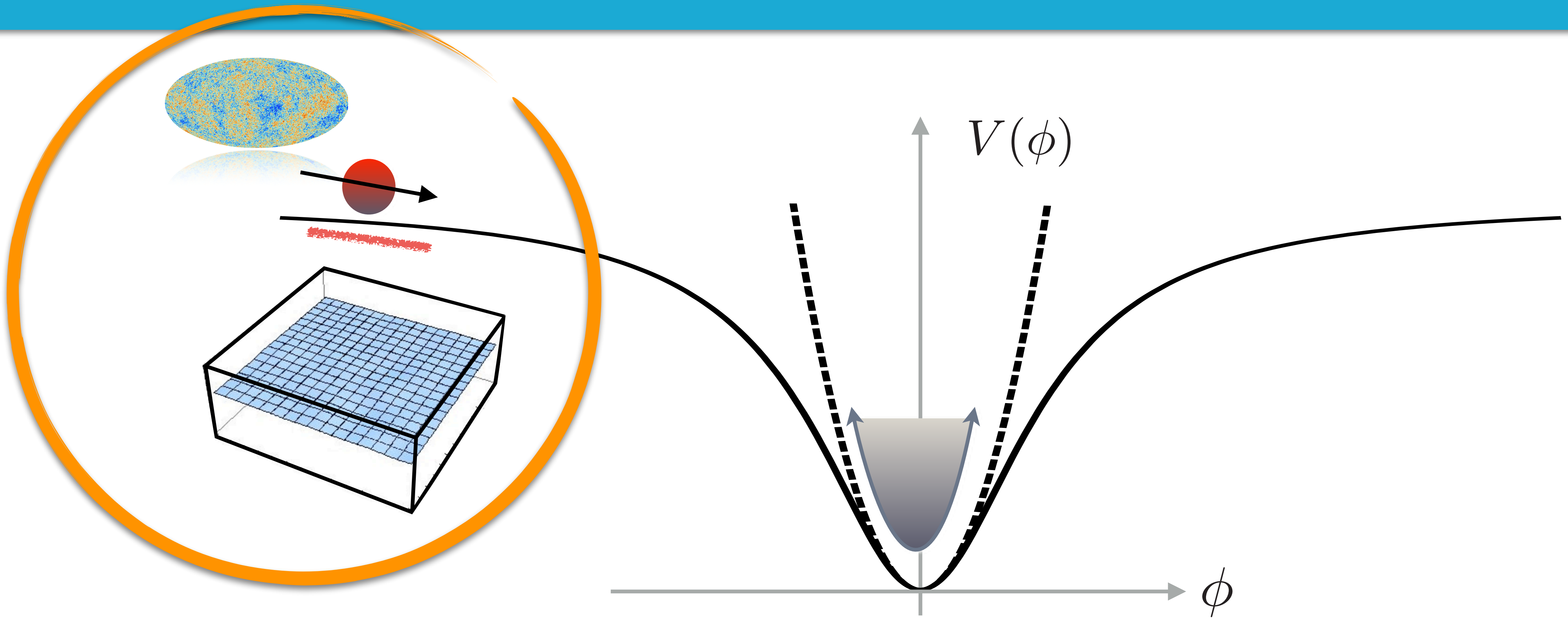


for example:

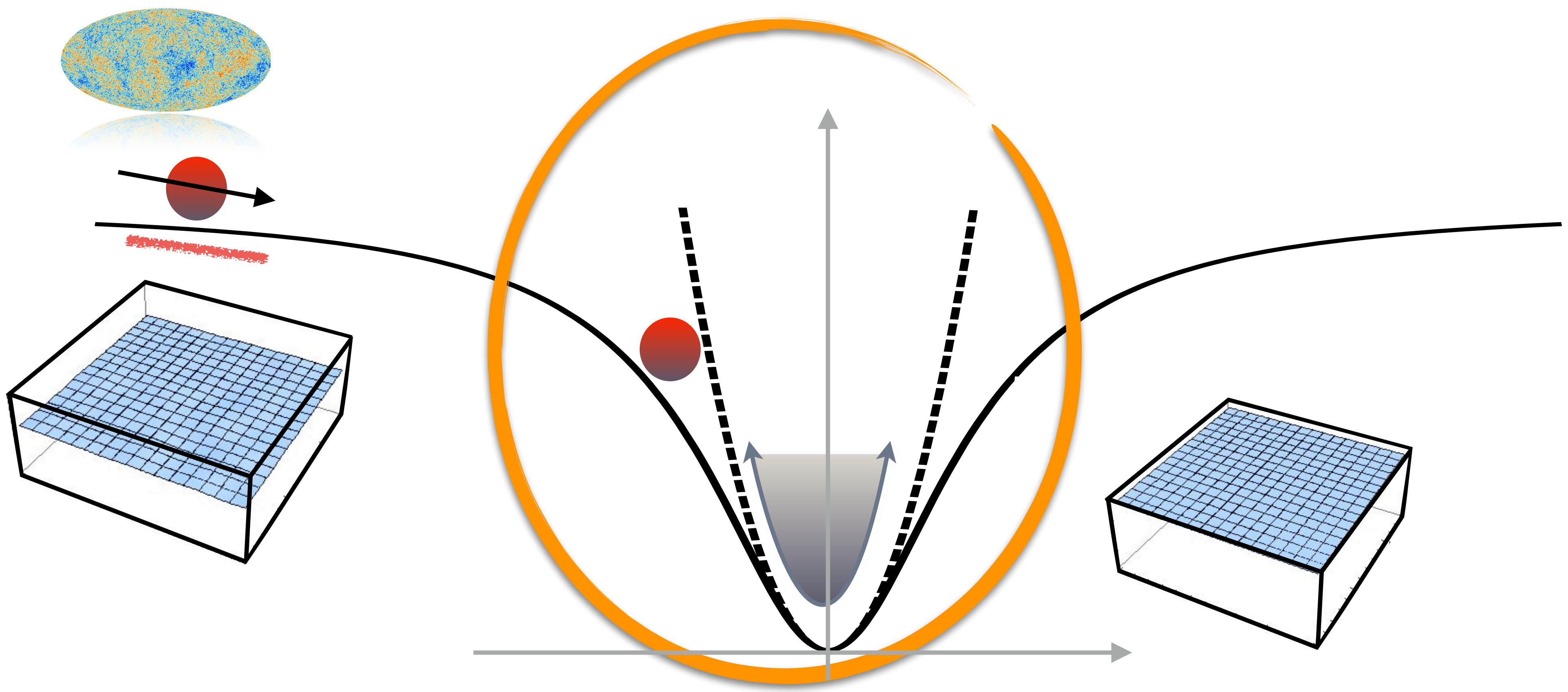
Starobinsky Inflation (1979/80)
Silverstein & Westhpal (2008)
Kallosh & Linde (2013)

but also see talk by Cumrun Vafa next

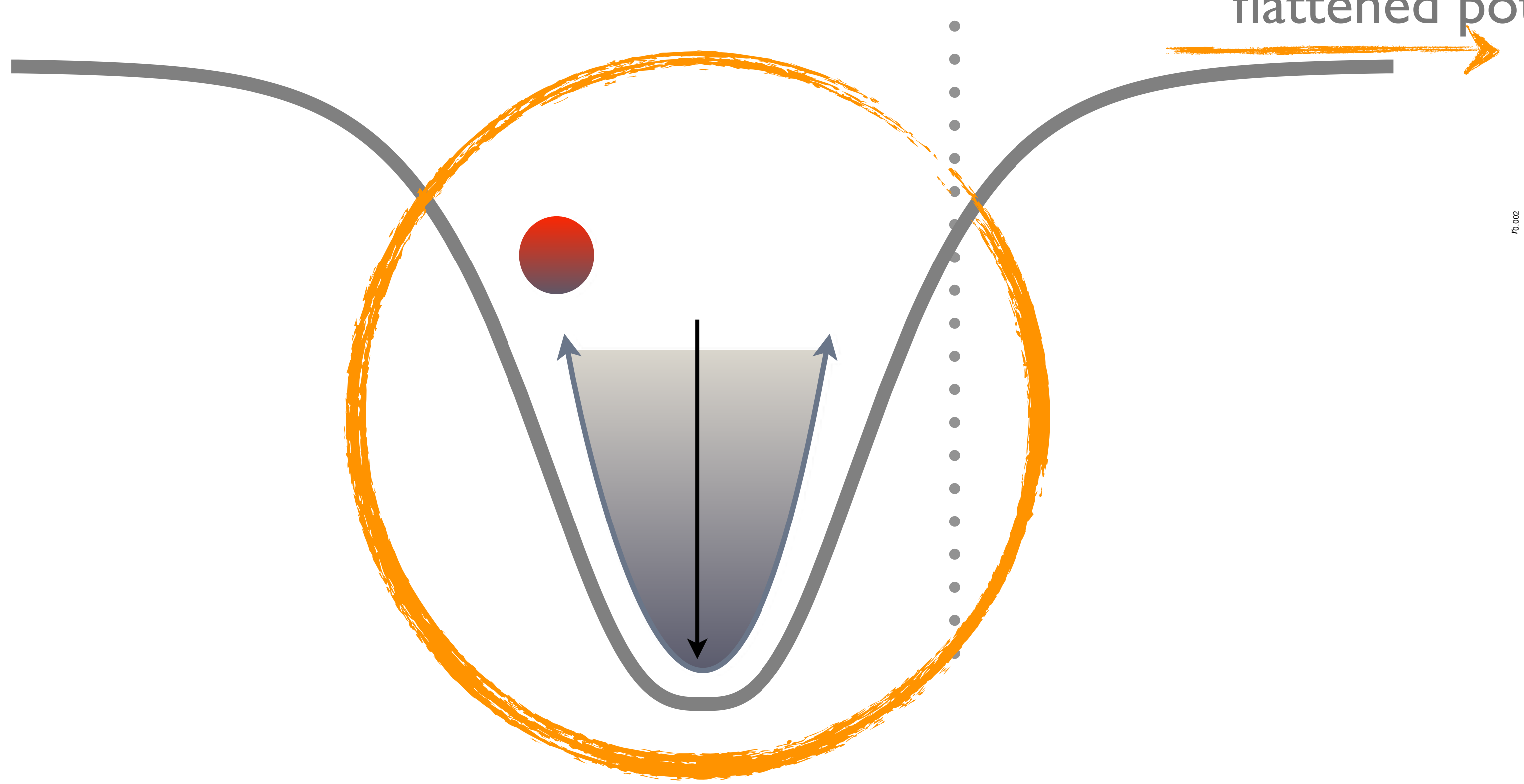
during inflation



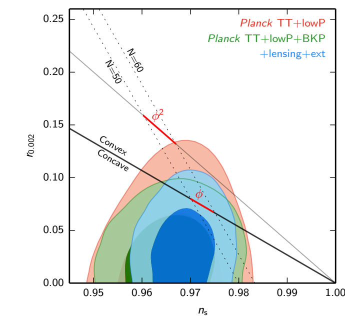
end of inflation



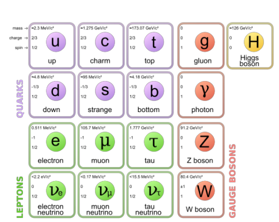
end of inflation



flattened potential $\phi^{p<2}$

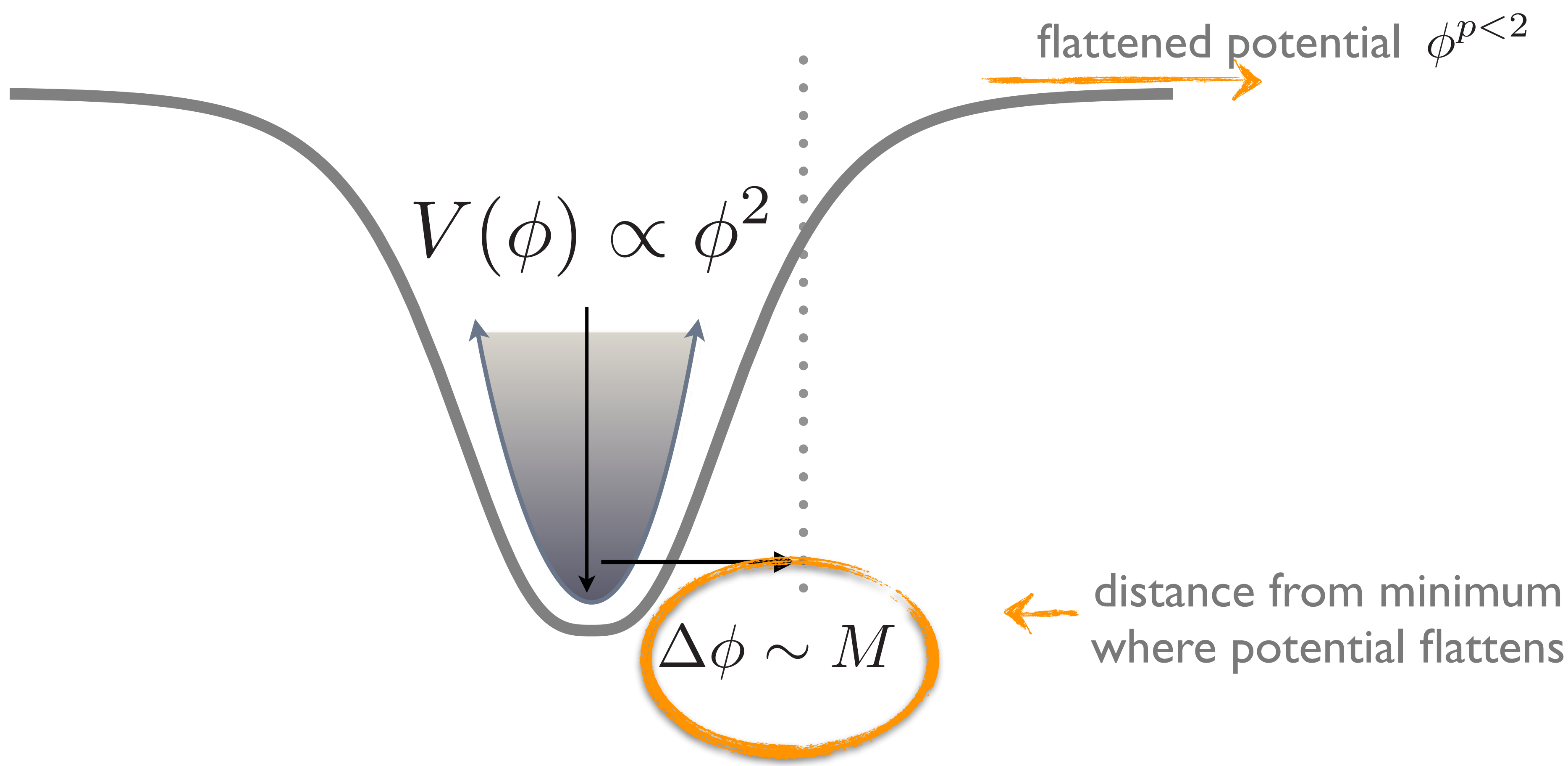


- shape of the potential (self couplings)
- couplings to other fields

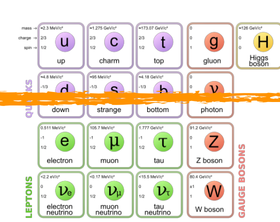


χ , ψ A_μ

end of inflation in "simple" models



- shape of the potential (self couplings)
- ~~couplings to other fields~~



$\chi \; , \psi \; A_\mu$

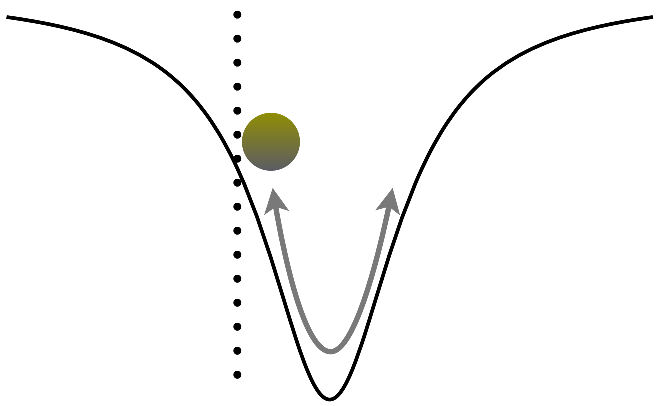
See **P. Adshead's** talk about very strong coupling to gauge fields

instability of oscillating fields

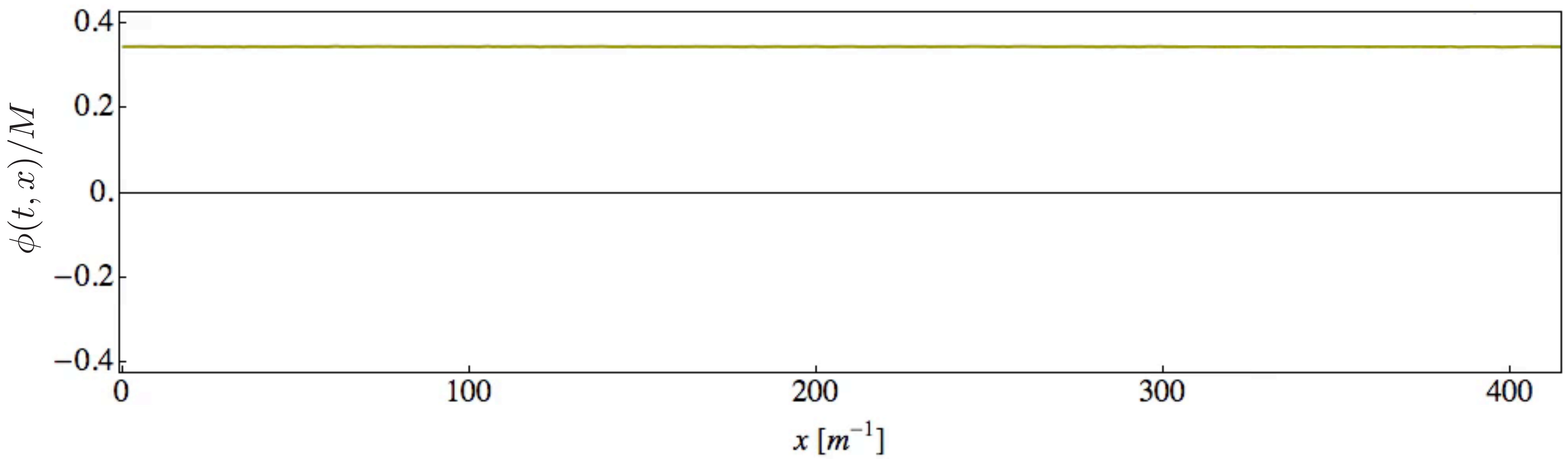
expansion ✓

self-interactions ✓

gravitational int. ✗



$$\Box\phi = V'(\phi)$$



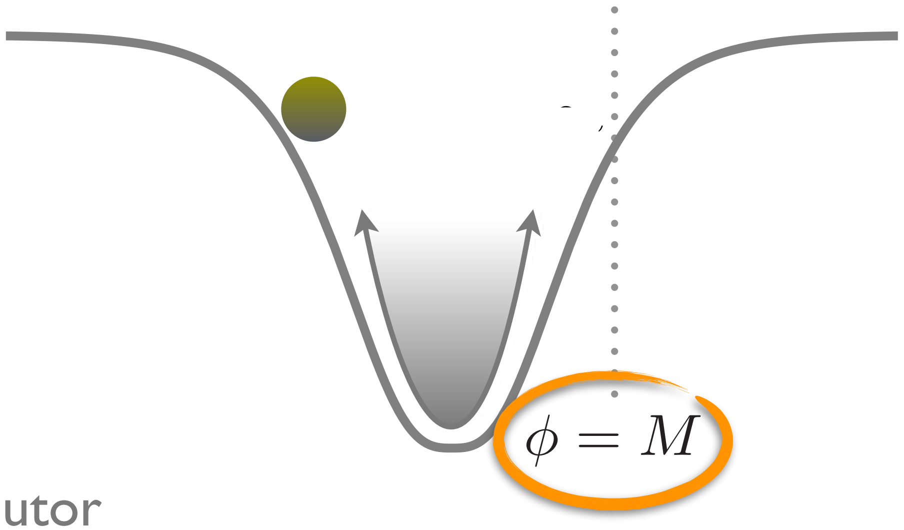
MA (2010)
1006.3075

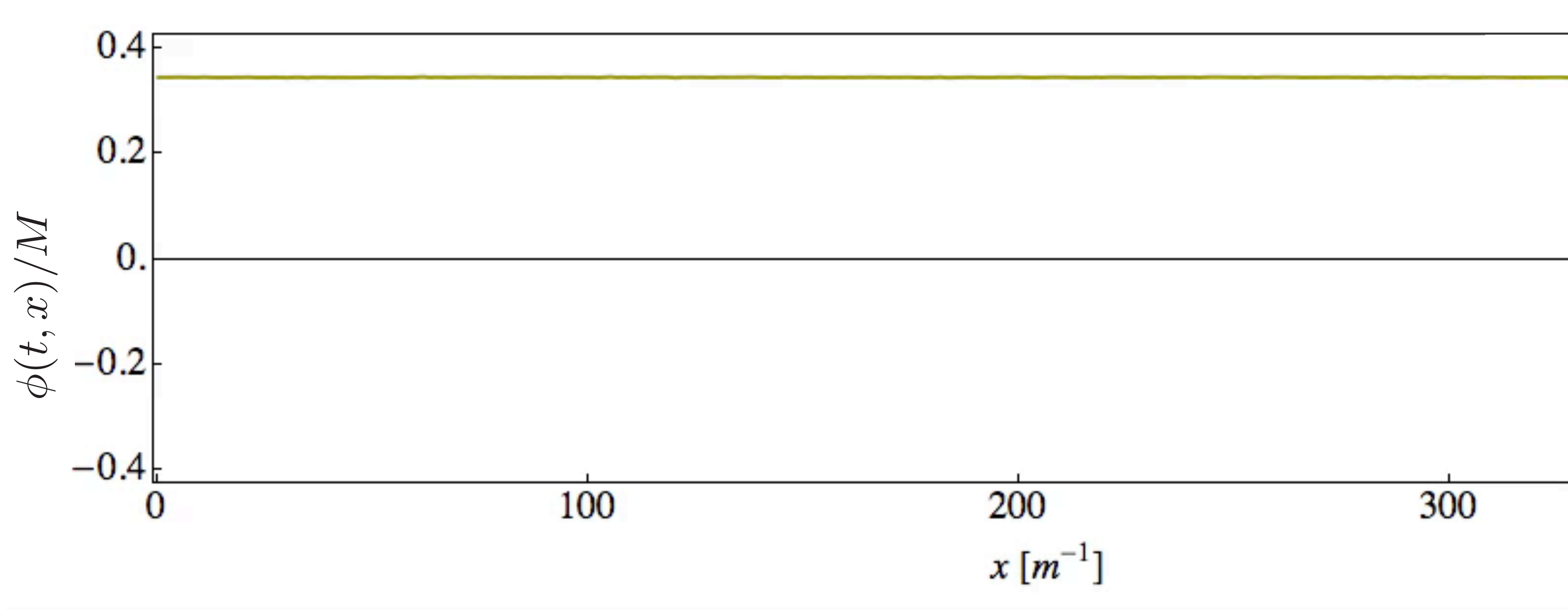
instabilities in an expanding universe

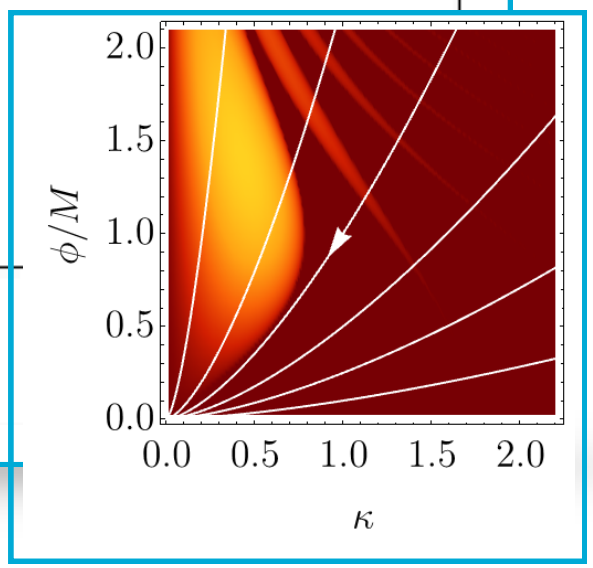
$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} = \frac{\Re[\mu_k]}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

general

dominant contributor to energy density







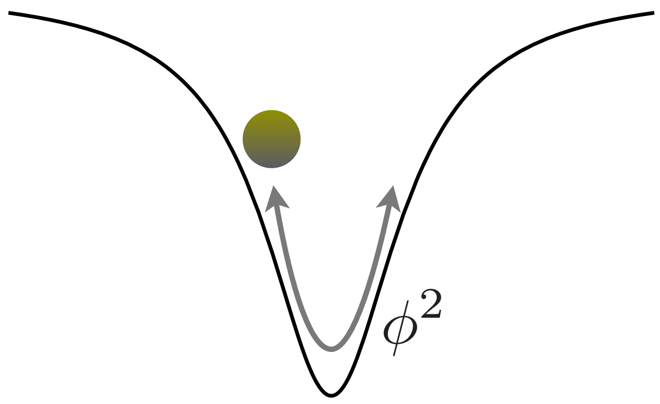
result 1: instability in oscillating fields

oscillating, cosmologically relevant, (almost) homogeneous scalar fields are often unstable to spatial perturbations

