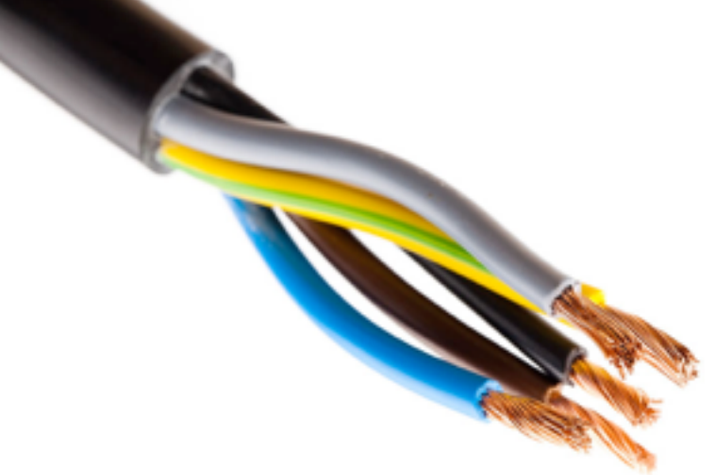
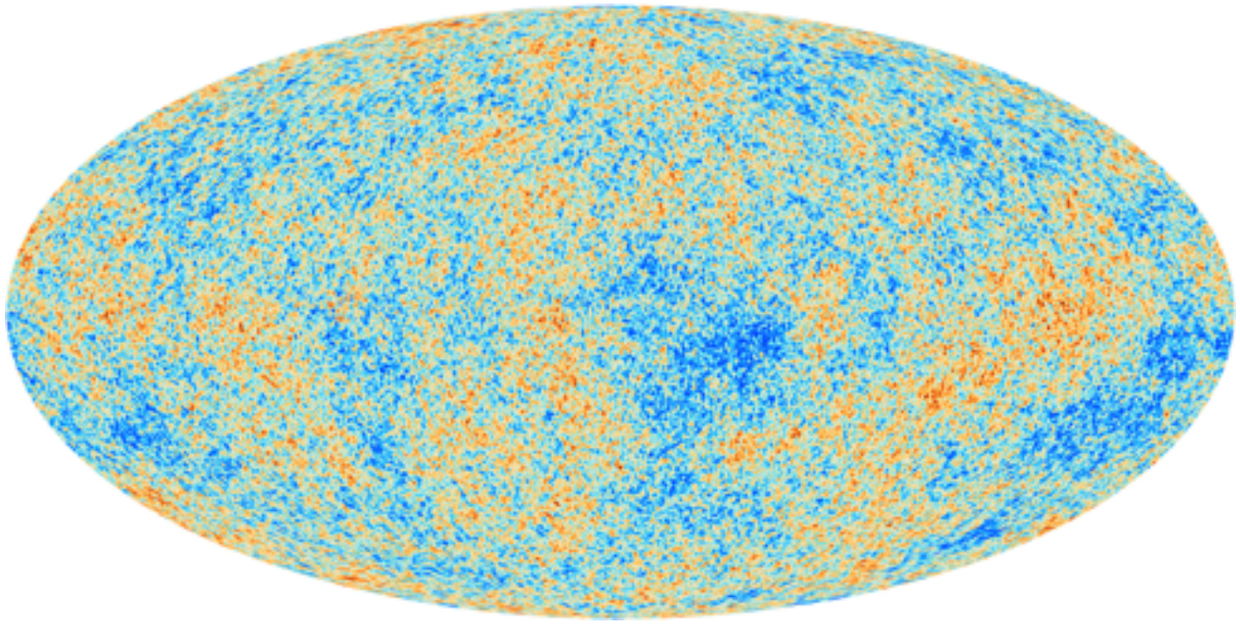


Stochastic Particle Production in the Early Universe



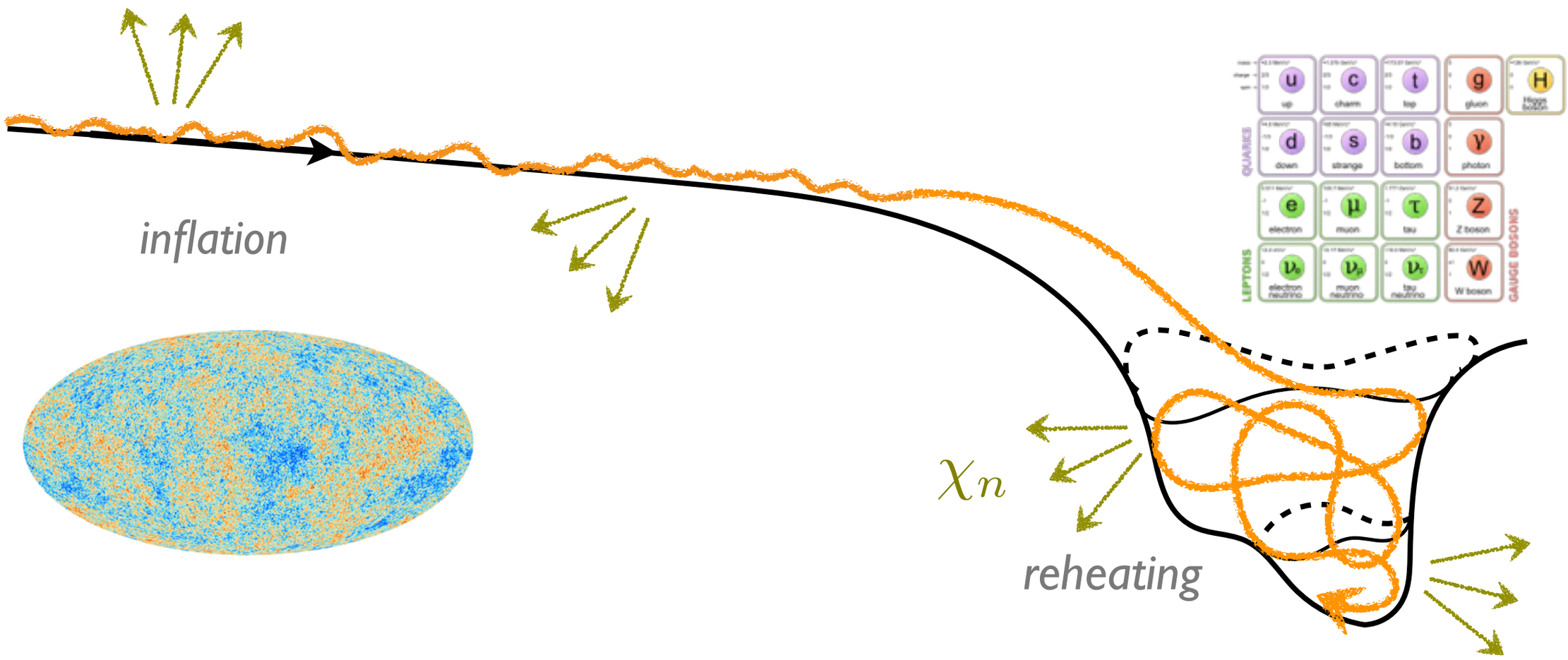
RICE

Mustafa A. Amin

synopsis

- theoretical tools for calculating particle production in *sufficiently complex* models of inflation and reheating

- hints of universality



work in progress

based on

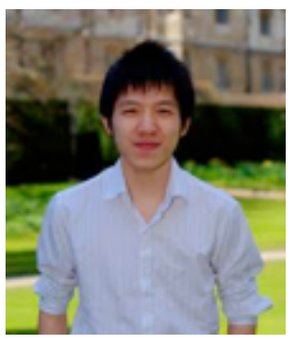
- MA & Baumann, *Wires to Cosmology (2016)*
- MA, Garcia, Xie & Wen, *Multifield Stochastic Particle Production (2017)*

+ ongoing work with Garcia, Carleston, Chia, Baumann & Green

RICE





related work: condensed matter & cosmology

Anderson
Absence of diffusion in certain random lattices
(1957)

Mello, Pereyra Kumar
Macroscopic approach to multichannel disordered wires
(1987)

C. Beenakker,
Random matrix theory of quantum transport
(1997)

C. Muller and D. Delande,
Disorder and interference: localization phenomena
(2010)

Brandenburger & Taschen
(1990)

Kofman, Linde & Starobinsky
(1994, 1997)

Traschen and Brandenberger
(1995)

Zanchin, Maia, Craig & Brandenberger
(1998)

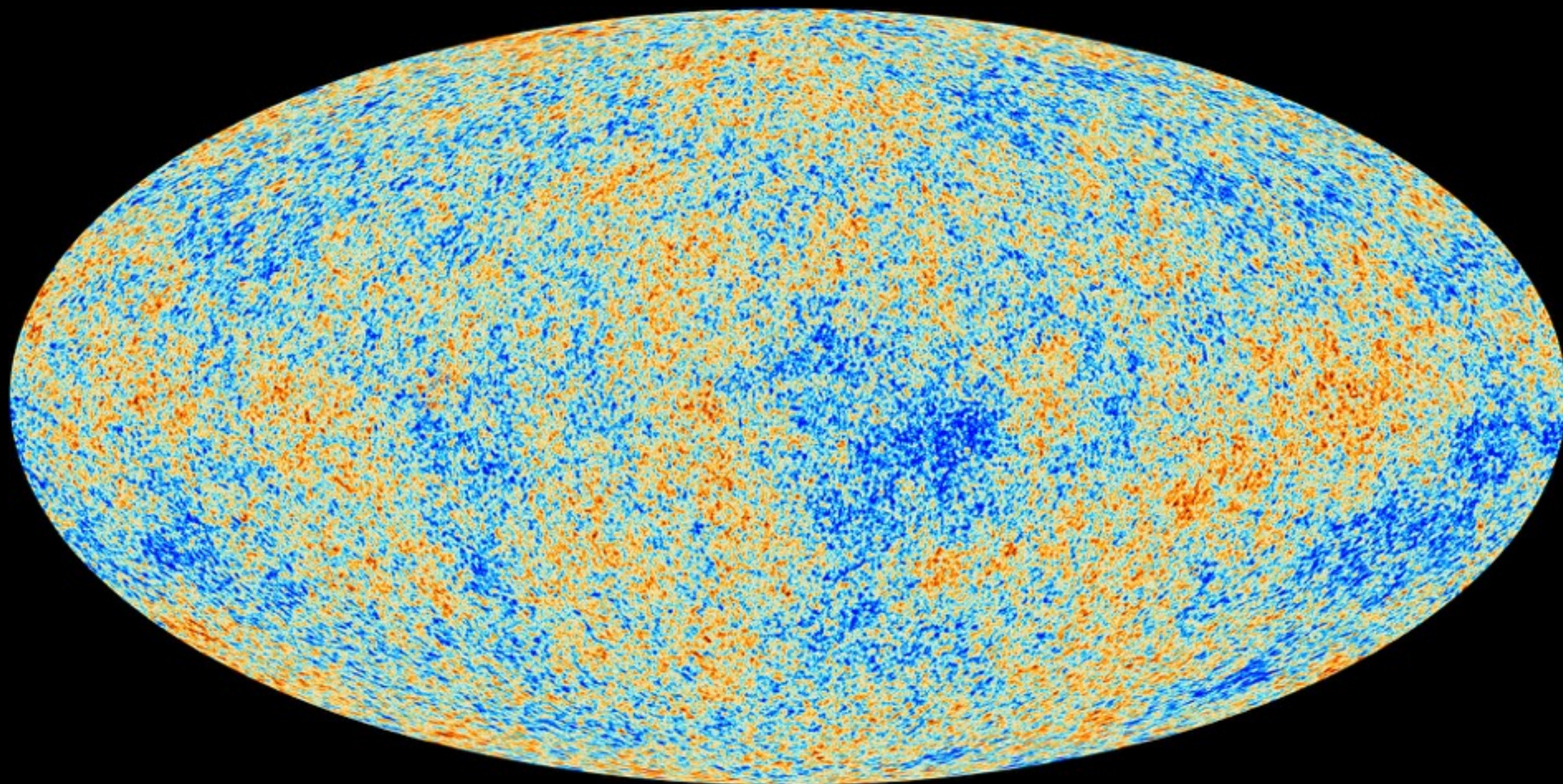
Bassett
(1998)

Green
(2015)

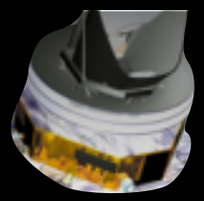
+ many works on particle production during and after inflation.

motivation

anisotropies:
cosmic microwave background



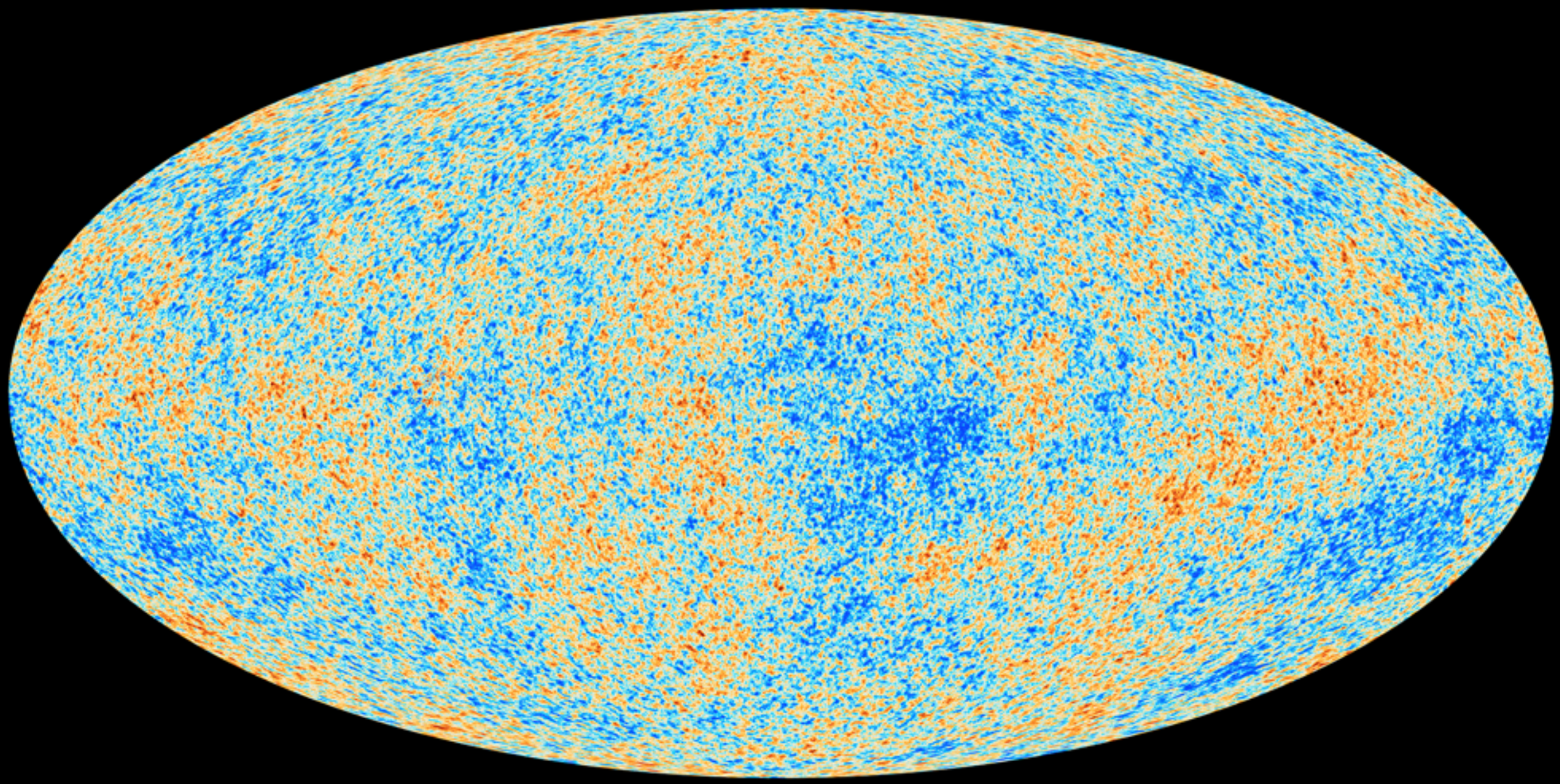
$$\delta T/T \sim 10^{-5}$$



Planck 2015

~ gaussian

almost scale invariant



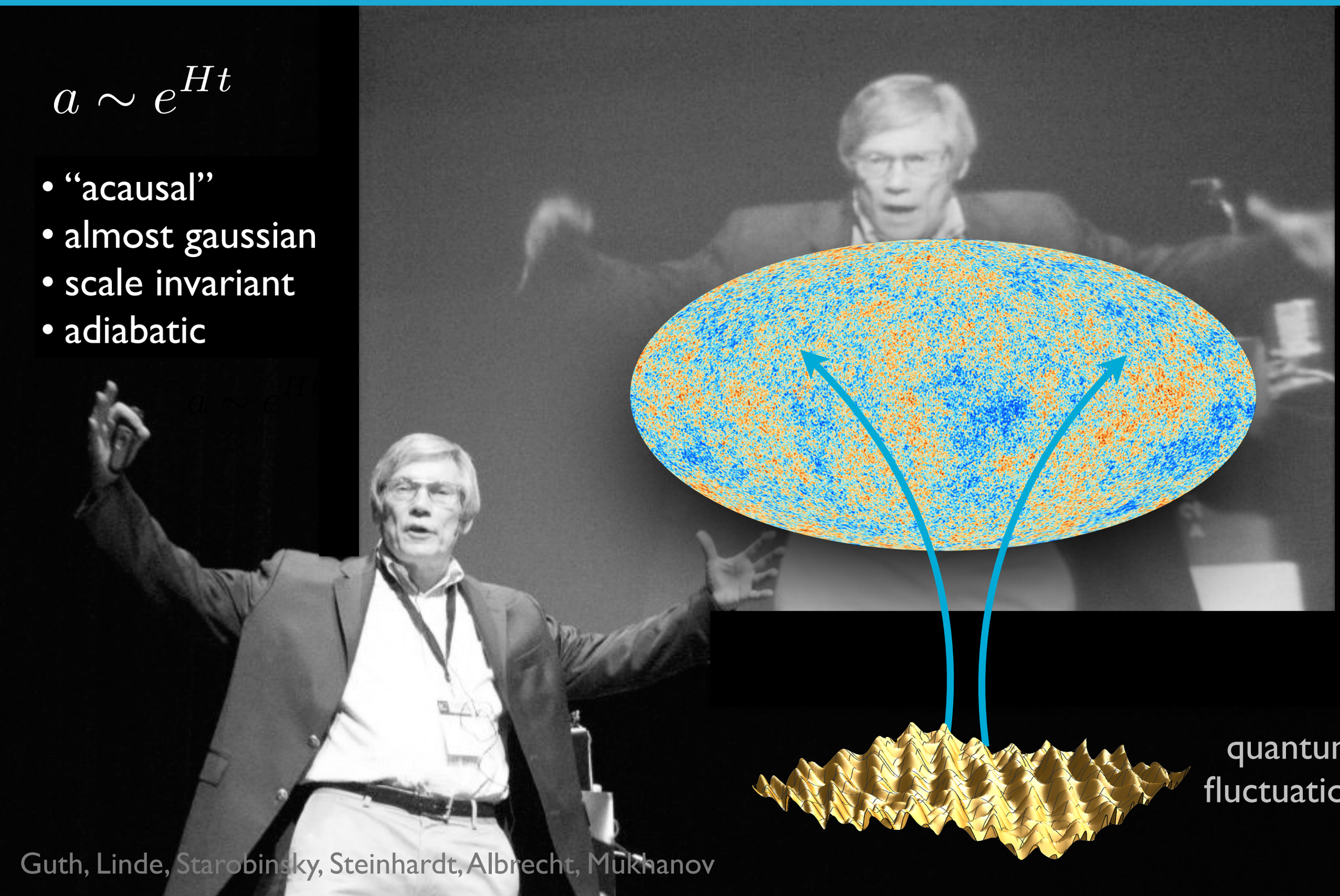
seemingly "acausal"

~ adiabatic

inflation: a simple explanation?

$$a \sim e^{Ht}$$

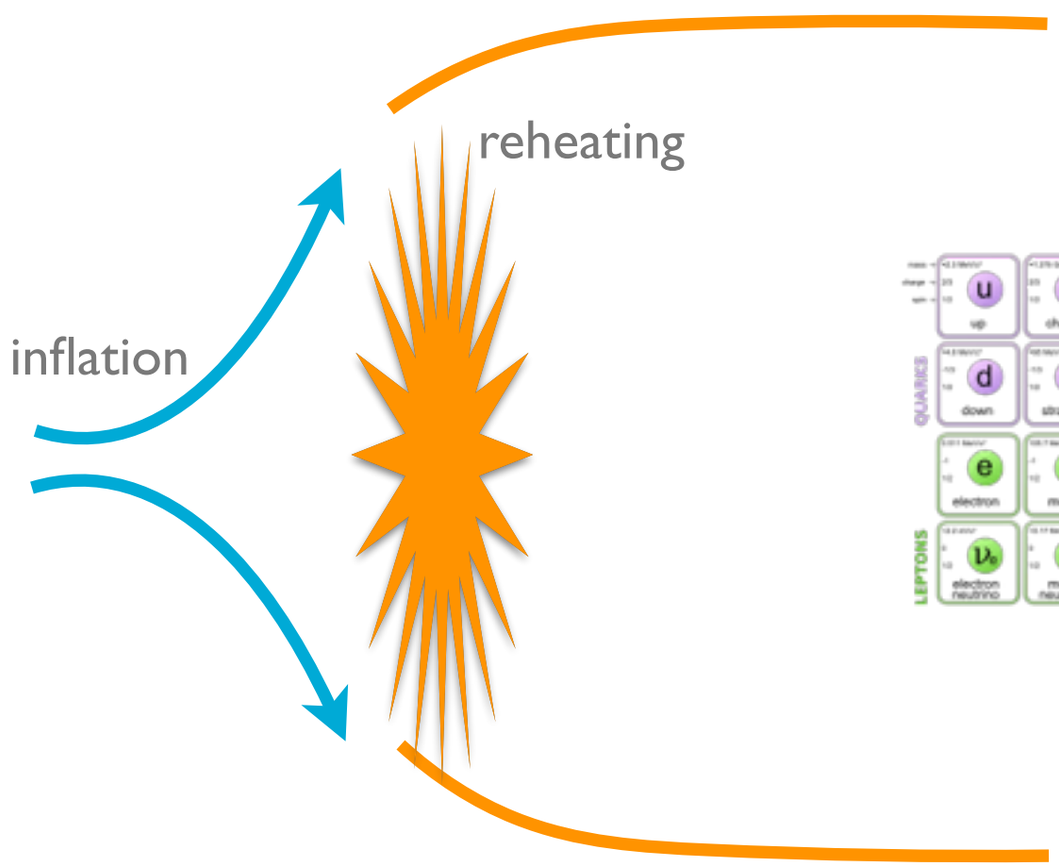
- "acausal"
- almost gaussian
- scale invariant
- adiabatic

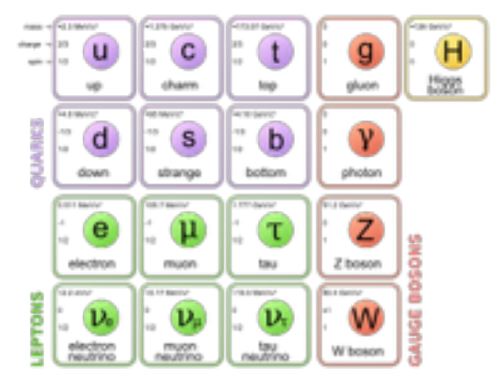


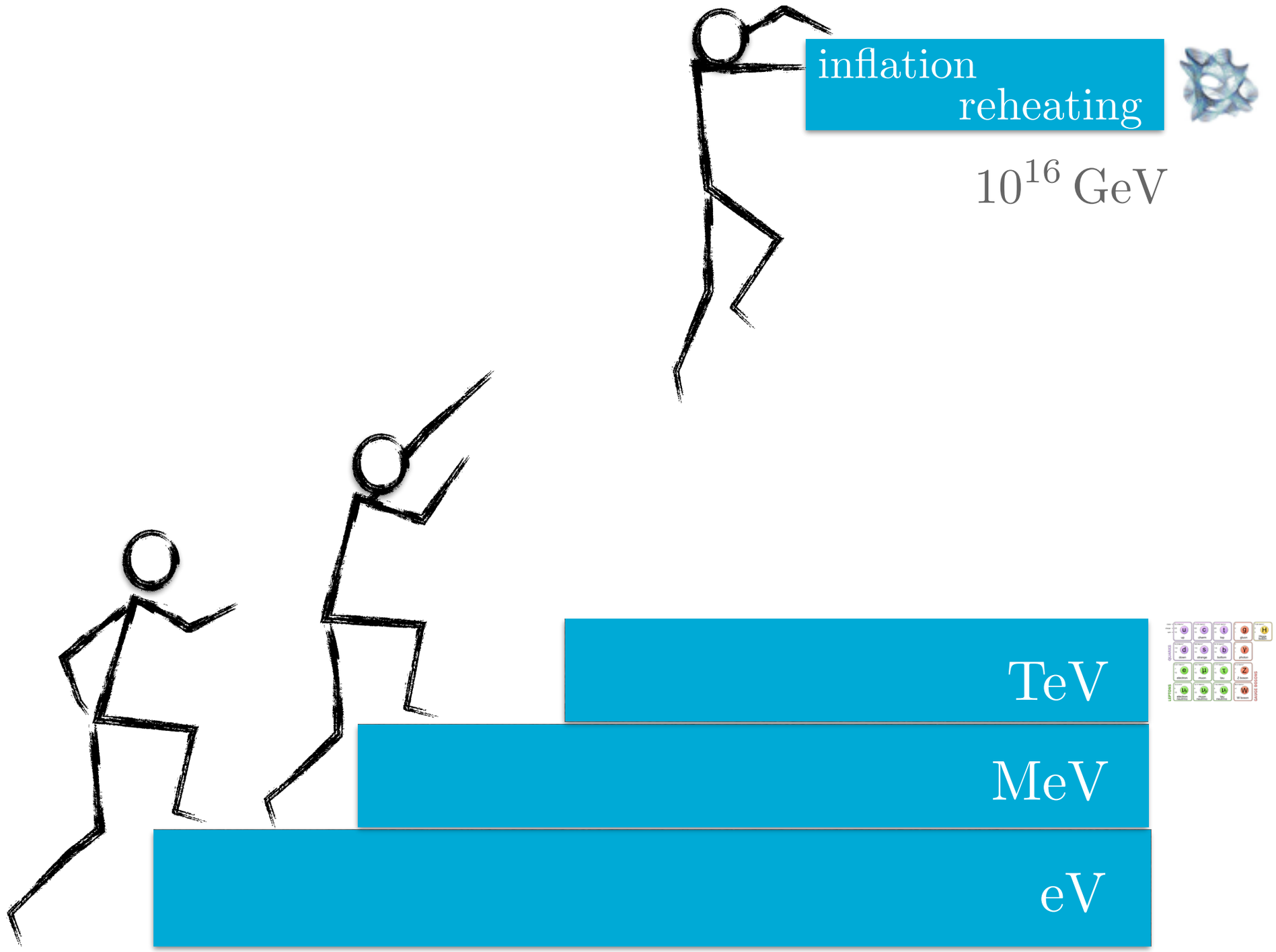
Guth, Linde, Starobinsky, Steinhardt, Albrecht, Mukhanov

physics of inflation and reheating?

