Gravitational Aspects of Solitons in Cosmology

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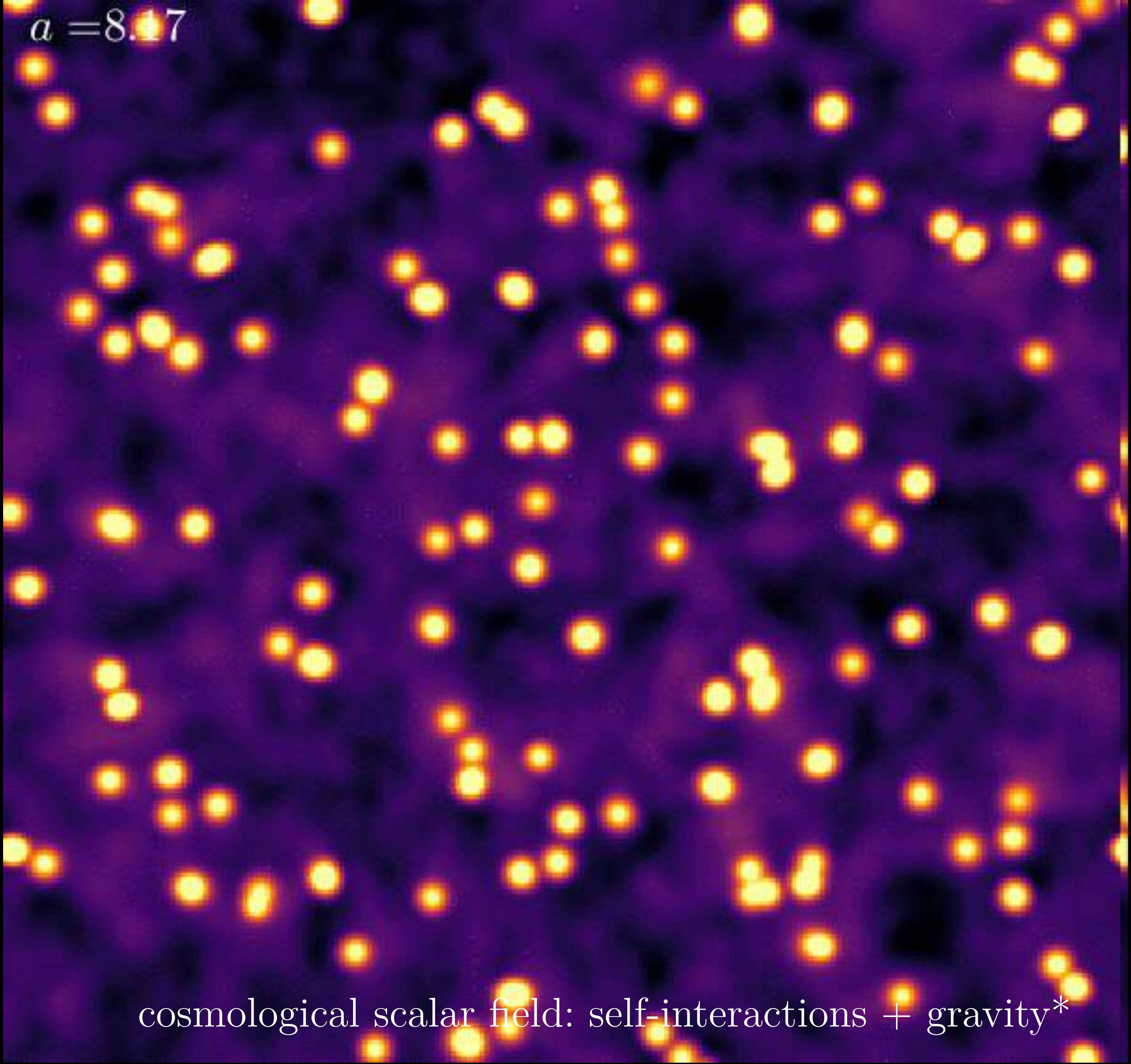
Feb 25, 2019, SITP Colloquium