Khlopov, Malomed, Zeldovich (1985)

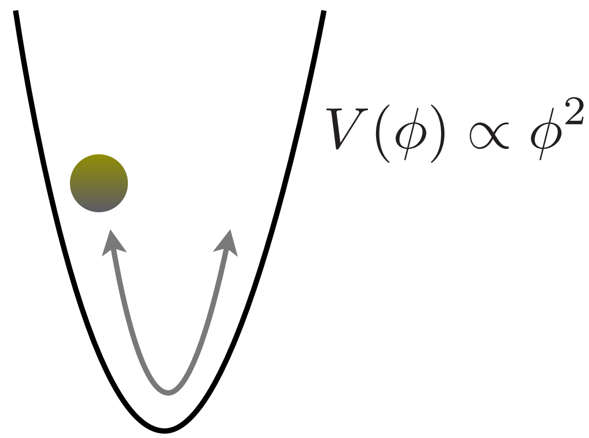
* there are timescales associated with the instability, typically the longest is Hubble time scale

what drives the instabilities ?



- **self-interactions**
  - fields can cluster/become inhomogeneous
  - (can be) much faster than Hubble (due to *self*-resonance)

- **gravity**
  - fields cluster, Hubble time scales



*for this talk, I will ignore interactions with other fields, which can also be important.

Review : MA, Kaiser, Karouby, Herzberg [1410.3808]

gravity only: Gilmore, Flauger & Easther (2012)

Early Works : Kofman, Linde, Starobinsky (1994/97)

Shatnov, Traschen & Brandenburger (1990/95)

Khlebnikov & Tkachev (1996), Bellido & Linde (1997)

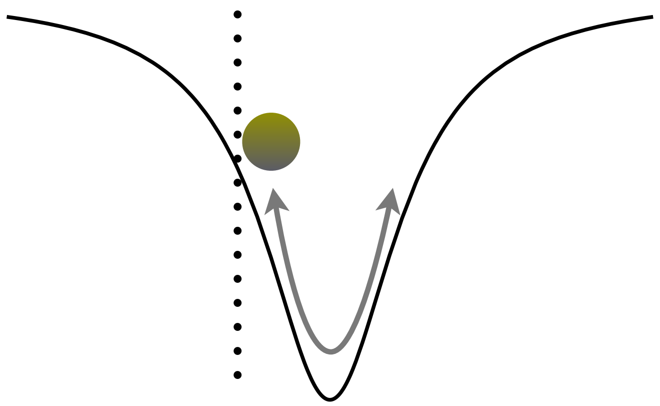
instability — formation of oscillons
(non-topological "solitons" in real scalar fields)

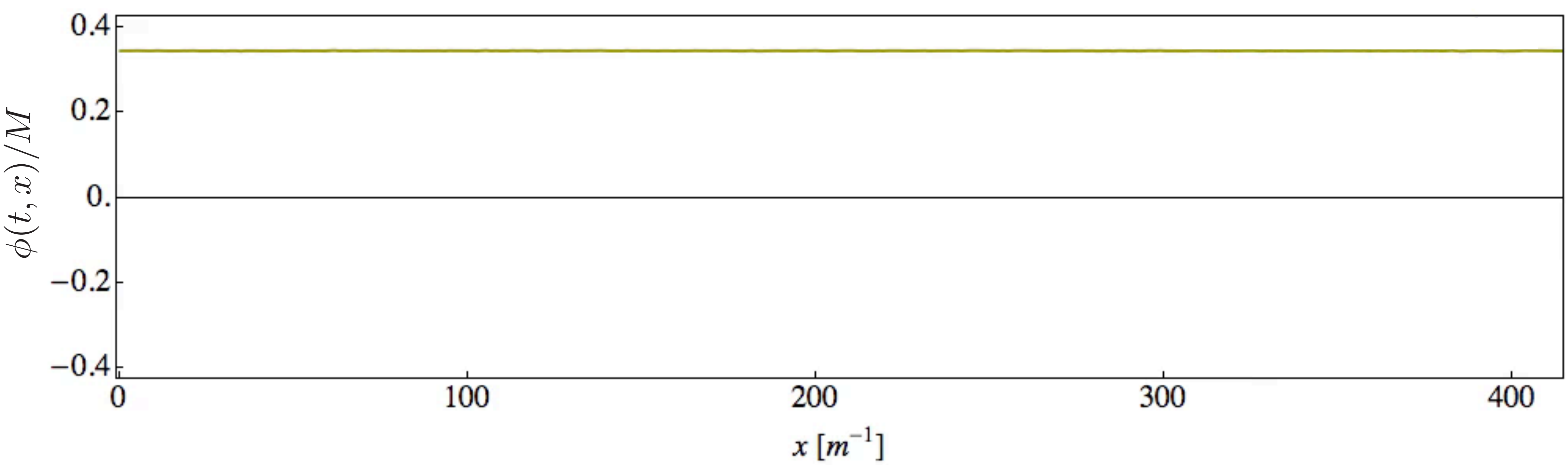
expansion ✓

self-interactions ✓

gravitational int. ✗

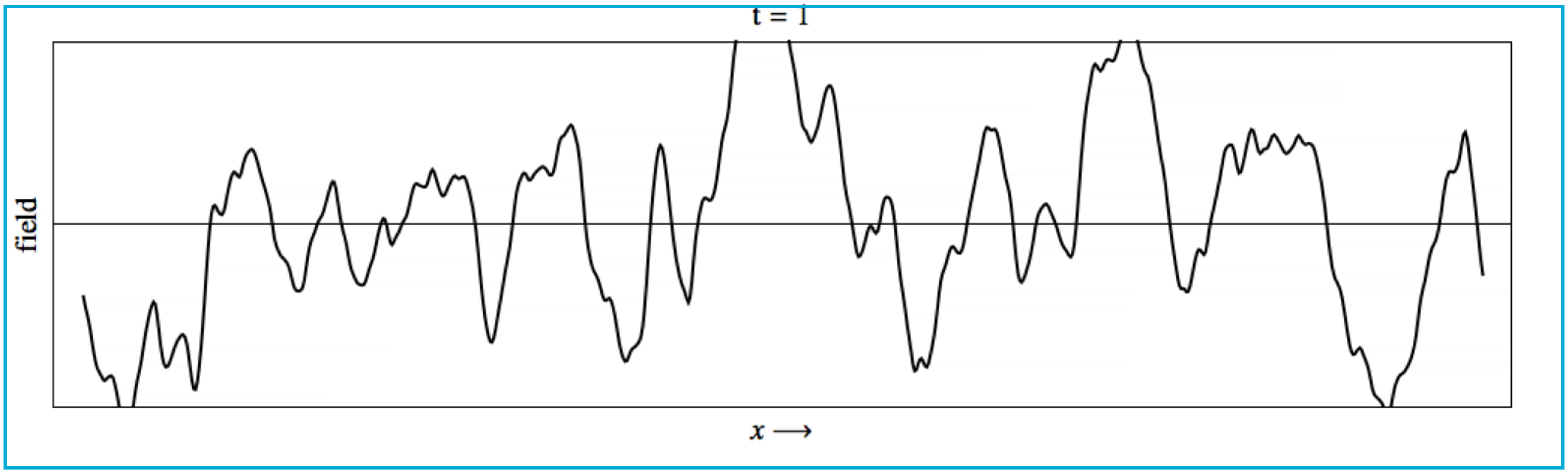
$$\Box\phi = V'(\phi)$$





MA (2010)
1006.3075

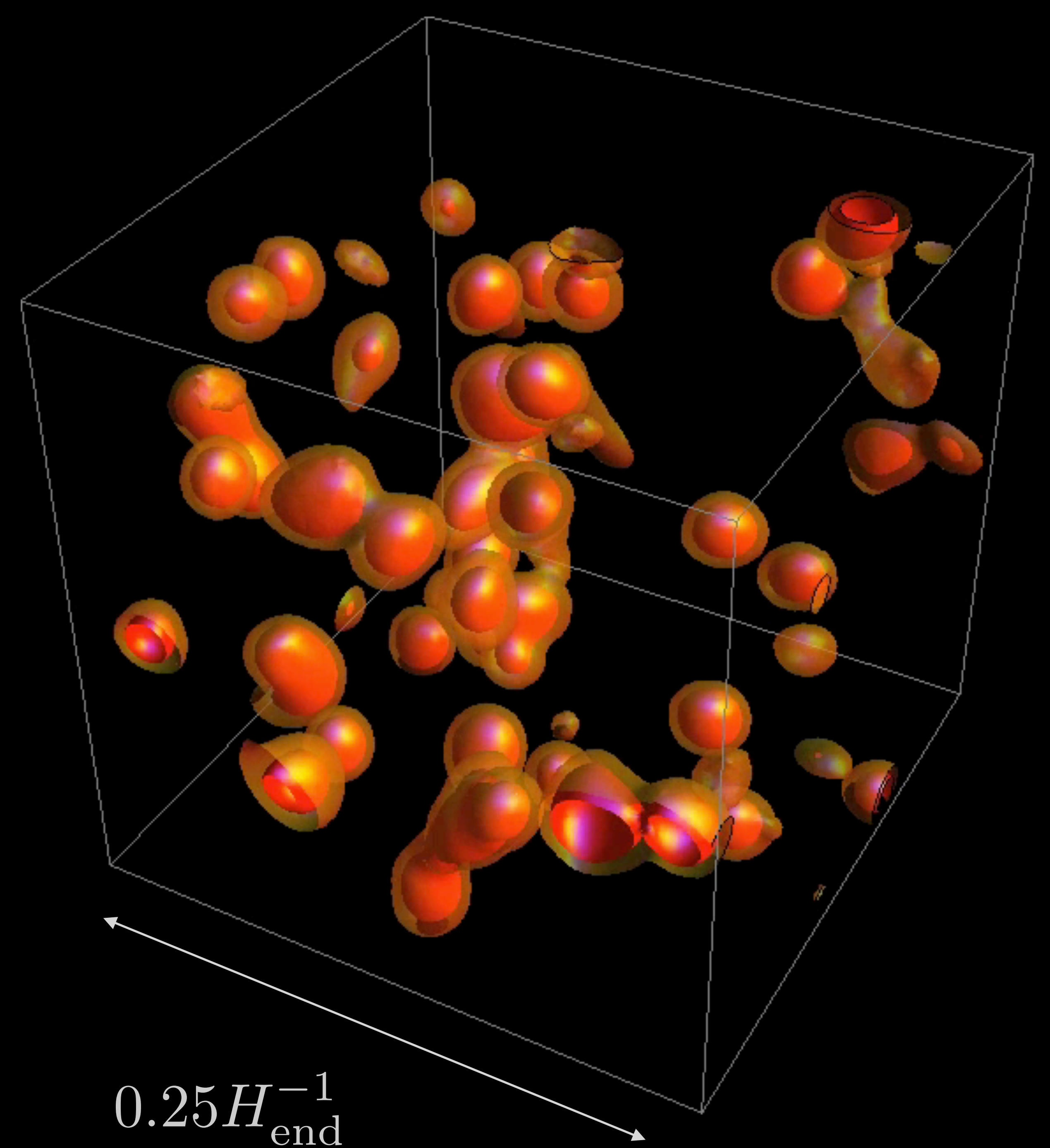
insensitive to initial conditions



simulation of "quasi-thermal" example in Farhi, Guth, Iqbal, Graham 2008

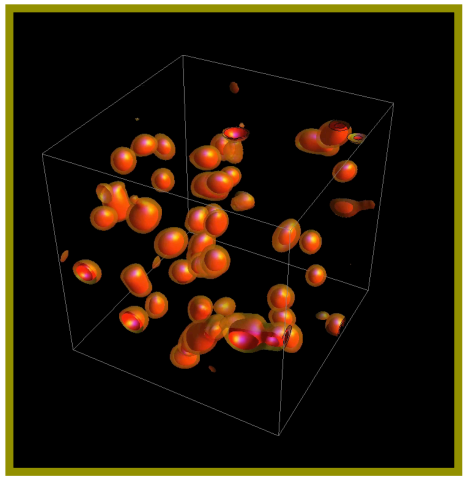
oscillon formation at the end of inflation

- expansion ✓
- self-interactions ✓
- gravitational int. ✗

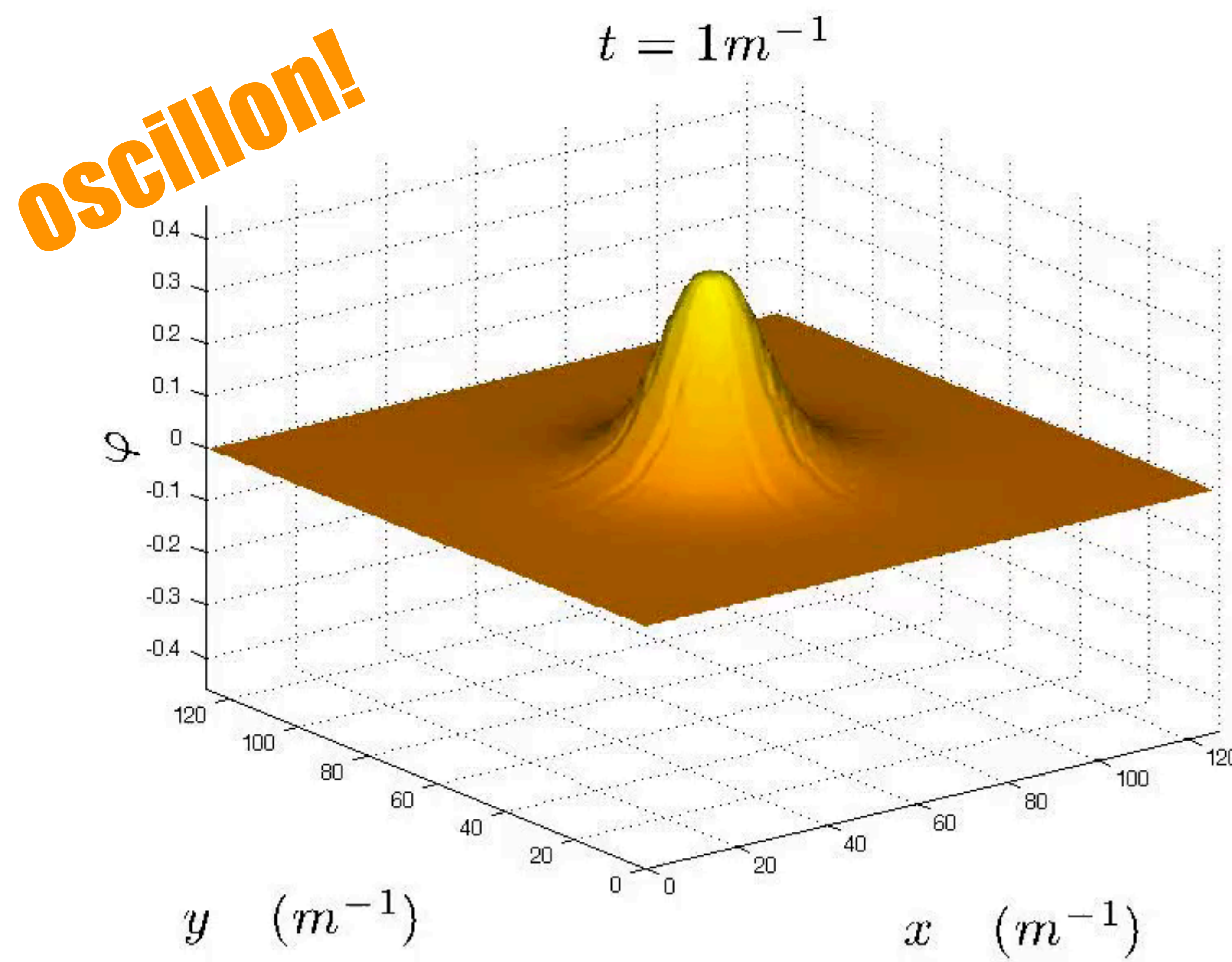


MA, Easther, Finkel, Flauger & Hertzberg (2011)
1106.3335

solitons ?