- what is the physics of inflation ?
- how did the universe get populated with particles after inflation ? (reheating)






inflation
reheating
10^{16} GeV
TeV
MeV
eV

two approaches

SIMPLE enough

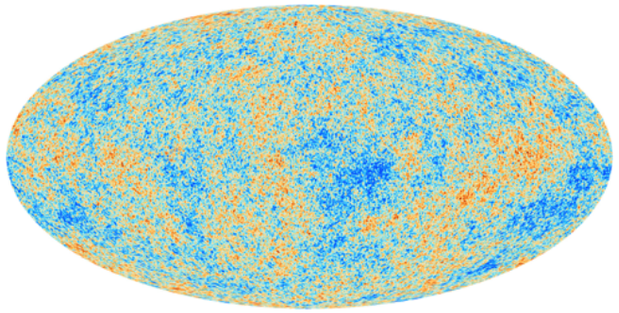
COMPLEX enough

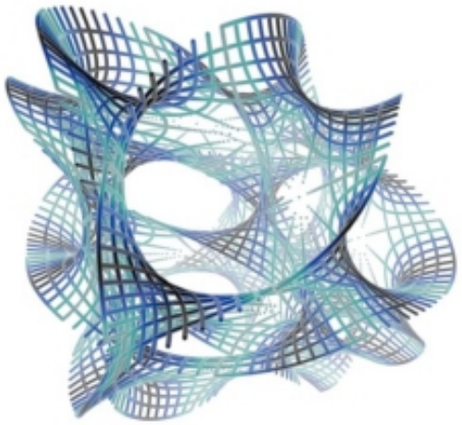
theory : its complicated (probably)

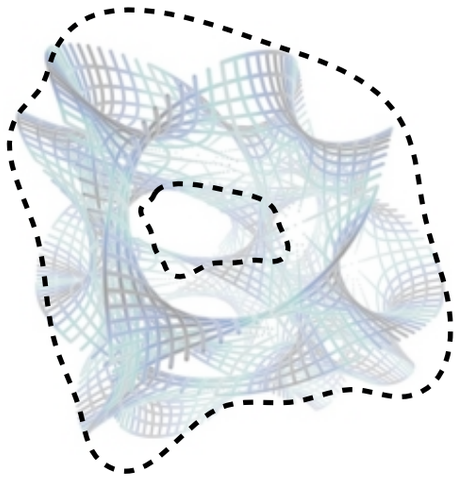
- inflation
- reheating after inflation

a coarse grained approach?

- **observations**: early universe is simple
- **theory**: not so much …
- **coarse grained view ?**
- **calculational tools ?**

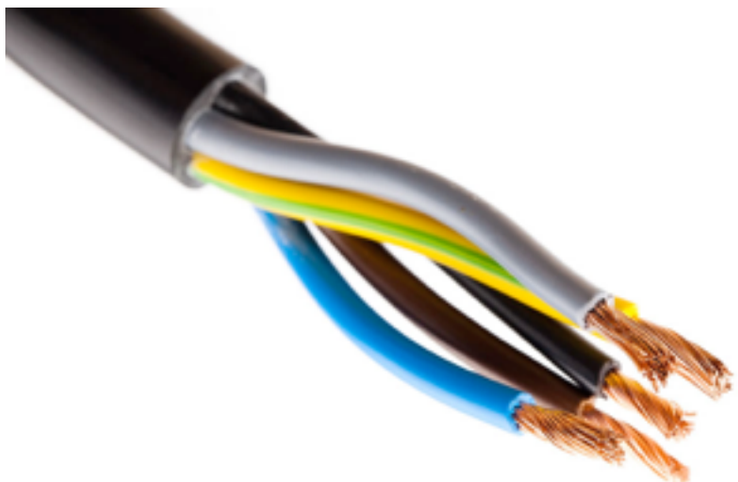


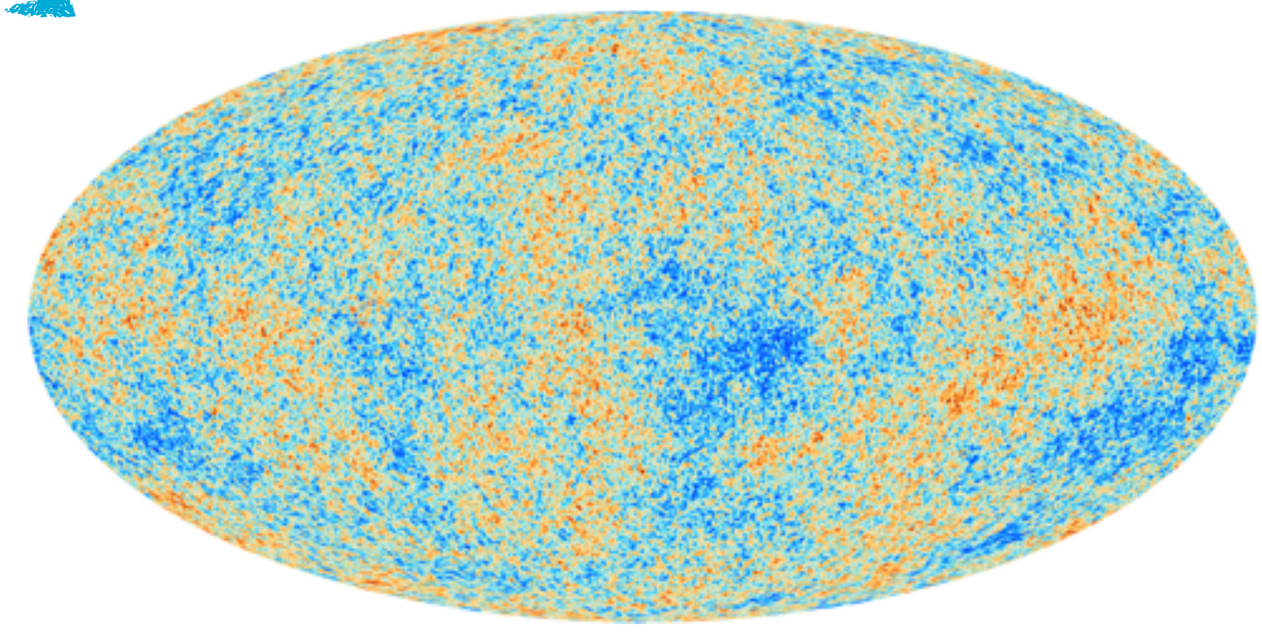




for related motivation, also see:
"EFT of inflation" (Cheung et. al 2007) &
"Towards EFT approach to reheating" (Giblin et. al 2016/17)

inspiration from disordered wires



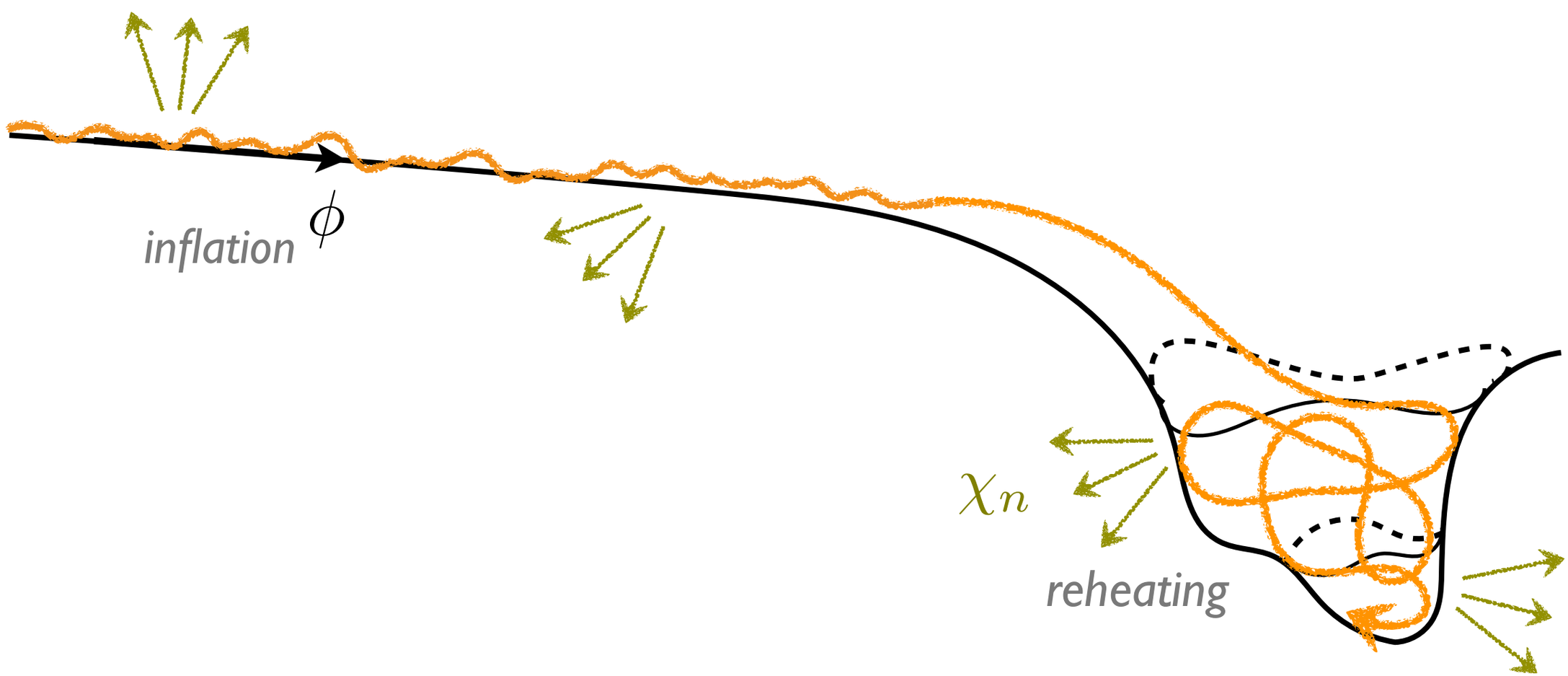


MA & Baumann 2015

the framework

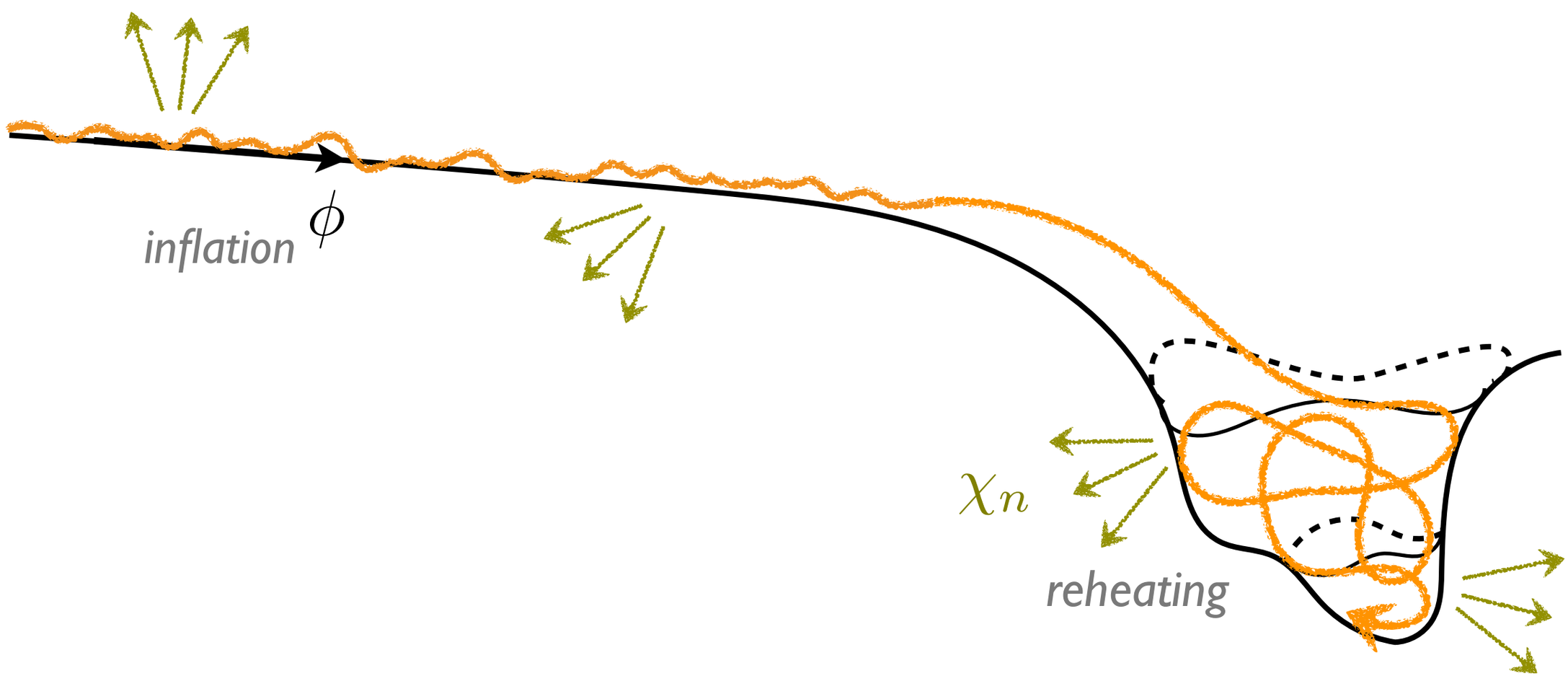
multifield inflation/reheating

- inflation/reheating: many interacting fields
- fluctuations: coupled, non-perturbative



multifield inflation/reheating

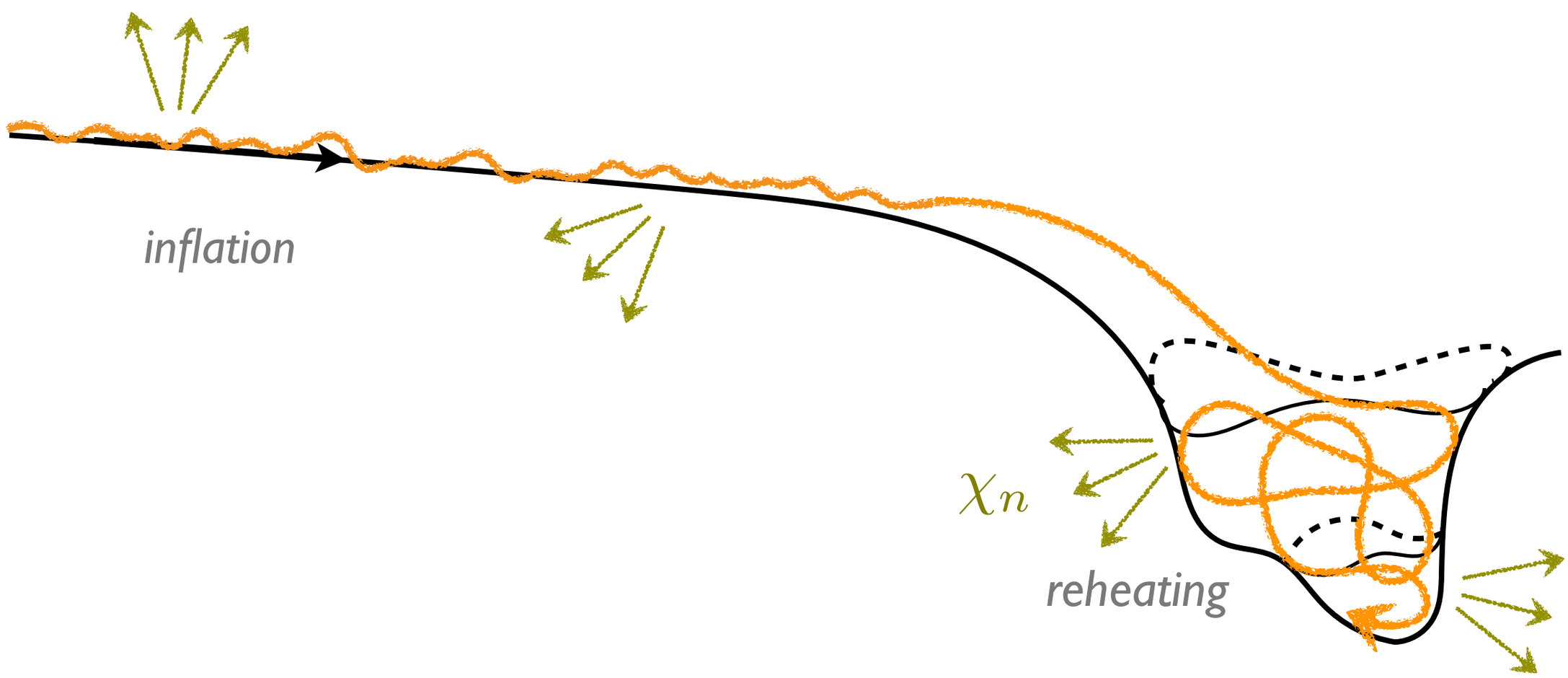
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}G_{ab}(\phi^c)\partial^\mu\phi^a\partial_\mu\phi^b - V(\phi^c) + \cdots\right]$$



focus on perturbations

$$S^{(2)} = \int d^4x\sqrt{-g}\left(\frac{1}{2}\delta_{ab}\partial\chi^a\partial\chi^b - \frac{1}{2}\mathcal{M}_{ab}(\tau)\chi^a\chi^b\right)$$

$$\mathcal{M}_{ab}(\tau) = a^2(\tau)\left[m_a^2\delta_{ab} + m_{ab}^{\mathrm{s}}(\tau)\right]$$

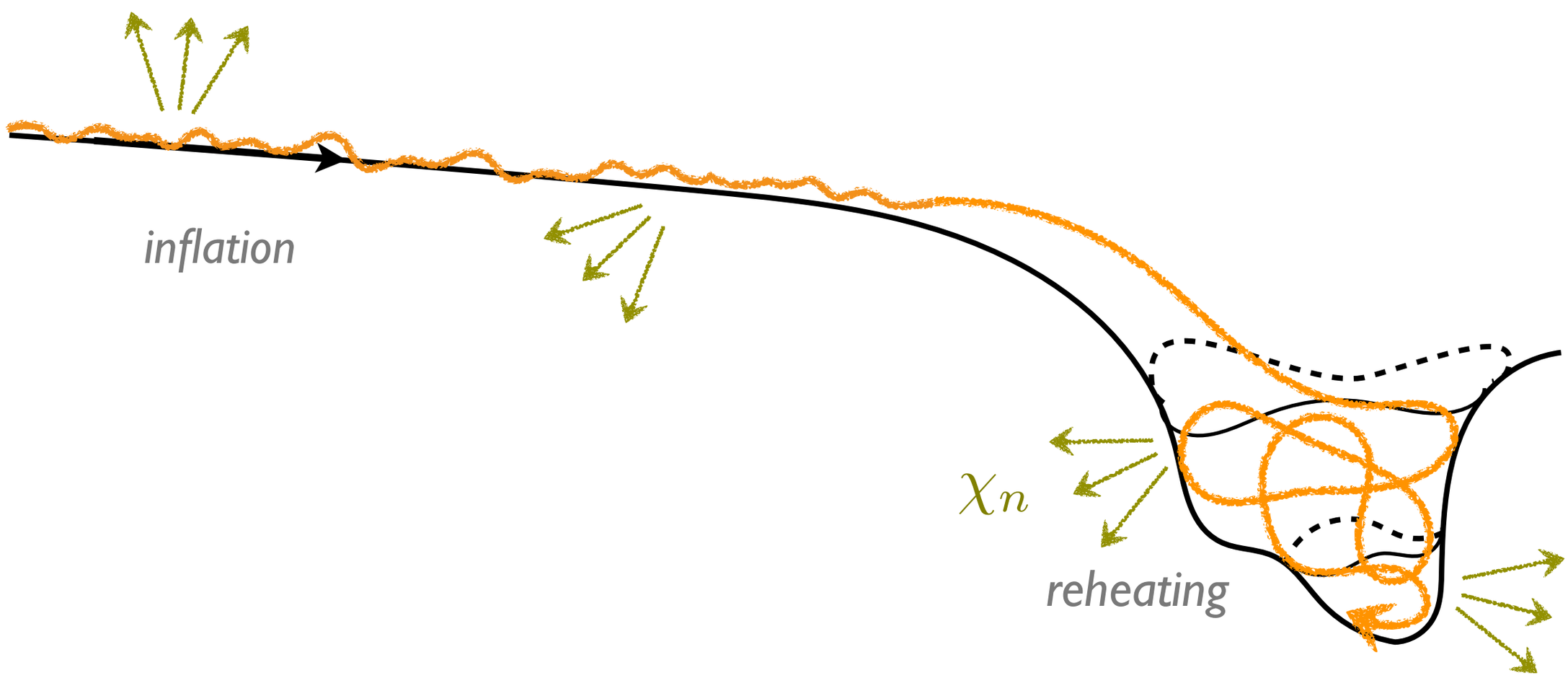


focus on perturbations

mode functions in Fourier space

$$\left[\frac{d^2}{d\tau^2} + 2\frac{\dot{a}(\tau)}{a(\tau)}\frac{d}{d\tau} + a^2(\tau)\omega_a^2(k)\right]\chi^a(\tau,k) + a^2(\tau)\sum_{b=1}^{N_{\rm f}} m^{\rm s}_{ab}(\tau)\chi^b(\tau,k) = 0$$

$$\omega_a^2(k) = k^2 + m_a^2\,,$$

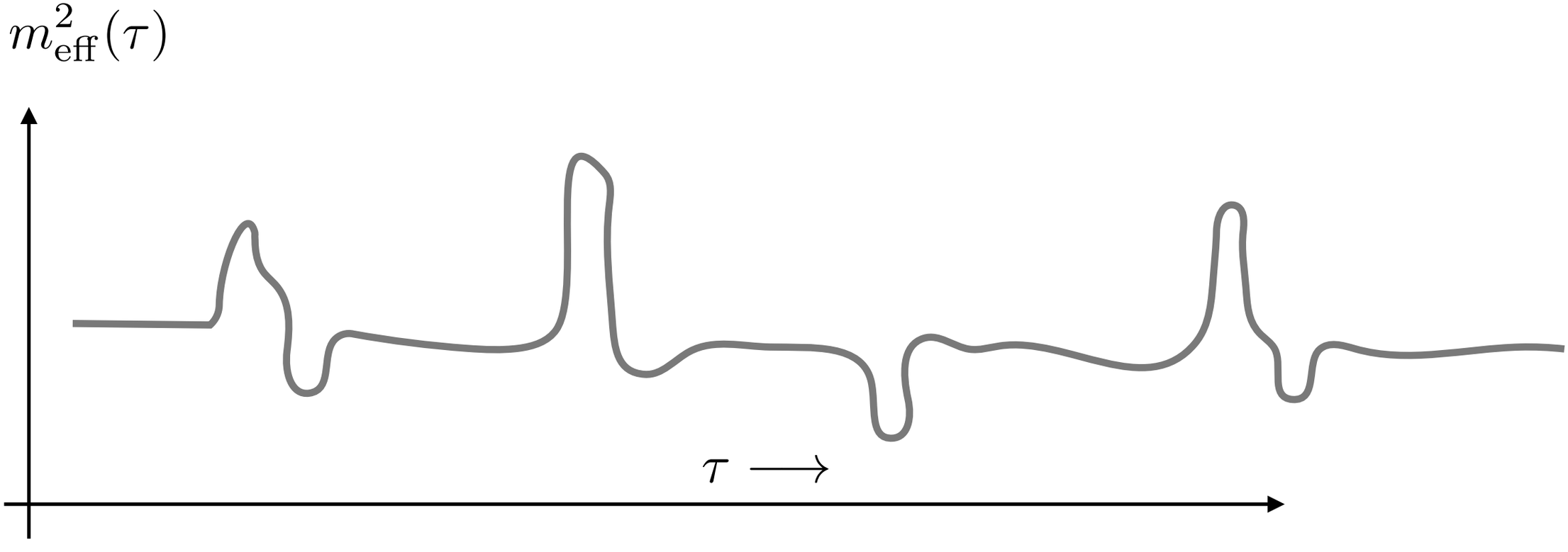


complexity in the "effective mass"/ interactions

simplified version!