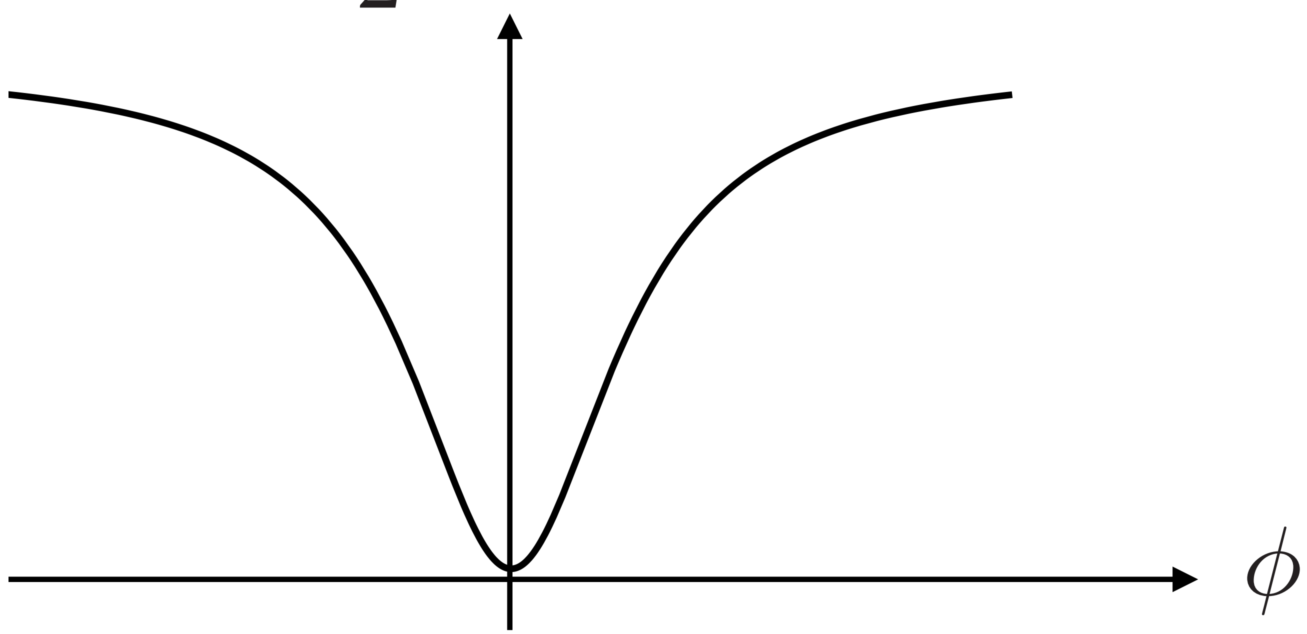


cosmological scalar field: self-interactions + gravity*

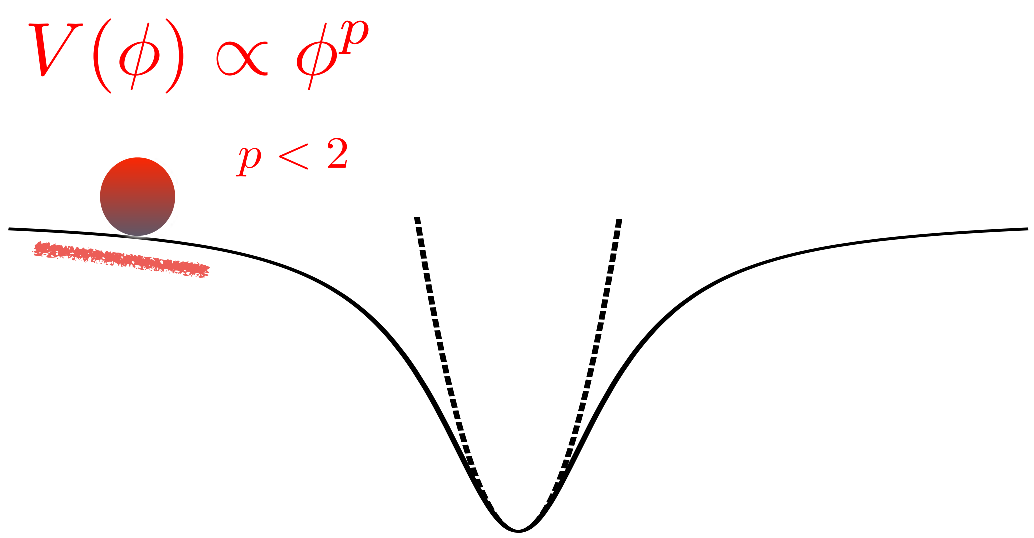
cosmological scalar fields
self-interaction + gravity