(1) oscillatory (2) spatially localized (3) **very long lived**



For example:
Segur & Kruskal (1987)
MA & Shirokoff (2010)
Hertzberg (2011)
MA (2013)
Mukaida et. al (2016)
Salmi & Hindmarsh (2014)
Sakstein & Trodden (2018)

Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland, Gleiser and Mueller et al. (1995) …

lifetimes (without gravity)

$$T_{\max} \sim 10^3\, m^{-1}$$

$$V(\phi) = m^2 M^2 \left[1 - \cos\frac{\phi}{M}\right]$$

$$T_{\max} \sim 10^6\, m^{-1}$$

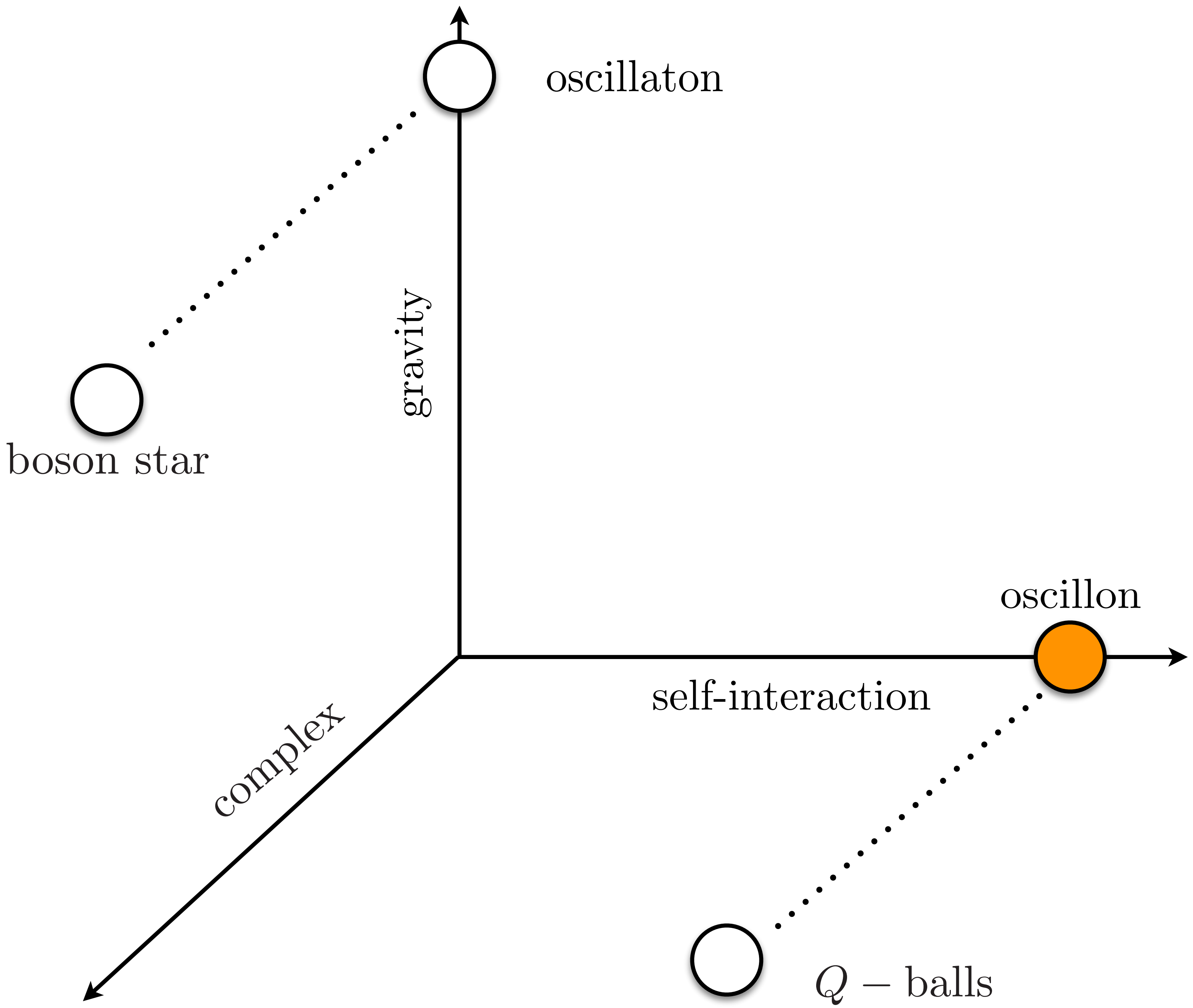
$$V(\phi) = \frac{m^2 M^2}{2} \tanh^2\left(\frac{\phi}{M}\right)$$

$$T_{\max} > 10^7\, m^{-1}$$

$$V(\phi) = m^2 M^2 \left[\sqrt{1 + \frac{\phi^2}{M^2}} - 1\right]$$

Zhang, MA, Copeland, Saffin, Lozanov (in progress)

family of scalar field solitons

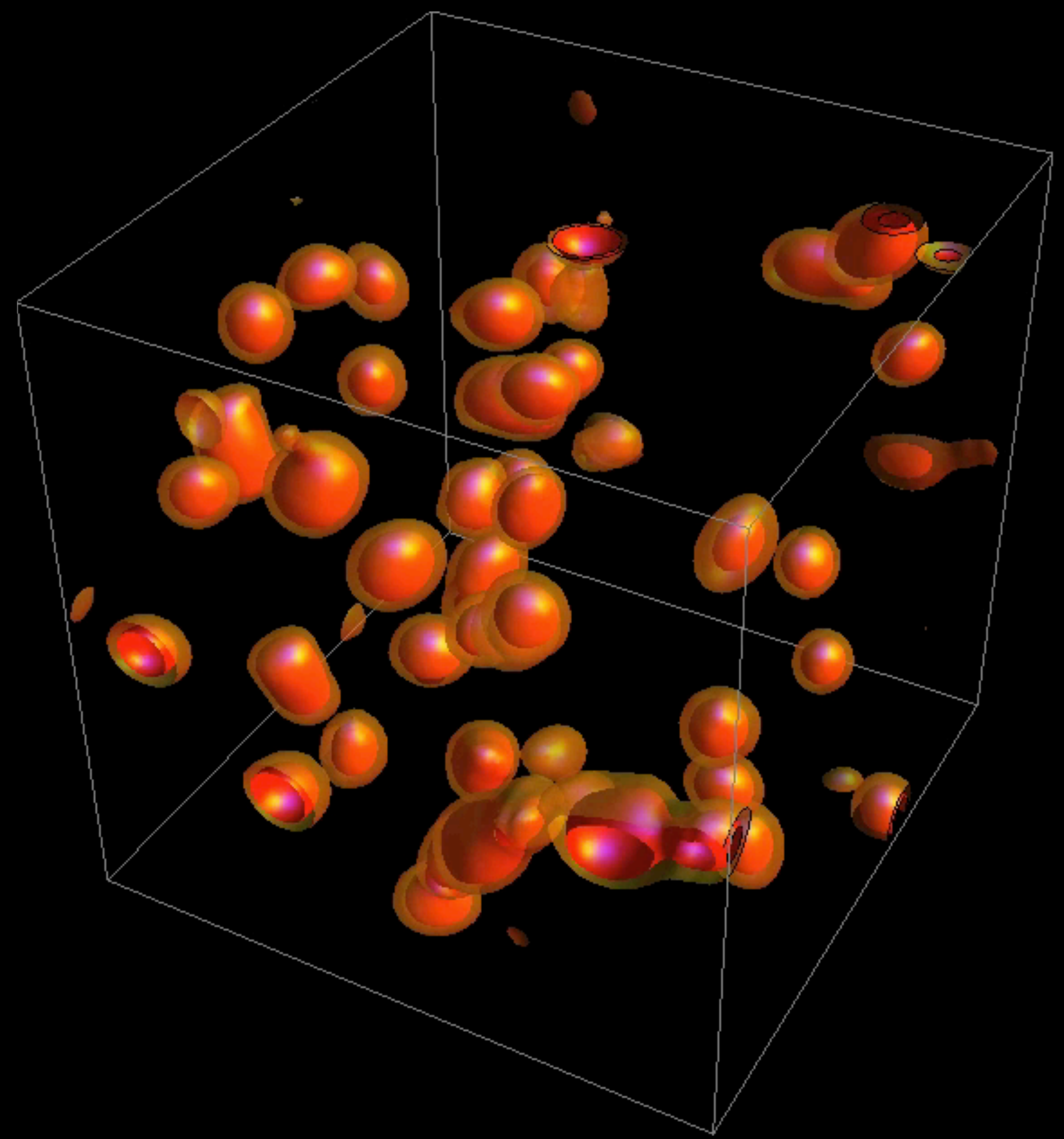


oscillon formation at the end of inflation

expansion ✓

self-interactions ✓

gravitational int. ✗



1. oscillons dominate the energy density of the field

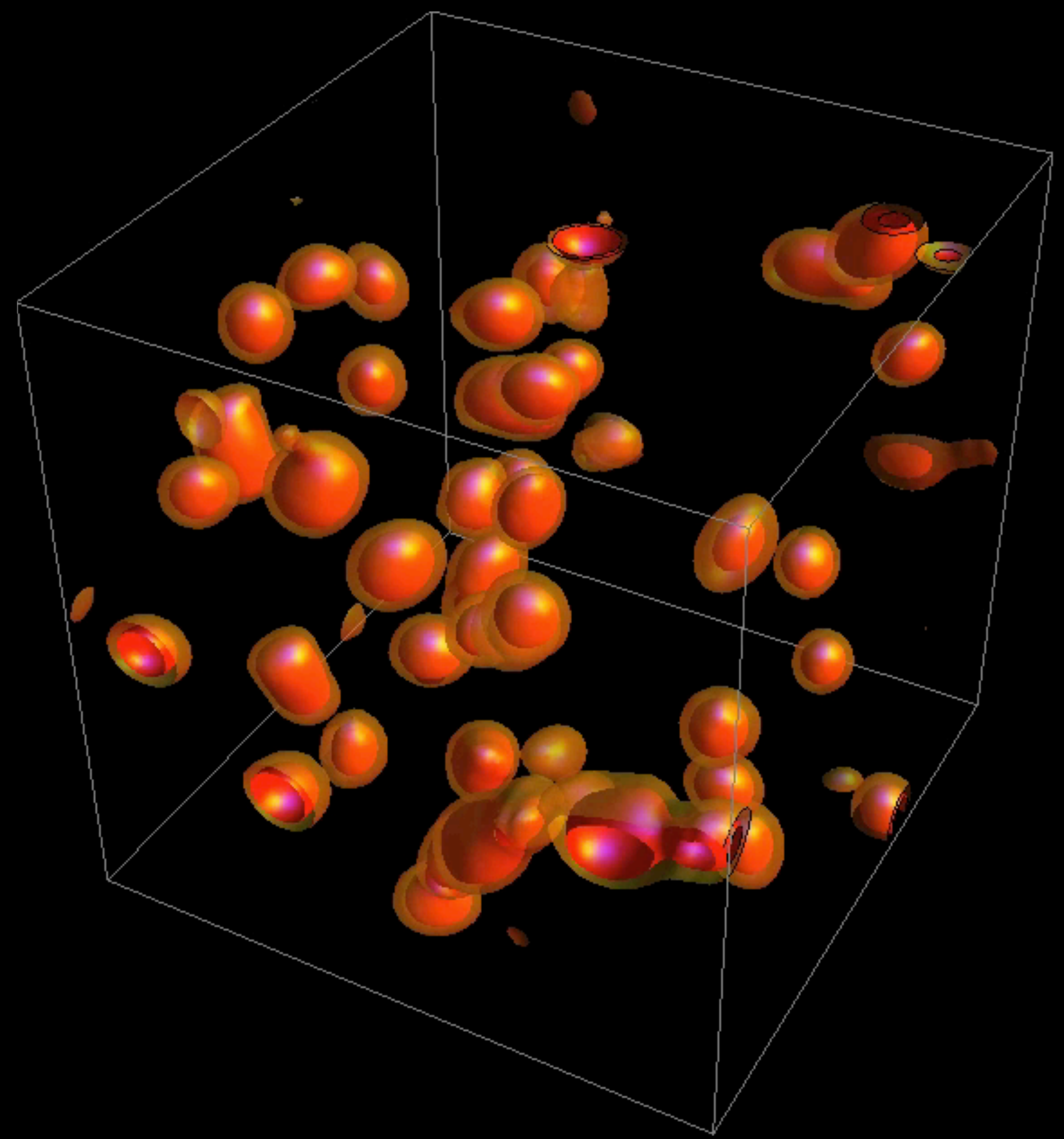
2. eq. of state $w = 0$

long term dynamics ?

expansion ✓

self-interactions ✓

gravitational int. ✗



assuming coupling to other fields is sufficiently weak

self-interactions

+ gravity*

(Schrodinger-Poisson)

- expansion ✓
- self-interactions ✓
- gravitational int. ✓
- relativistic? ✗

MA & Mocz (2019)
1902.07261

qualitative comparison
with relativistic system
Lozanov & MA (2019) 1902.06736

self interaction more important than gravity initially

self-interactions

+ gravity*

(Schrodinger-Poisson)

expansion ✓

self-interactions ✓

gravitational int. ✓

relativistic? ✗

MA & Mocz (2019)
1902.07261

qualitative comparison
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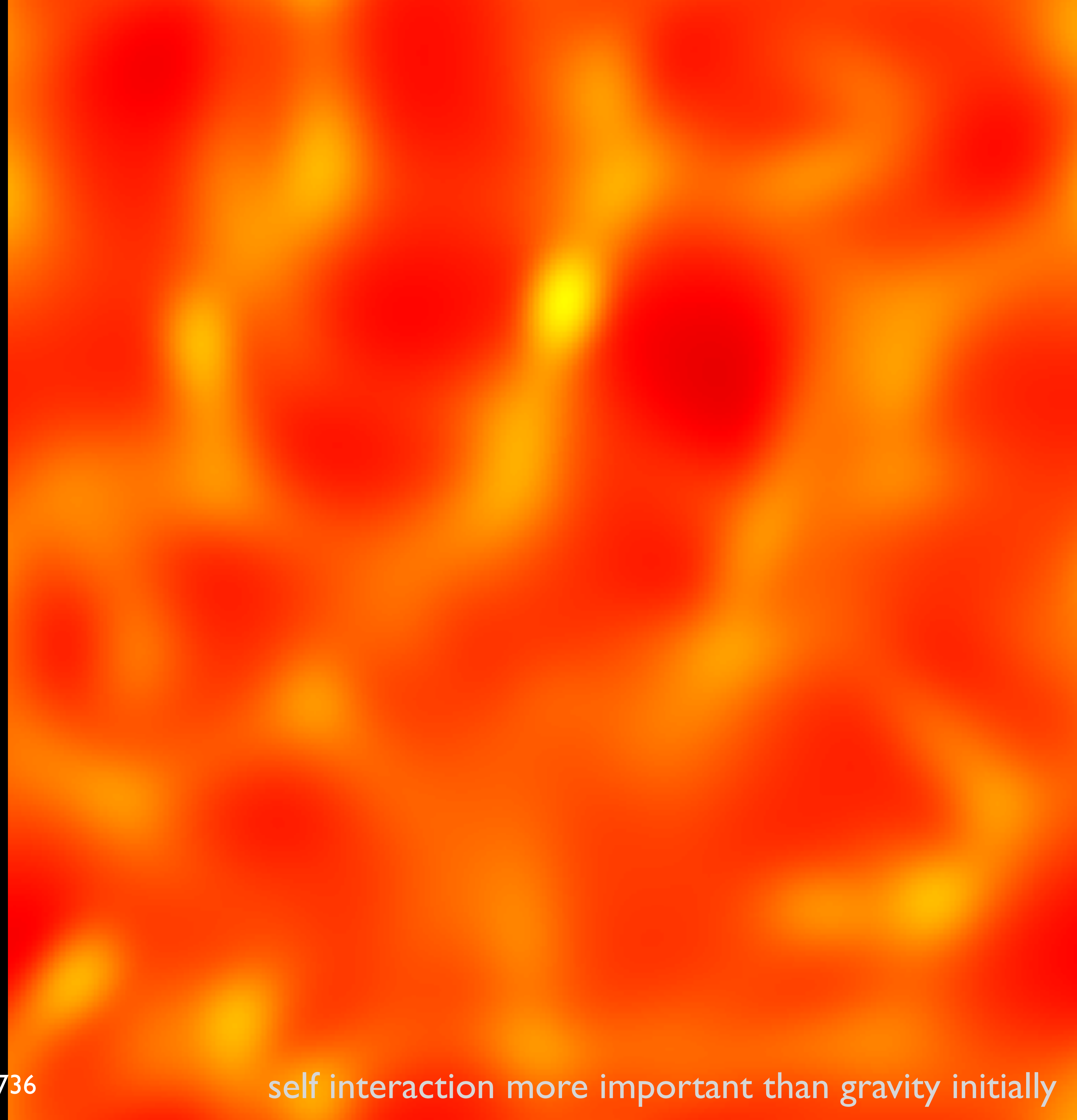

self-interactions

$+$ gravity*

(Schrodinger-Poisson)

- expansion ✓
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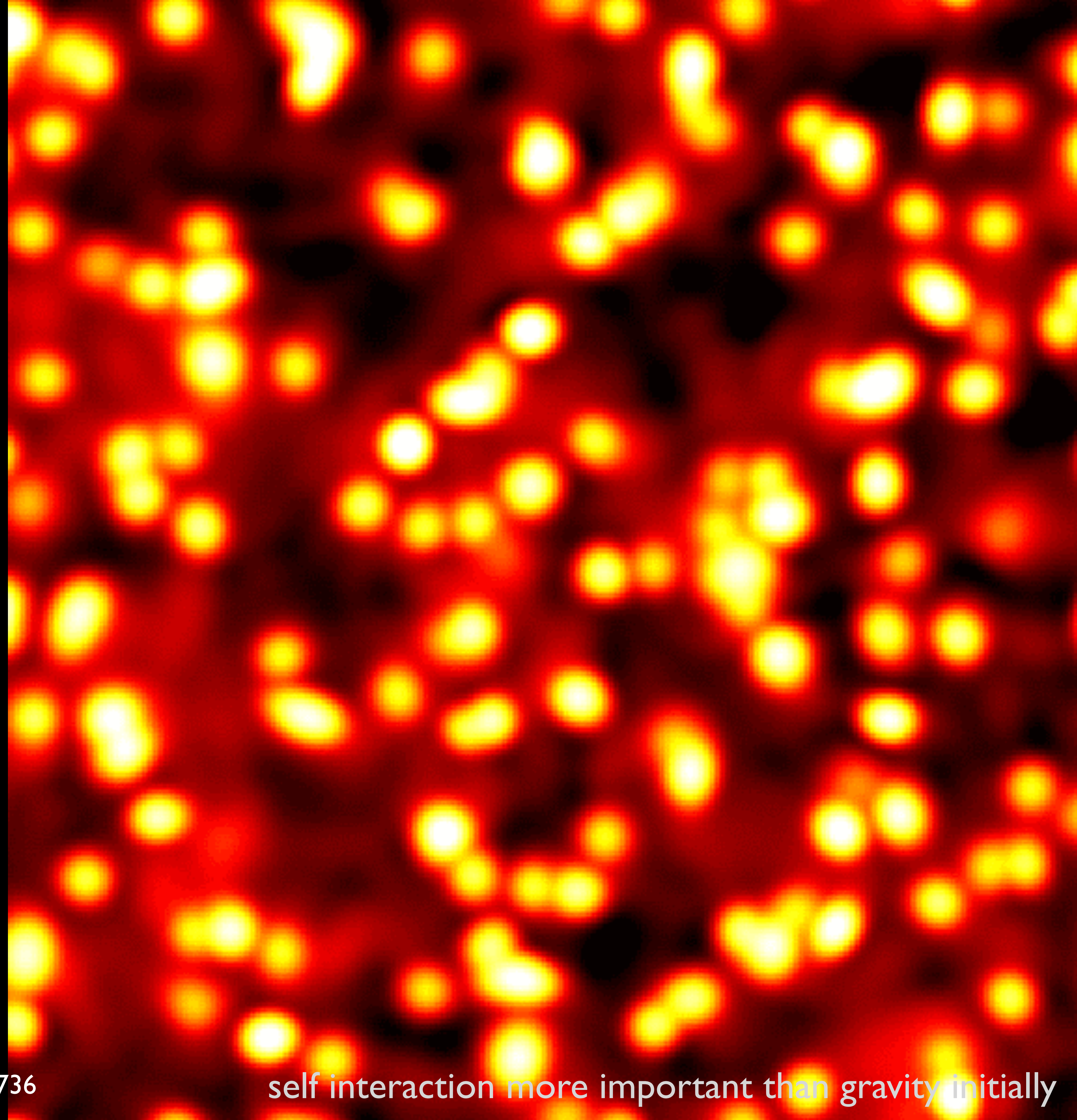

self-interactions
+ gravity*
(Schrodinger-Poisson)

- expansion ✓
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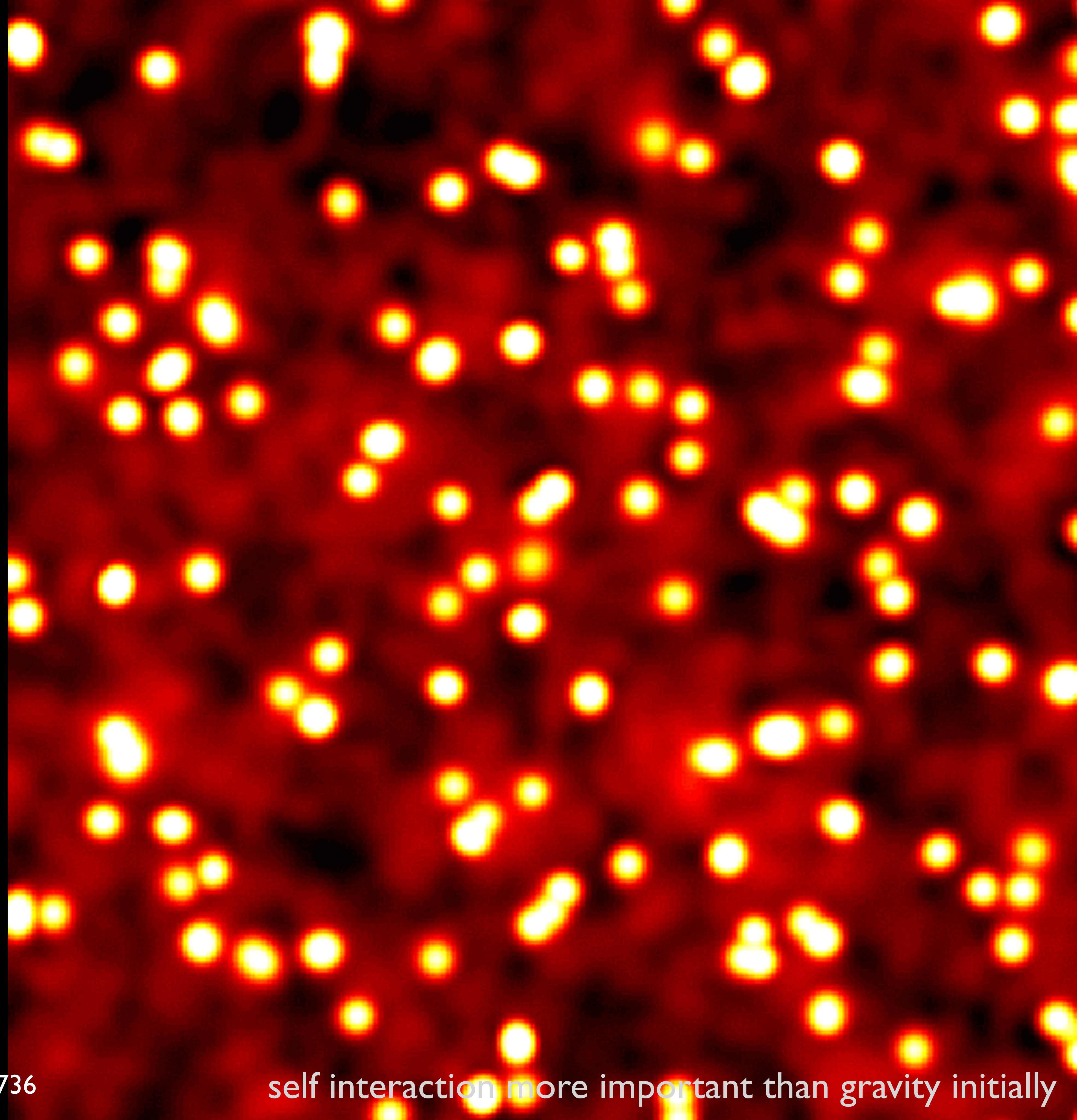
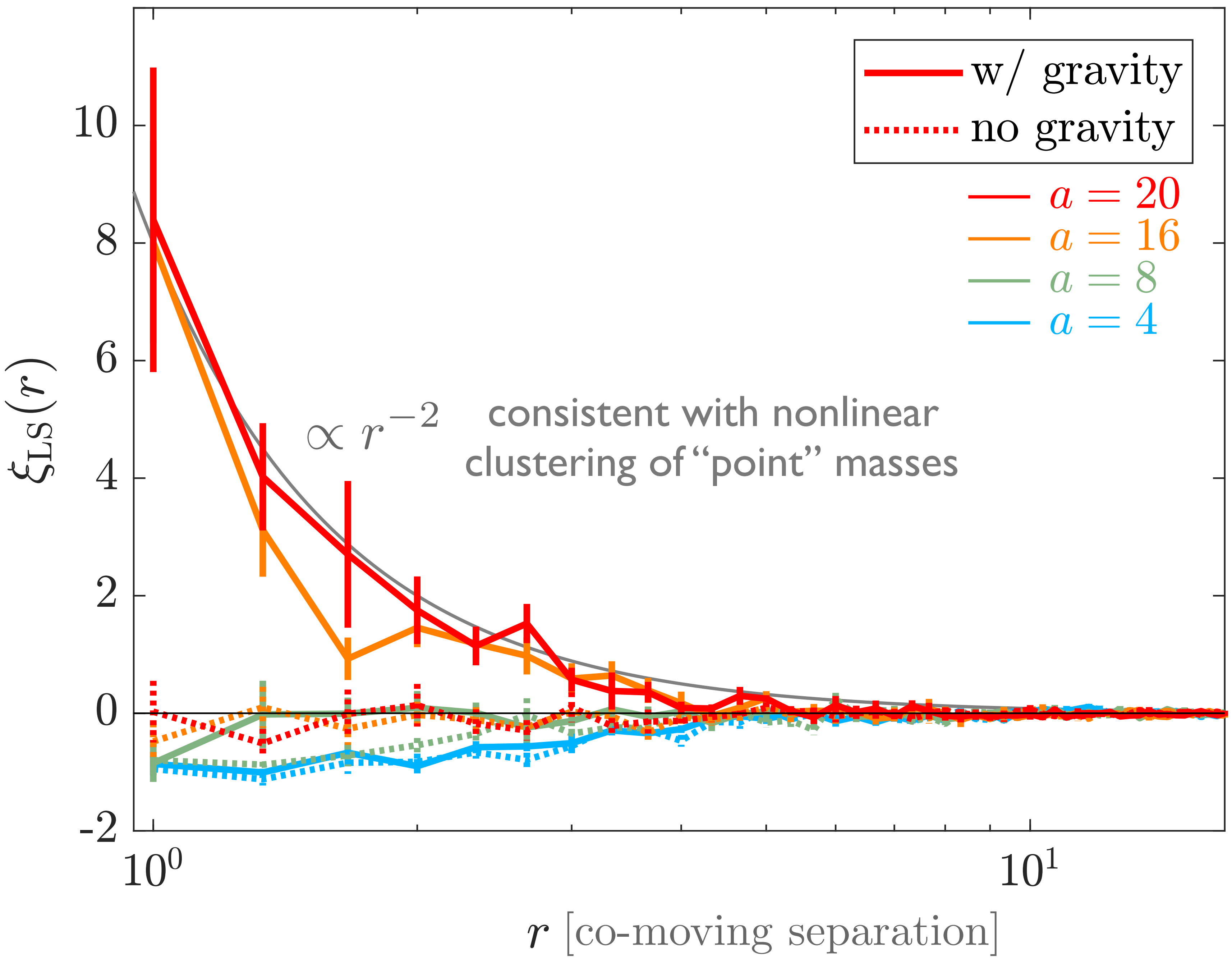