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$$m_{\rm eff}^2(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)m_\varphi^2 + a^2(\tau)g^2(\phi(\tau)-\phi_*)^2 + \ldots$$

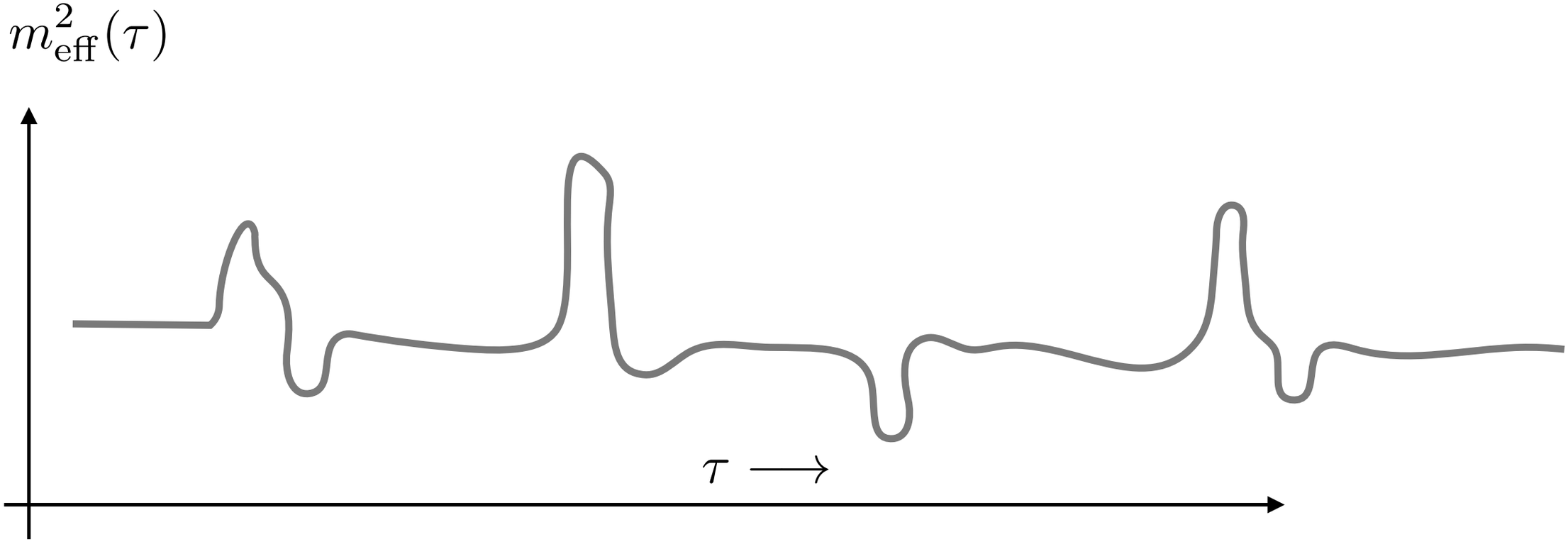


complexity in the "effective mass"/ interactions

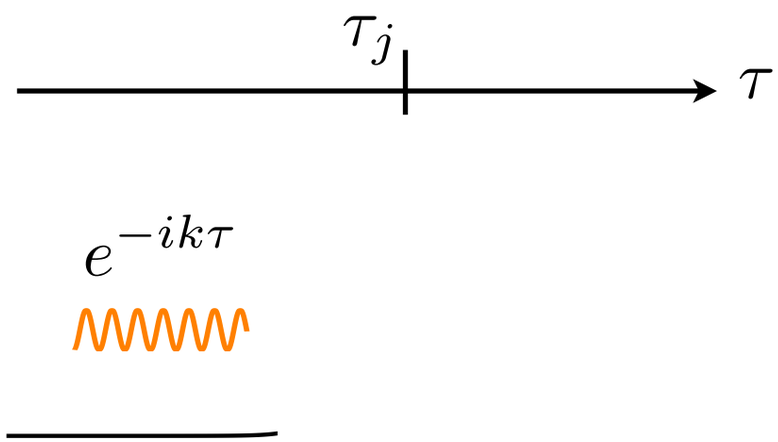
simplified version!

$$\ddot{\chi}_k(\tau) + \left[k^2 + m^2_{\rm eff}(\tau)\right]\chi_k(\tau) = 0$$

$$m^2_{\rm eff}(\tau) = \underbrace{-\frac{\ddot{a}(\tau)}{a(\tau)}}_{0} + \underbrace{a^2(\tau)m^2_\varphi}_{1} + \underbrace{a^2(\tau)g^2(\phi(\tau)-\phi_*)^2}_{1} + \ldots$$



particle production as "scattering"



$$|\chi_k(\text{after})\rangle = \underbrace{\begin{pmatrix} 1/t_j^* & -r_j^*/t_j^* \\ -r_j/t_j & 1/t_j \end{pmatrix}}_{\mathsf{M}_j} |\chi_k(\text{before})\rangle$$

occupation number per mode

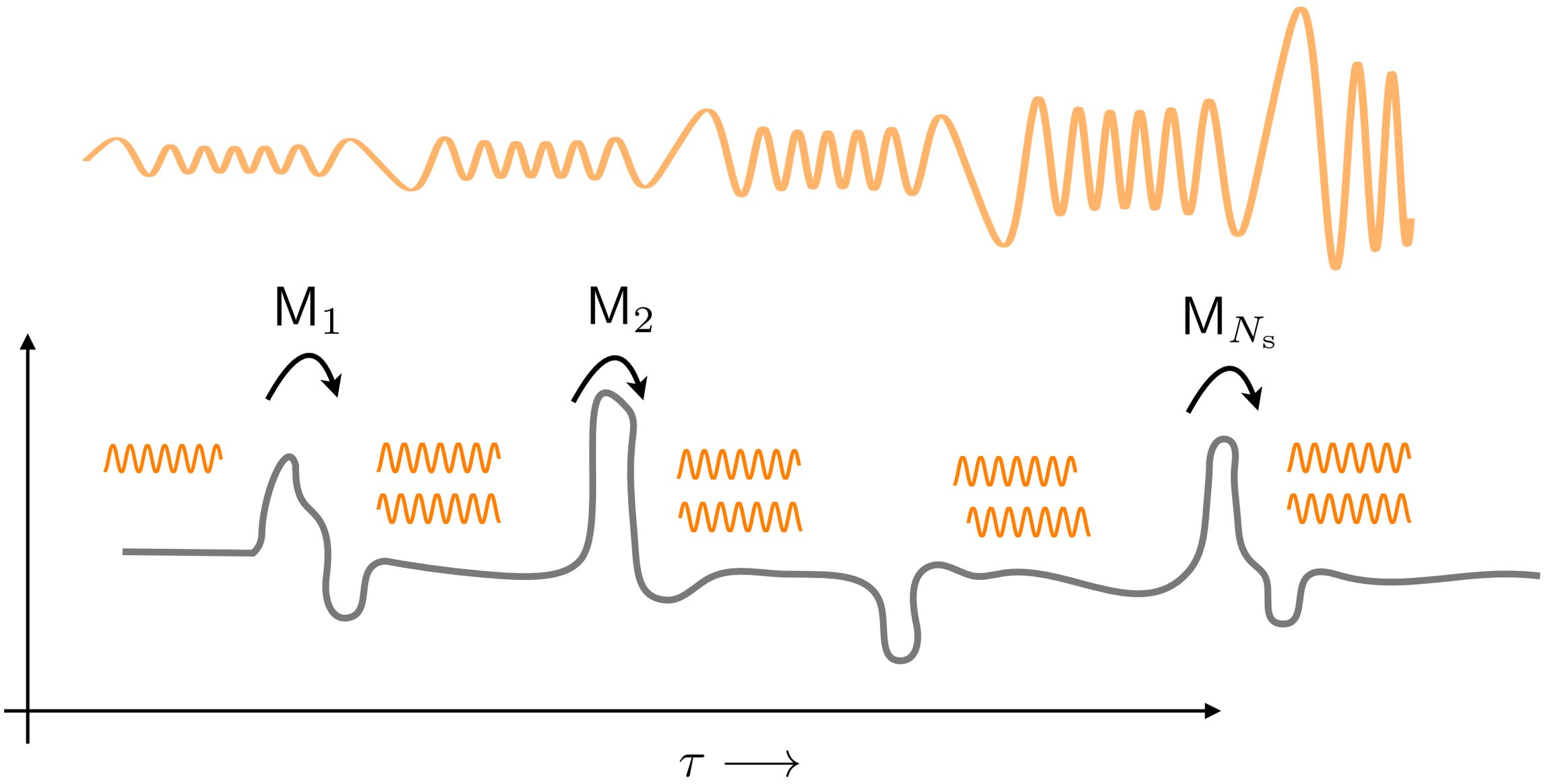
$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

$$T_j = |t_j|^2 \qquad n_j \equiv \frac{|r_j|^2}{|t_j|^2} = T_j^{-1} - 1$$

Kofman, Linde & Starobinsky 1997

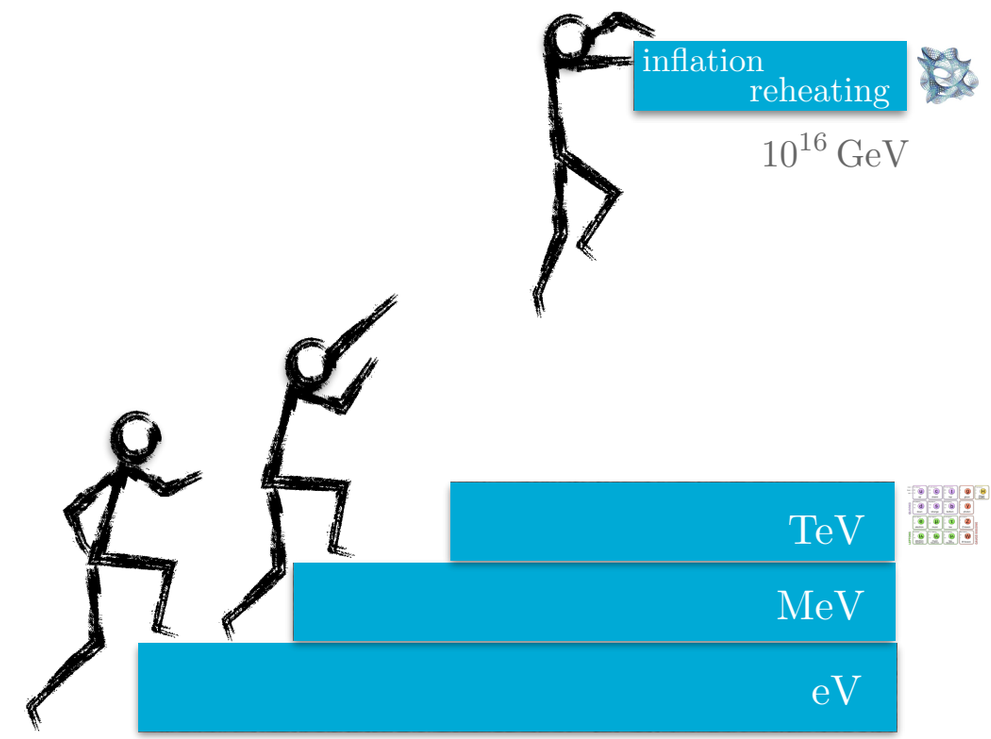
chaining transfer matrices

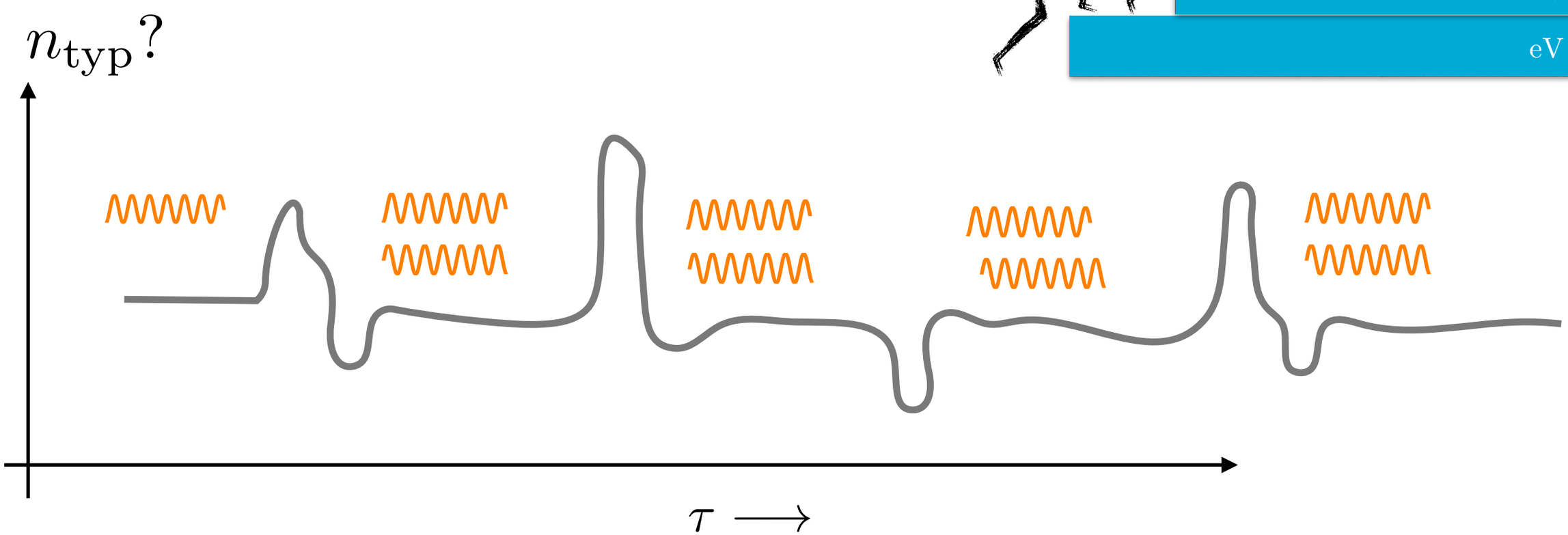
$$|\chi_k(N_\mathrm{s})\rangle = \mathsf{M}|\chi_k(0)\rangle \qquad \text{where} \qquad \mathsf{M} \equiv \mathsf{M}_{N_\mathrm{s}} \cdots \mathsf{M}_2 \mathsf{M}_1$$

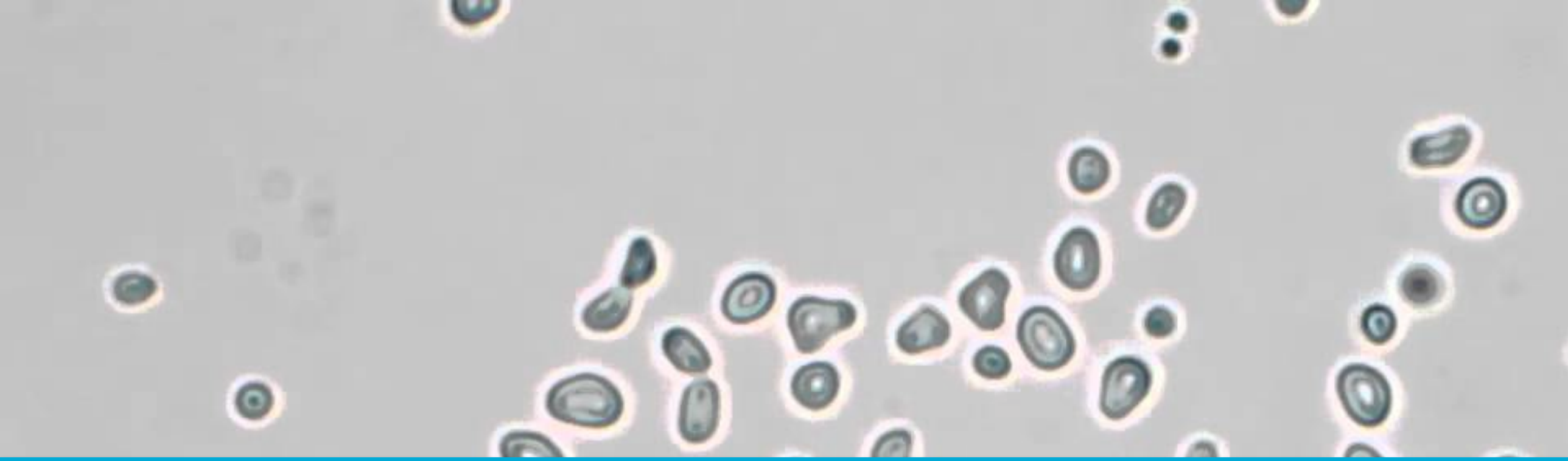


typical behavior in spite of ignorance of details ?

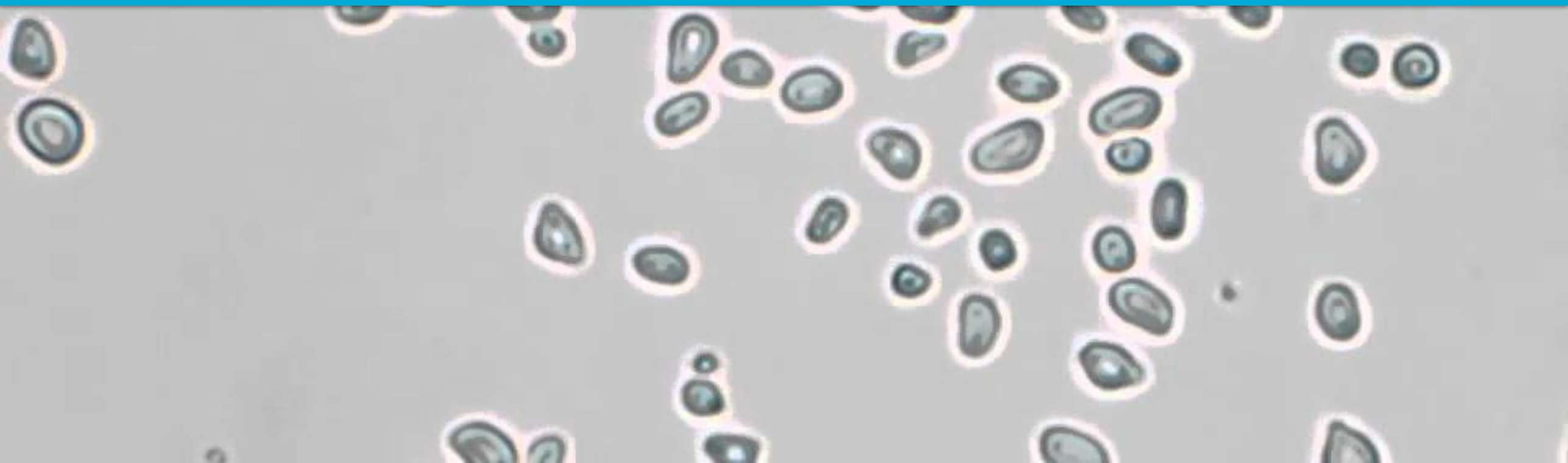
- details not known
- "typical" behavior ?





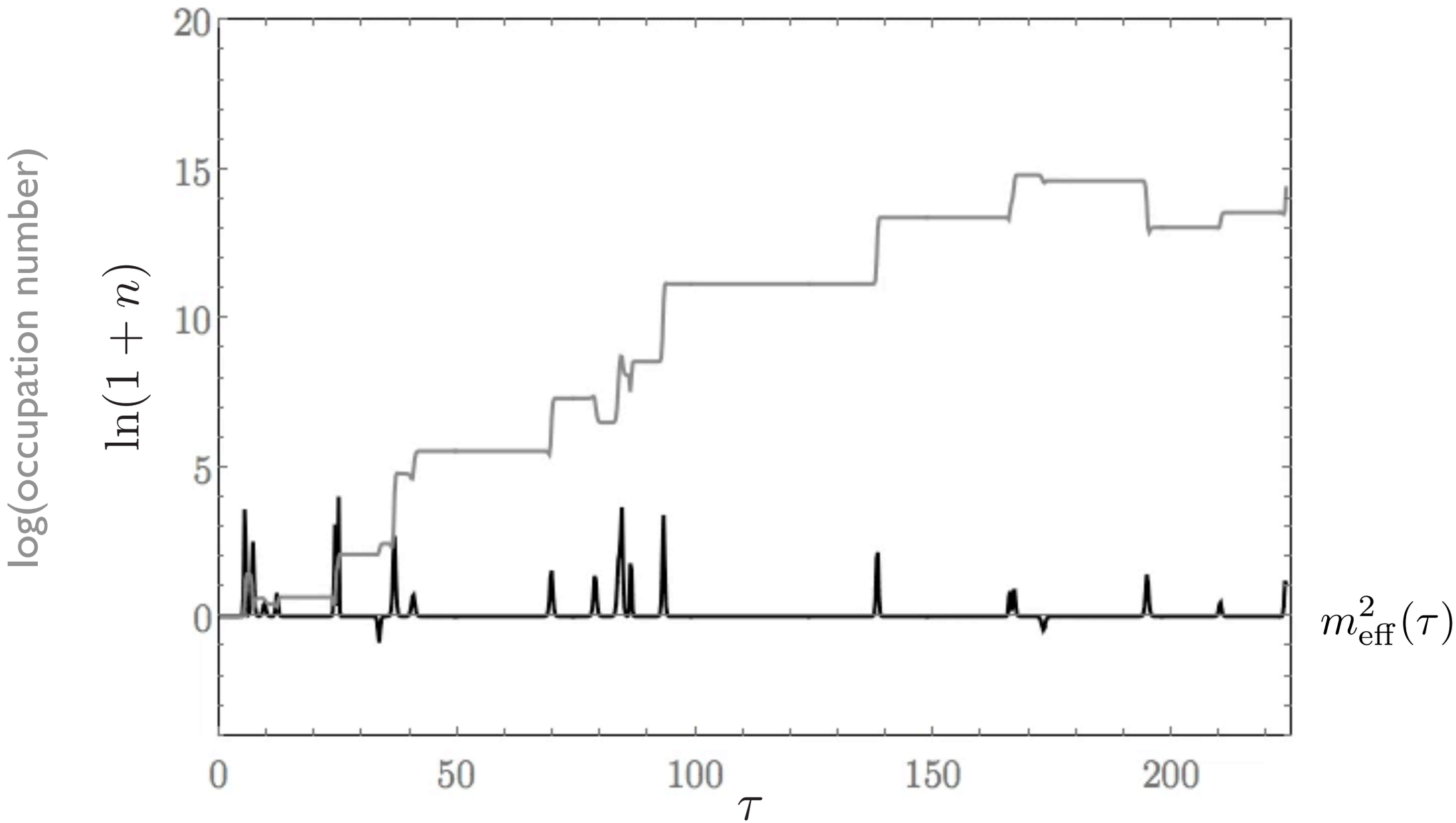


“brownian motion” of the occupation number



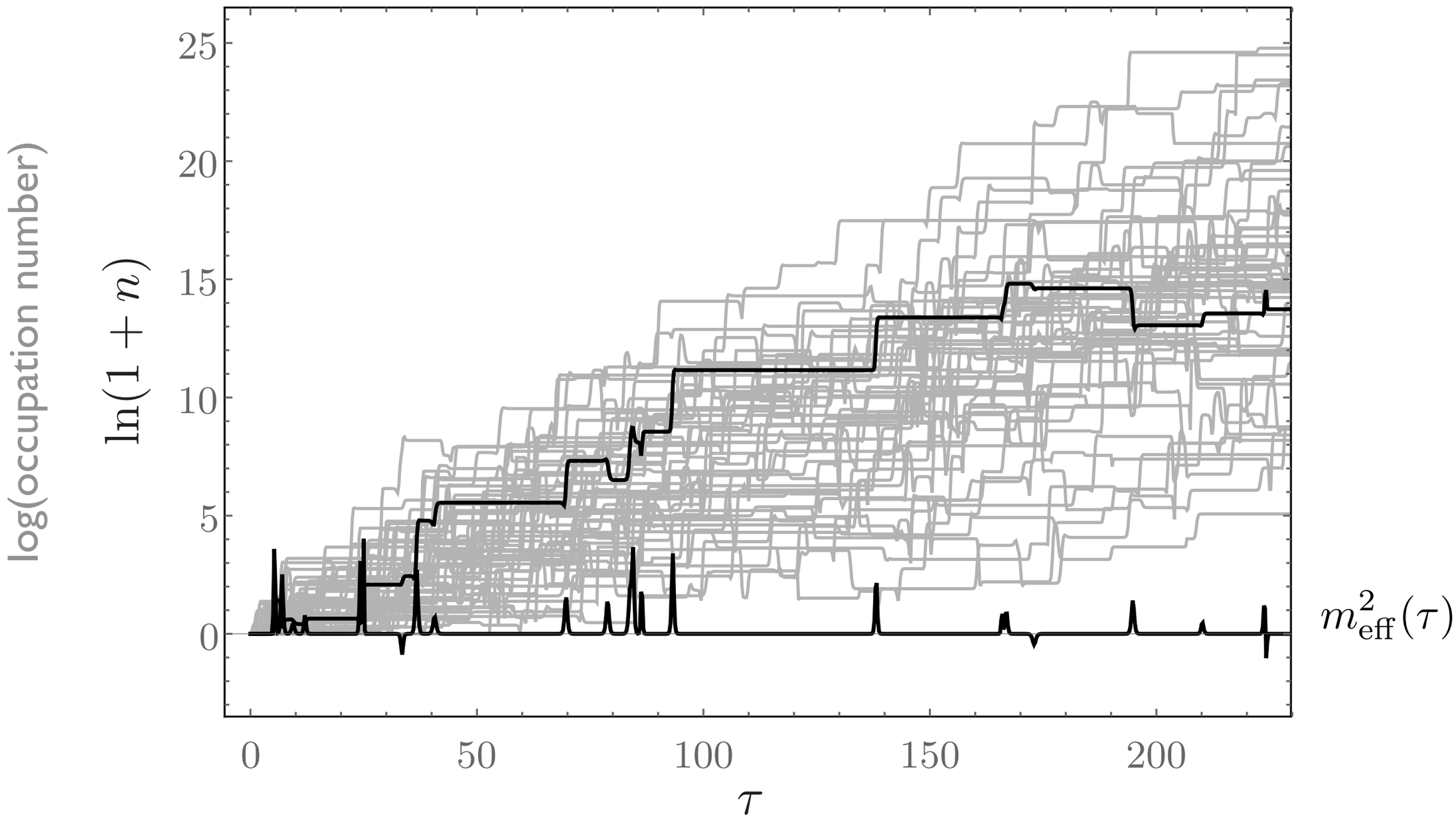
occupation number performs a drifted random walk

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

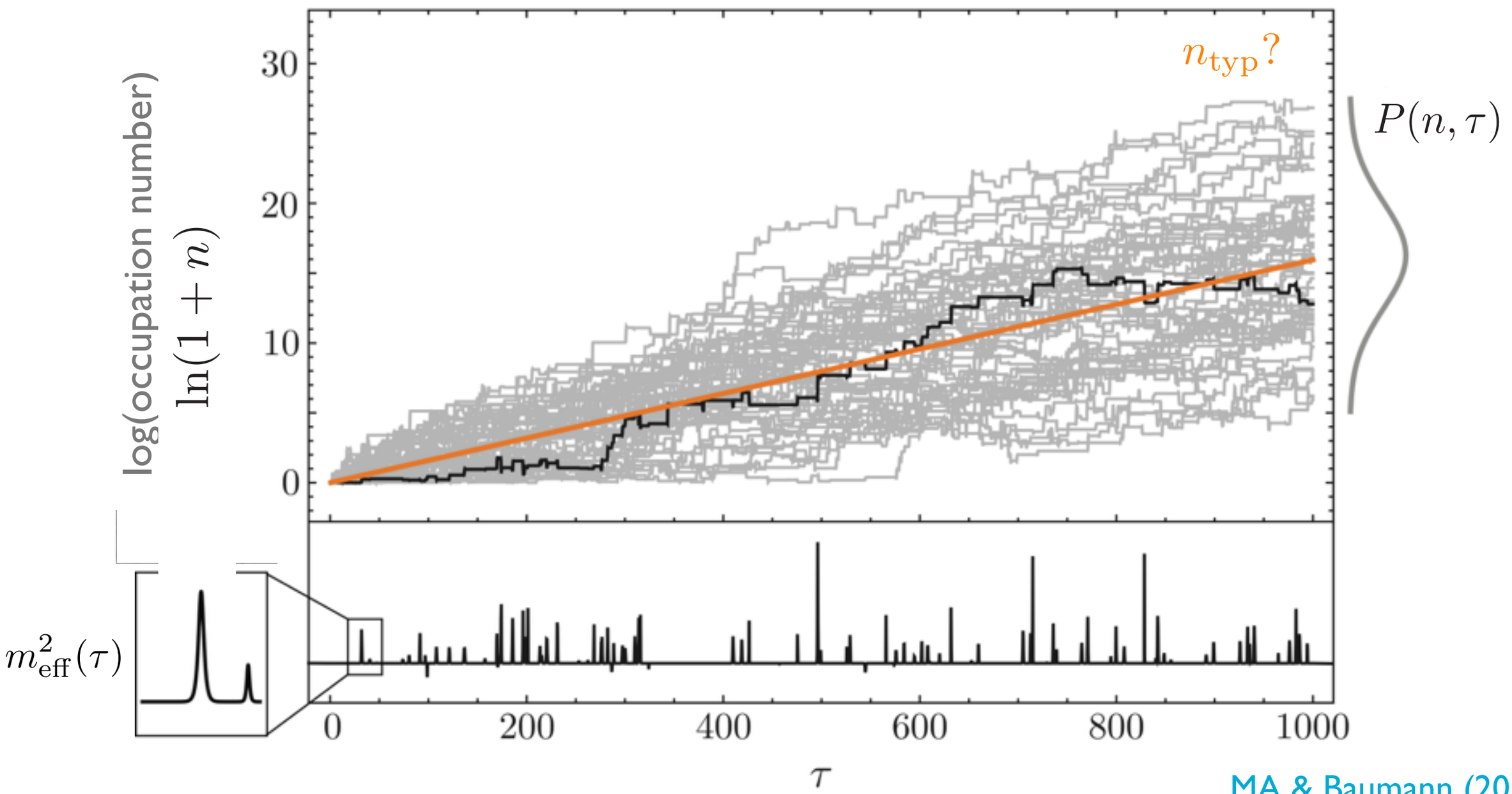


a drifted random walk different realizations

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$



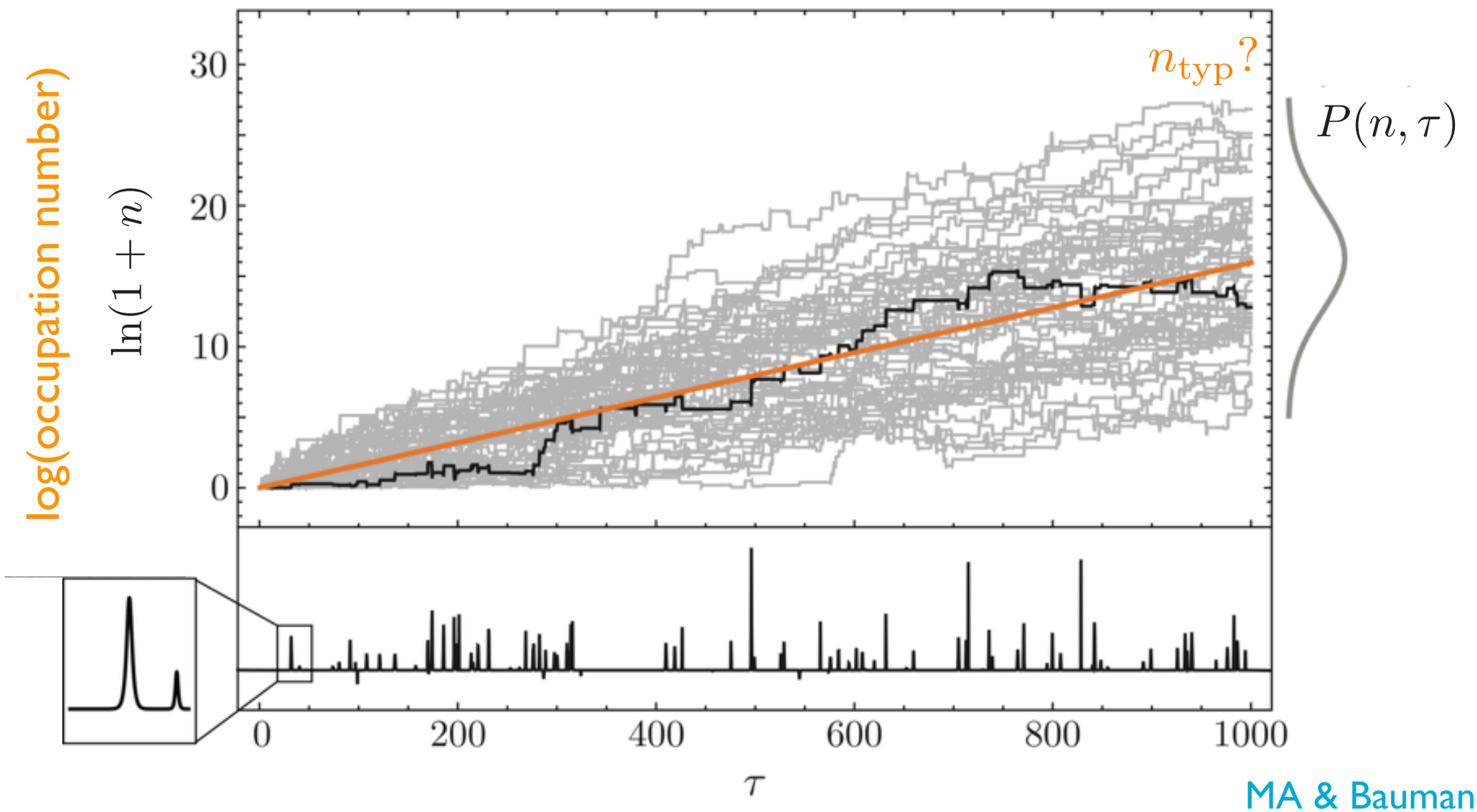
probability distribution ?
typical occupation number ?



MA & Baumann (2015)

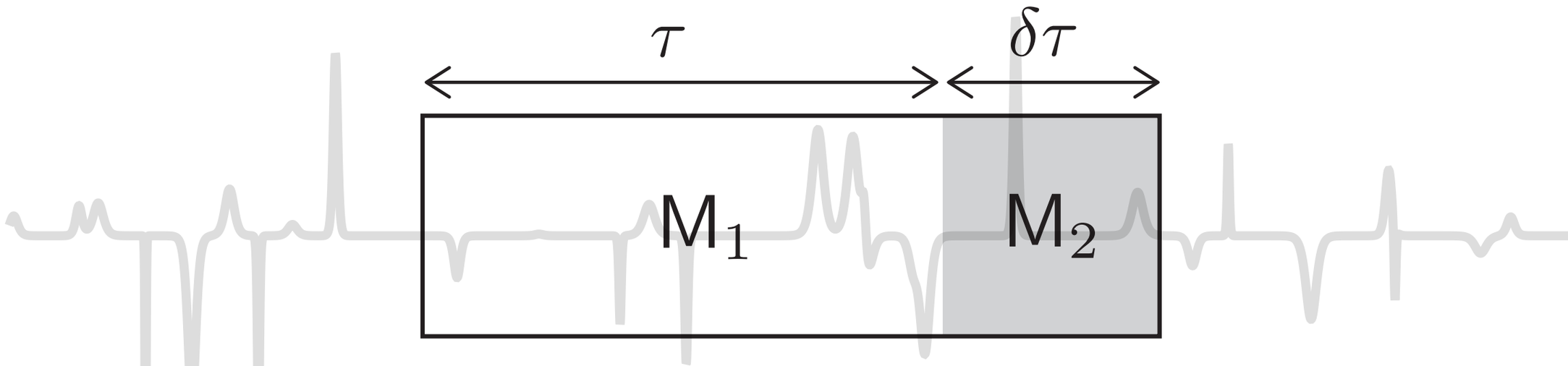
a Fokker Planck equation

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$



MA & Baumann (2015)

Fokker Planck eq: heuristic derivation



$$\mathsf{M} = \mathsf{M}_2 \cdot \mathsf{M}_1$$

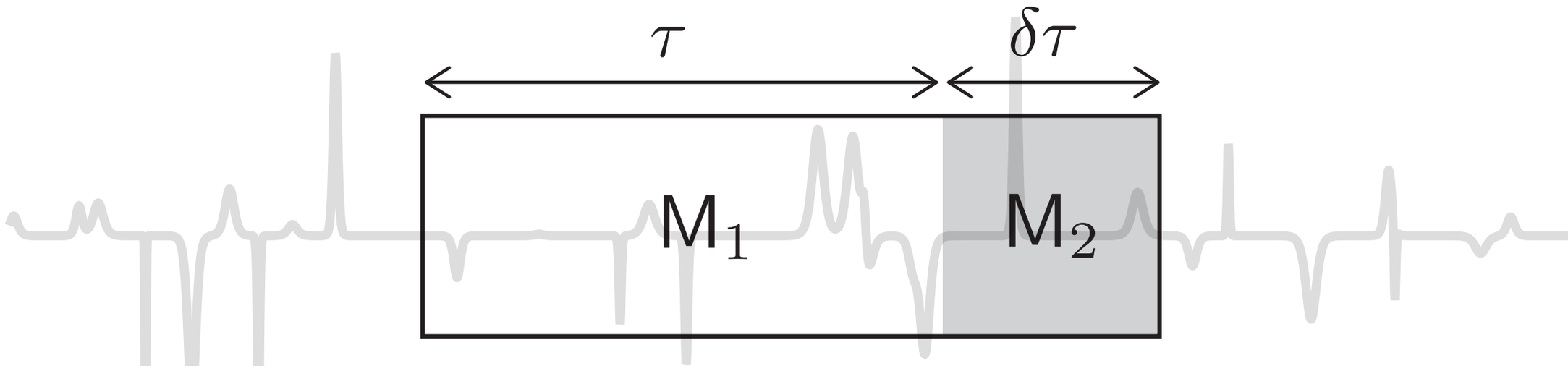
$$P(\mathsf{M}, \tau + \delta\tau) = \int d\mu(\mathsf{M}_1)\, d\mu(\mathsf{M}_2)\, P(\mathsf{M}_1, \tau) P(\mathsf{M}_2, \delta\tau)\, \delta(\mathsf{M} - \mathsf{M}_2\mathsf{M}_1)$$

Smoluchowski eq.

$$\partial_\tau P(\mathsf{M}; \tau) = \frac{\langle \delta\mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau} \partial_{\mathsf{M}} P(\mathsf{M}; \tau) + \frac{\langle \delta\mathsf{M}\delta\mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau} \partial_{\mathsf{M}}\partial_{\mathsf{M}} P(\mathsf{M}; \tau) + \cdots$$

“formal” Fokker Planck eq.

Fokker Planck eq: heuristic derivation



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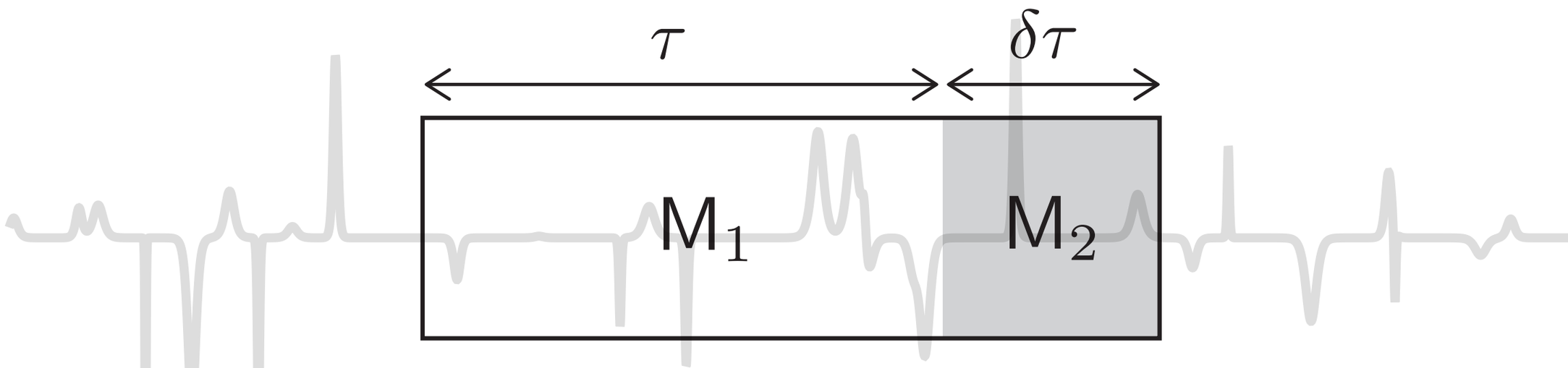
these needs to be calculated (usually possible)

weak scattering assumption

Fokker Planck eq: heuristic derivation



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Smoluchowski eq.

$$\partial_\tau P(\mathsf{M}; \tau) = \frac{\langle \delta\mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau} \partial_{\mathsf{M}} P(\mathsf{M}; \tau) + \frac{\langle \delta\mathsf{M}\delta\mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau} \partial_{\mathsf{M}}\partial_{\mathsf{M}} P(\mathsf{M}; \tau) + \cdots$$

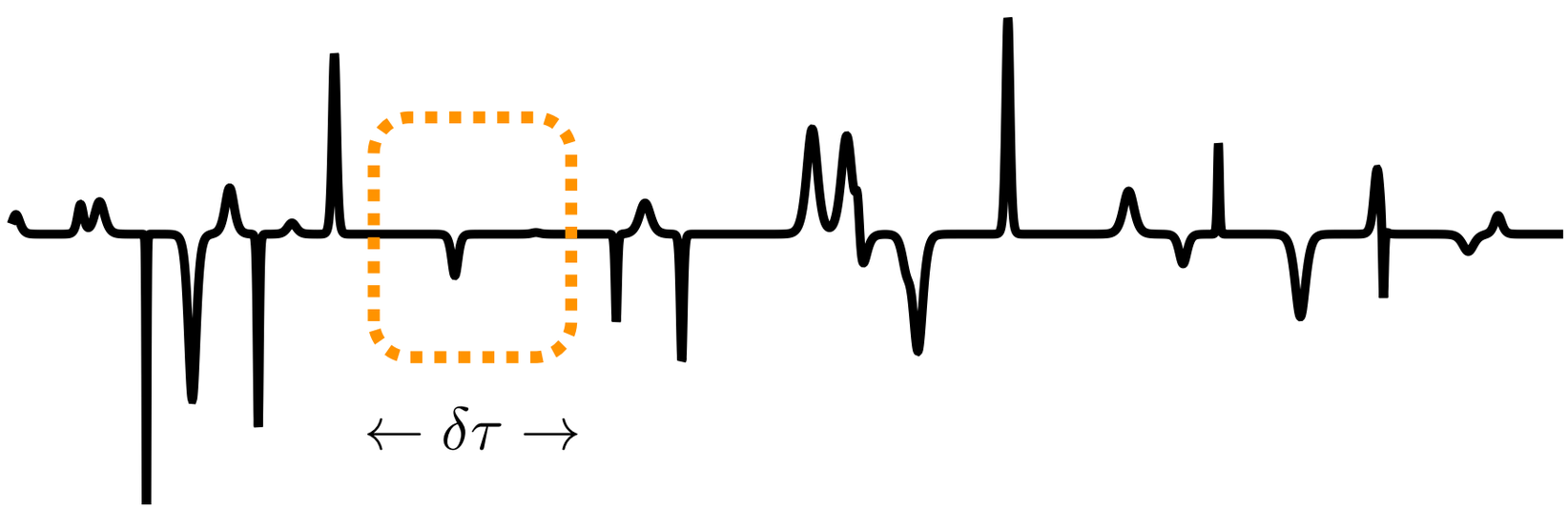
weak scattering assumption

“local” mean particle production rate

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$

μ_k local mean particle production rate

$$\mu_k = \frac{\langle n\rangle_{\delta\tau}}{\delta\tau}$$



“local” mean particle production rate

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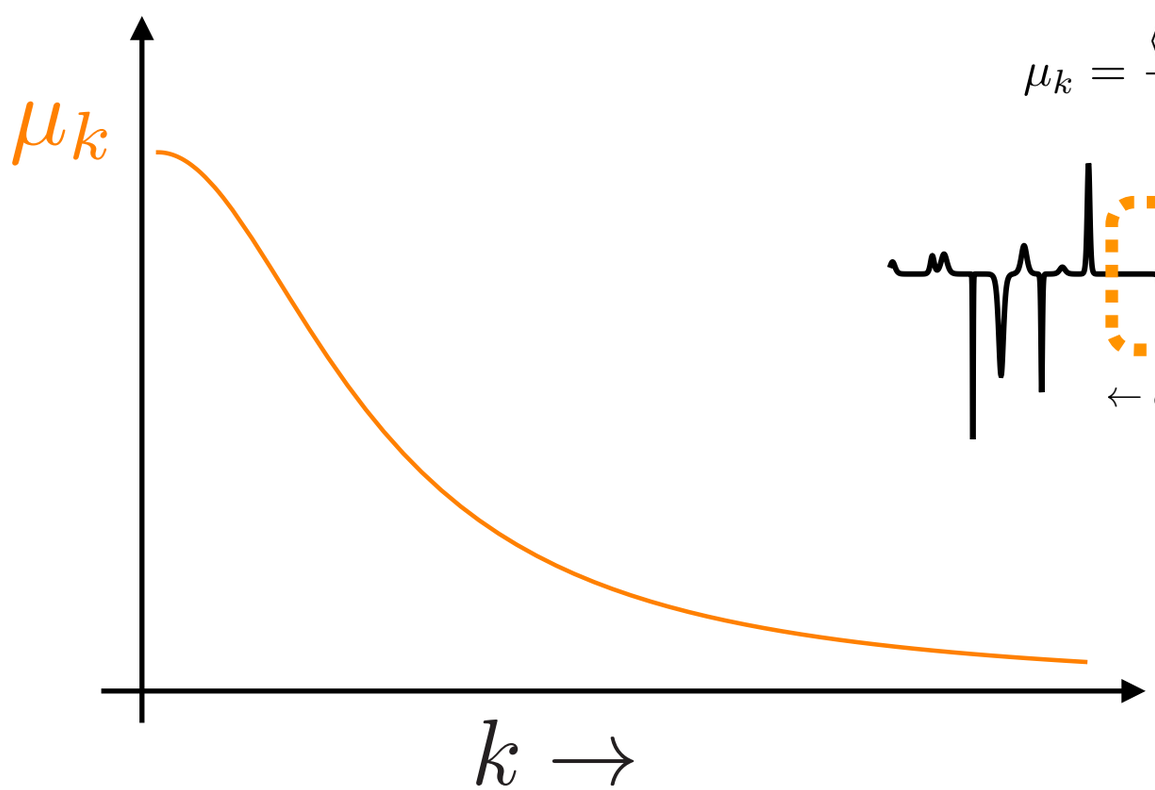
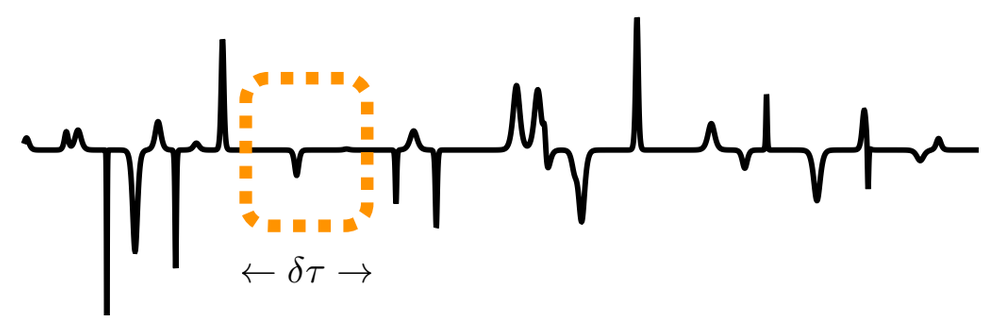
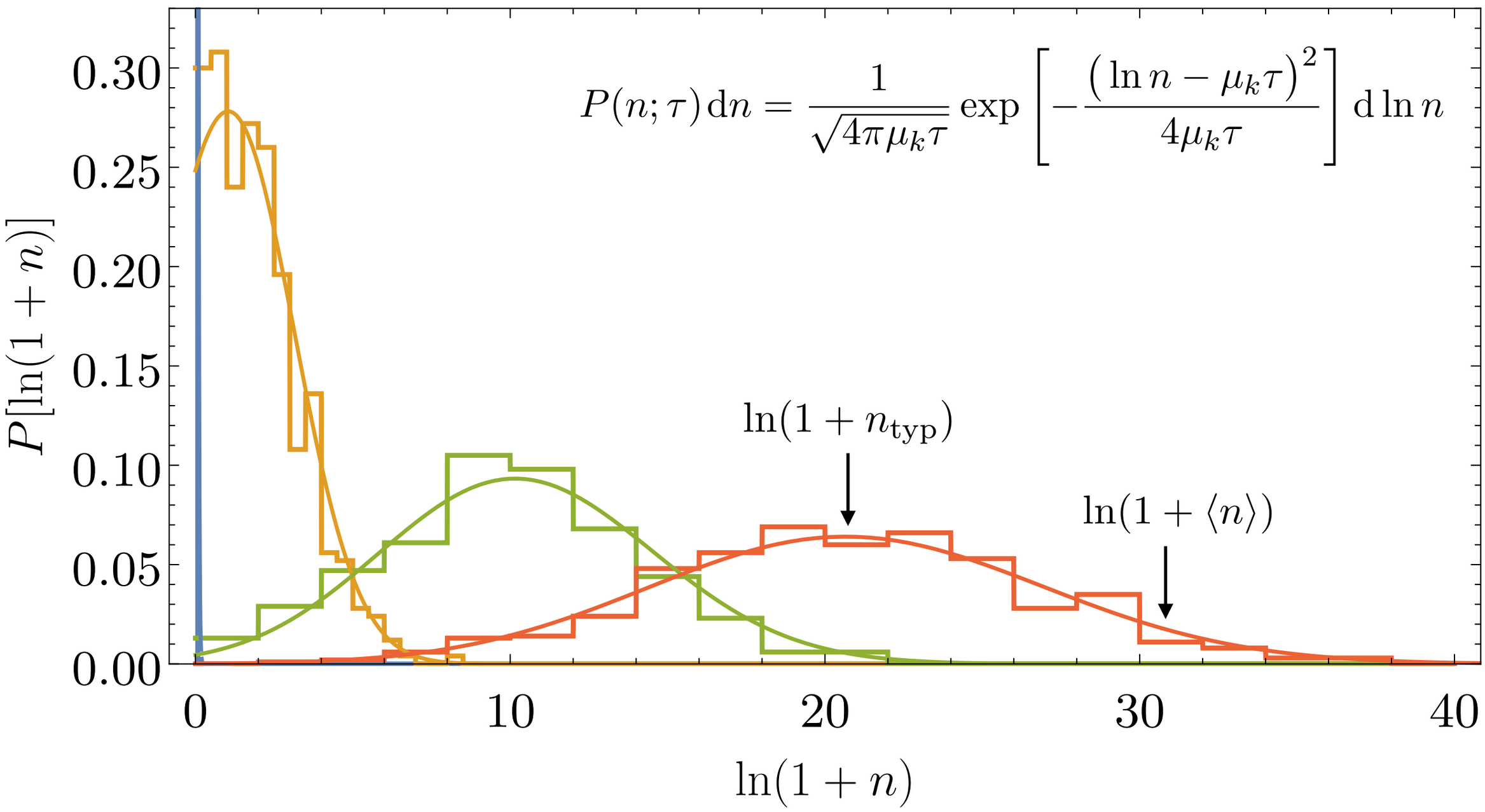
μ_k local mean particle production rate



$$\mu_k = \frac{\langle n \rangle_{\delta\tau}}{\delta\tau}$$



solution: “universal” distributions



moments: Fokker Planck equation

$$\langle n \rangle = \frac{1}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\frac{\mathrm{Var}(n)}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} e^{2\mu_k \tau}$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