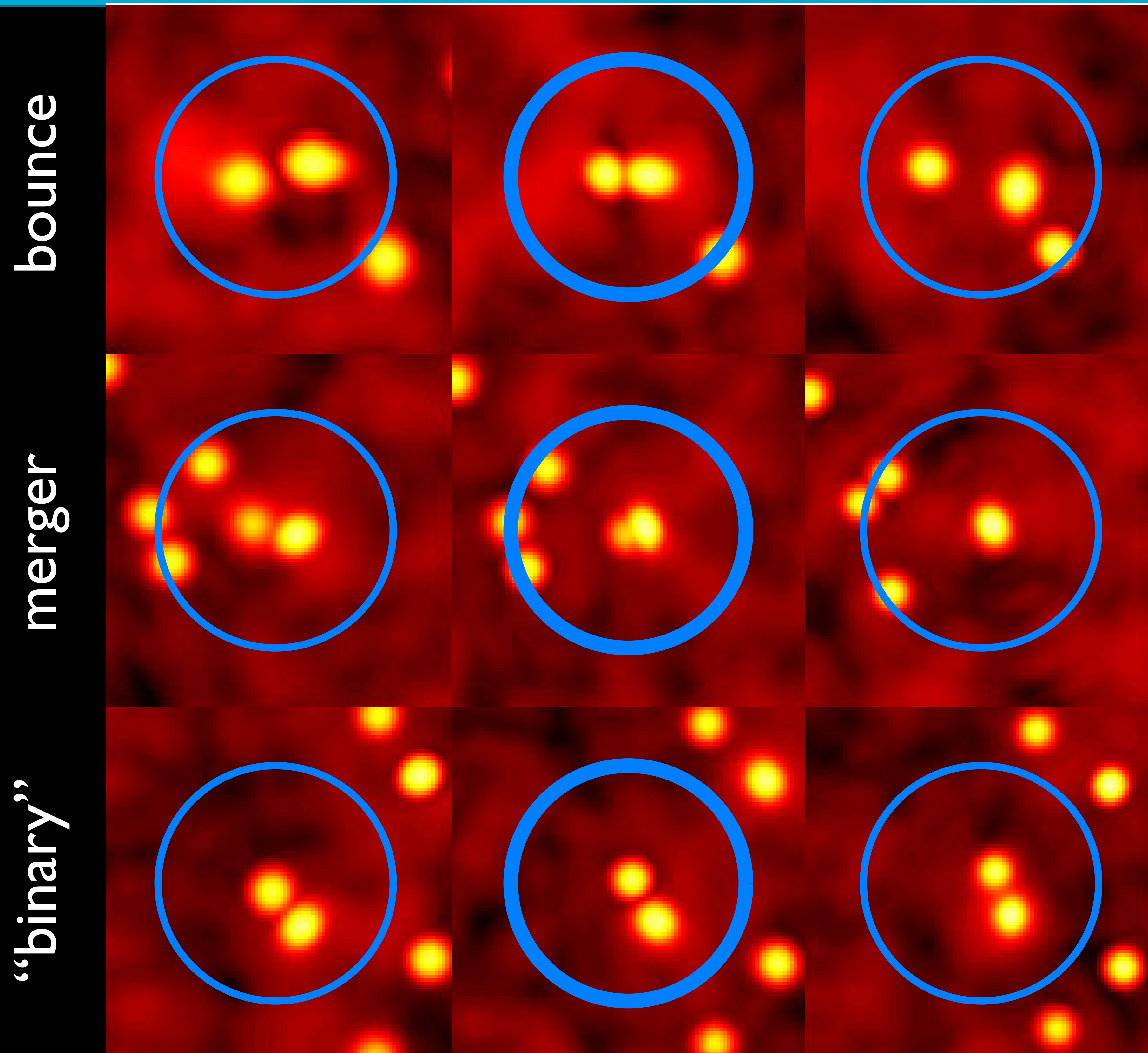
gravitational clustering of solitons



*theoretical arguments for r^{-2} in Saslaw 1980

*we don't fully understand the velocity distribution

phase dependent interactions



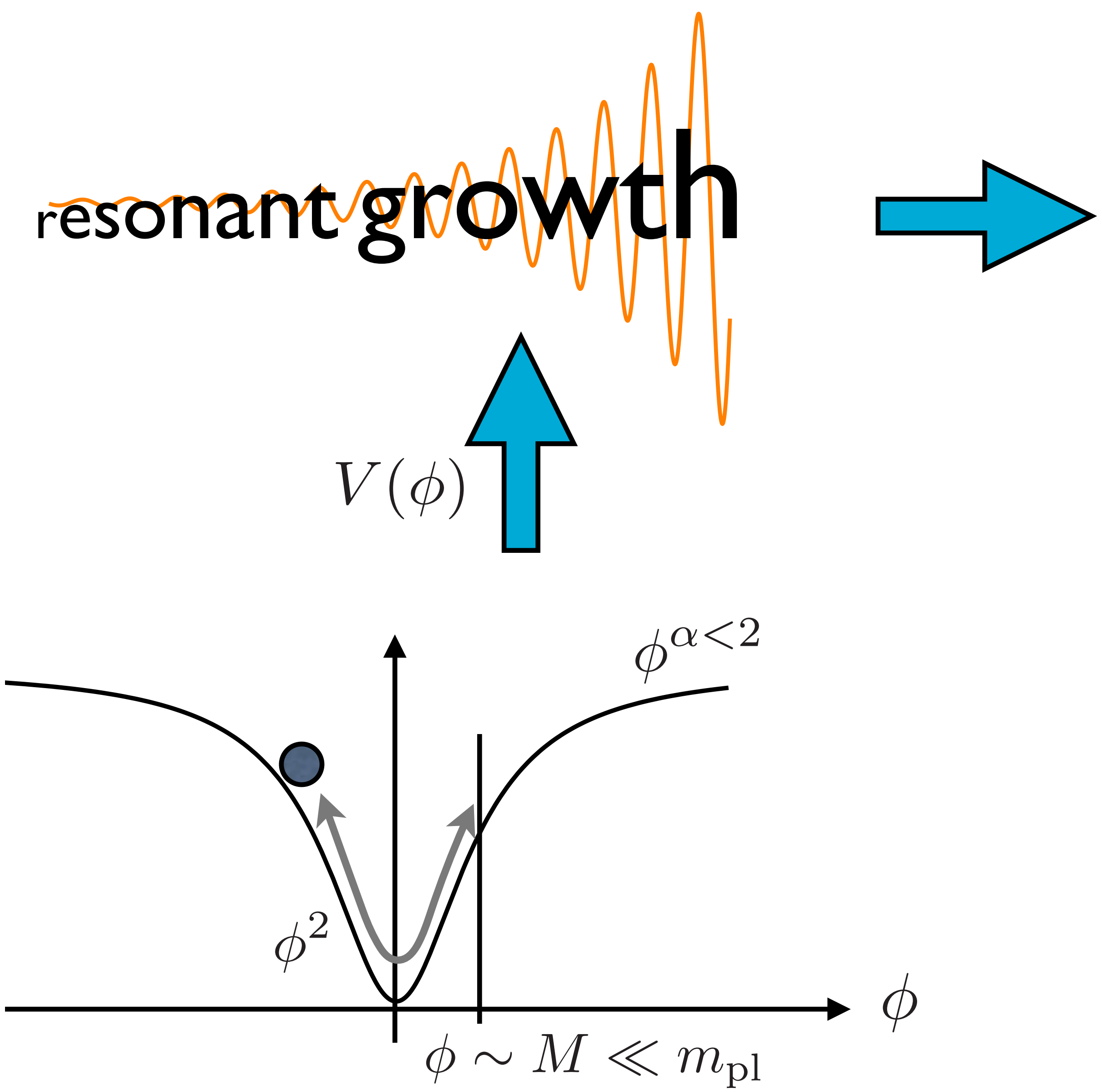
$\phi \propto \Re[\psi]$

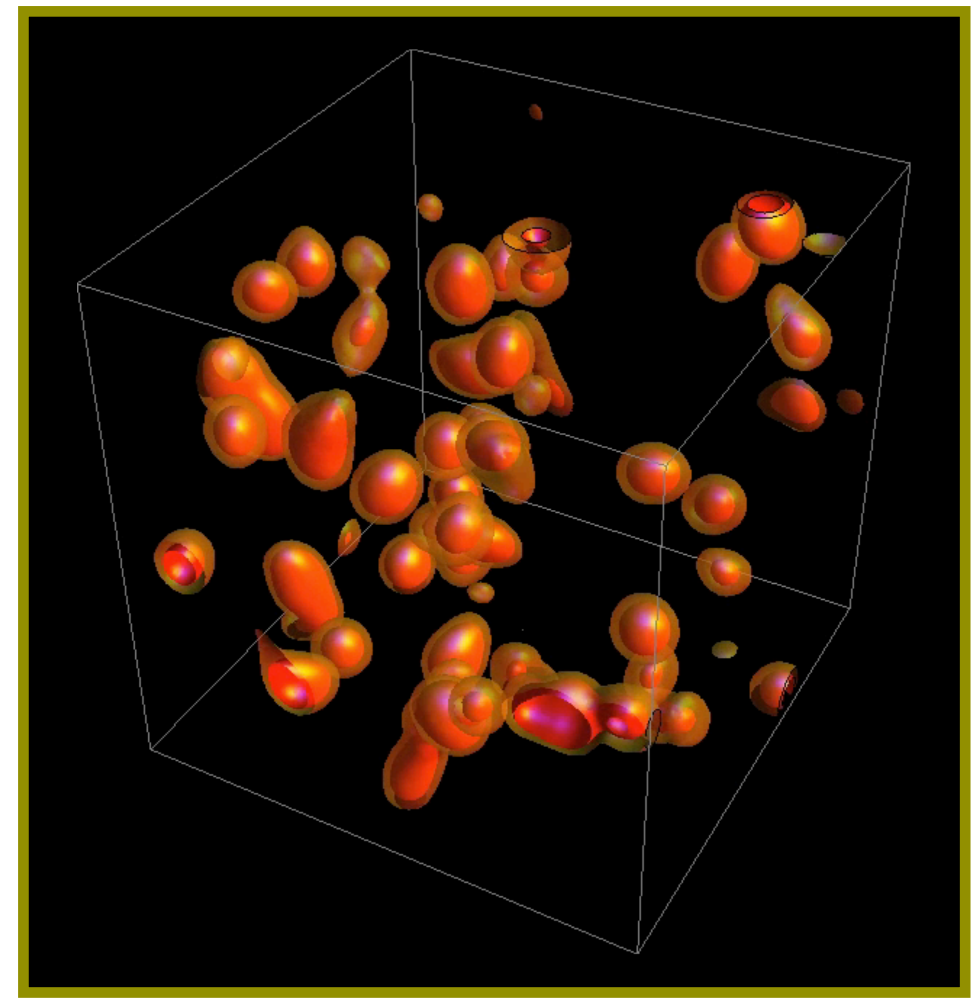
$\psi_a(t, \mathbf{x}) = \Psi_a(\mathbf{x}) e^{-i\nu_a t + \theta_a}$

$|\theta_1 - \theta_2| \simeq \pi$

$|\theta_1 - \theta_2| \simeq 0$

result 2: oscillon formation (solitons)



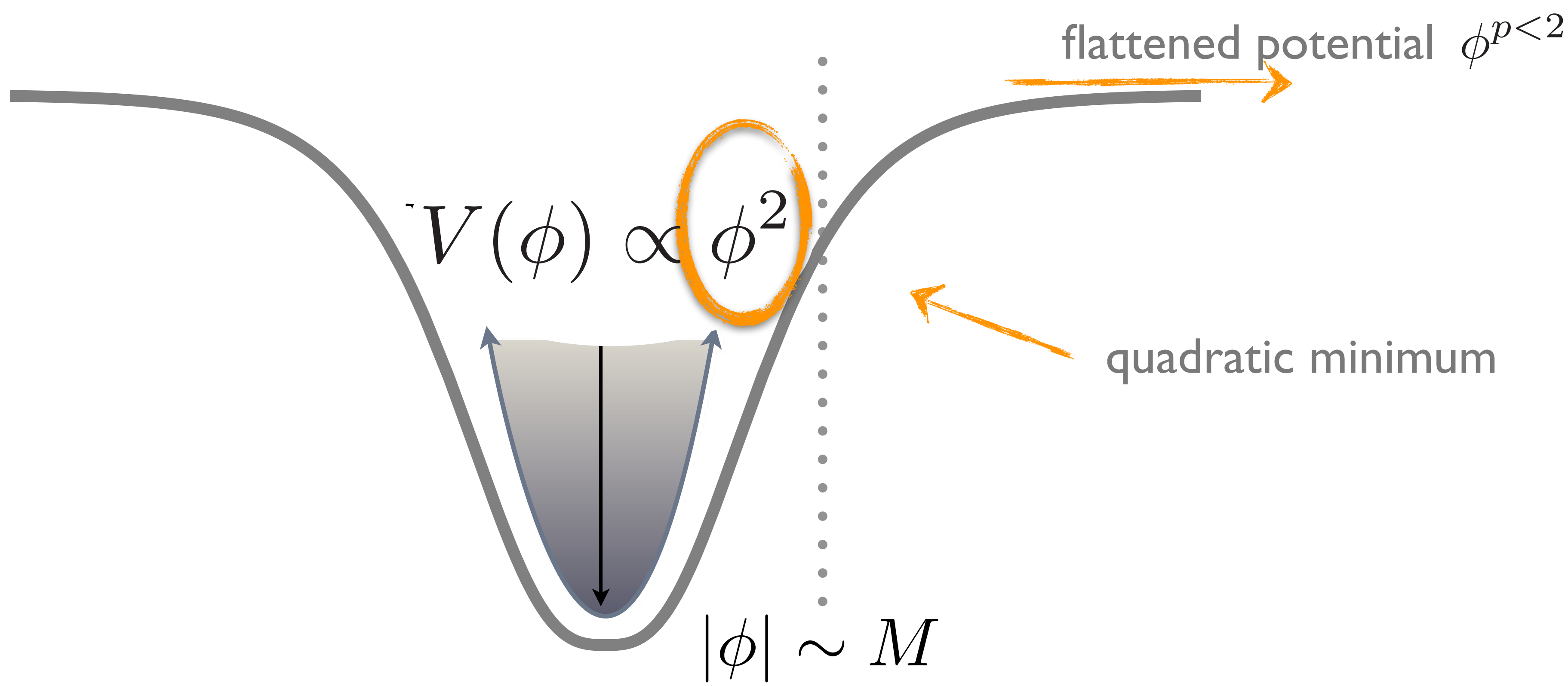


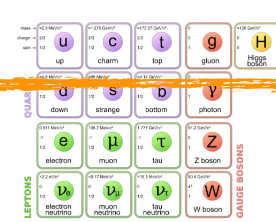
1.oscillons dominate the energy density

2.they cluster gravitationally

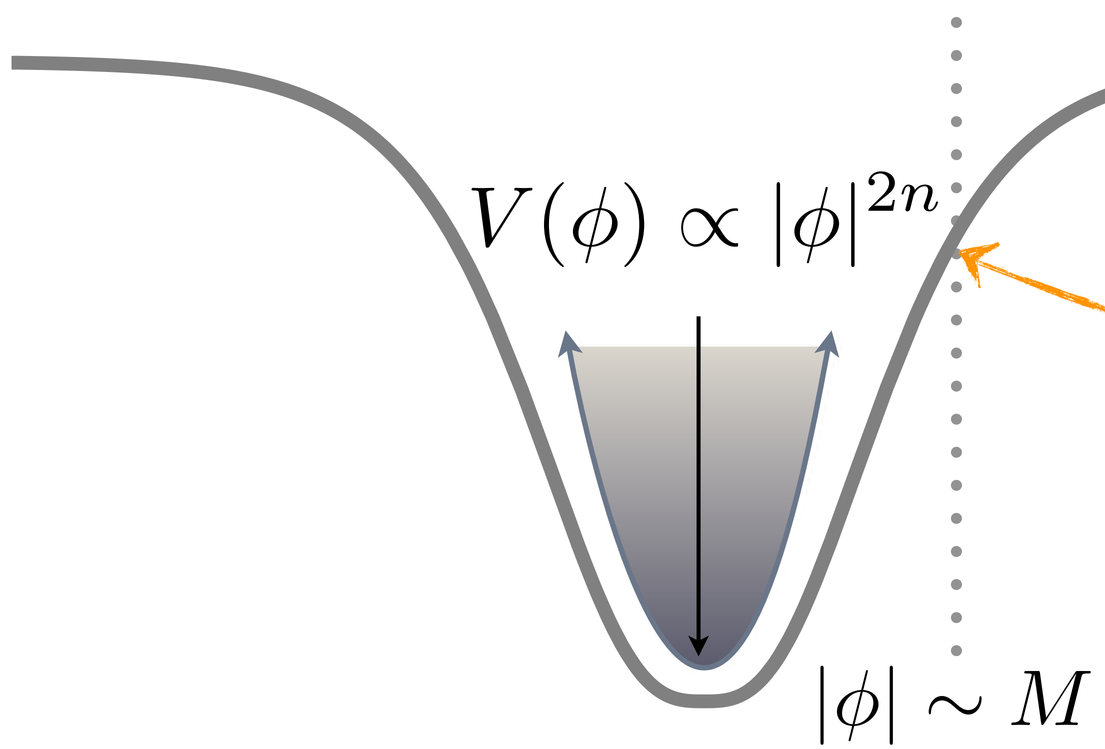
3. can undergo complex scattering

so far, quadratic minima with wings …

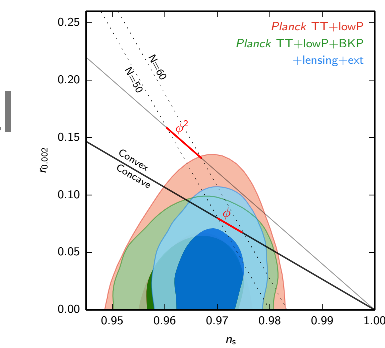


- shape of the potential (self couplings)
- ~~couplings to other fields~~  $\chi\ ,\psi$

non-quadratic, power-law minima ?