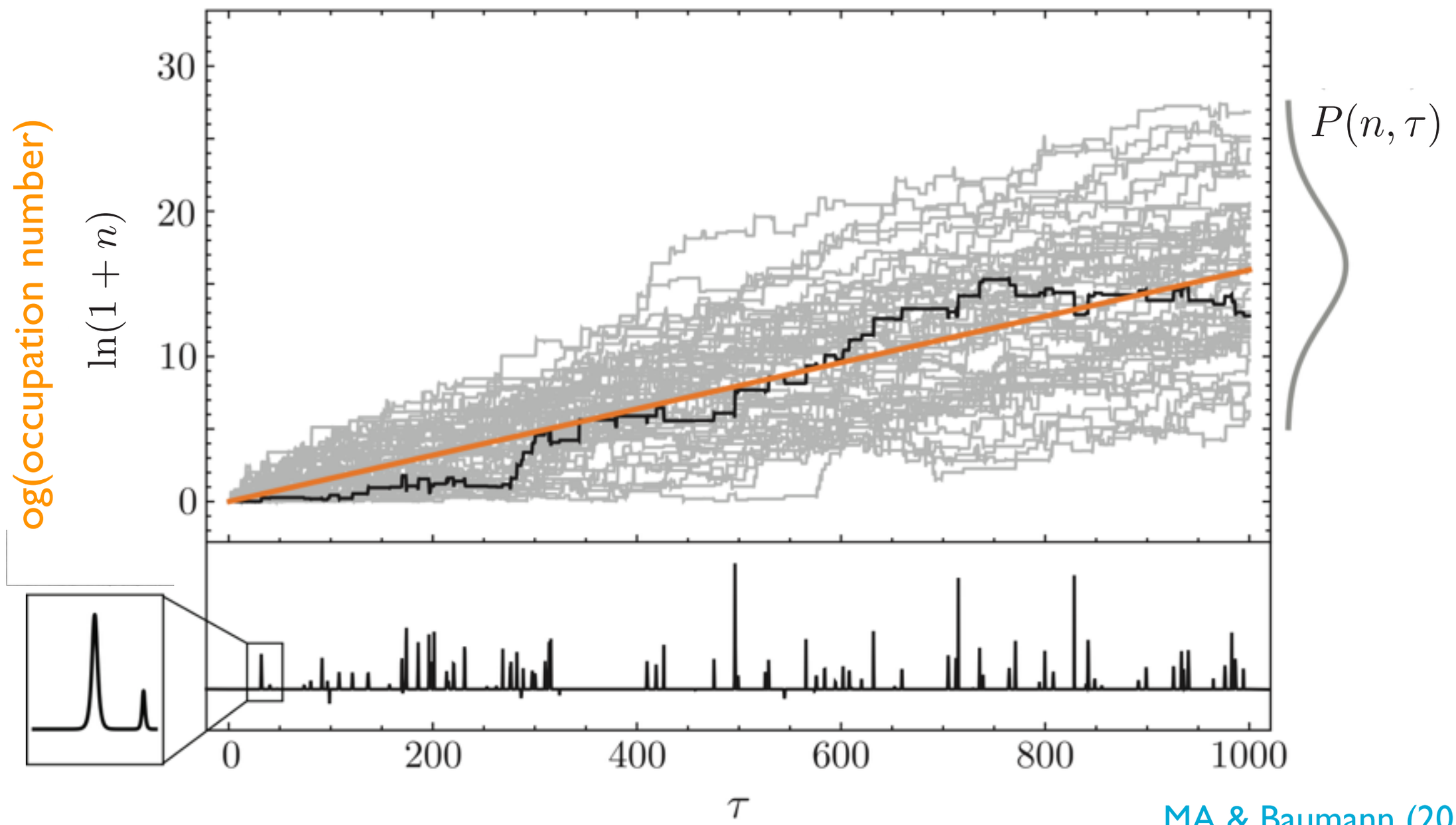
$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \longrightarrow \frac{2}{\mu_k \tau}$$

The most probable value of the occupation number

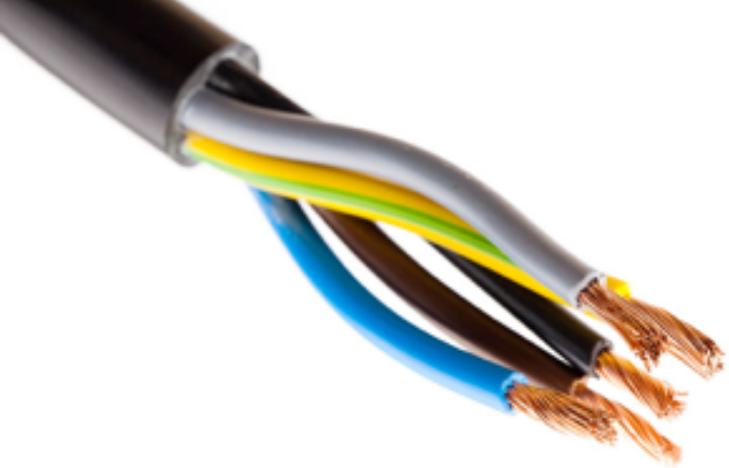
$$\boxed{n_{\mathrm{typ}} \equiv \exp\langle \ln(1+n) \rangle = e^{\mu_k \tau}}$$

the typical occupation number

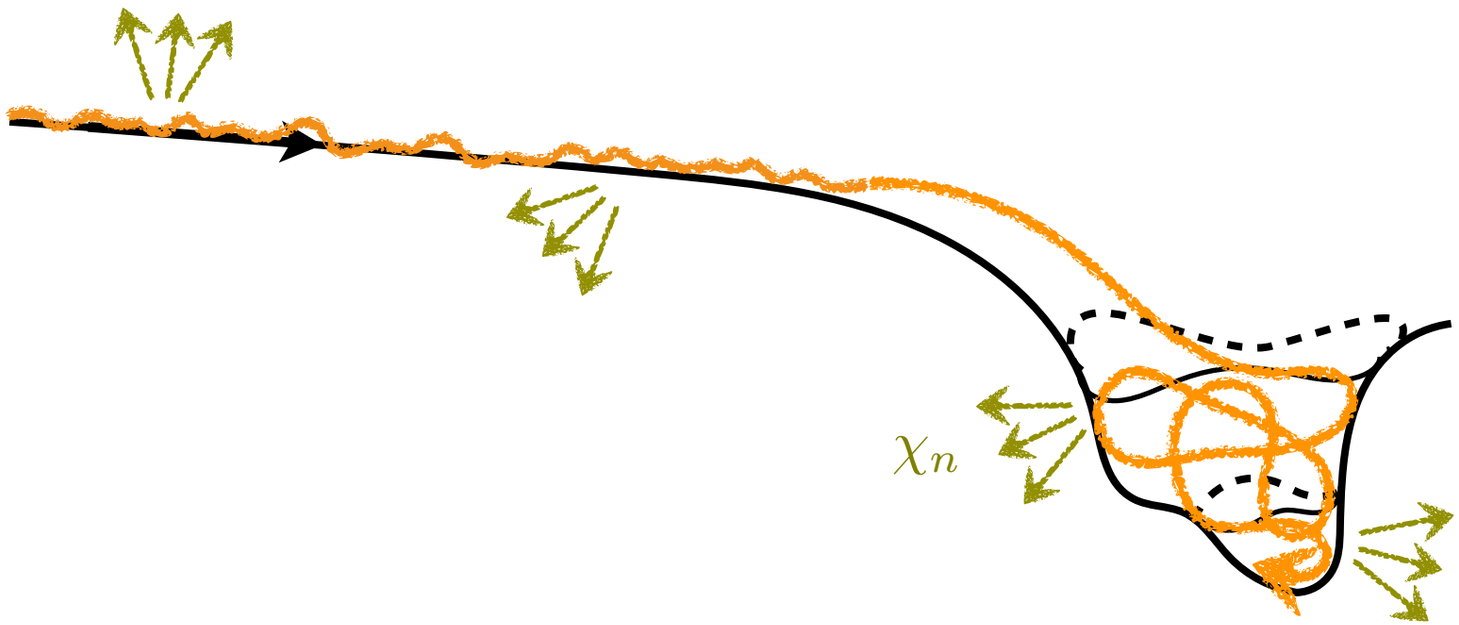
$$n_{\mathrm{typ}} \equiv \exp\langle \ln(1+n) \rangle = e^{\mu_k \tau}$$



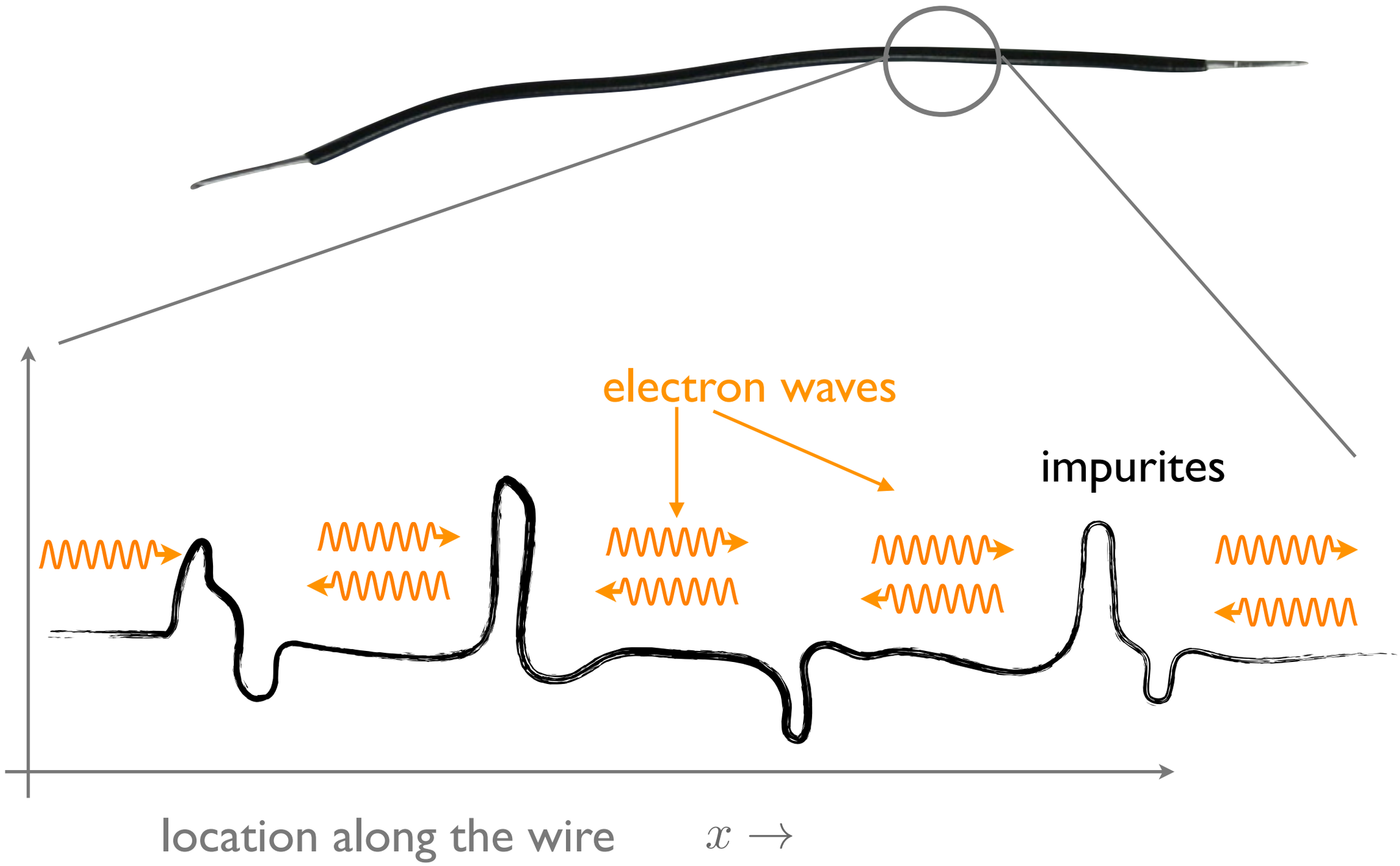
MA & Baumann (2015)



what is the connection to wires?

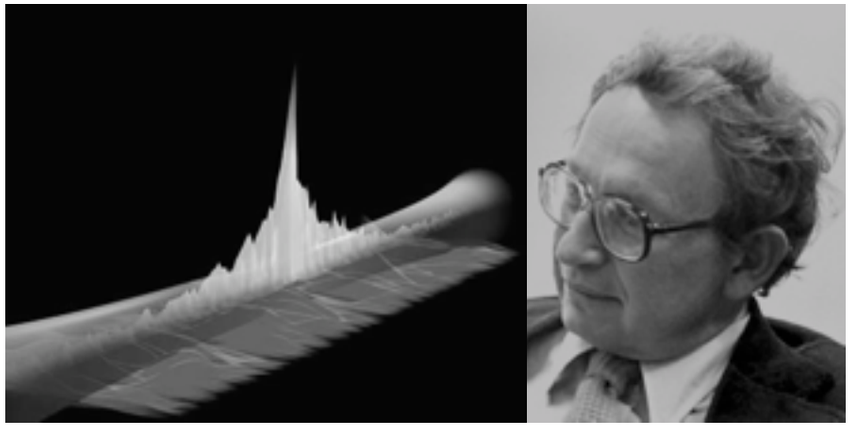


electron wave function: disordered wires

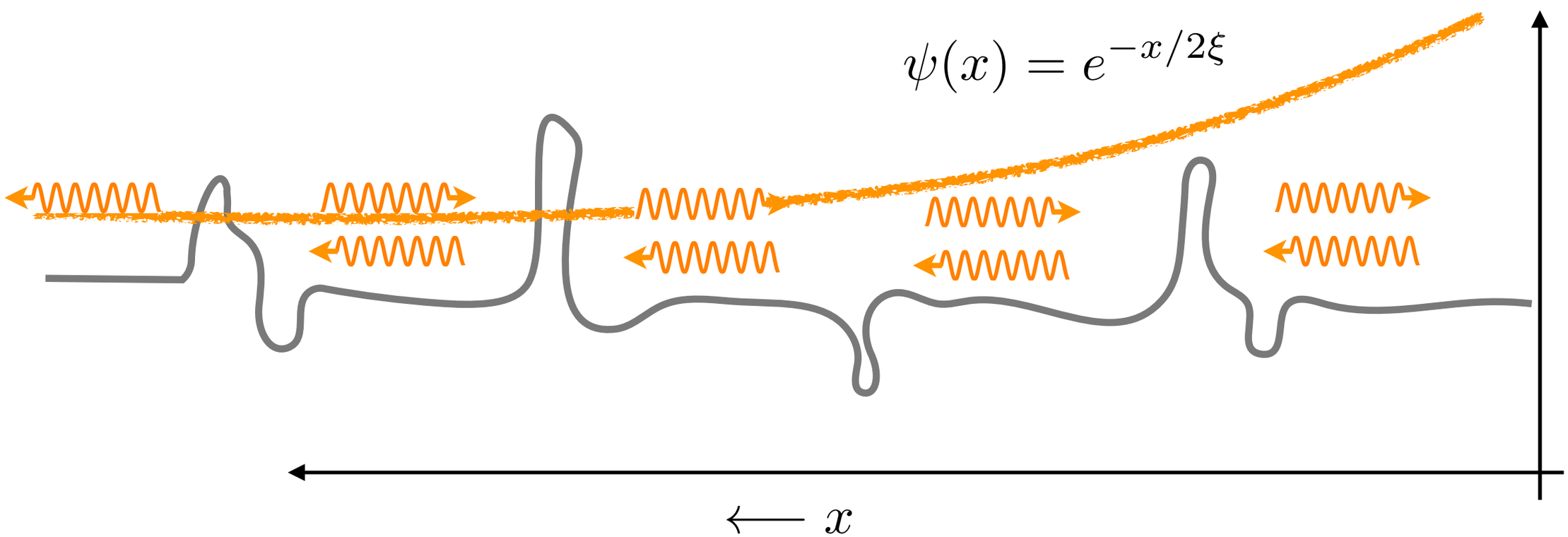


Anderson localization !

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

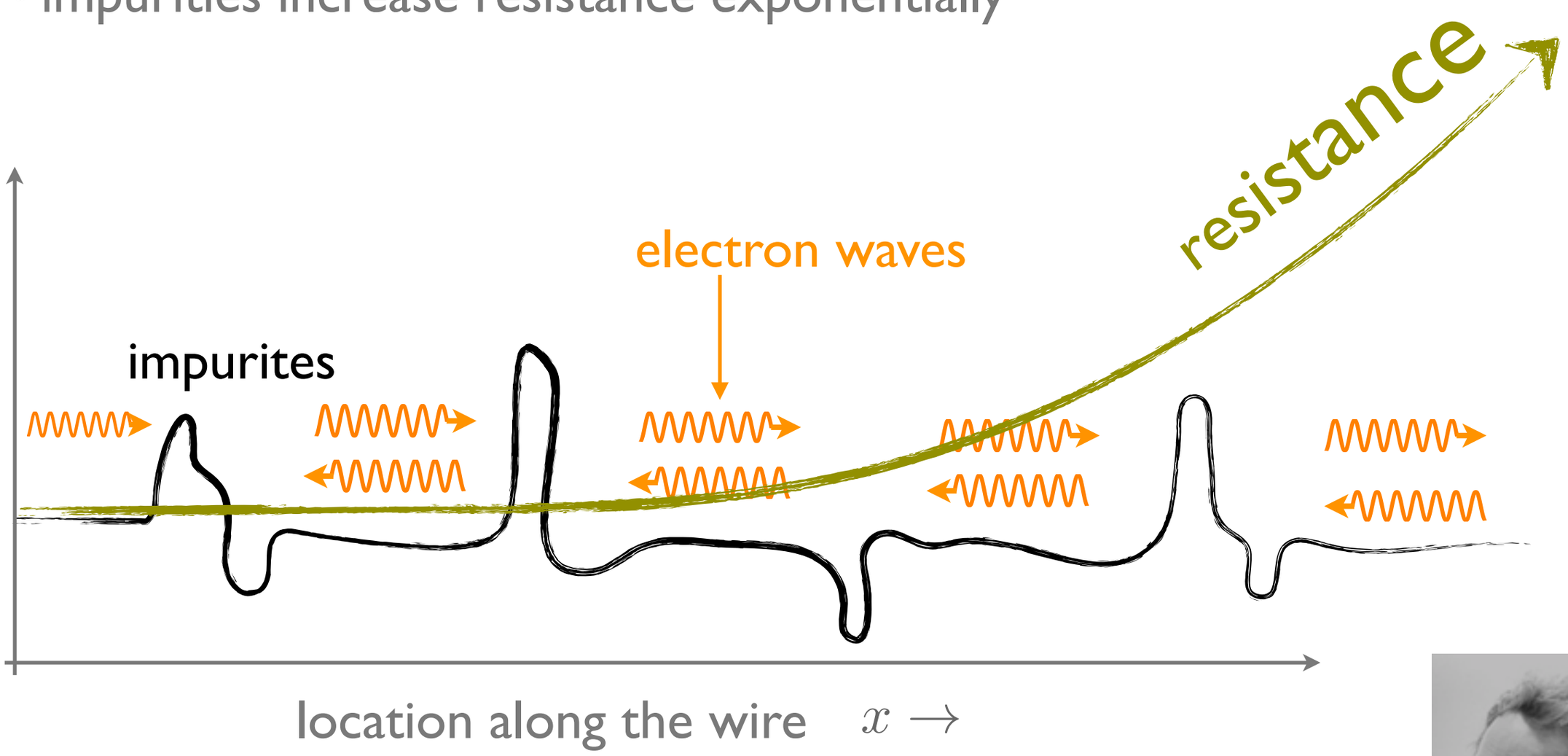


Anderson 1957



universal behavior

- impurities increase resistance exponentially



at low temperatures, one dimensional wires are insulators

complexity in time cosmology $\longleftrightarrow$ complexity in space wires

exponential growth in occupation number

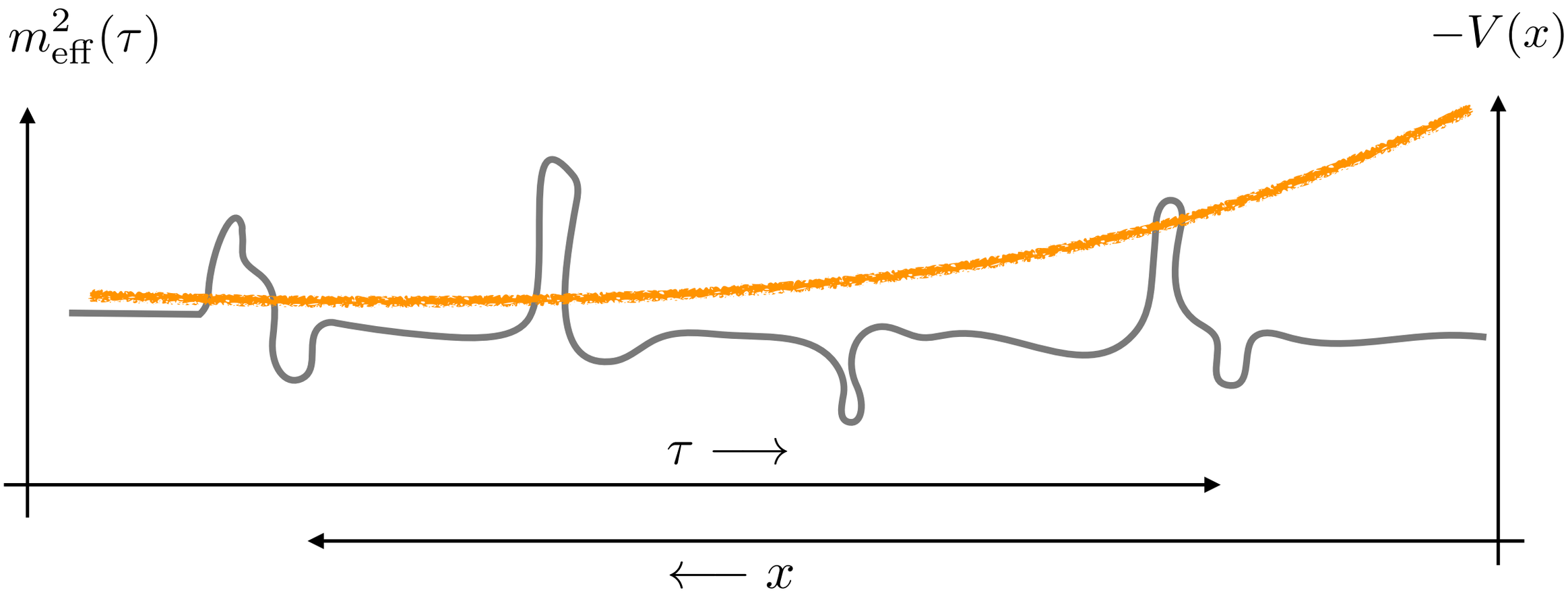
$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$\longleftrightarrow$

Anderson localization

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

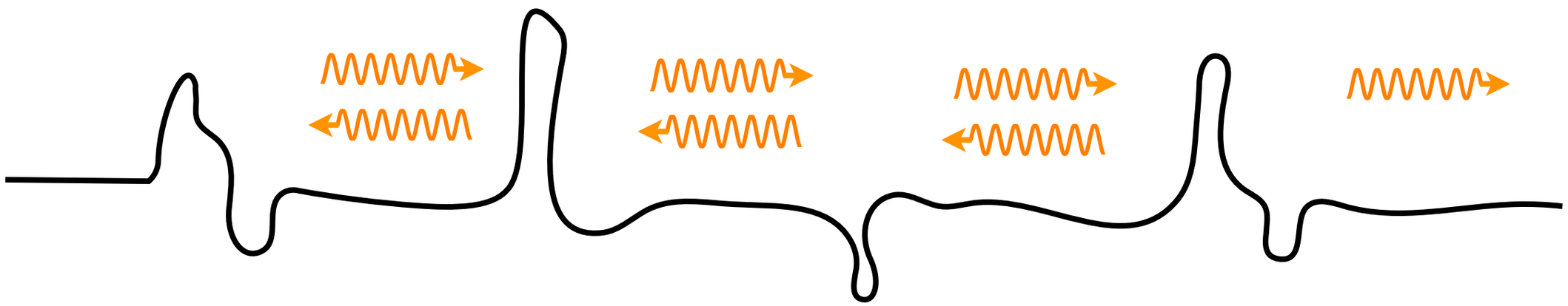
simplified version!