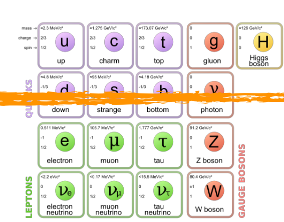
flattened potential



power law at the minimum

(fractional powers also)

- shape of the potential (self couplings)
- ~~couplings to other fields~~



$\chi\ ,\psi$

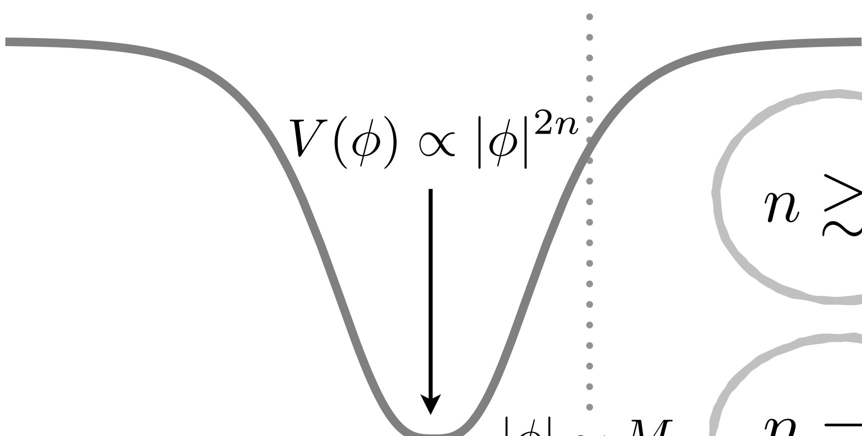
dynamics in different power law minima

Homogeneous oscillations

$$w = \frac{n-1}{n+1}$$

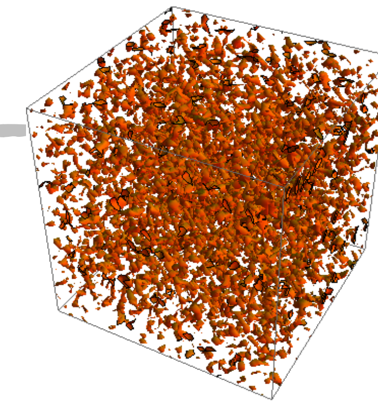
Turner (1983)

inflaton potential



$n \gtrsim 1$

field eventually fragments, but no solitons



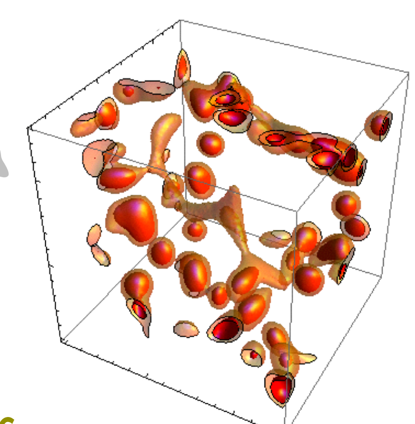
eq. of state

$$w \equiv \frac{\text{pressure}}{\text{density}} \to 1/3$$

radiation domination

$n = 1$

field fragments into solitons (for $M << m_{\rm pl}$)



eq. of state

$$w \to 0$$

matter domination

Lozanov & MA (2016/17)

1608.01213, 1710.06851

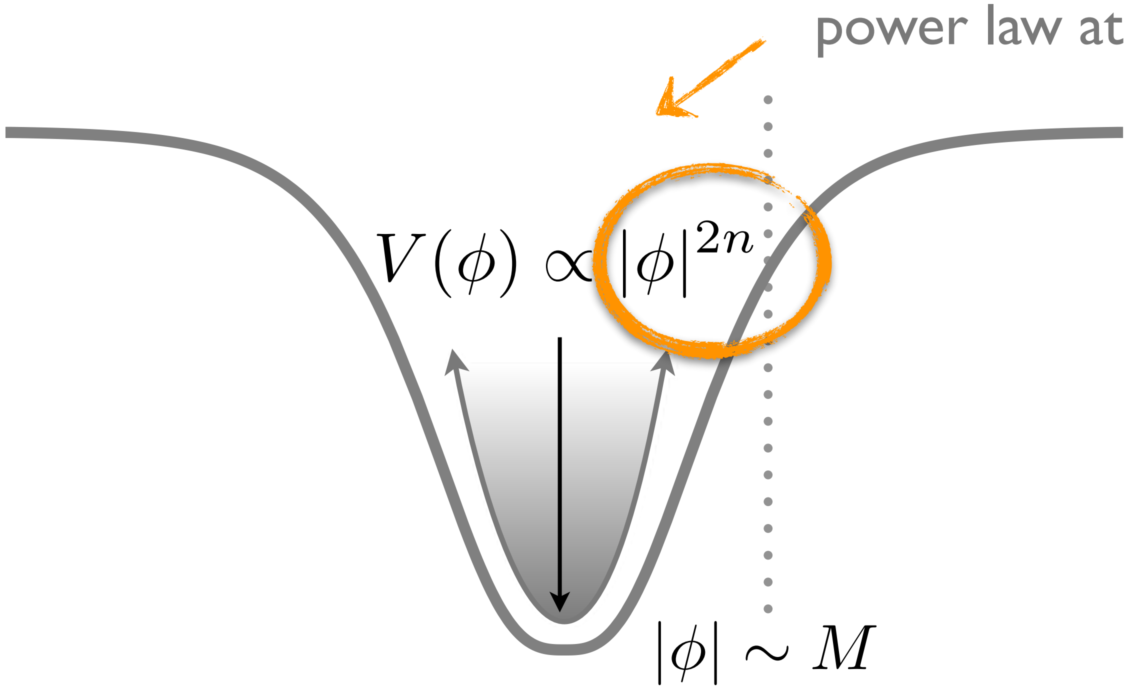
result 3: “equation-of-state”

the *spatially averaged* equation-of-state of fields

- $(n = 1)$ quadratic minima $w = 0$
- $(n > 1)$ non-quadratic minima $w = 1/3$ (after sufficient time)

why? $\mu_k/H \propto \phi^{-1}$

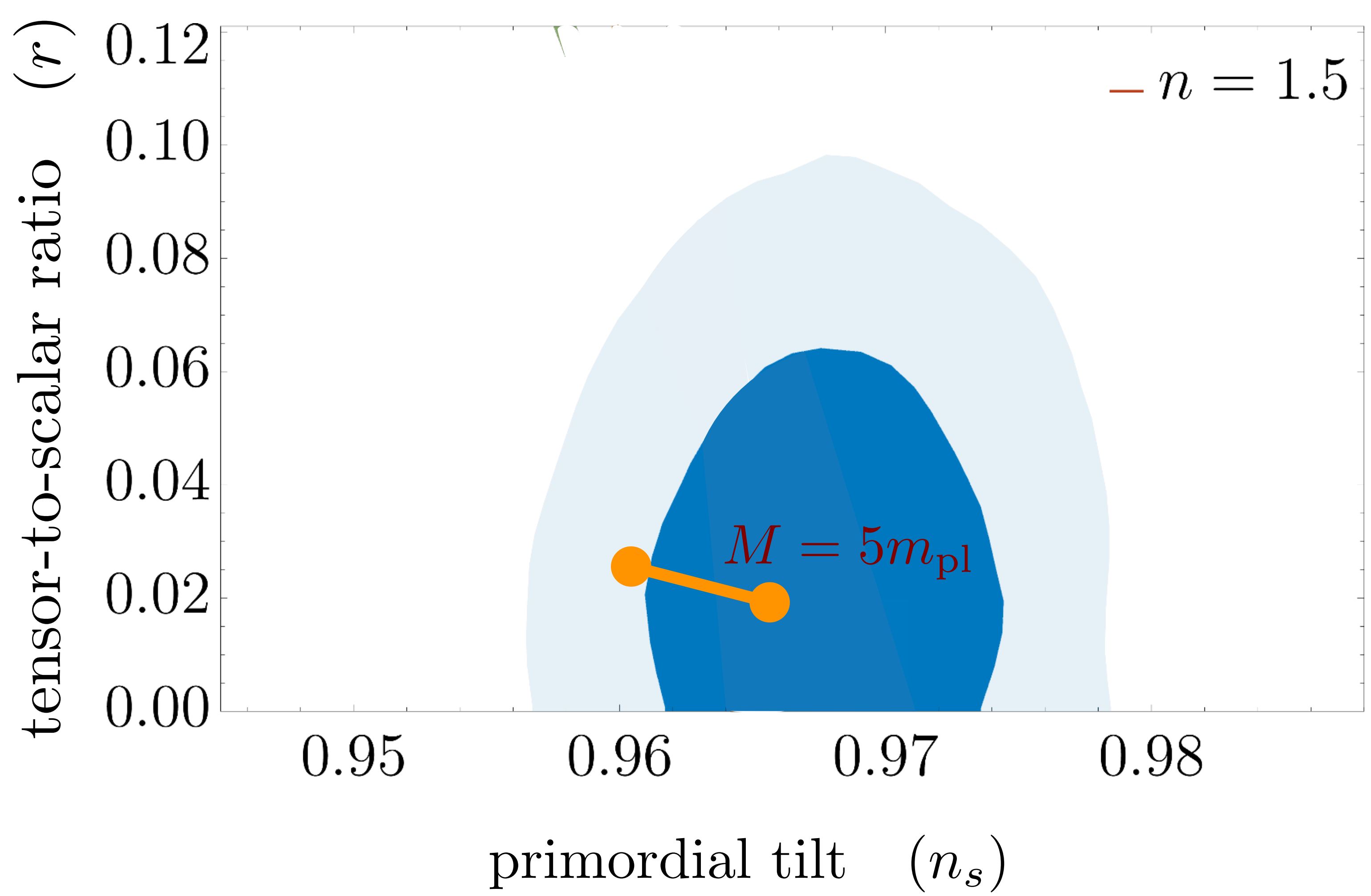
power law at the minimum

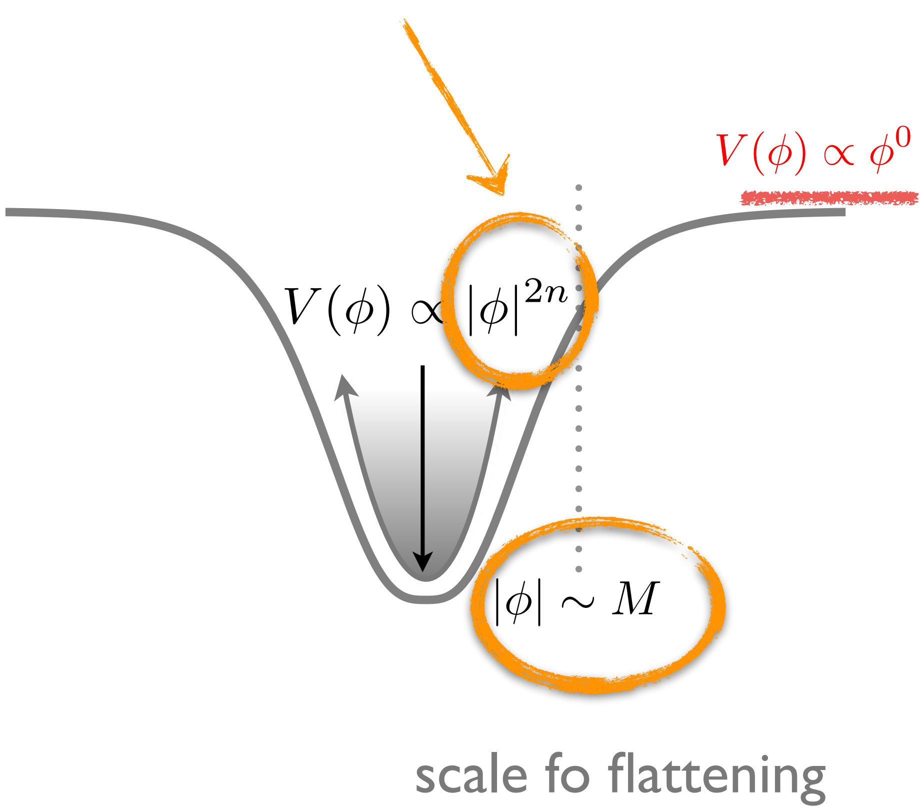


Lozanov & MA (2016/17)

1608.01213, 1710.06851

eq. of state & CMB observables





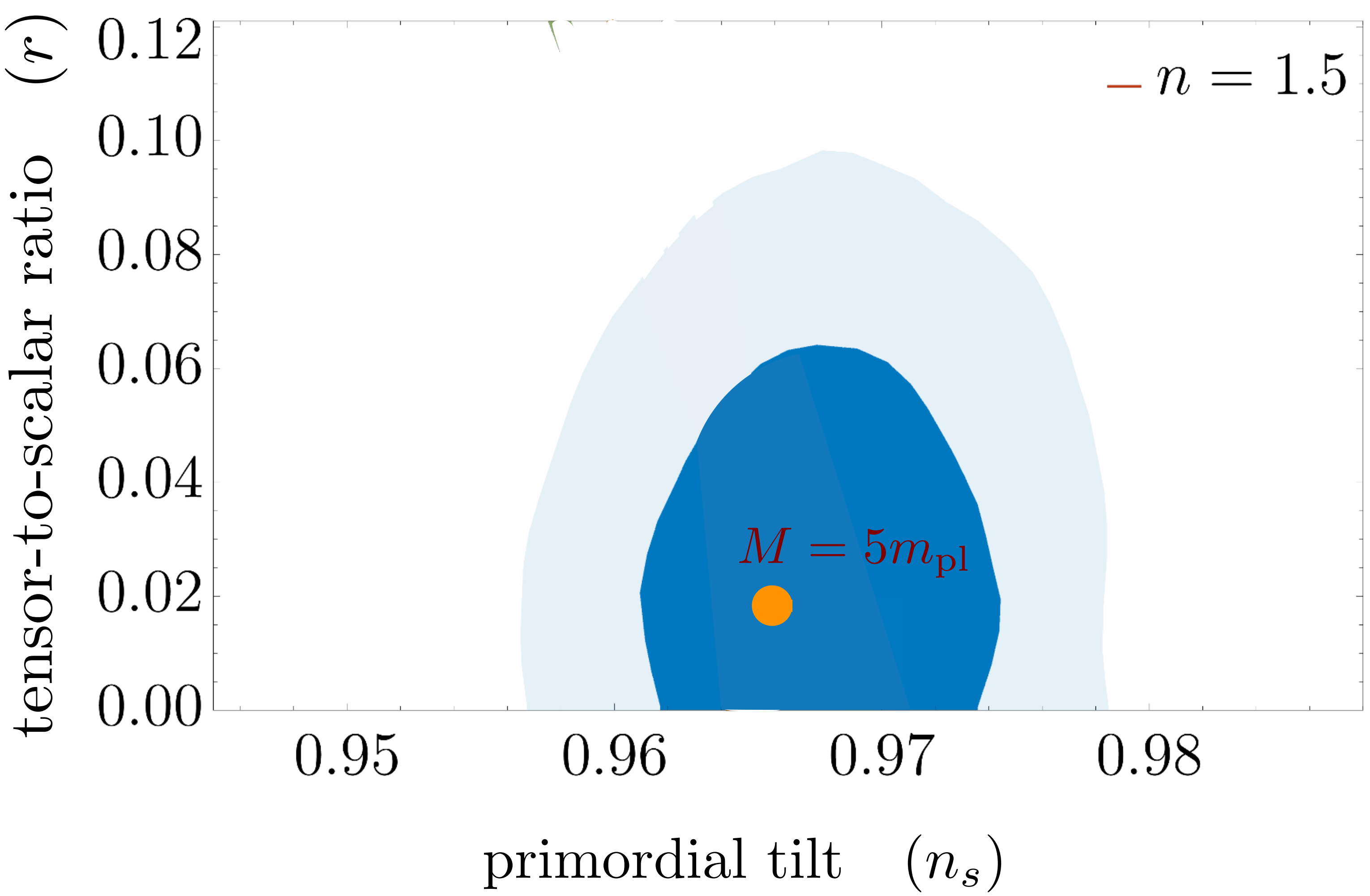
MA & Lozanov 2017 [1608.01213, 1710.06851]

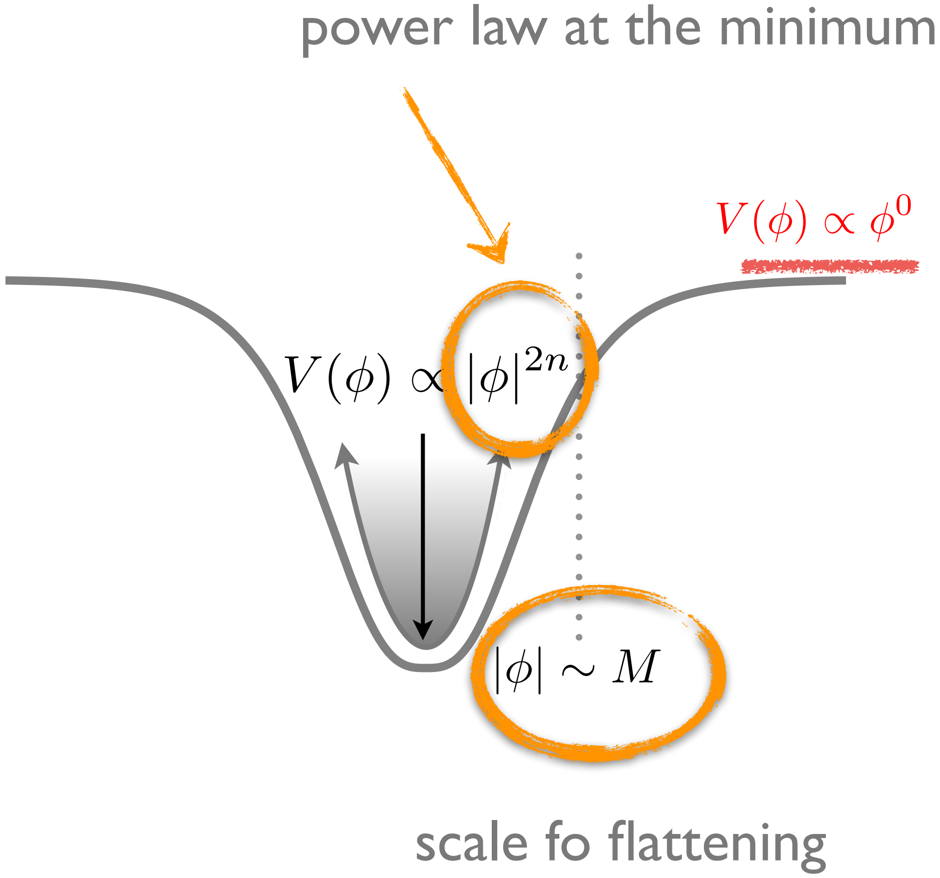
also see: Kamionkowski & Munoz (2014)

$n \neq 1$

* non-quadratic minimum

reduction in uncertainty





$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

$n \neq 1$

* non-quadratic minimum

Lozanov & MA (2017) [1608.01213, 1710.06851]

*all other fields are assumed to be light and massless.

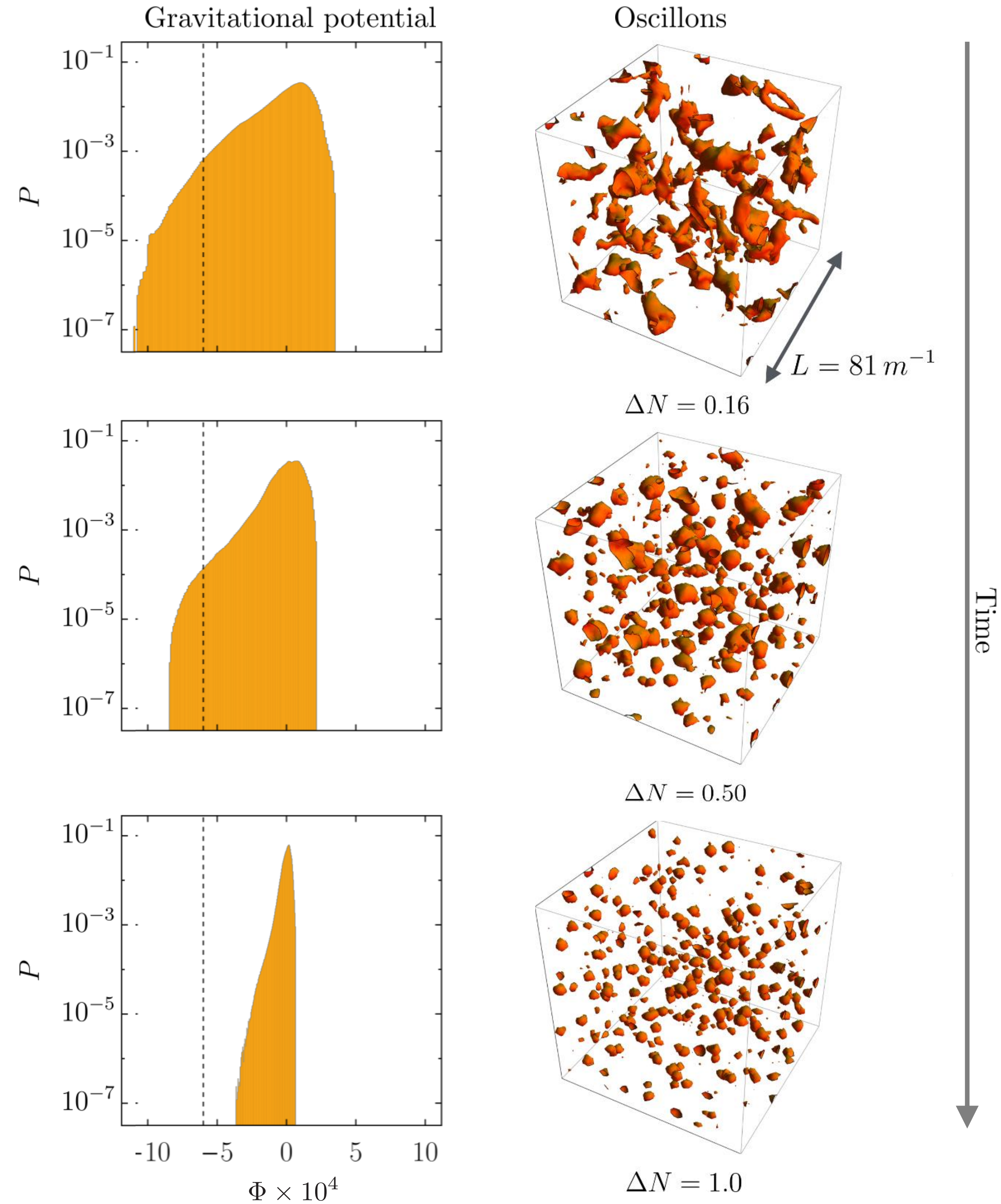
primordial black hole formation from solitons?