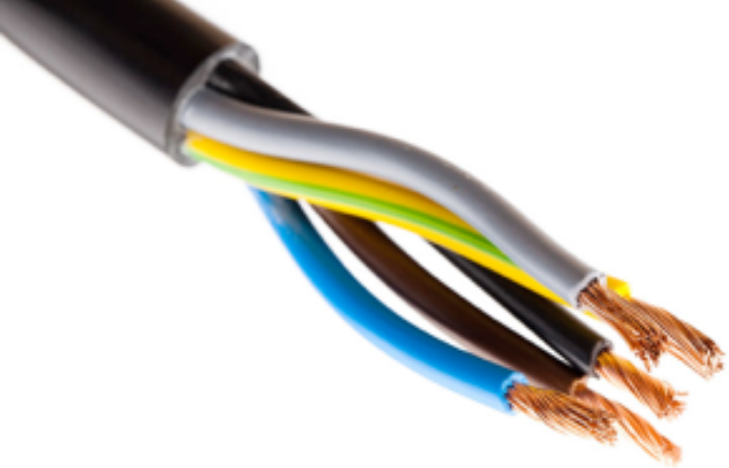
for periodic case with noise see Zanchin et. al 1998, Brandenburger & Craig 2008

dictionary

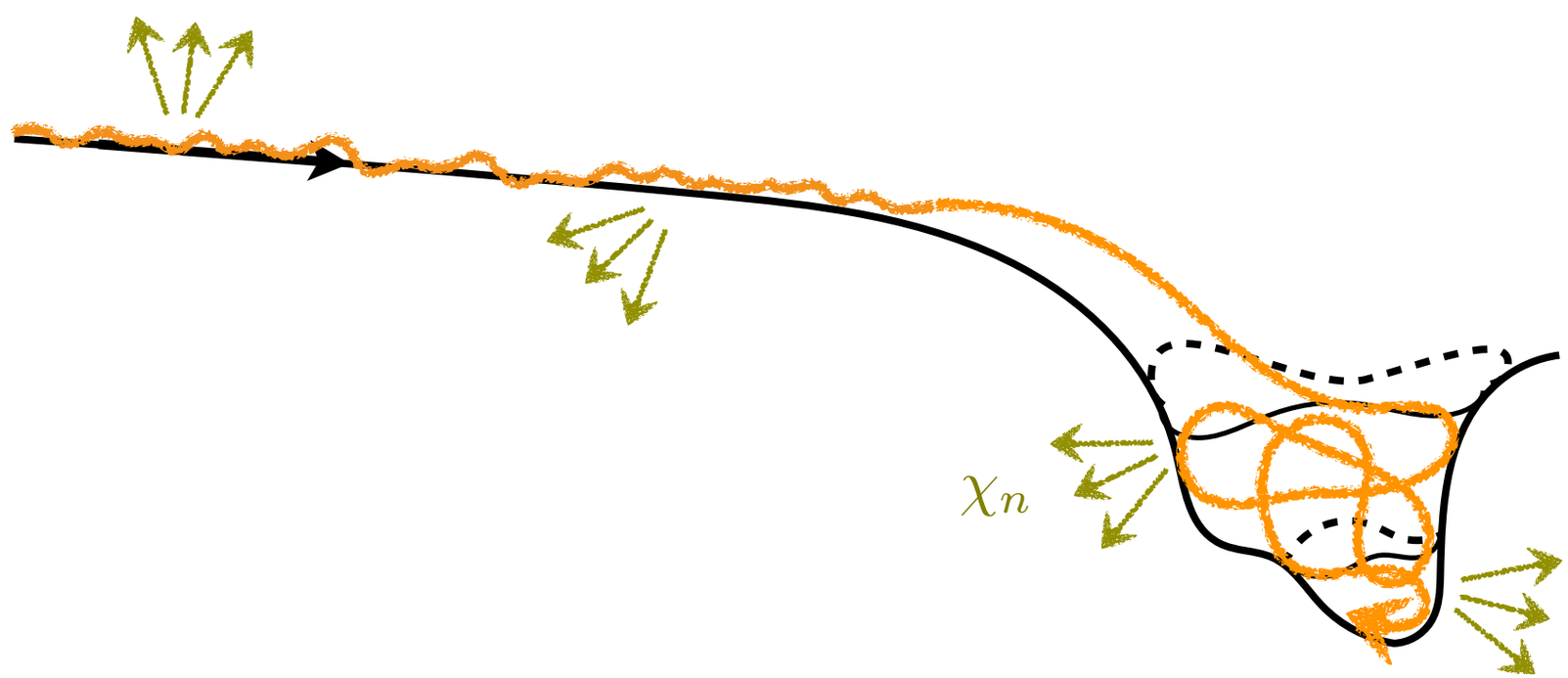
Time-dependent “Klein-Gordon”	Time-independent Schrödinger
$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$	$\frac{d^2\psi}{dx^2} + \big(E - V(x)\big)\psi = 0$



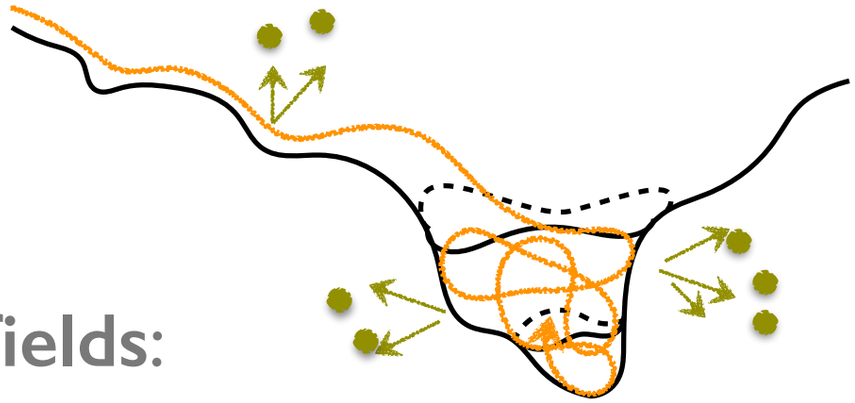
occupation number	resistance
(local) particle production rate	mean free path
multiple fields	**multiple channels**



multifield dynamics



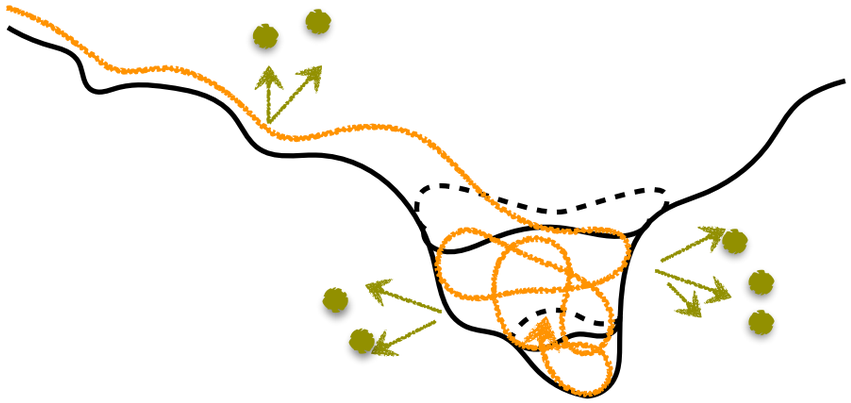
many fields



early universe: **multiple interacting fields**:

$$\left[\frac{d^2}{d\tau^2} + \omega_a^2(k)\right]\chi^a(\tau,k) + \sum_{b=1}^{N_{\rm f}} m^{\rm s}_{ab}(\tau)\chi^b(\tau,k) = 0$$

many fields

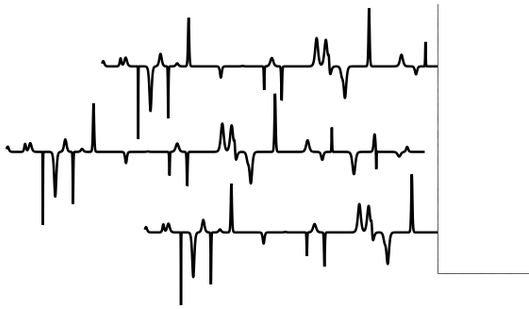


early universe: **multiple fields**:

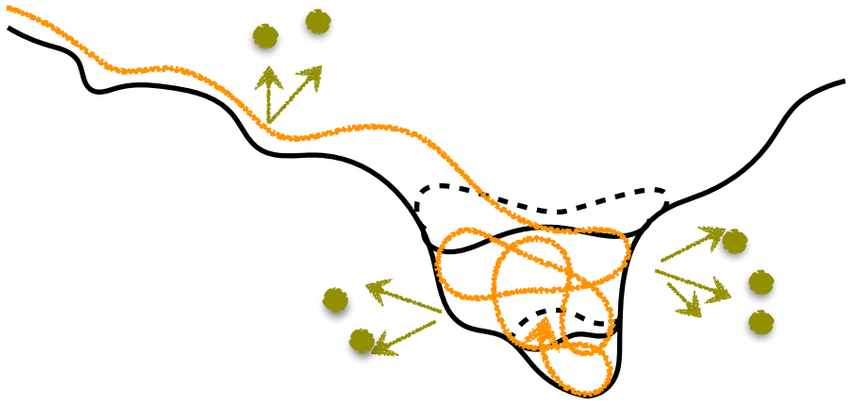
$$\left[\frac{d^2}{d\tau^2} + \omega_a^2(k)\right]\chi^a(\tau,k) + \sum_{b=1}^{N_\mathrm{f}} m^\mathrm{s}_{ab}(\tau)\chi^b(\tau,k) = 0$$

$$m^\mathrm{s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_\mathrm{s}}\Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)$$

"thin" scatterers



many fields



early universe: **multiple fields**:

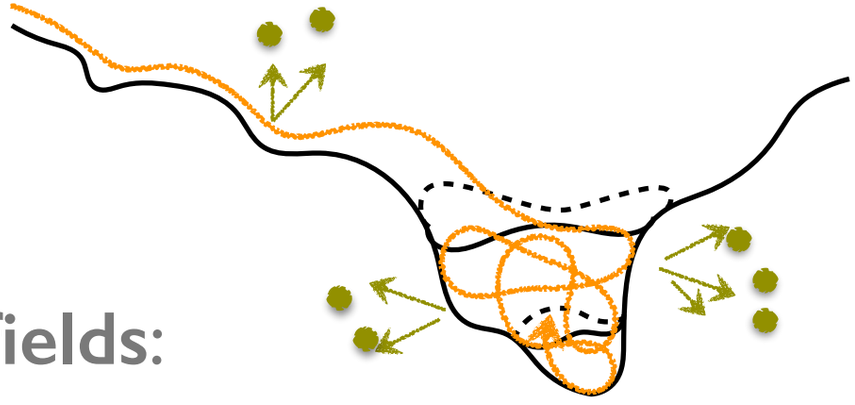
$$\left[\frac{d^2}{d\tau^2} + \omega_a^2(k)\right]\chi^a(\tau,k) + \sum_{b=1}^{N_\mathrm{f}} m^\mathrm{s}_{ab}(\tau)\chi^b(\tau,k) = 0$$

$$m^\mathrm{s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_\mathrm{s}}\Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)$$

$$\langle\Lambda_{ab}(\tau_j)\rangle = 0\,,$$

$$\langle\Lambda_{ab}\Lambda_{cd}\rangle = \sigma^2_{ab}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$$

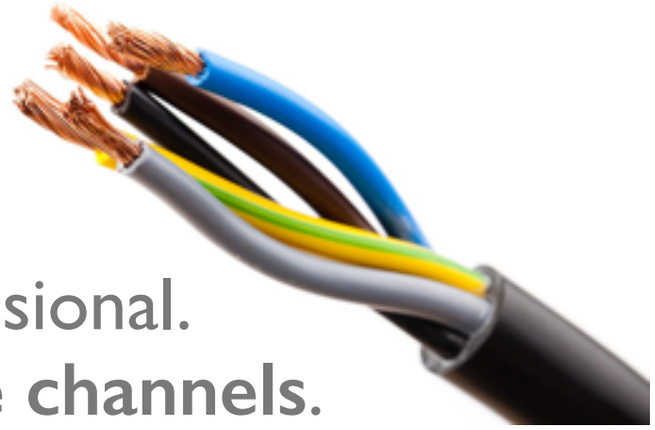
many coupled fields



early universe: **multiple interacting fields**:

$$\left[\frac{d^2}{d\tau^2} + \omega_a^2(k)\right]\chi^a(\tau,k) + \sum_{b=1}^{N_\mathrm{f}} m^\mathrm{s}_{ab}(\tau)\chi^b(\tau,k) = 0$$

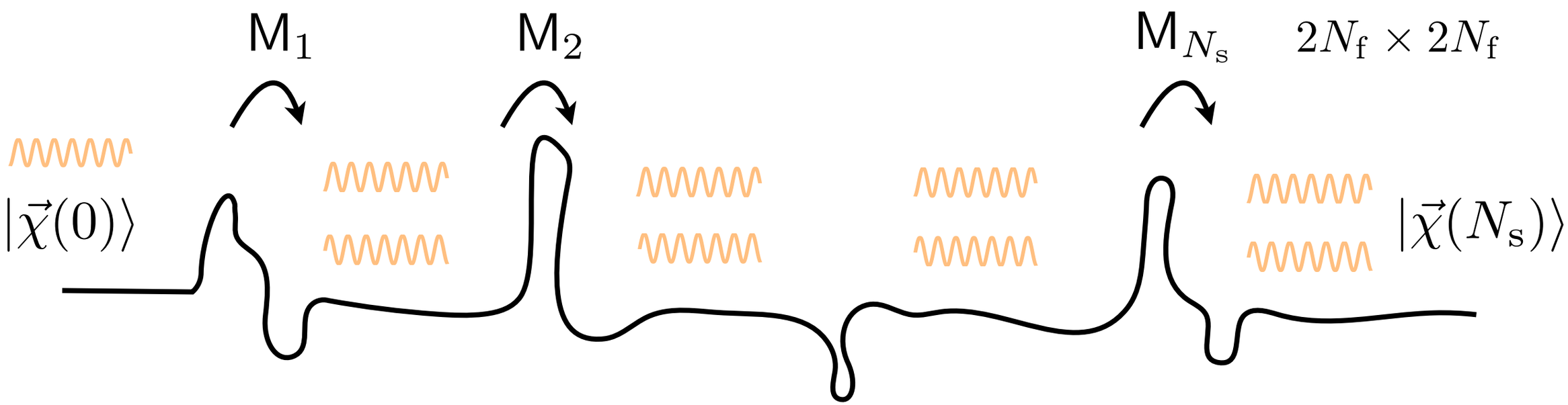
$$m^\mathrm{s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_\mathrm{s}}\Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)$$



real wires are not one-dimensional.
current conduction: **multiple channels**.

multifield particle production as scattering

$$|\vec{\chi}(N_\mathrm{s})\rangle \;=\; \mathsf{M}\,|\vec{\chi}(0)\rangle \quad \text{where} \quad \mathsf{M} \equiv \mathsf{M}_{N_\mathrm{s}} \cdots \mathsf{M}_2 \mathsf{M}_1$$



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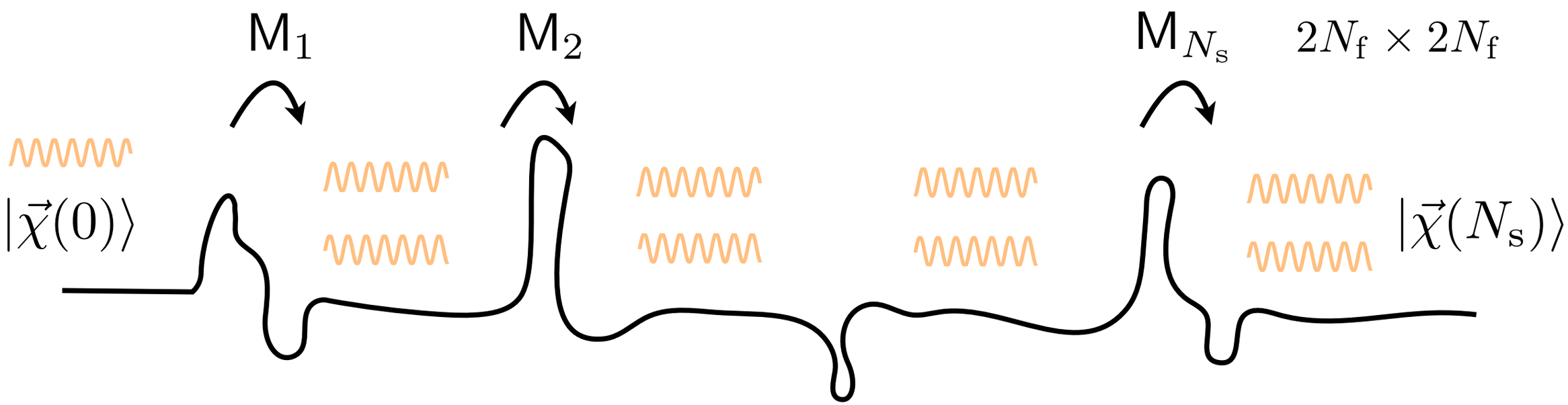


total occupation number of fields:

$$n \sim \mathrm{Tr}(\mathsf{M}\mathsf{M}^\dagger)$$

multi-dimensional Fokker Planck Eq.

$$|\vec{\chi}(N_{\rm s})\rangle \;=\; \mathsf{M}\,|\vec{\chi}(0)\rangle \qquad \text{where} \qquad \mathsf{M} \equiv \mathsf{M}_{N_{\rm s}} \cdots \mathsf{M}_2 \mathsf{M}_1$$

Fokker Planck Equation:

$$\partial_\tau P(\mathsf{M};\tau) = \frac{\langle \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau}\,\partial_{\mathsf{M}} P(\mathsf{M};\tau) + \frac{\langle \delta \mathsf{M} \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta\tau}\,\partial_{\mathsf{M}}\partial_{\mathsf{M}} P(\mathsf{M};\tau)$$

exponential growth in typical occupation number

$$|\vec{\chi}(N_{\rm s})\rangle \;=\; \mathsf{M}\,|\vec{\chi}(0)\rangle \qquad \text{where} \qquad \mathsf{M} \equiv \mathsf{M}_{N_{\rm s}} \cdots \mathsf{M}_2 \mathsf{M}_1$$

Fokker Planck Equation:

$$\partial_\tau P(\mathsf{M};\tau) = \frac{\langle \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta \tau} \partial_{\mathsf{M}} P(\mathsf{M};\tau) + \frac{\langle \delta \mathsf{M} \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta \tau} \partial_{\mathsf{M}} \partial_{\mathsf{M}} P(\mathsf{M};\tau)$$

total occupation number of fields: $n \sim \mathrm{Tr}(\mathsf{M}\mathsf{M}^\dagger)$

$$\boxed{n_{\rm typ} \sim \exp\left[\frac{2}{1+N_{\rm f}} \langle \mathrm{Tr}\,\Lambda^2 \rangle\; \tau\right]}$$

MA, Garcia, Xie & Wen (2017)

robust results in large N_f limit

MA, Garcia, Xie & Wen (2017)

- simple *Trace formula* for estimating particle production rate when the number of fields is large (*without being statistically similar*)

$$n_{\rm typ} \sim \exp\left[\frac{2}{1+N_{\rm f}}\langle {\rm Tr}\,\Lambda^2\rangle\ \tau\right]$$

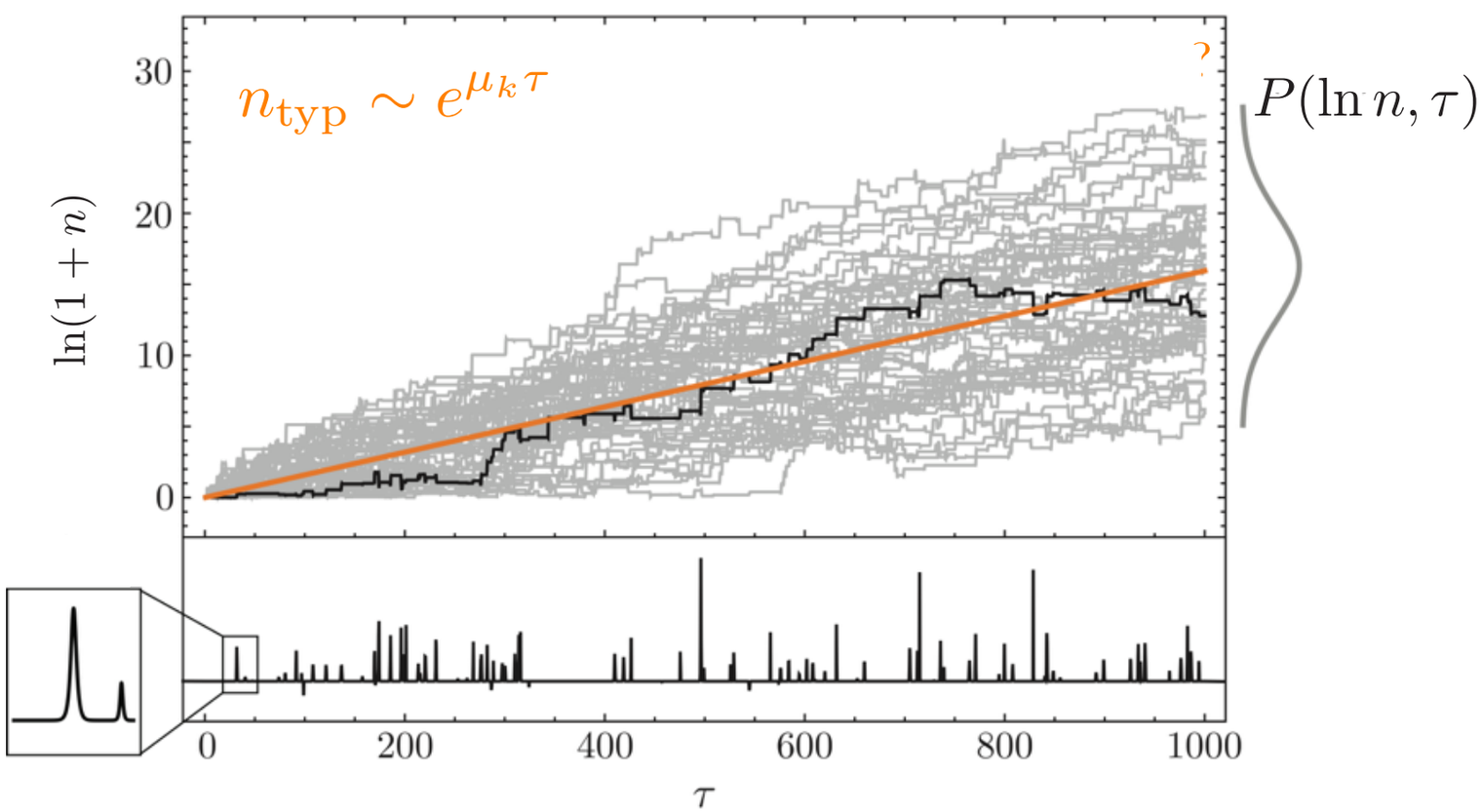
- Λ contains all the information about the strengths of interactions

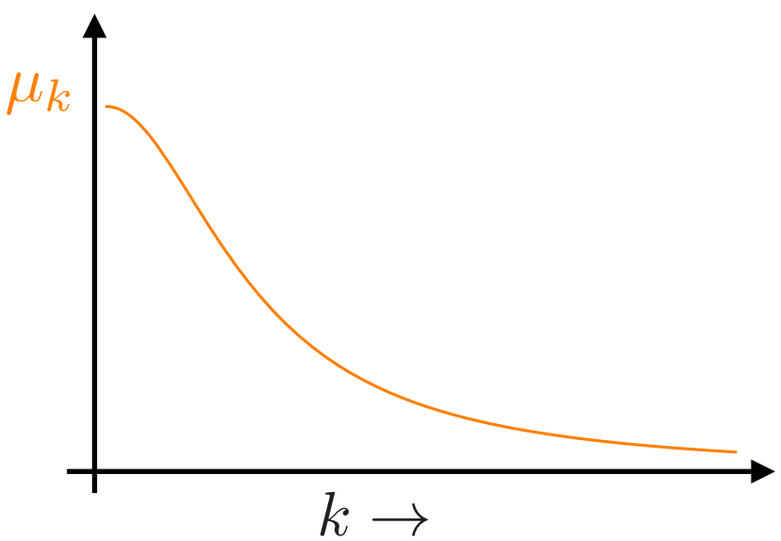
summary so far

- repeated non-perturbative particle production (ignoring expansion)

  1. *typical* occupation number grows exponentially with time

  2. the distribution of occupation numbers is log-normal





next up

- repeated non-perturbative particle production (ignoring expansion)
  1. *typical* occupation number grows exponentially with time
  2. the distribution of occupation numbers is log-normal
- effect of expansion ?
- applications to inflation and reheating ?

expansion effects ?

stochastic particle production in expanding spacetime

$$\ddot{\chi}(t,k) + 3H\dot{\chi}(t,k) + \left[\frac{k^2}{a^2} + m_{\rm eff}^2(t)\right]\chi(t,k) = 0$$

$$m_{\rm eff}^2(t) = m^2 + m_{\rm s}^2(t)$$

$$m_{\rm s}^2(t) = \sum_{j=1}^{N_{\rm s}} m_j \delta(t - t_j) \qquad \langle m_j^2 \rangle \equiv \sigma_{\rm s}^2$$

- competition between growth from particle production and dilution due to expansion
- Useful parameter: $\mathcal{N}_s \frac{\sigma_{\rm s}^2}{H^2}$

  ↓

  # of scatters per Hubble time

stochastic particle production in expanding spacetime

$$\ddot{\chi}(t,k) + 3H\dot{\chi}(t,k) + \left[\frac{k^2}{a^2} + m_{\rm eff}^2(t)\right]\chi(t,k) = 0$$

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- competition between growth from particle production and dilution due to expansion
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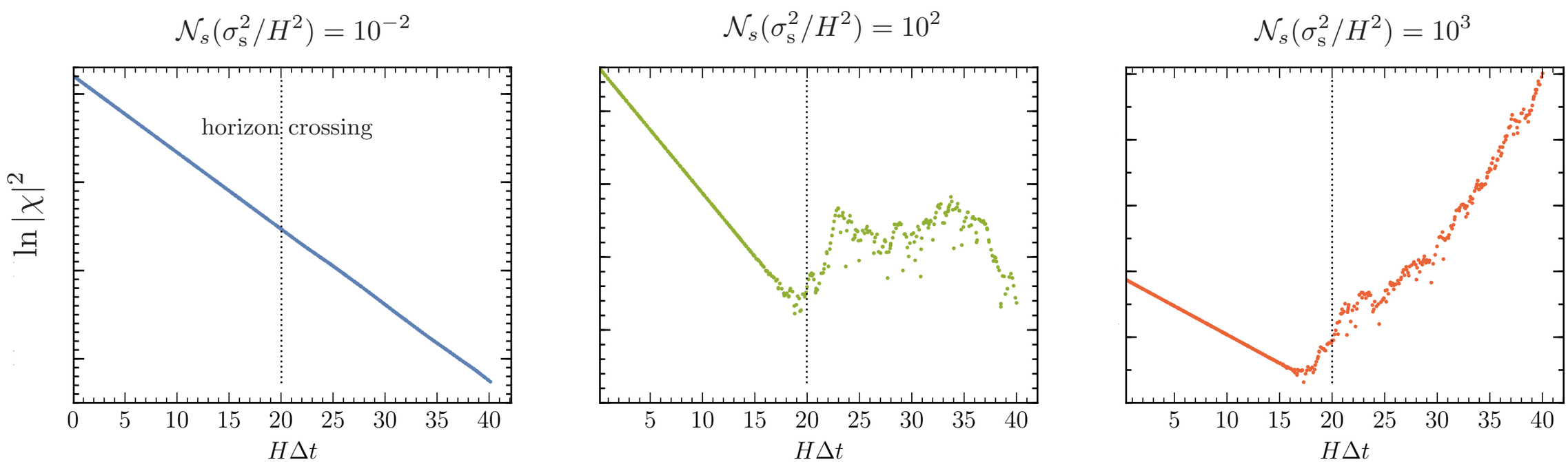
  ↓

  # of scatters per Hubble time

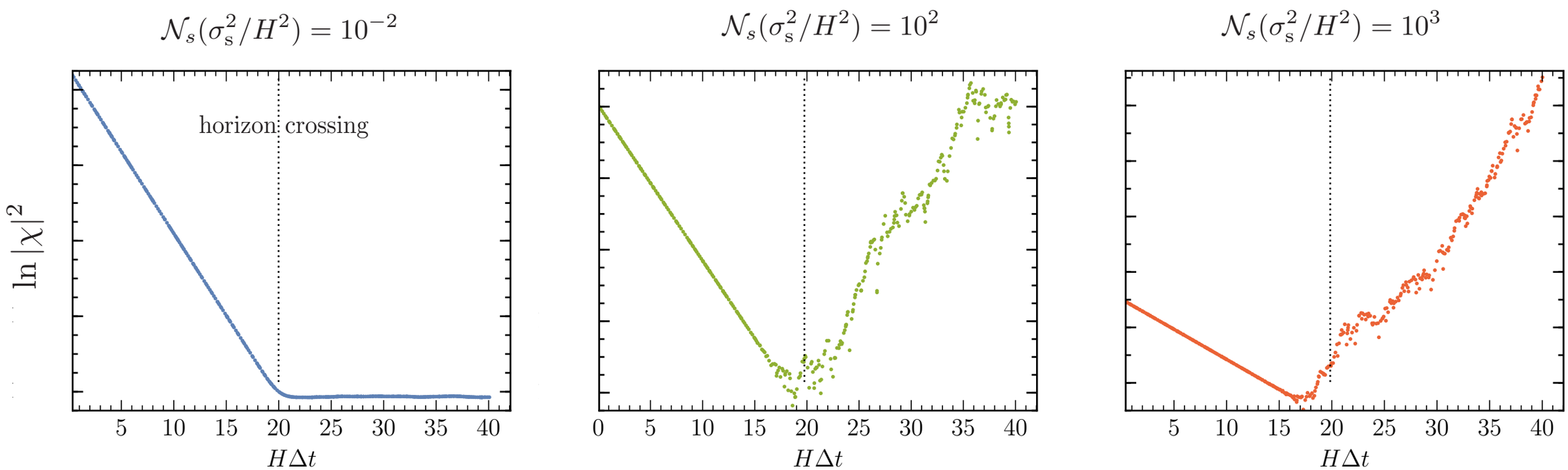
field evolution in deSitter conformal case: $m^2 = 2H^2$



- in absence of interactions the field amplitude decays inside and outside the horizon
- interaction (depending on their strength and frequency) can arrest, and reverse this dilution.