Lozanov & MA
1902.06736

$$\Phi \lesssim \text{few} \times 10^{-3}$$

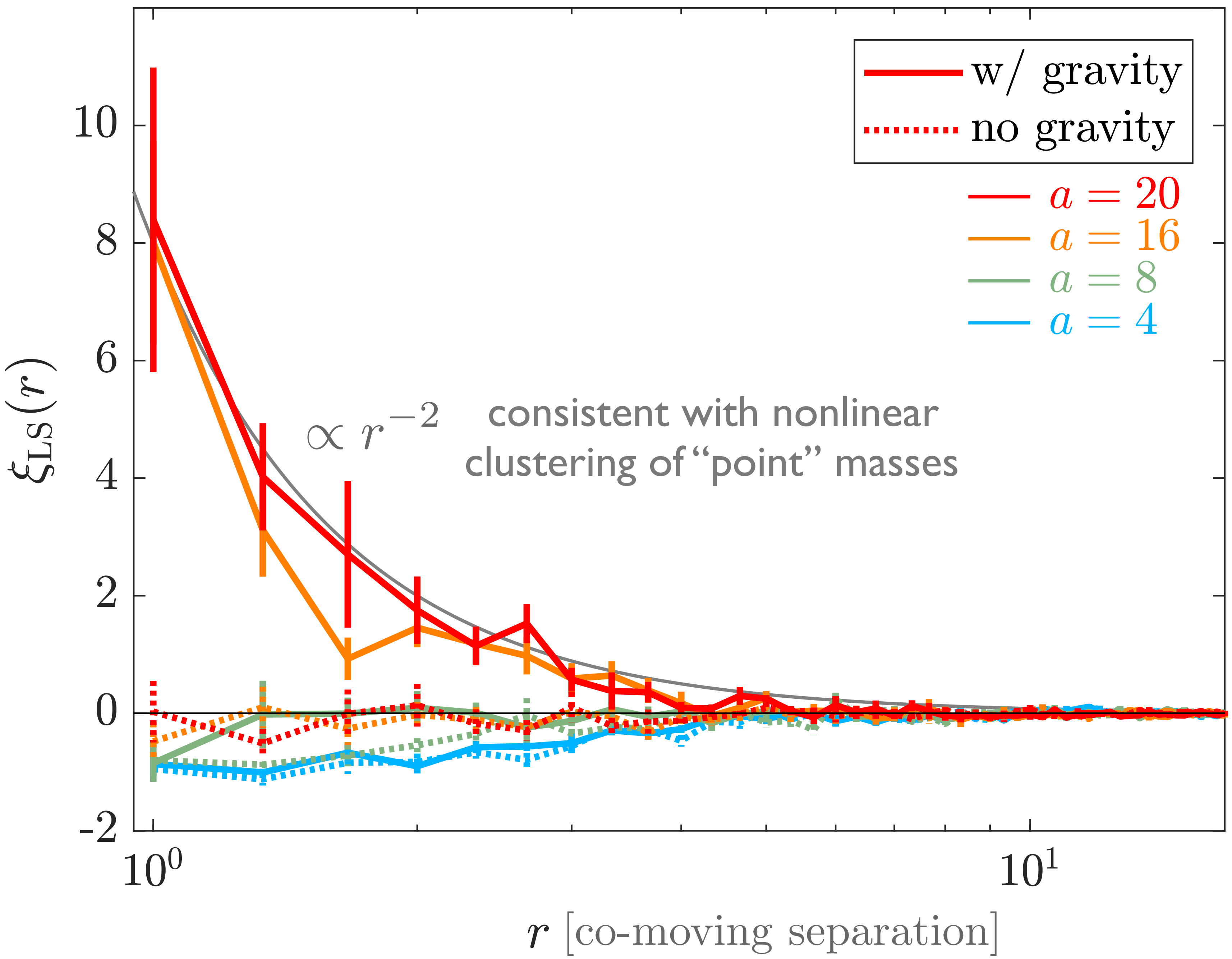
Not easy to form *PHBs* from *individual solitons* from self resonance

However *accidental over-densities* in solitons *more likely* to form PBHs (Cotner et. al 2018/19)

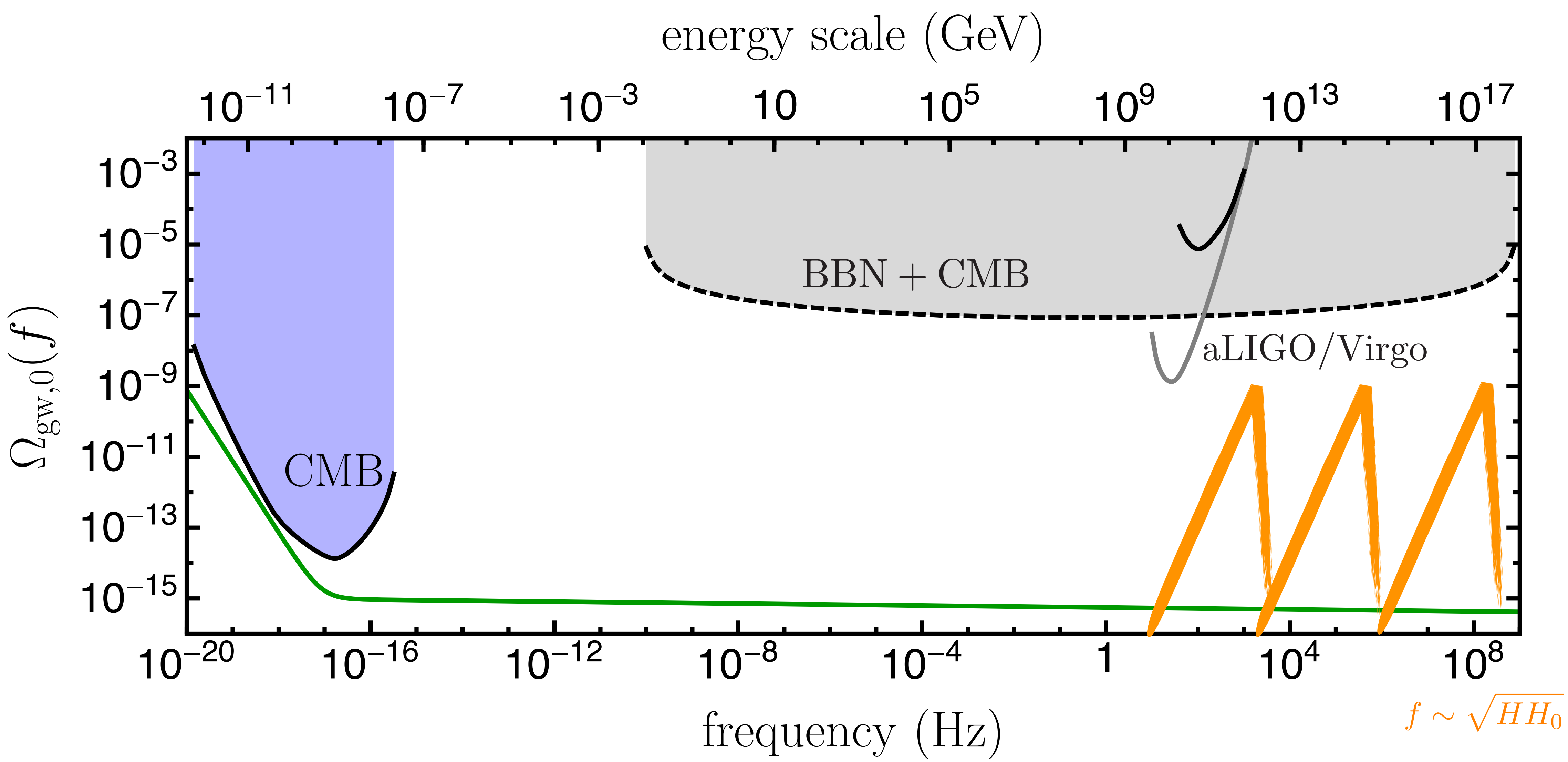


Also see talk by Juan Garcia Bellido on Monday on PBHs from inflation

quantitative?
gravitational clustering of solitons



implications: gravitational waves



also potential constraints from Neff from CMB S4

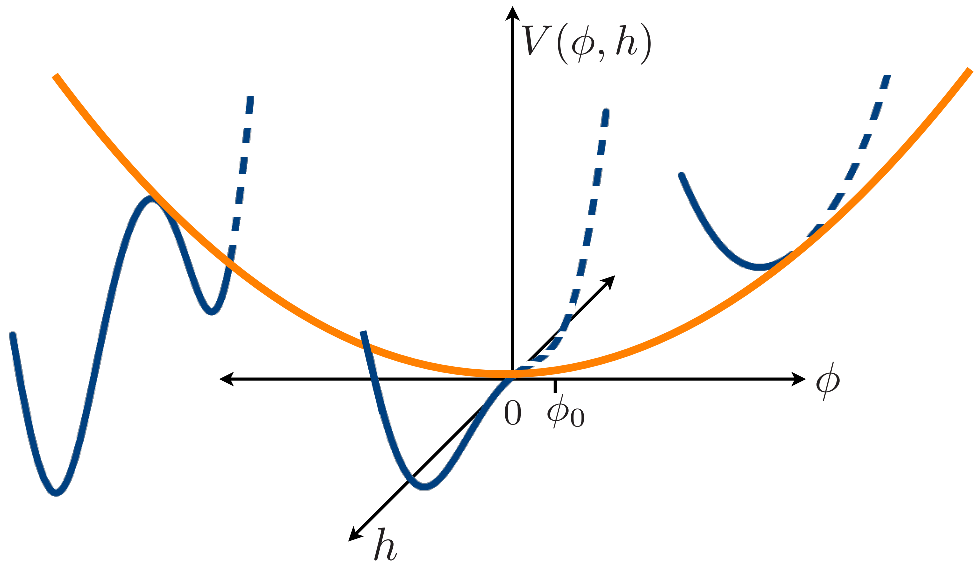
limits adapted from Lasky et. al (2015)

caveat* early universe g-waves amplitude depend on assumptions of expansion history

Earlier work on g-waves from end of inflation: Khlebnikov & Tkachev (1996), Easther, Giblin, Lim (2006/07), Dufaux et. al (2007)

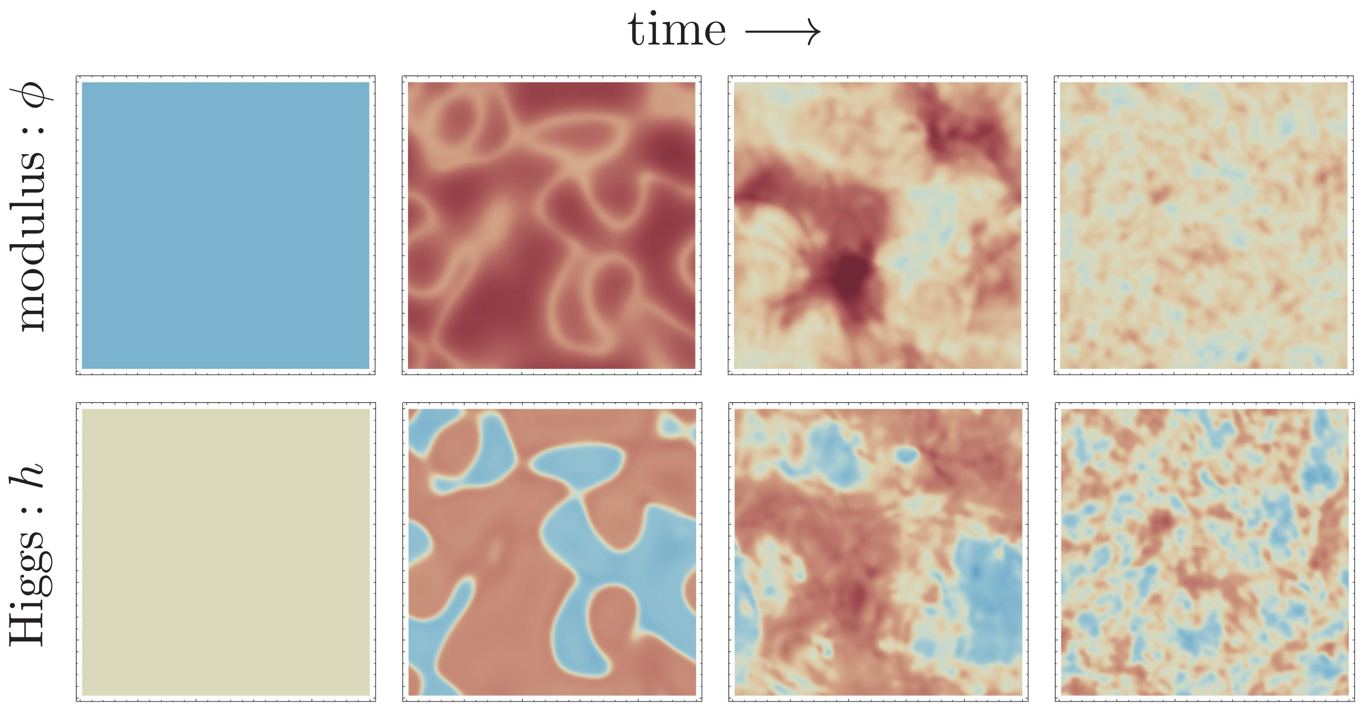
another example: Higgs - modulus system

MA, J. Fan, K. Lozanov & M. Reece (2018) [1802.00444]



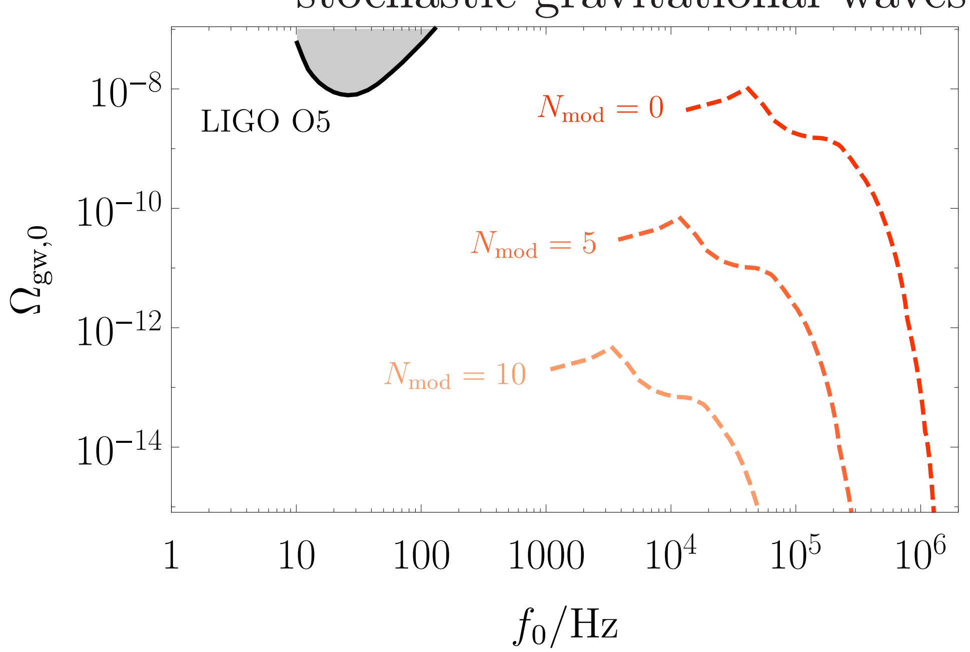
$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}(\phi - \phi_0)\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2$$

$$\text{fine tuning} \Leftrightarrow \frac{\phi_0}{f} \ll 1$$

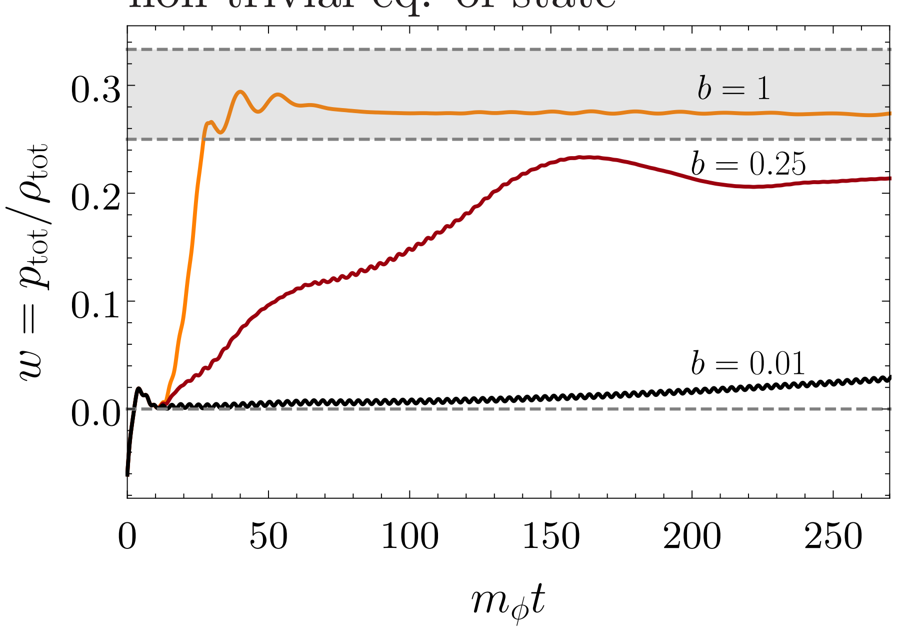


$$\frac{M^4}{2\lambda f^2 m_\phi^2} \to 1 \Leftrightarrow \text{rapid fragmentation}$$

stochastic gravitational waves



non-trivial eq. of state



end of inflation & dark matter abundance?

Can the abundance of dark matter depend on the non-thermal conditions at the end of inflation ?

* assuming only gravitational strength couplings between inflaton to dark matter

Yes, If:

$$\text{inflaton} \longrightarrow \text{radiation} \longleftrightarrow \text{dark matter}$$

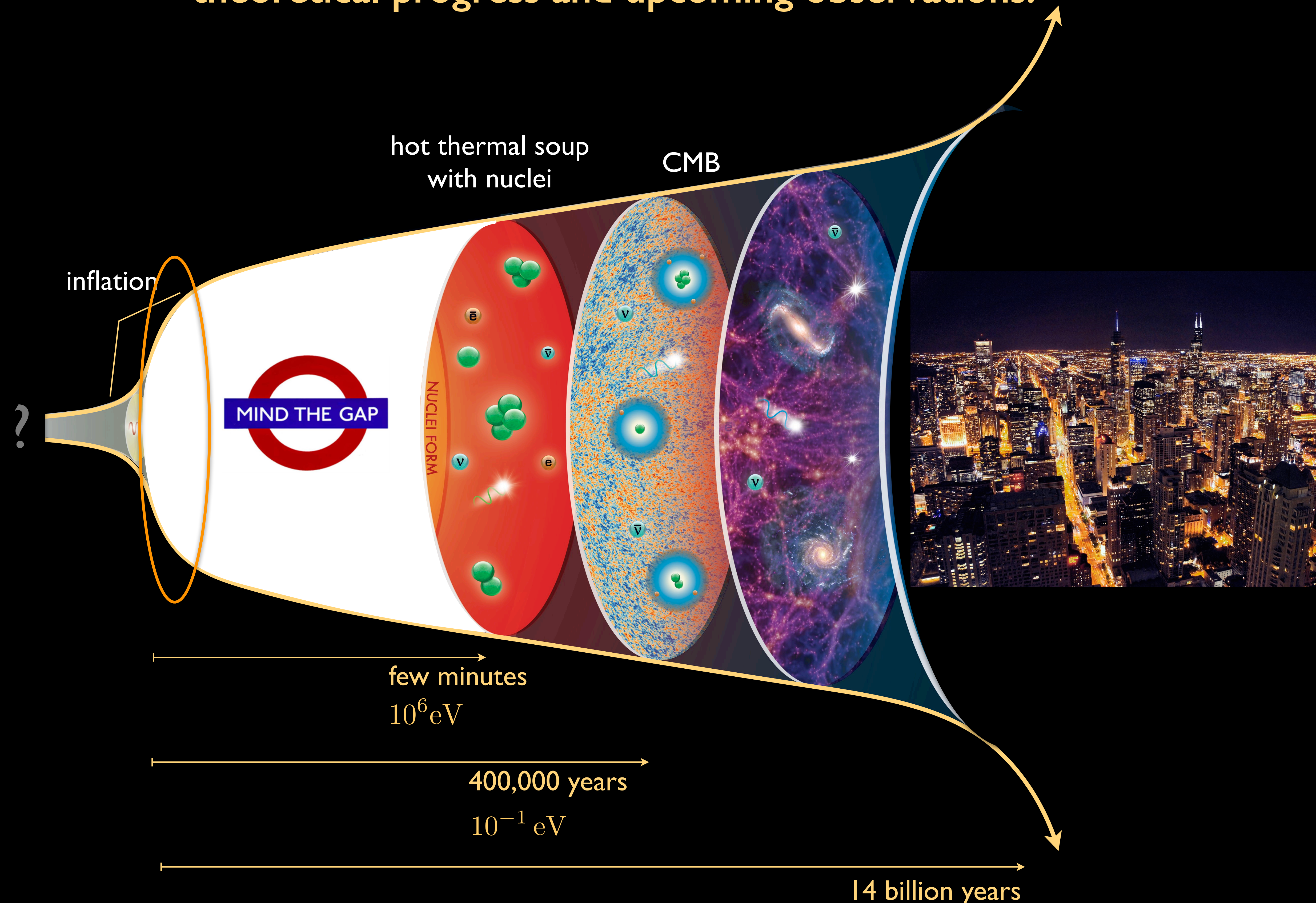
$$\sigma(s) \propto s^n \qquad n > 2$$

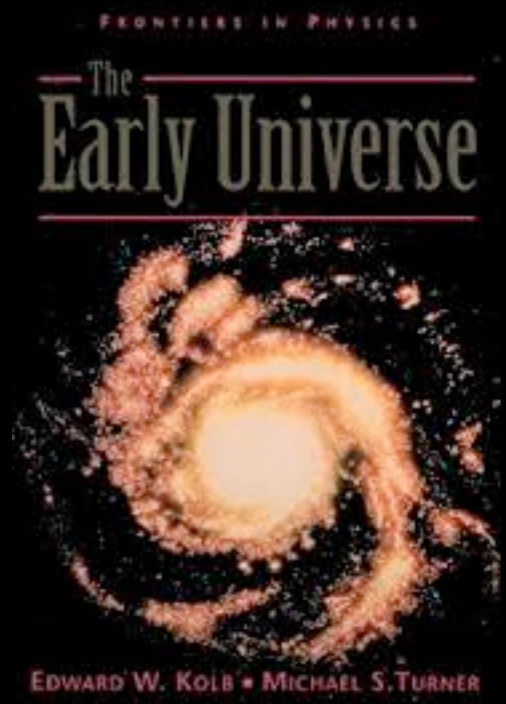
* radiation is SM particles

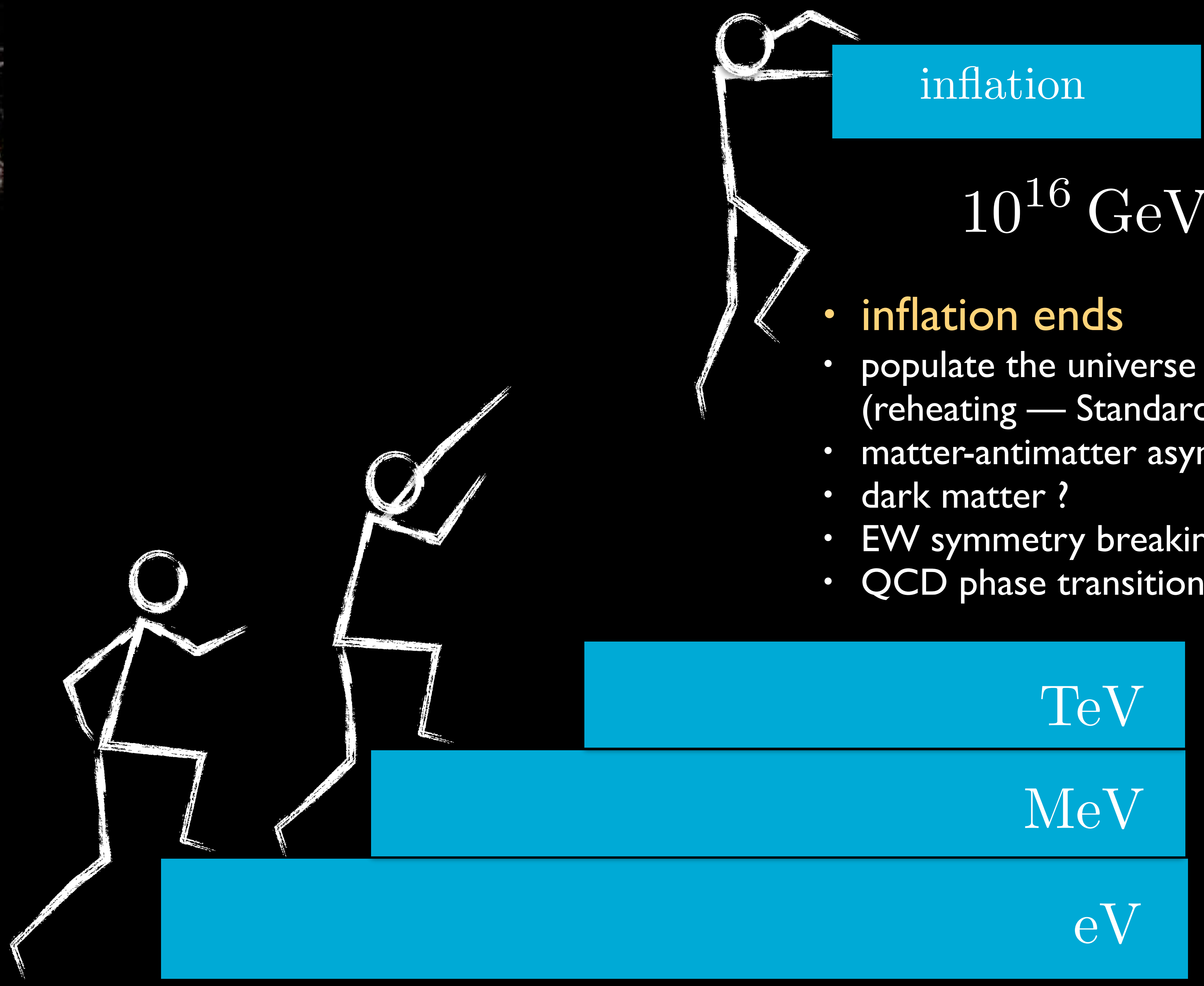
Garcia & MA (2018)