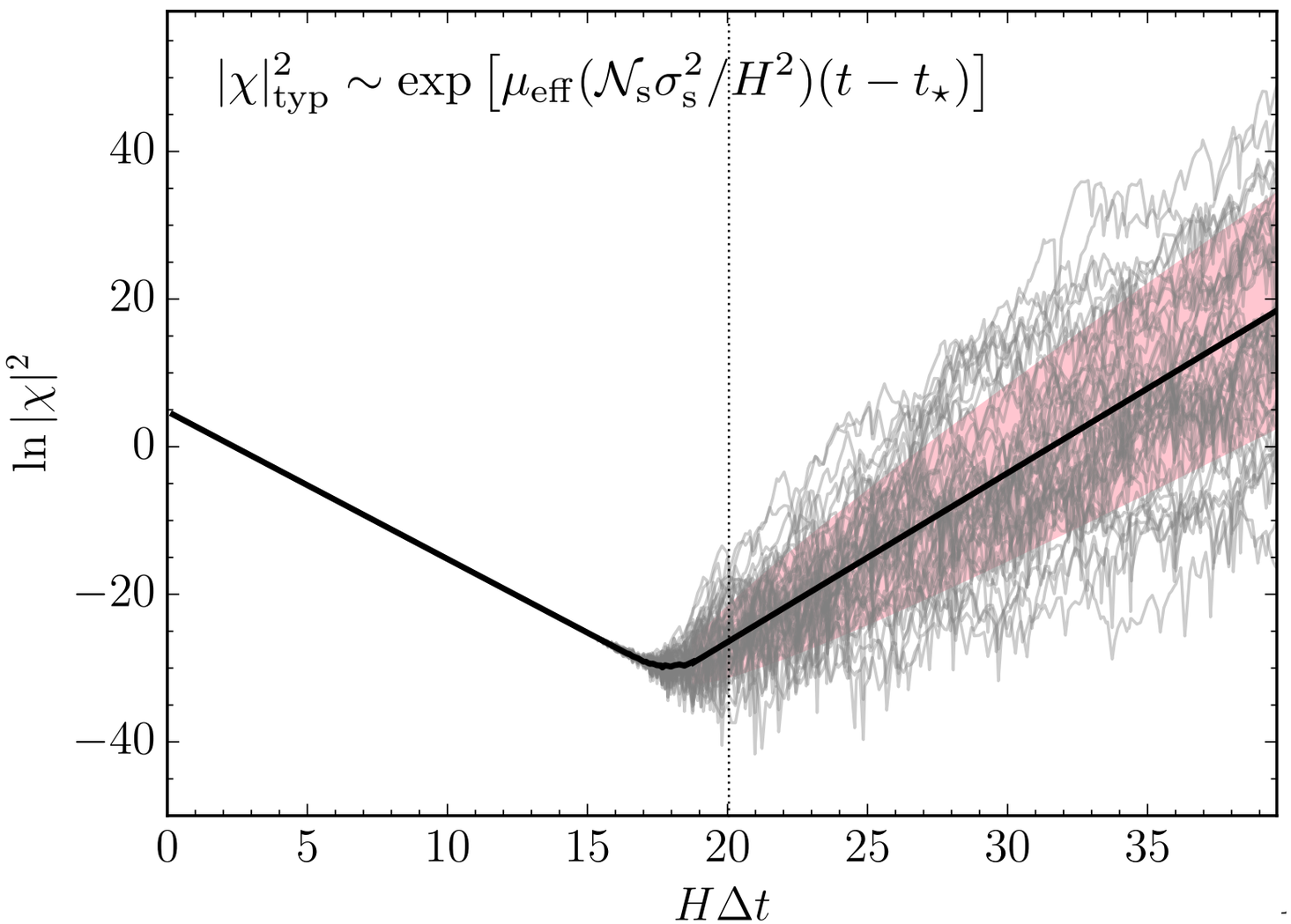
* better to deal with fields rather than occupation numbers of superhorizon scales

field evolution in deSitter massless case: $m^2 = 0$

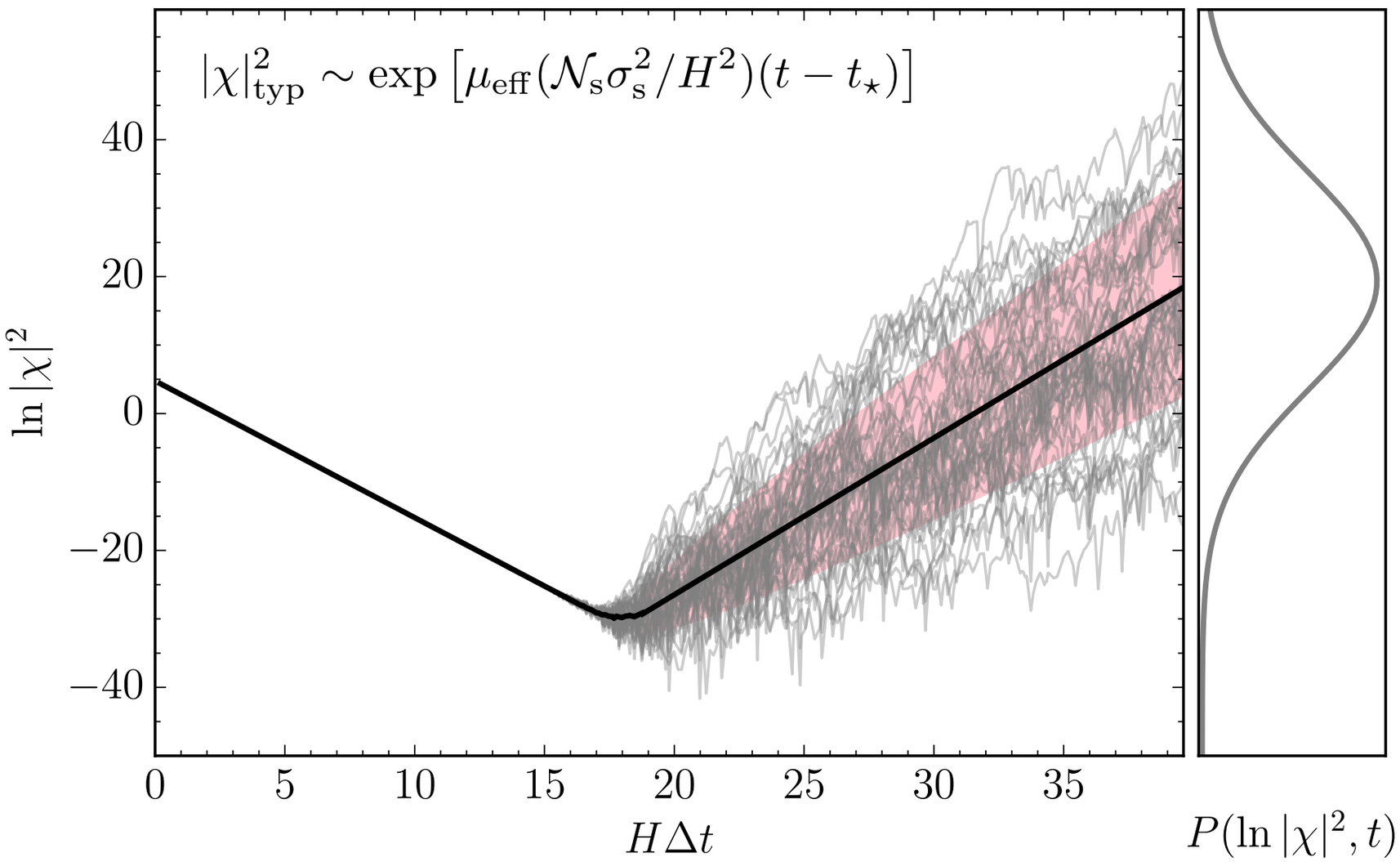


- in absence of interactions the field amplitude decays inside the horizon, and is constant outside
- interaction (depending on their strength and frequency) can arrest, and reverse this dilution.

typical superhorizon behavior of field amplitude



log-normal distribution



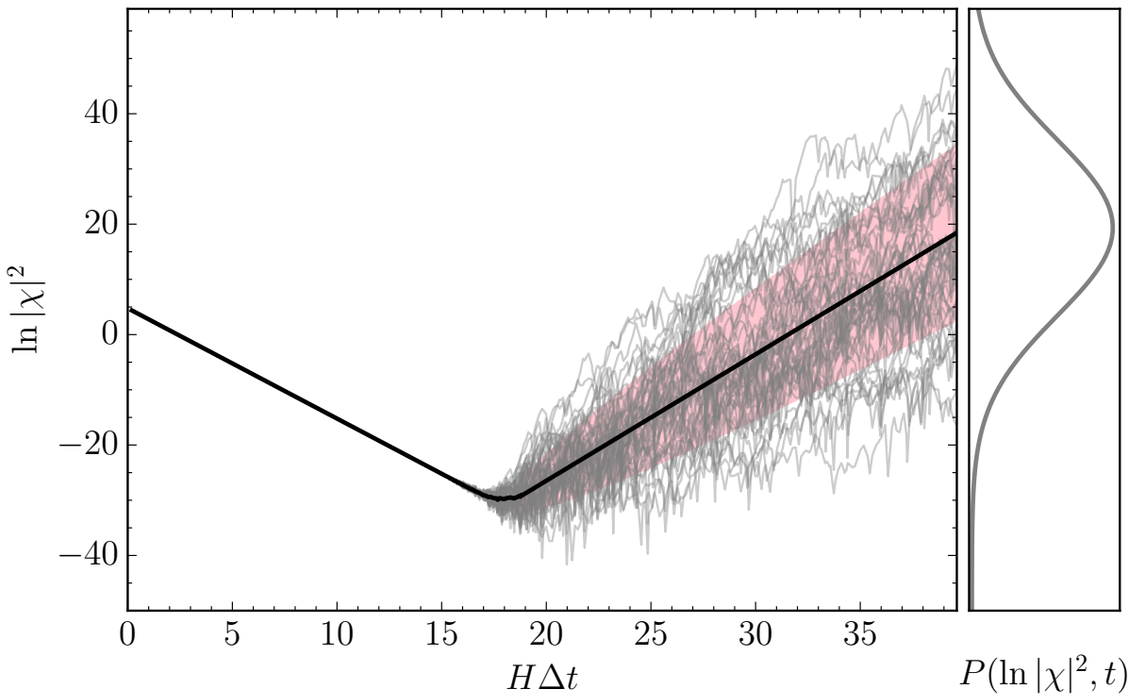
Mean

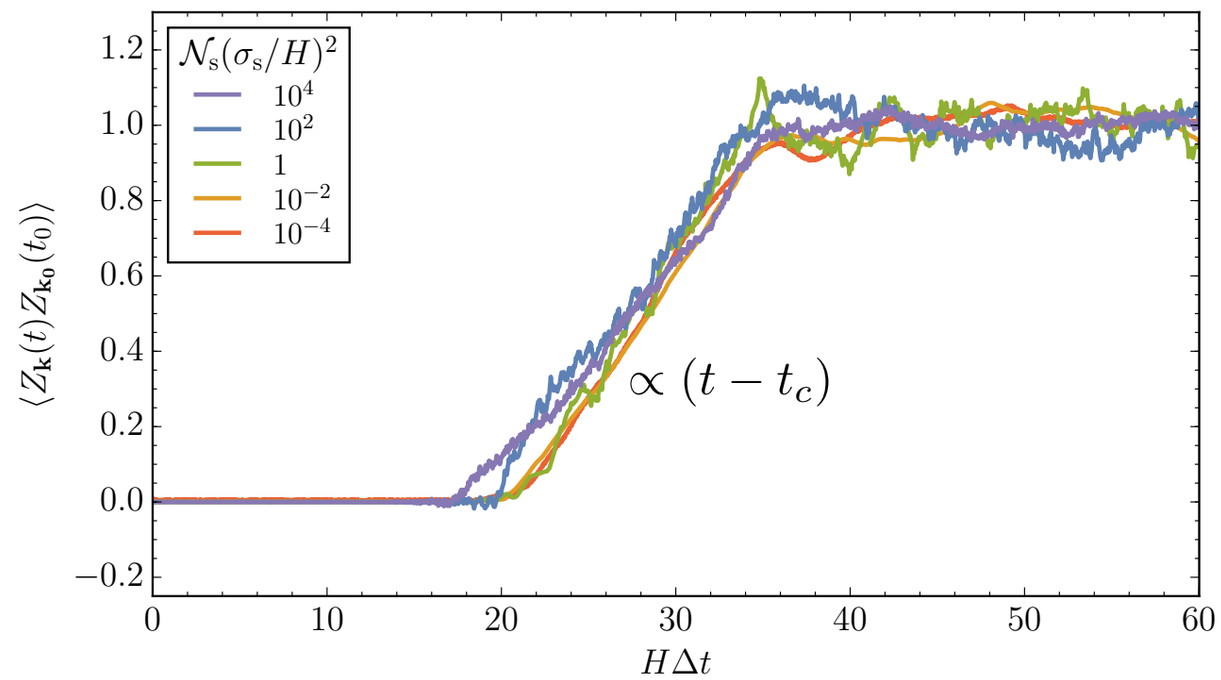
$\langle \ln|\chi|^2\rangle \propto (t-t_\star)$

Variance

$\mathrm{Var}[\ln|\chi|^2] \propto (t-t_\star)$

geometric random walk & universality





On superhorizon scales the behavior of the fields is that of a geometric random walk:

$$\langle Z_{\mathbf{k}}(t) Z_{\mathbf{k_0}}(t_0)\rangle \propto \theta(t - t_0) \qquad \text{where} \qquad Z_{\mathbf{k}}(t) = \ln|\chi|^2 - \langle \ln|\chi|^2\rangle$$

the shape is universal (up to known/calculable scalings)

caveat

caveats

- weak scattering ?
- exponential growth: linearity/ backreaction ?

assumptions

Fokker Planck equation relied on weak scattering assumption

$$\partial_\tau P(\mathsf{M};\tau) = \frac{\langle \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta \tau} \partial_{\mathsf{M}} P(\mathsf{M};\tau) + \frac{\langle \delta \mathsf{M} \delta \mathsf{M} \rangle_{\mathsf{M}_2}}{\delta \tau} \partial_{\mathsf{M}} \partial_{\mathsf{M}} P(\mathsf{M};\tau) + \cdots$$

possible resolution:
Random Matrix Theory

two large N's to make life easier:

- large number of fields:

$$N_{\mathrm{f}}$$

$$\updownarrow$$

- large number of scatterings:

$$N_{\mathrm{s}}$$

$$\updownarrow$$

from RMT:

- eigenvalue spectrum of M_j
- non-random limit of $\mathsf{M} = \prod_{j=1}^{N_{\mathrm{s}}} \mathsf{M}_j$

. A. Crisanti, G. Paladin, and A. Vulpiani (1993).

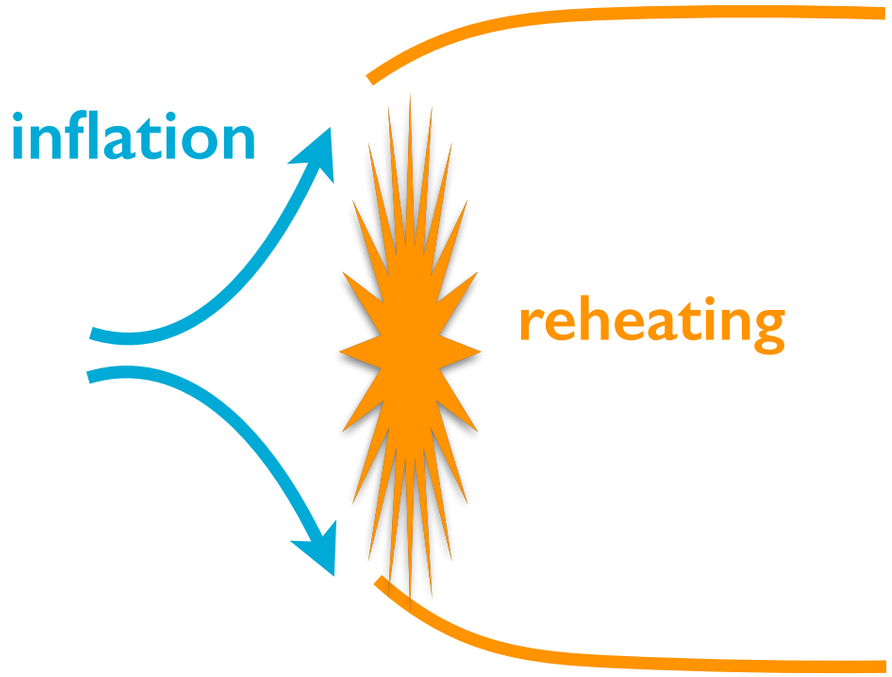
prediction for exponential behavior in time

non-random behavior of the exponent

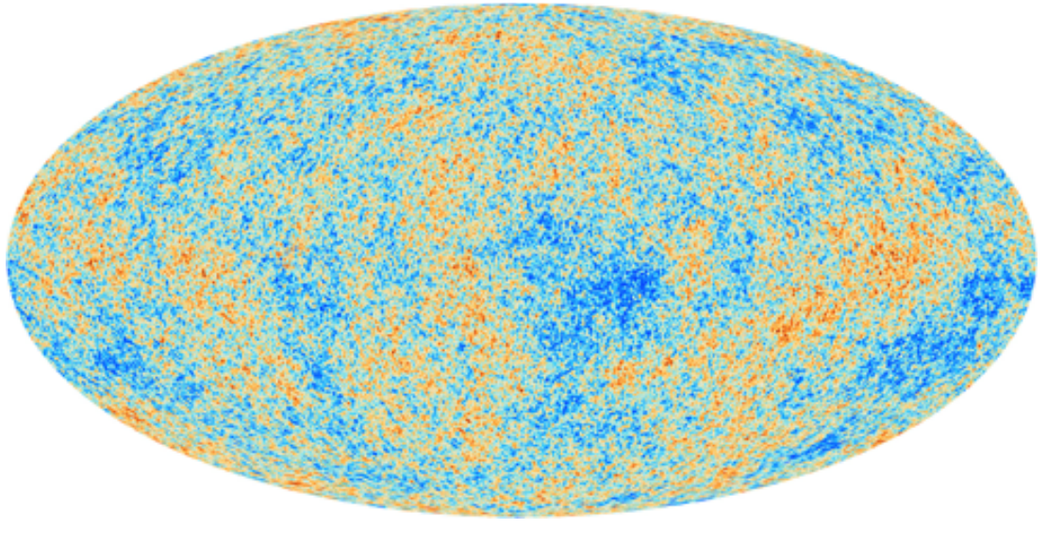
FP equation relied on weak scattering per interaction, RMT does not!

caveats

- weak scattering ?
- exponential growth — nonlinearity ?
  - nonlinearity/backreaction *can be* avoided for sufficiently weak scattering or finite duration of scattering
  - however, for reheating, eventual lattice simulations will likely needed [another talk]



applications



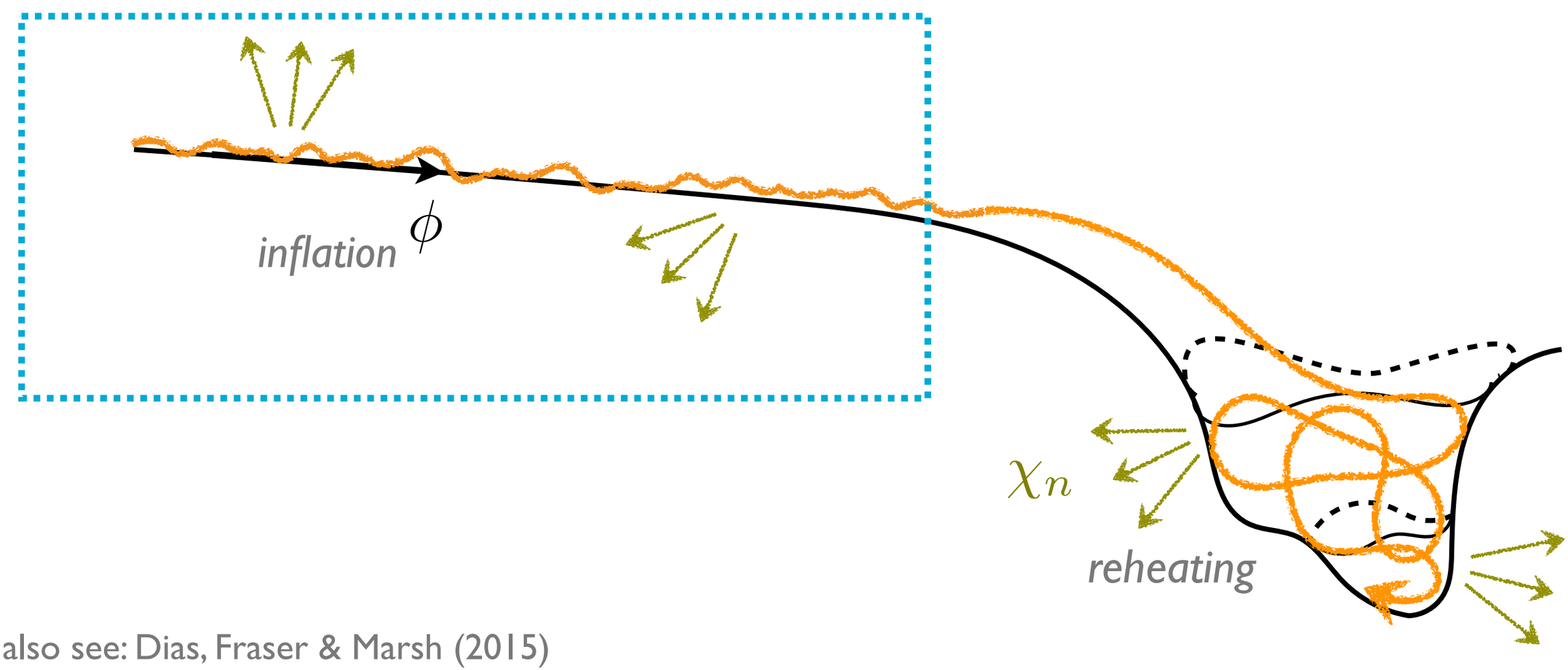
WORK IN PROGRESS

applications: inflation

MA, Baumann, Carlsten, Garcia & Green

background dynamics → particle production ↔ curvature fluctuations

$\langle \chi_{k_1} \chi_{k_2} \ldots \rangle$ $\langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$



also see: Dias, Fraser & Marsh (2015)

WORK IN PROGRESS

effects on curvature perturbations ?

background dynamics → particle production ↔ curvature fluctuations

$\langle \chi_{k_1} \chi_{k_2} \ldots \rangle$ $\langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$

$$\zeta_k = -H \pi_k$$

$$\ddot{\pi}_k + [3H + \mathcal{O}_{\rm d}]\, \pi_k + \frac{k^2}{a^2} \pi_k = \mathcal{O}_{\rm s}(\langle \chi \chi \ldots \rangle_k)$$