arXiv:1806.01865

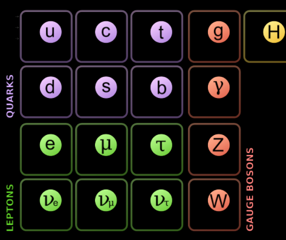
There is a lot to learn here — theoretical progress and upcoming observations.



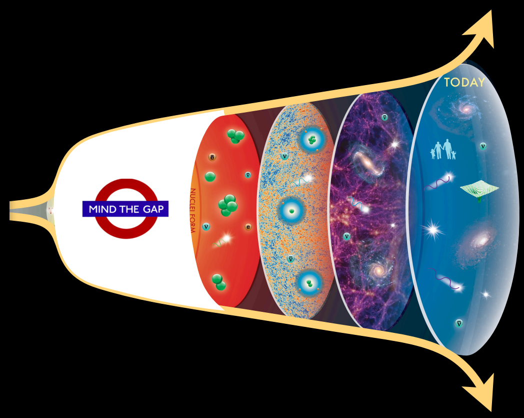




- inflation ends
- populate the universe (reheating — Standard Model) ?
- matter-antimatter asymmetry ?
- dark matter ?
- EW symmetry breaking
- QCD phase transition

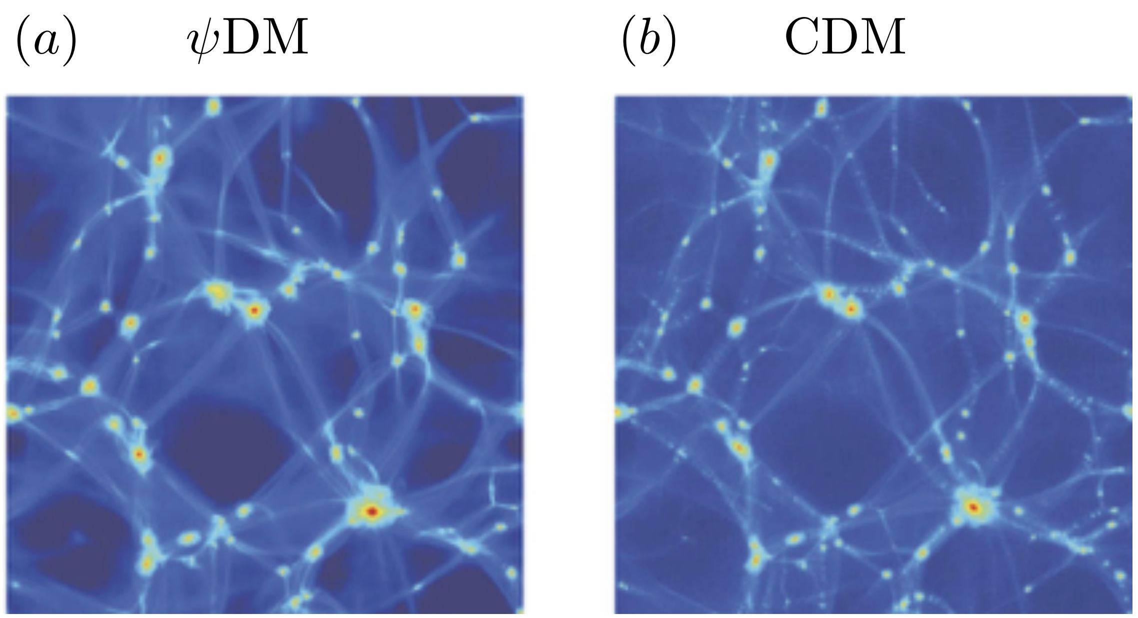


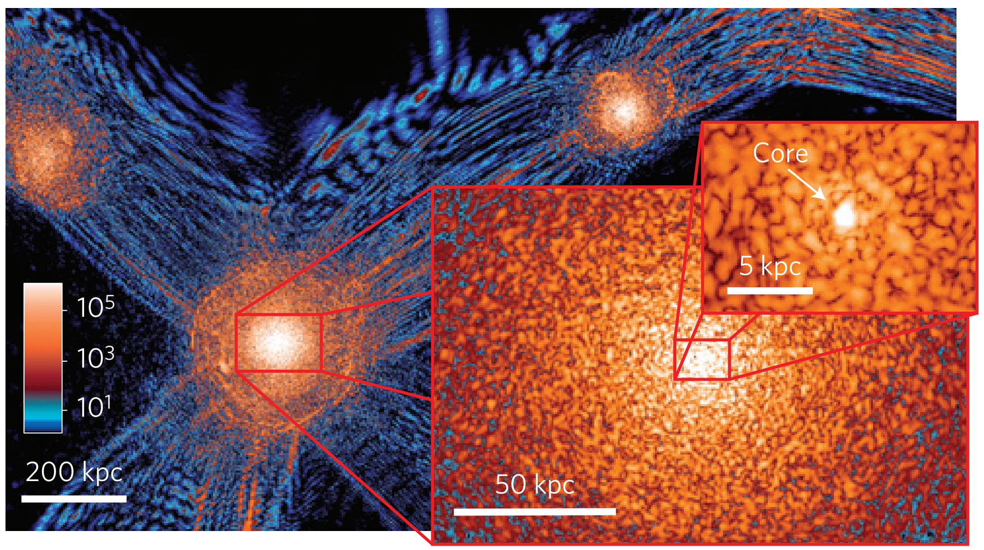
generality & novel connections



- Axionic dark matter [Jens Neimeyer's talk on Monday]
- Hubble tension ? [seeTristan Smith's talk on Monday]
- Condensed matter solitons
- Stochastic particle production (and connection to Anderson localization)

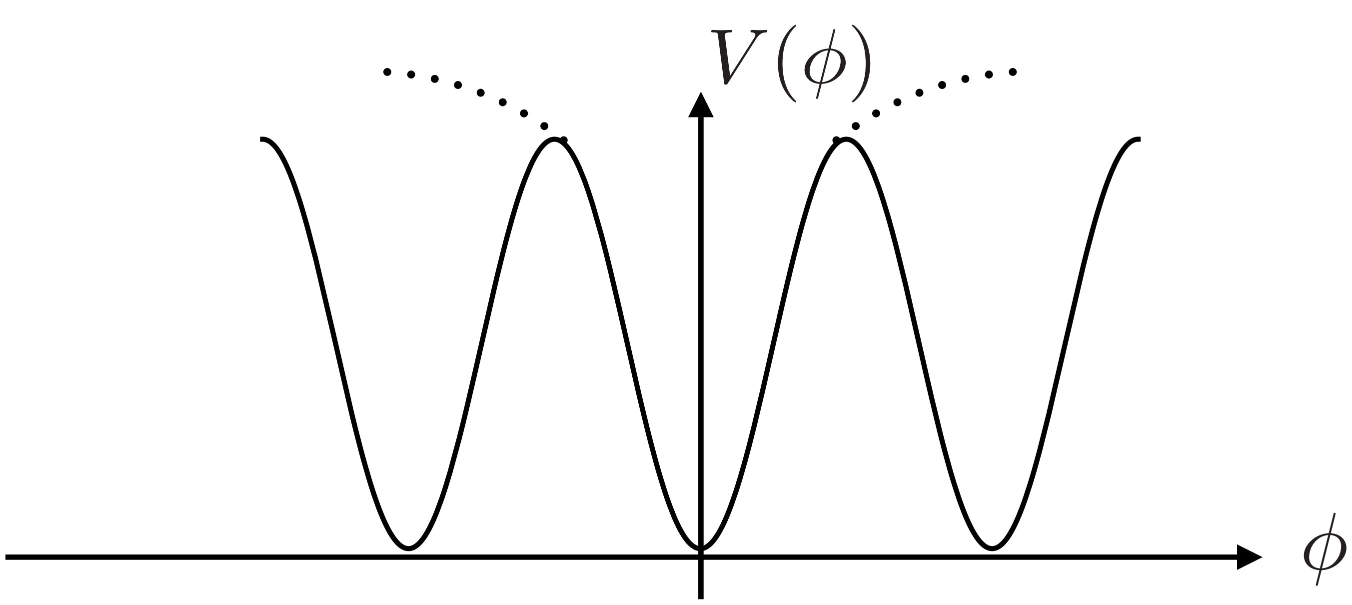
dark matter:
axion-like fields





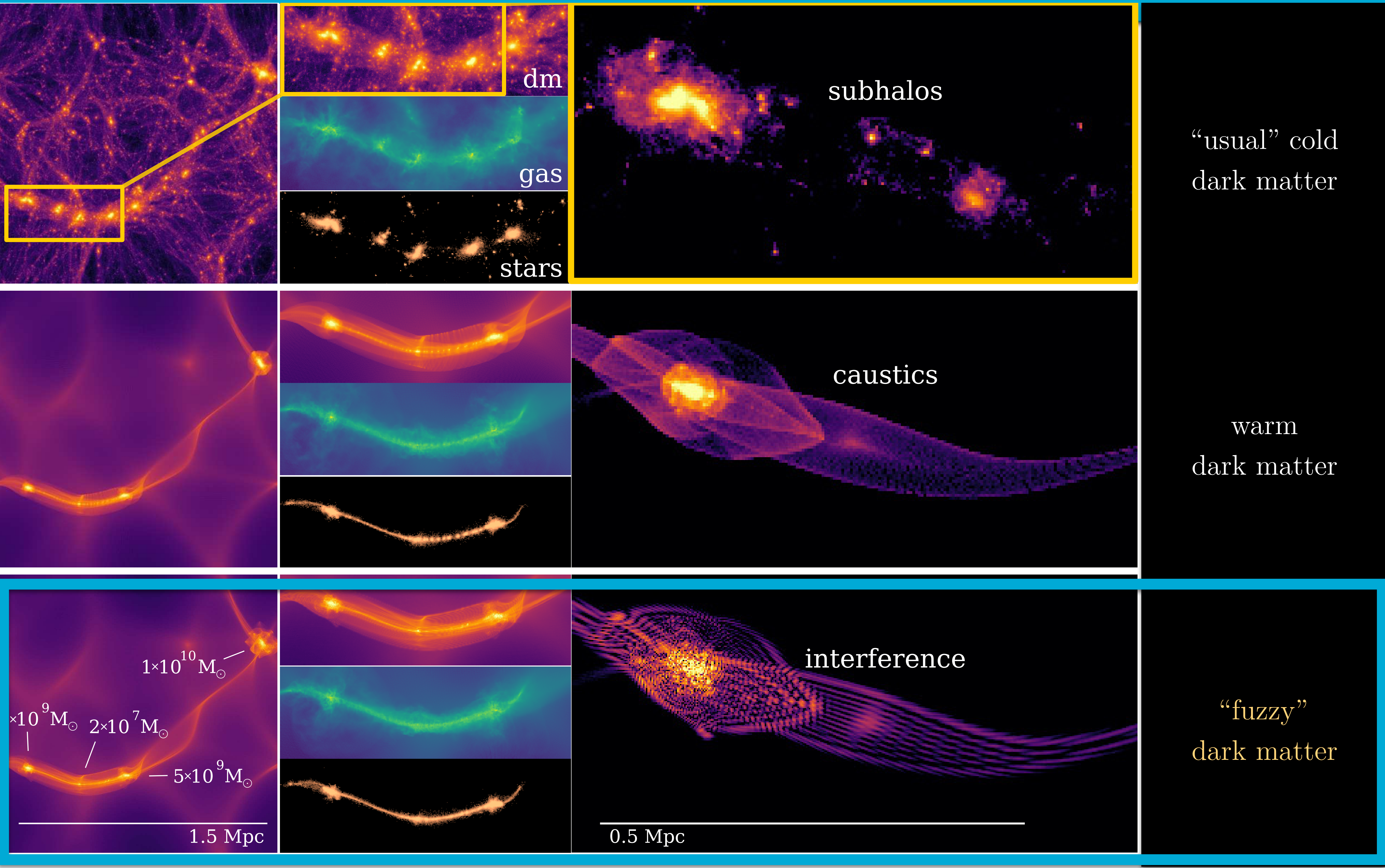
Schive et. al (2014)

for example:
Peccei & Quinn (1977)
Hogan & Reece (1988)
Kolb & Tkachev (1994)
Hu, Barkana & Gruzinov (2000)
Marsh & Silk (2014)
Niemeyer & Engels (2016)
Hui et. al (2016)
Arvanitaki et. al (2009/19)
Mocz et. al (2019)

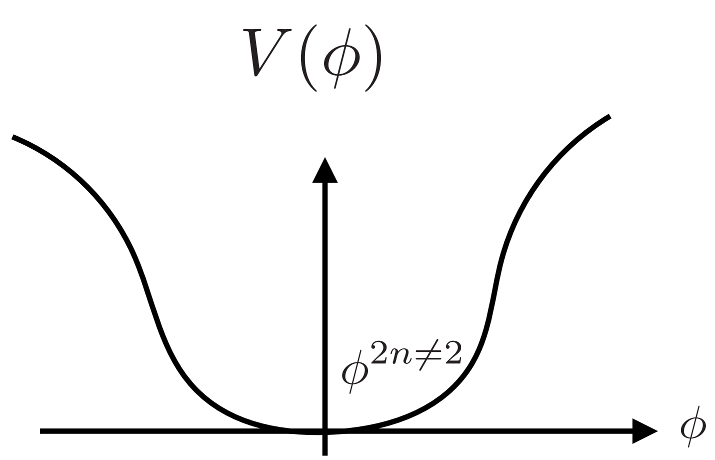


structure formation with light scalar fields

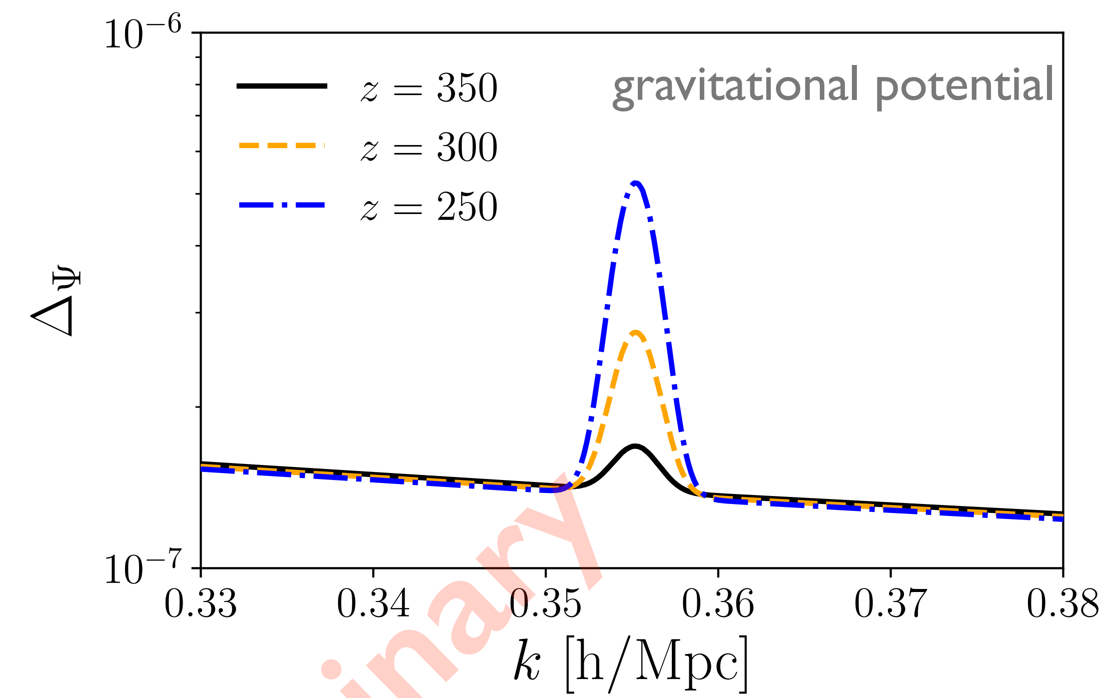
Mocz, +MA, et. al (2019)

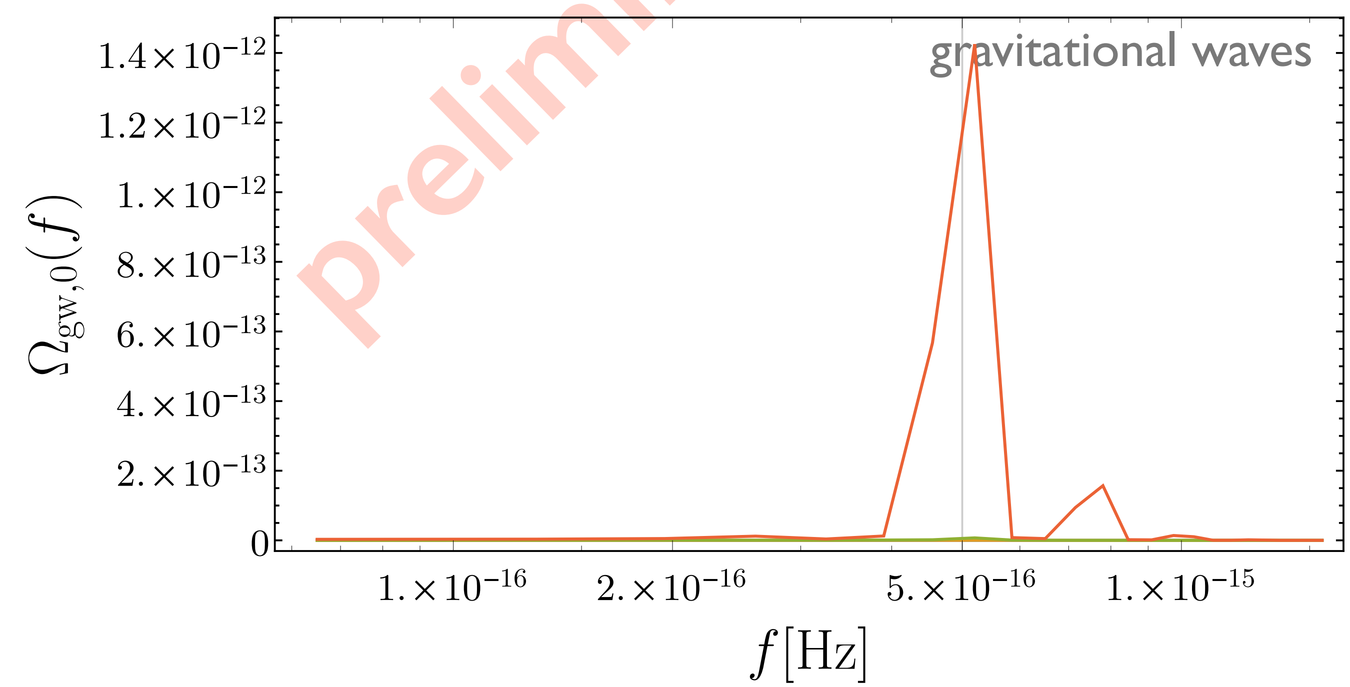


“Hubble Tension” resolution — some novel implications



*no-solitons

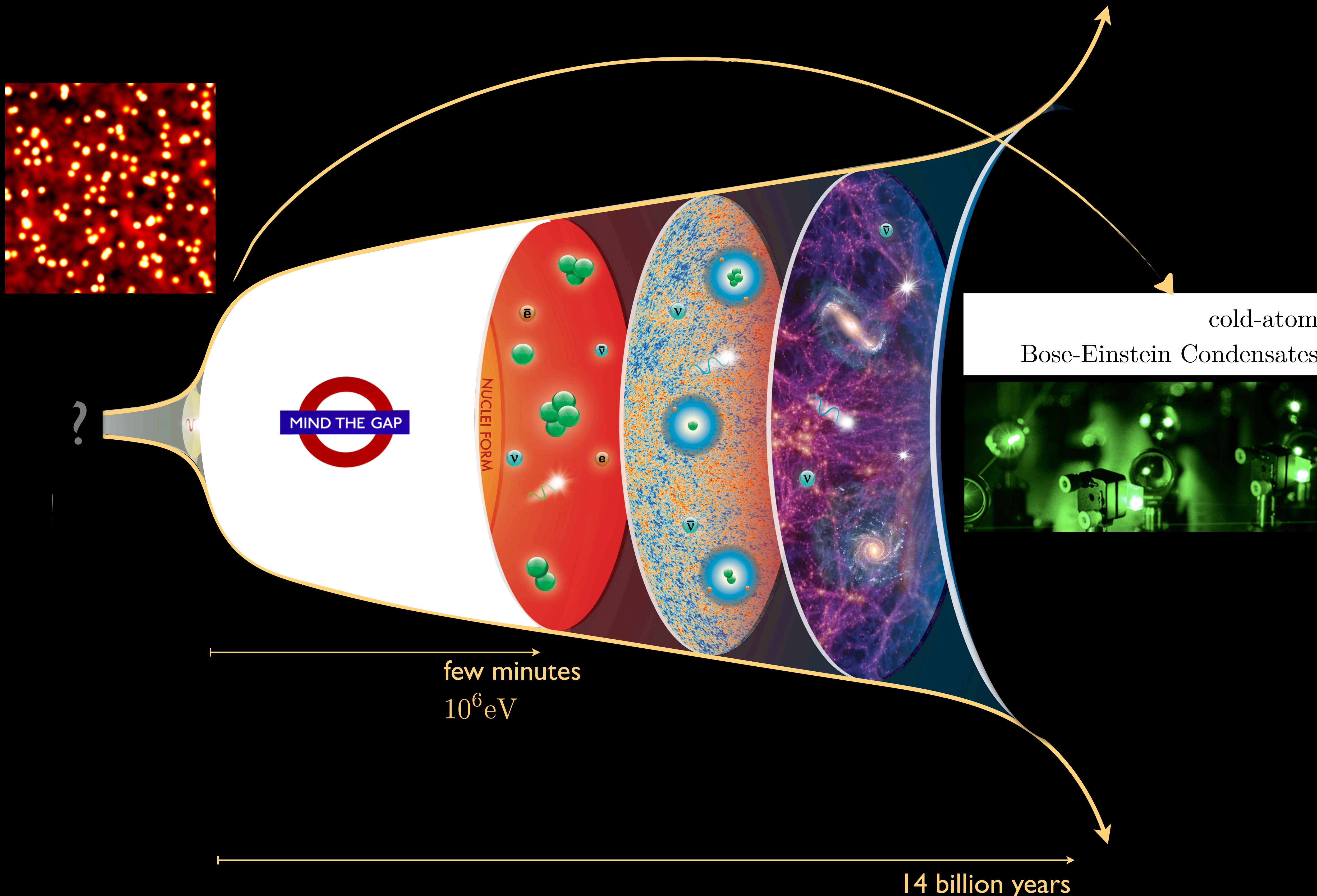




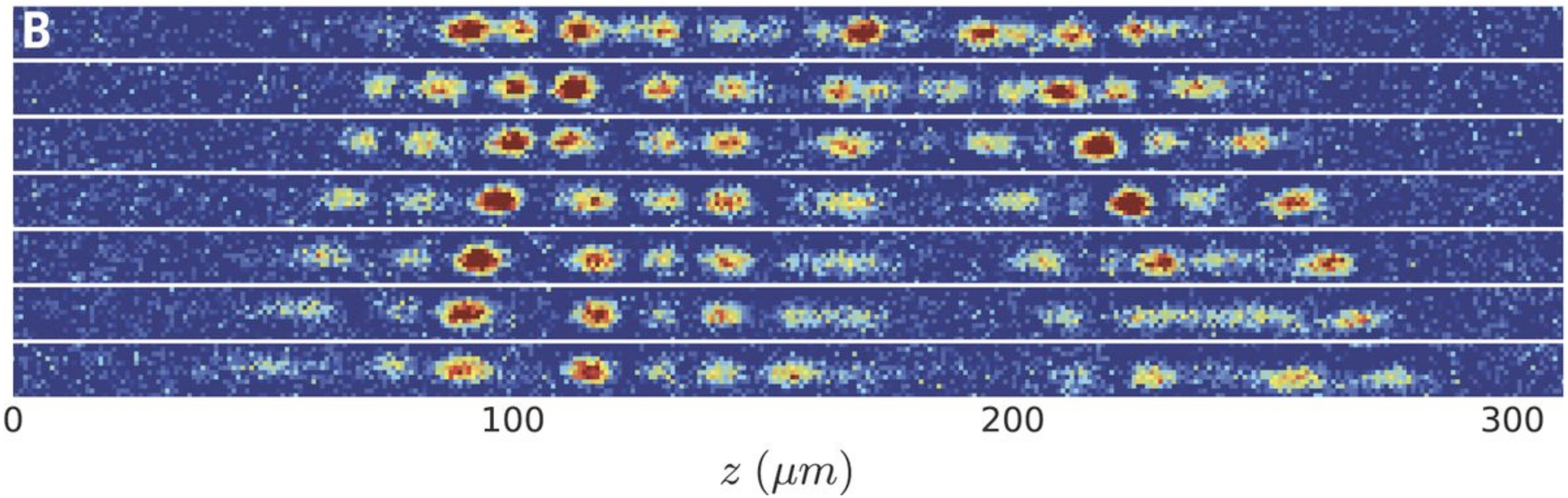
Smith, Poulin & MA (2019)
MA, Lozanov & Smith (in progress)

Also see: Karwal & Kamionkowski (2016), Poulin et. al (2018), Agrawal et. al (2019).

A Novel Connections



related solitons in BECs



Nguyen, Luo & Hulet (2017)

nonlinear Klein Gordon — nonlinear Schrodinger eq.

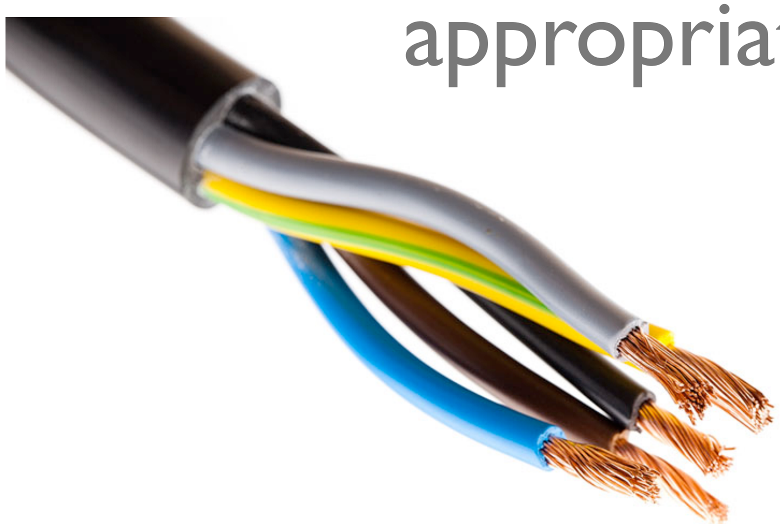
$$\partial_t^2 \phi - c^2 \nabla^2 \phi + \partial_\phi V(\phi) = 0$$

$$\partial_t^2 \varphi - c_{\rm s}^2 \nabla^2 \varphi + \partial_\varphi \mathcal{V}(\varphi) = 0 \quad \longleftrightarrow \quad i\partial_t \psi = \left[-\frac{1}{2m} \nabla^2 + U'(|\psi|^2) \right] \psi$$

relative phase between different condensates

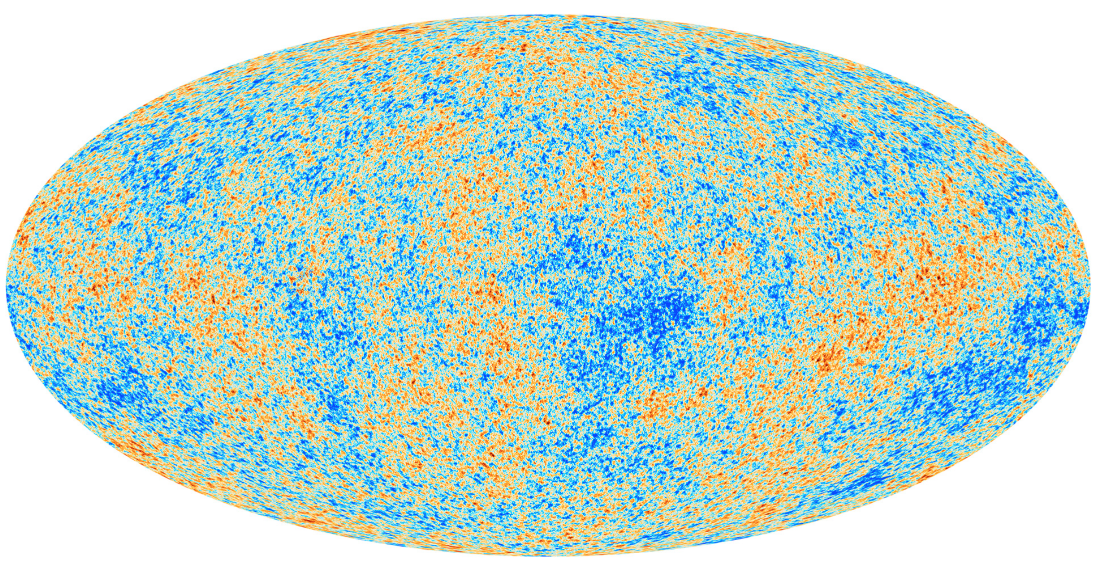
non-relativistic

(Stochastic) Particle Production in Cosmology



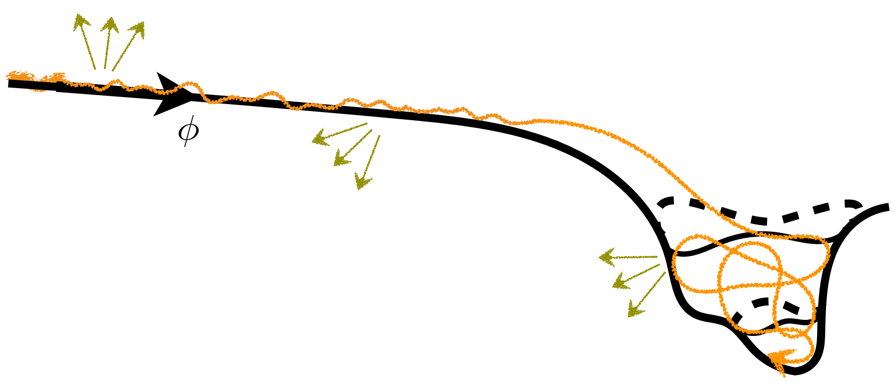
appropriate for sufficiently complex models of inflation

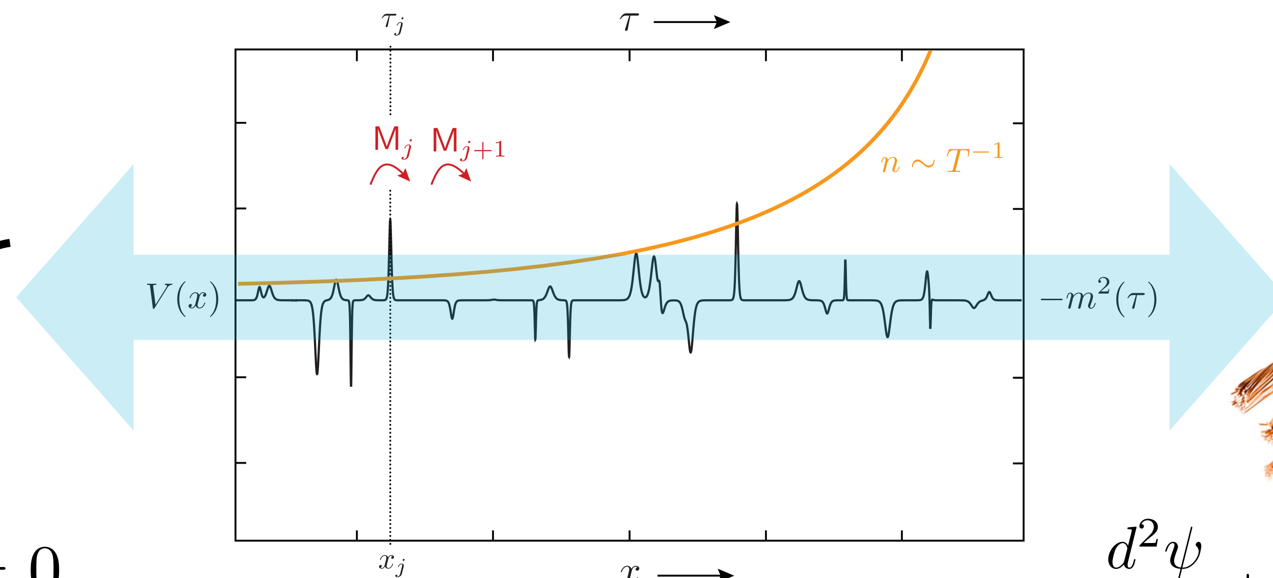
- Wires to Cosmology
  (w/ Baumann 1512.02637)
- Multifield Stochastic Particle Production
  (w/ Garcia, Wen & Xie 1706.02319)
- Stochastic Particle Production in deSitter Space
  (w/ Garcia, Carlsten & Green 1902.06736)
- Curvature Perturbations from Stochastic Particle Production
  (in progress)

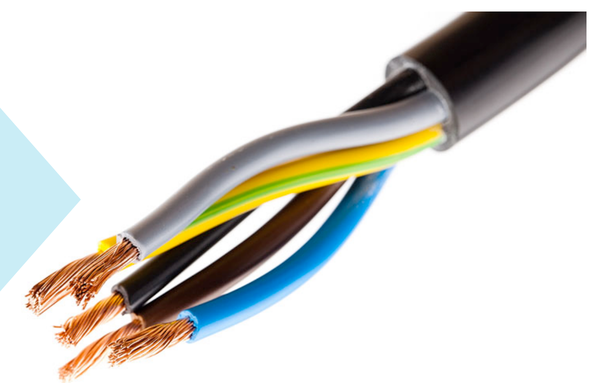


And related works by Brandenberger & B, Basset

temporal complexity ⟷ spatial complexity







$$\frac{d^2\chi_k}{d\tau^2} + \left[k^2 + m^2(\tau)\right]\chi_k = 0$$

$$\frac{d^2\psi}{dx^2} + \left[k^2 - V(x)\right]\psi = 0$$

- time
- number of particles
- complicated temporal behavior

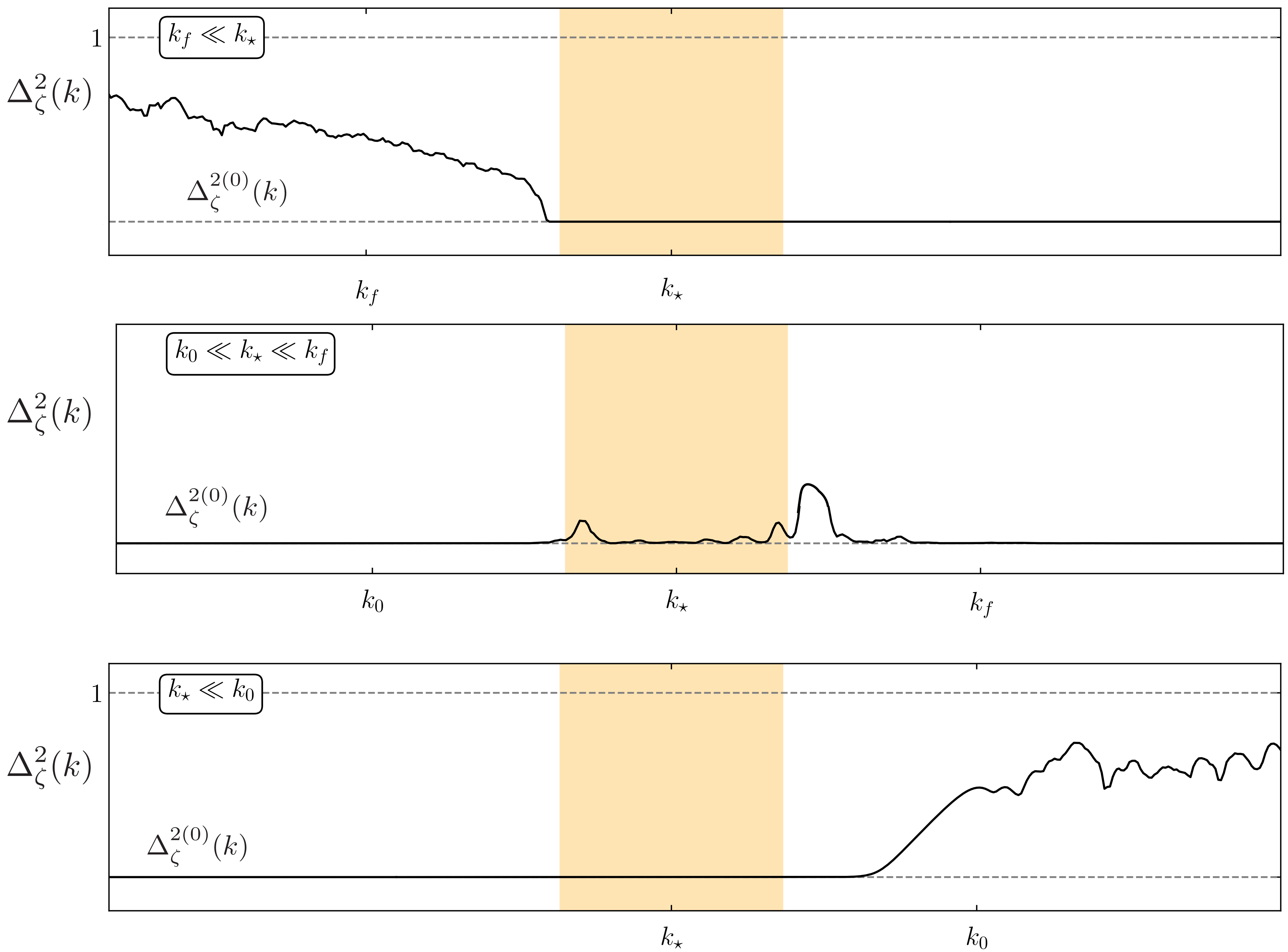
- position along wire
- resistance
- impurities in wires

curvature perturbations from particle production

Garcia, MA, Green, Baumann &Chia (in progress)

$$\Delta_\zeta^2(k) = \Delta_\zeta^{2(0)}(k) + \delta\Delta_\zeta^2(k)$$

observable range



observable?

Features ?

PBH ?

Spectral Distortions ?

Also see: Flauger, Mirbabayi, Senatore, Silverstein (2016)

Nonlinear Dynamics of Cosmological Fields (and novel connections)

3 theoretical/numerical results

✓ 1. instability in oscillating fields

✓ 2. formation of solitons

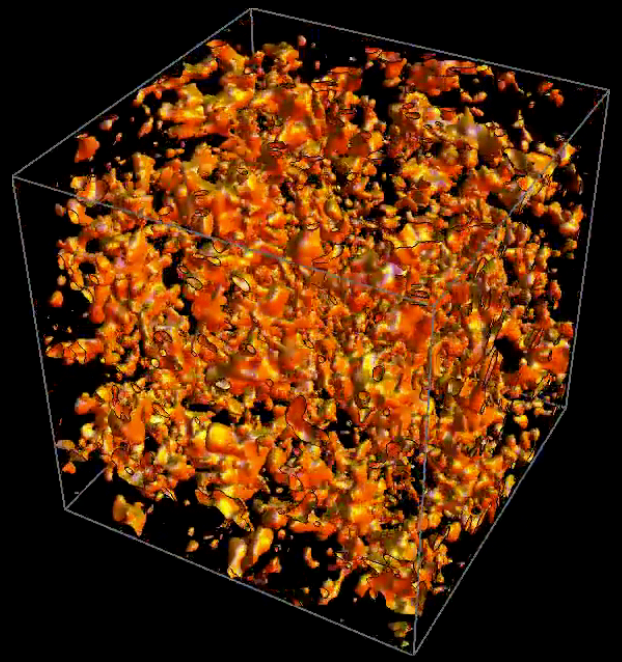
✓ 3. eq. of state

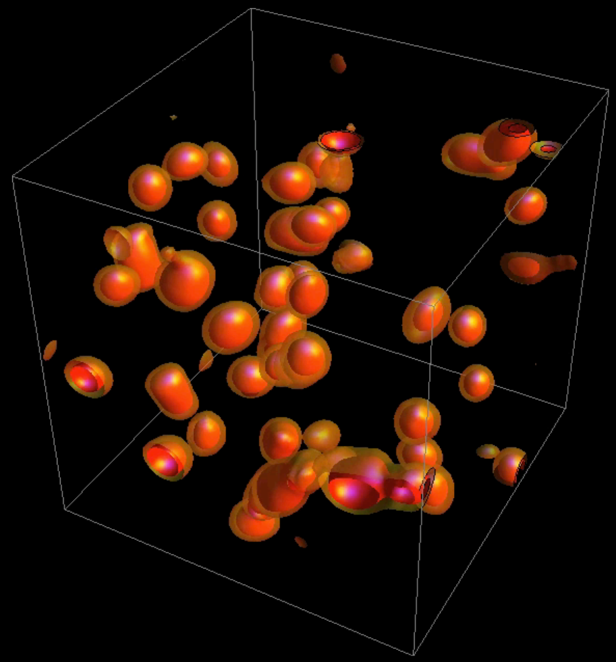
3 obs. implications

✓ 1. gravitational waves

✓ 2. structure formation

✓ 3. expansion history





thanks