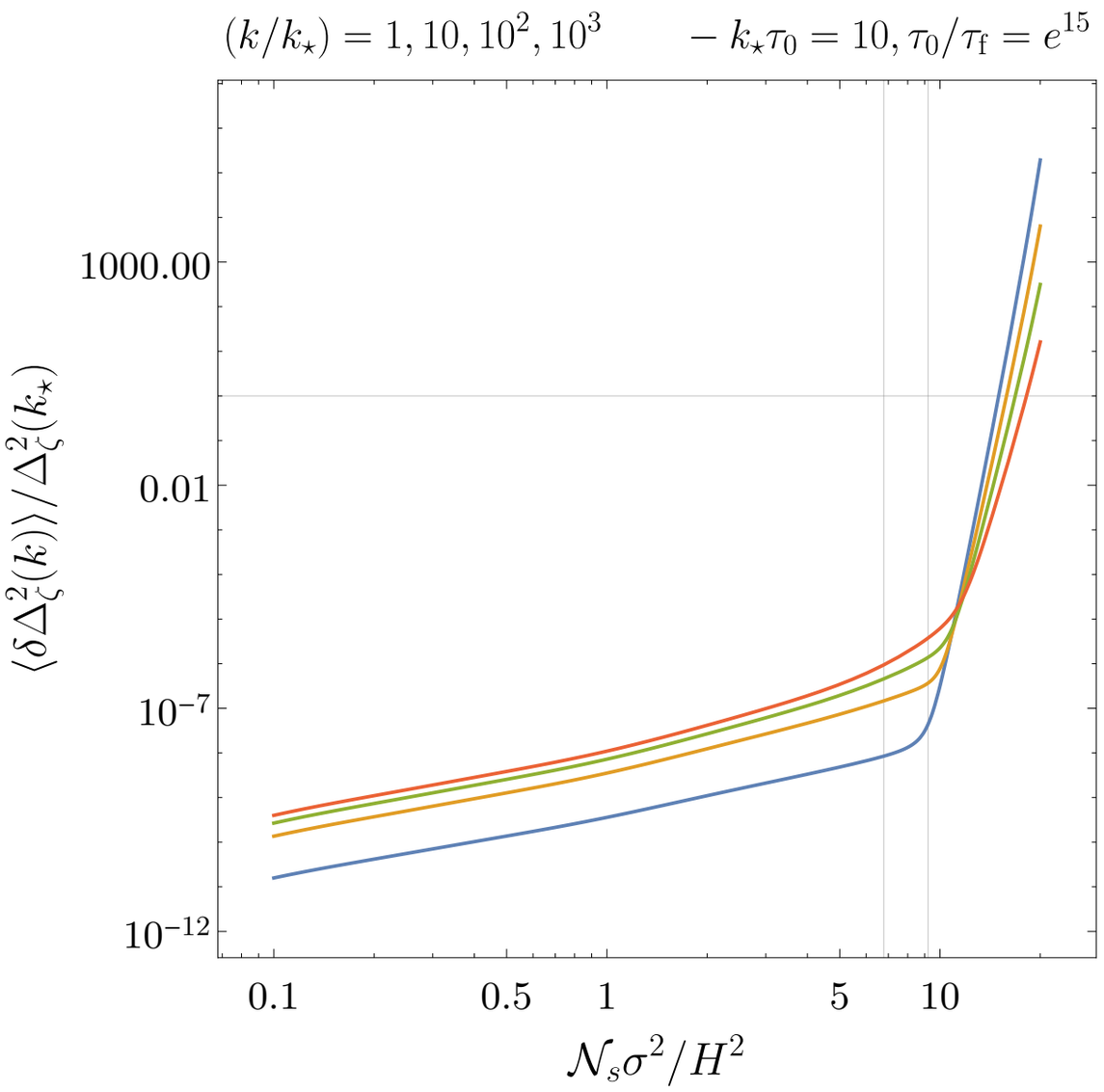
dissipation driving

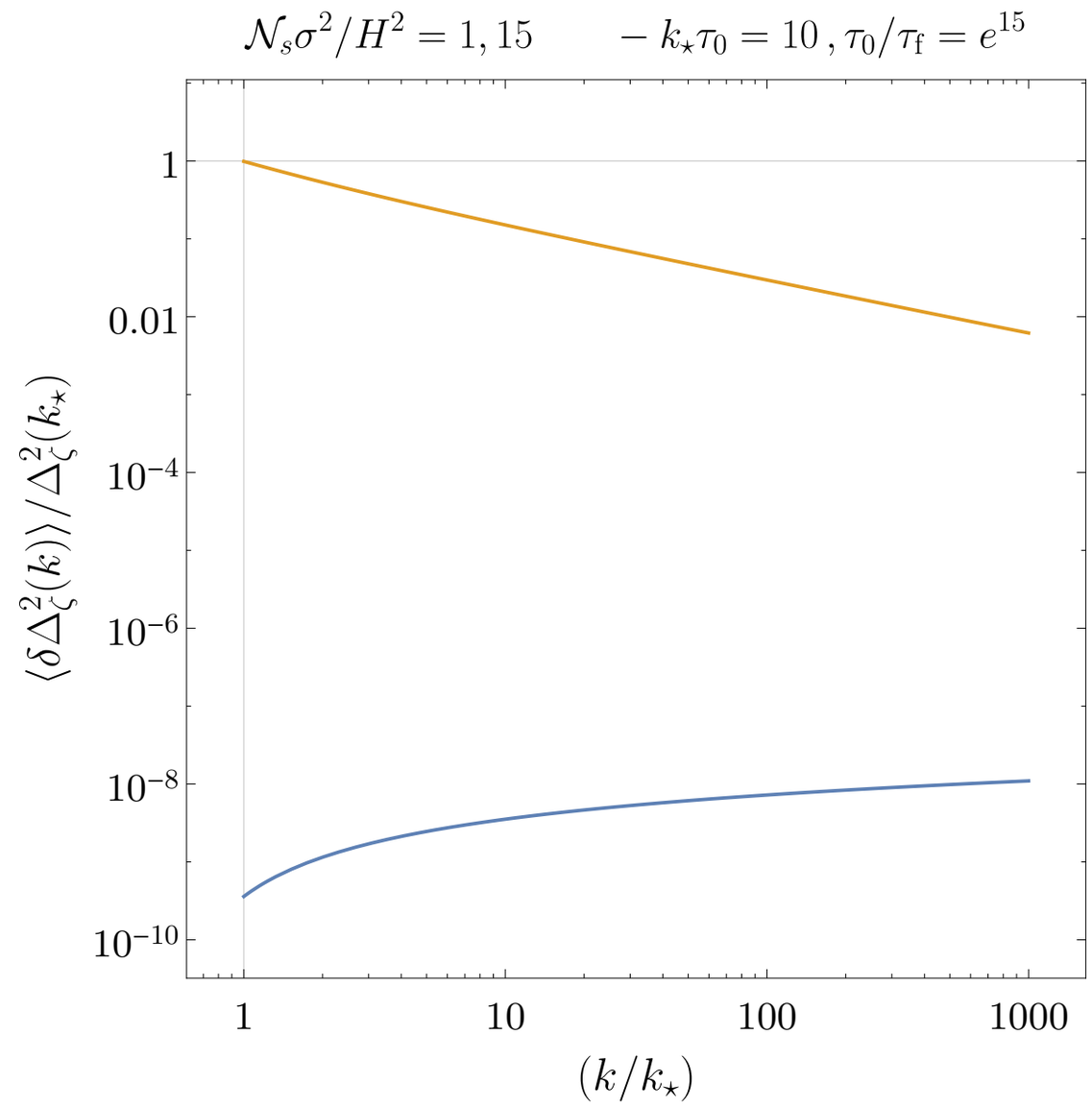
Green, Horn, Senatore, and Silverstein (2009)
Nacir, Porto, Senatore, and Zaldarriaga (2012)
Flauger, Mirbabayi, Senatore, Silverstein (2016)

MA, Baumann, Carlsten, Garcia & Green (in progress)

WORK IN PROGRESS

effects on curvature perturbations ?

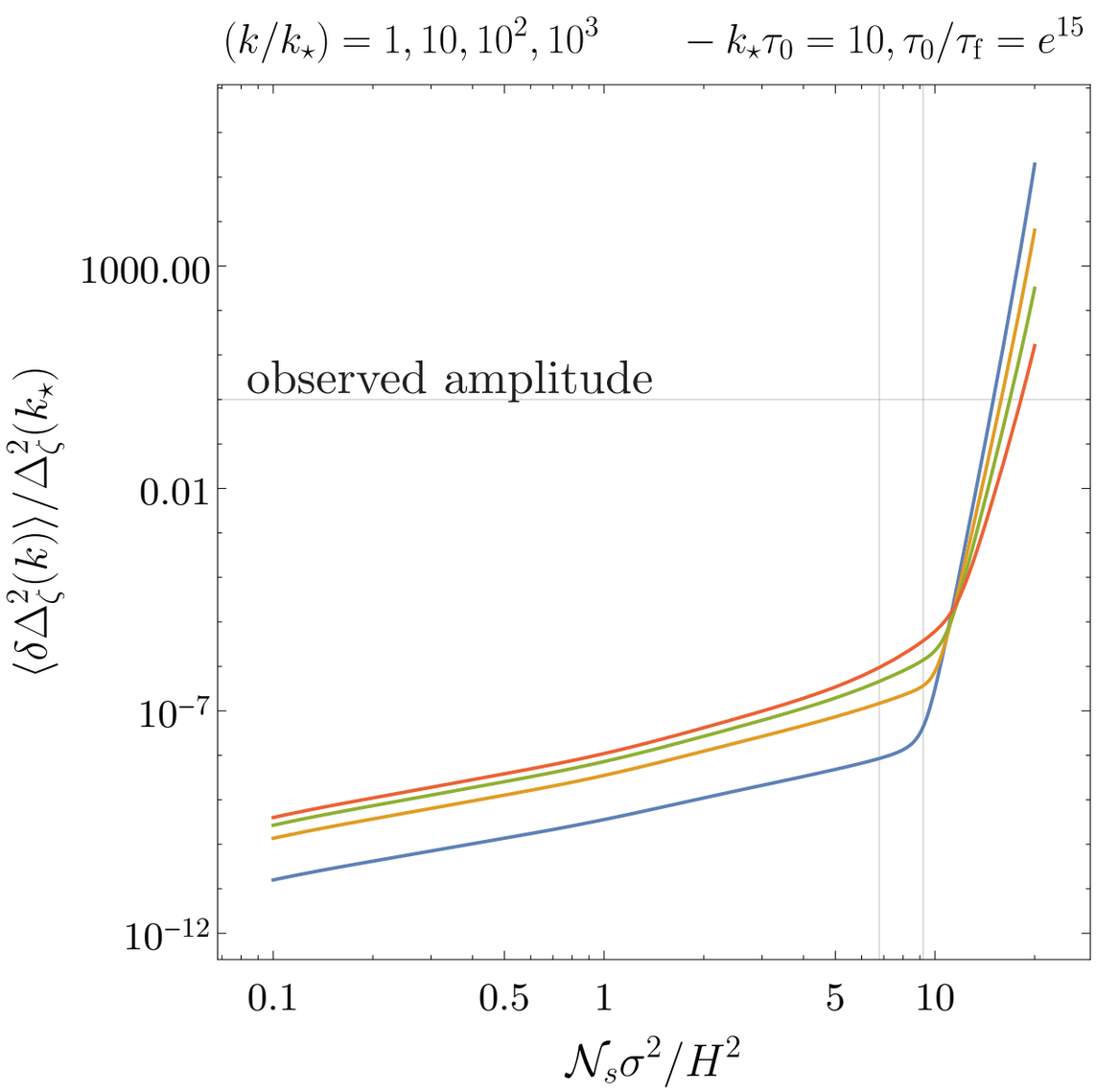


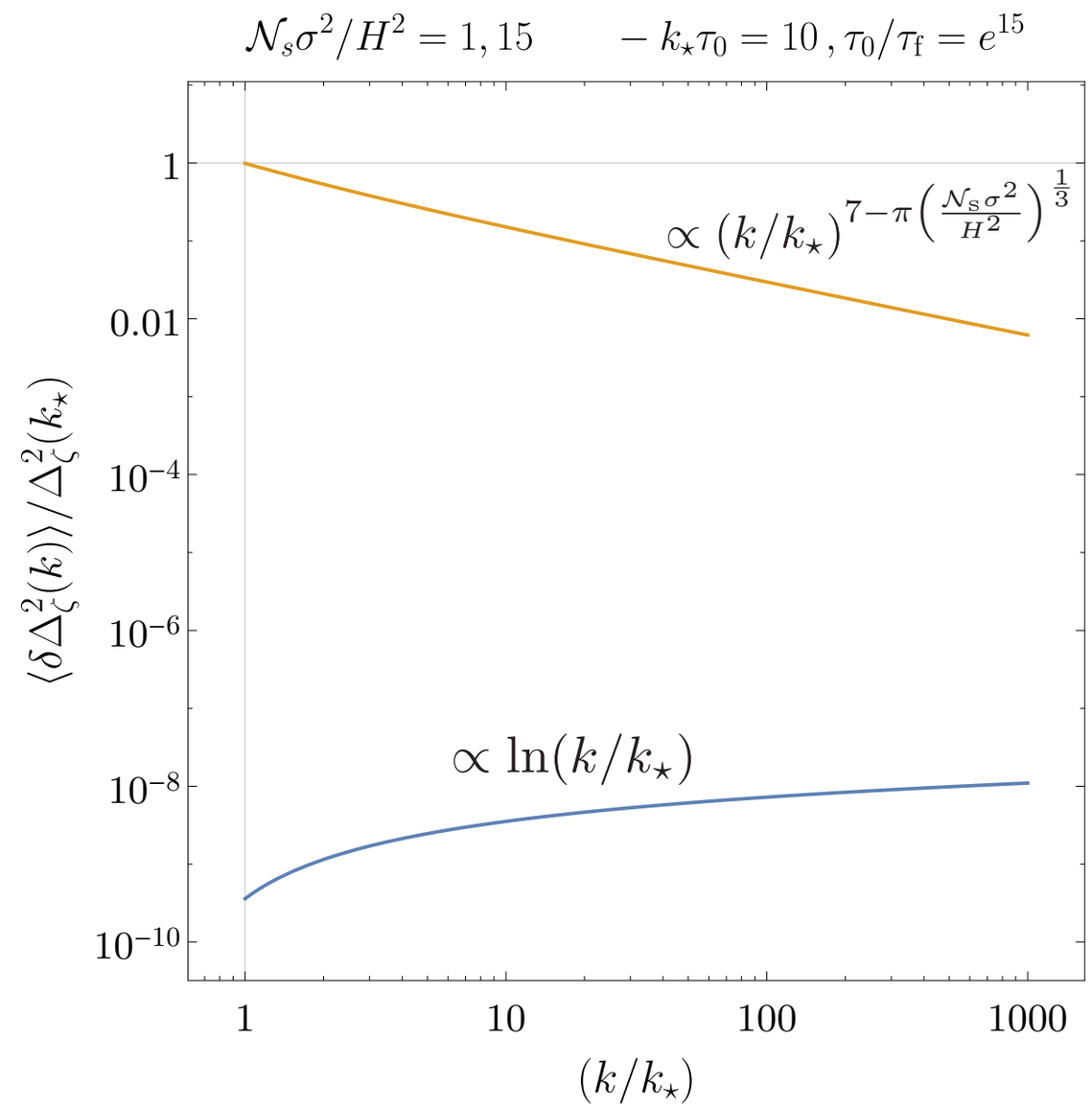


MA, Baumann, Carlsten, Garcia & Green (in progress)

WORK IN PROGRESS

Power Spectrum





MA, Baumann, Carlsten, Garcia & Green (in progress)

WORK IN PROGRESS

Higher n-point functions ?

background dynamics → particle production ↔ curvature fluctuations

$\langle \chi_{k_1} \chi_{k_2} \ldots \rangle$ $\langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$

from log normality

$$\langle \zeta^n \rangle \sim \langle \zeta^n \rangle_{\chi=0} + \langle \zeta^2 \rangle^n_{\chi=0} \times \exp\left[\frac{n^2}{2} F\left(\mathcal{N}_{\rm s} \frac{\sigma_{\rm s}^2}{H^2}\right)\right]$$

$n \gg 1$

MA, Baumann, Carlsten, Garcia & Green (in progress)

WORK IN PROGRESS

Large Higher n-point Functions ?

background dynamics → particle production ↔ curvature fluctuations

$$\langle \chi_{k_1} \chi_{k_2} \ldots \rangle \qquad \langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$$

- how to measure ?
- increased probability of rare events ?

WORK IN PROGRESS

application to gravitational waves?

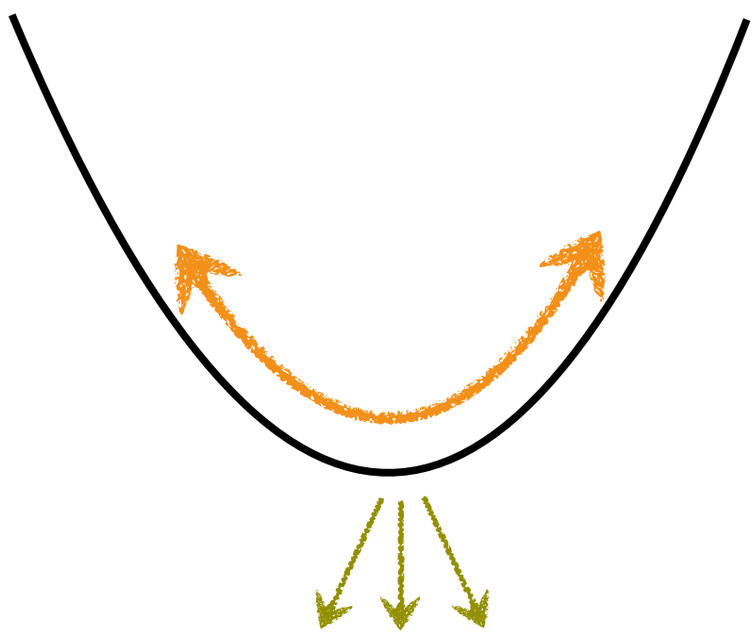
background dynamics → particle production ↔ gravitational waves?

$$\langle \chi_{k_1} \chi_{k_2} \ldots \rangle \qquad \langle h_{k_1} h_{k_2} \ldots \rangle$$

breaking the inflationary energy scale - *r* relation
Peloso & Sorbo, Silverstein et. al, Mirabayi et. al etc

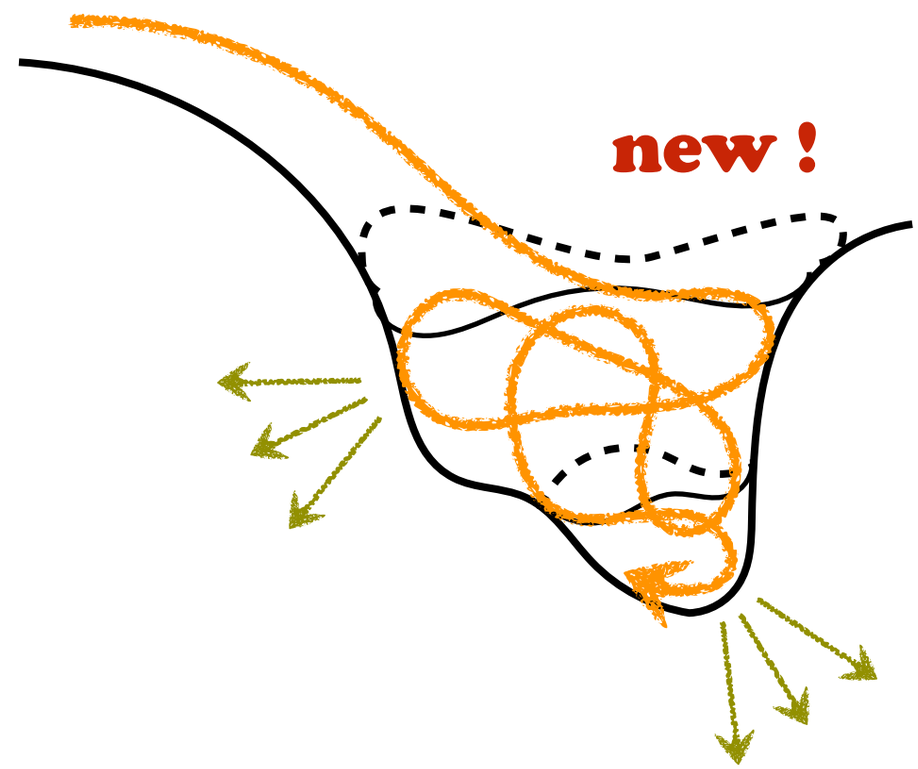
applications : reheating

WORK IN PROGRESS



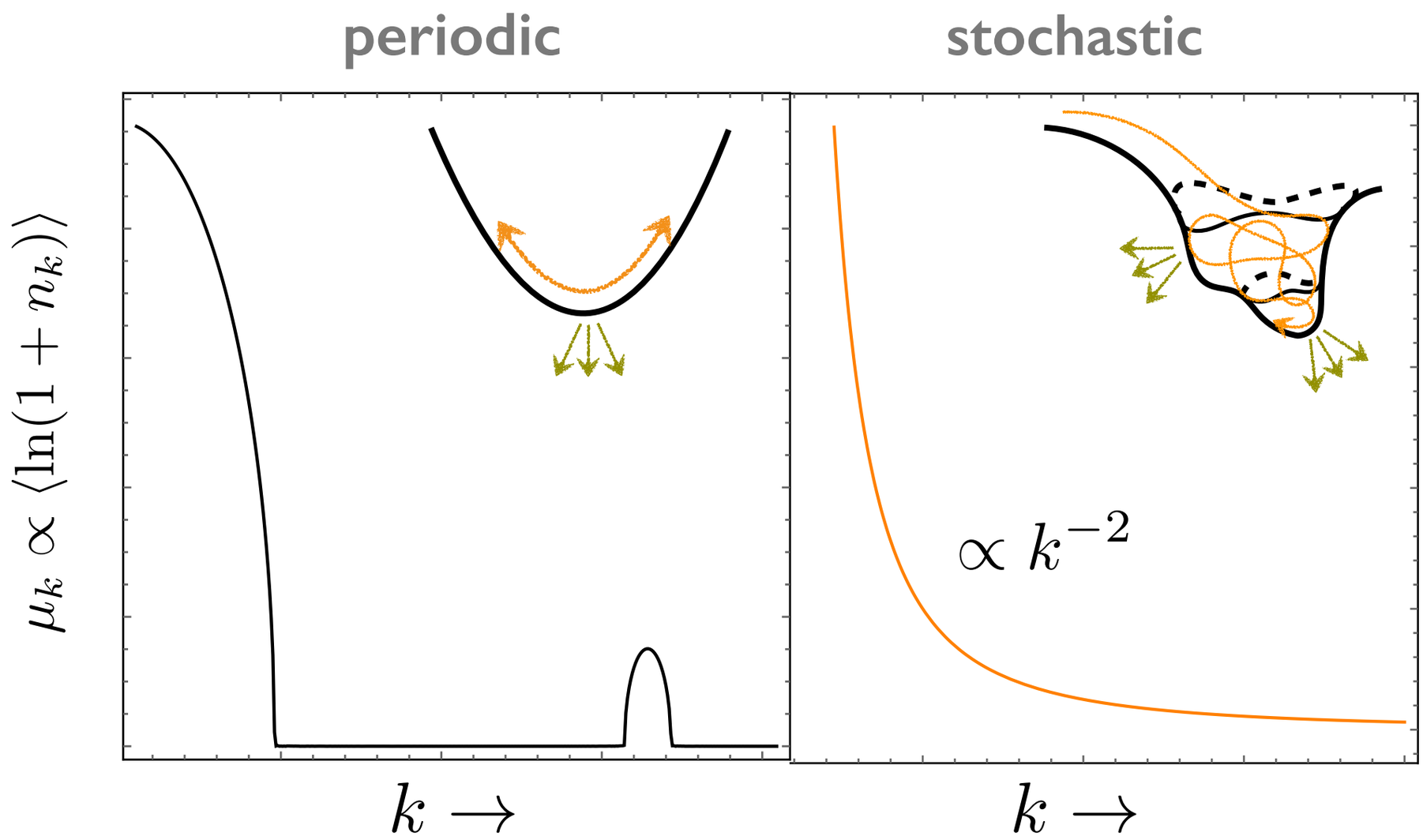
Kofman, Linde & Starobinsky (1997)
Traschen & Brandenberger (1997)
Zanchin et. al (1998) & Bassett (1998) [with noise]
Giblin, Nesbit, Ozsoy, Sengor & Watson (2016-17)

multichannel — multifield — statistical

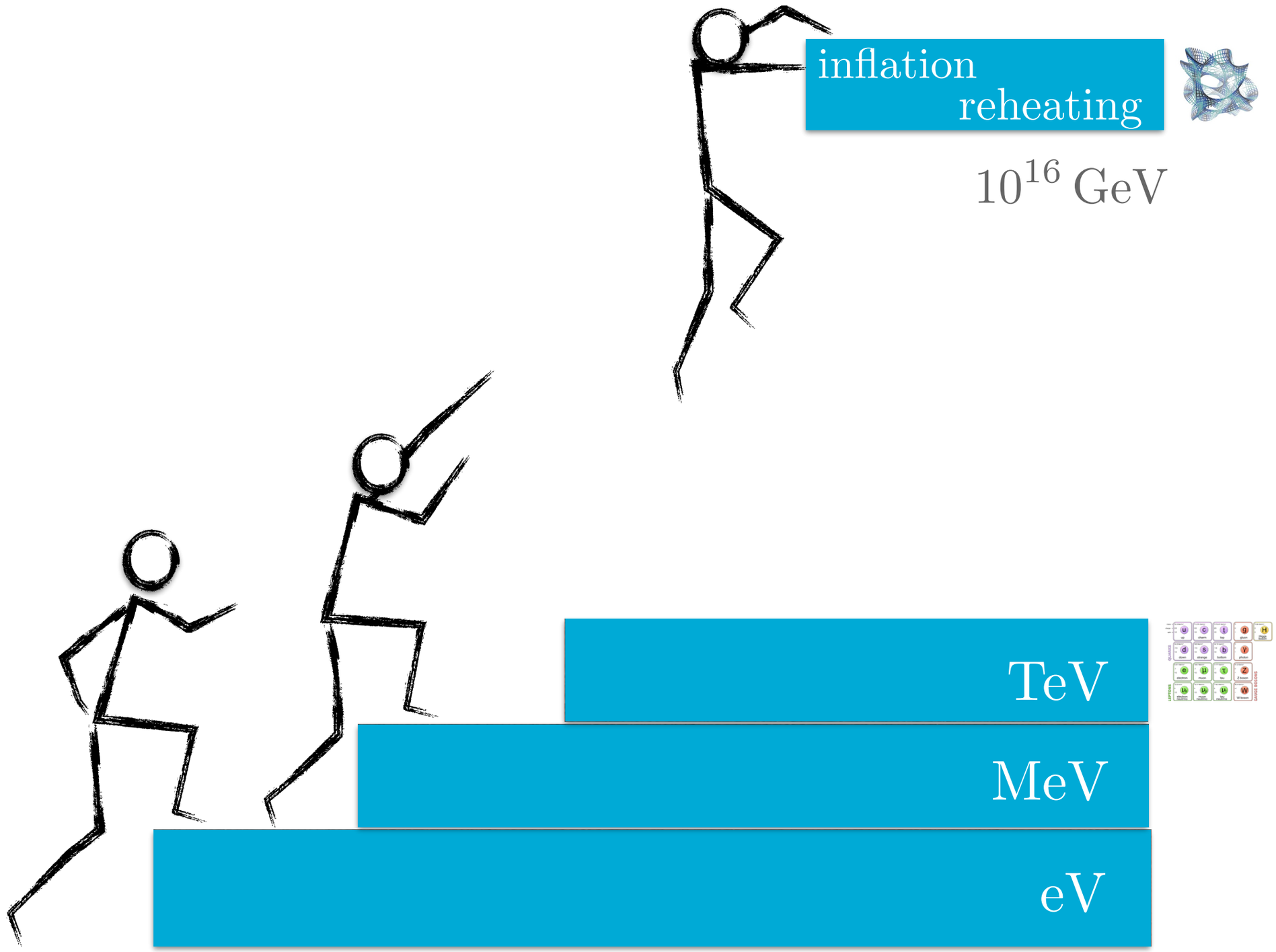


model-insensitive description of a complicated reheating process.

simplicity from stochasticity



see hints in: Bassett (1998), Barnaby, Kofman & Braden et. al 2010


inflation
reheating
10^{16} GeV
TeV
MeV
eV

summary

- *statistical* tools for theoretical complexity
- hints of universality
- *observed simplicity* in spite of underlying complexity + hints in higher point correlations?