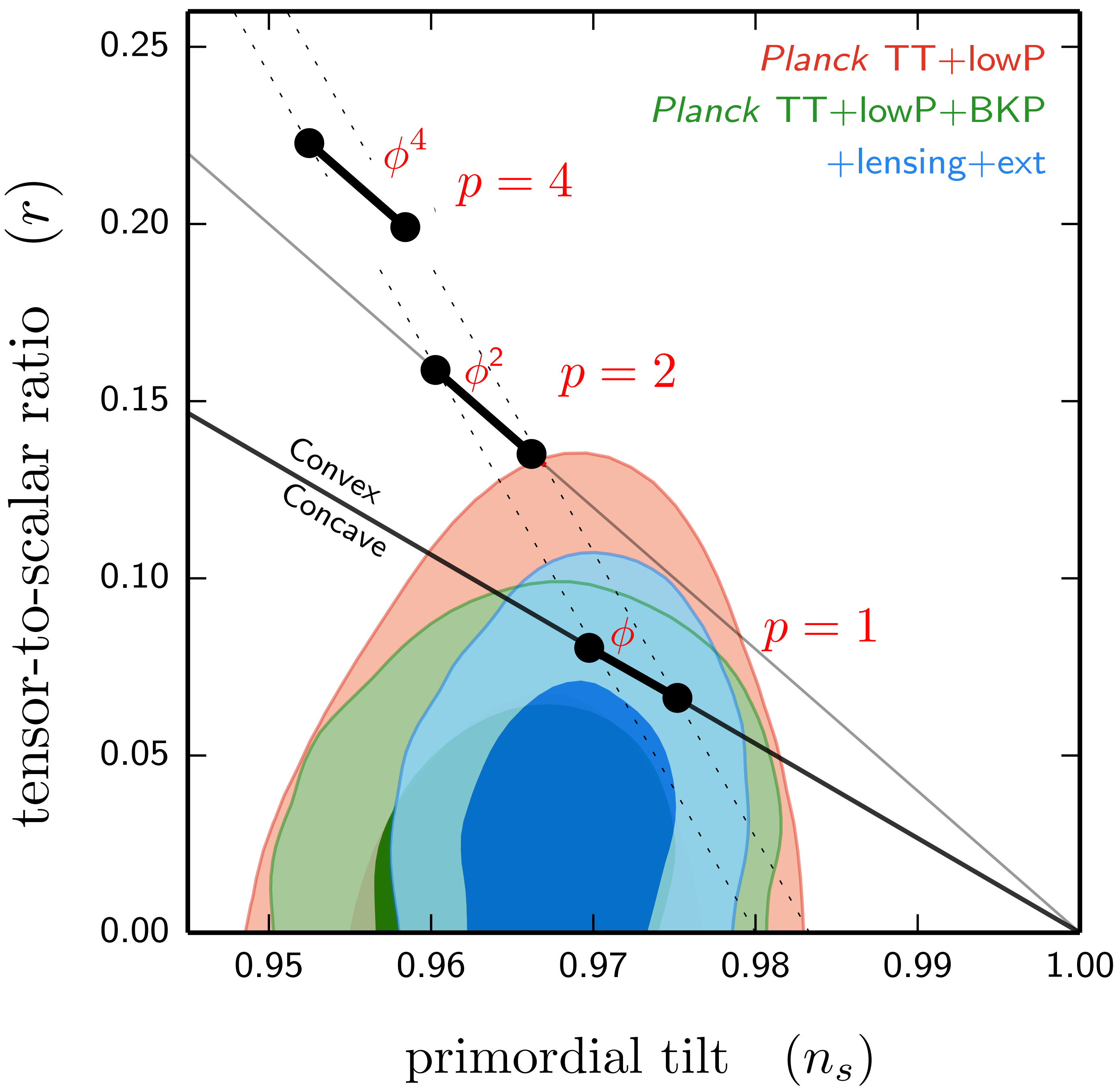
$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\rm pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + V_{\rm nl}(\phi)$$



inflation:
post-inflationary dynamics

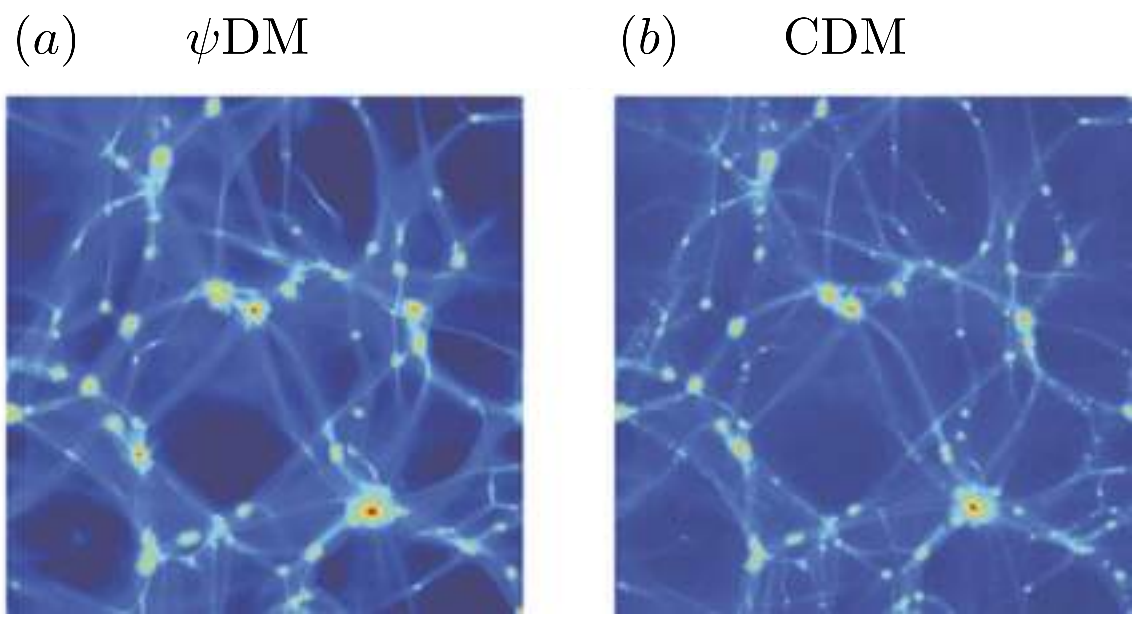


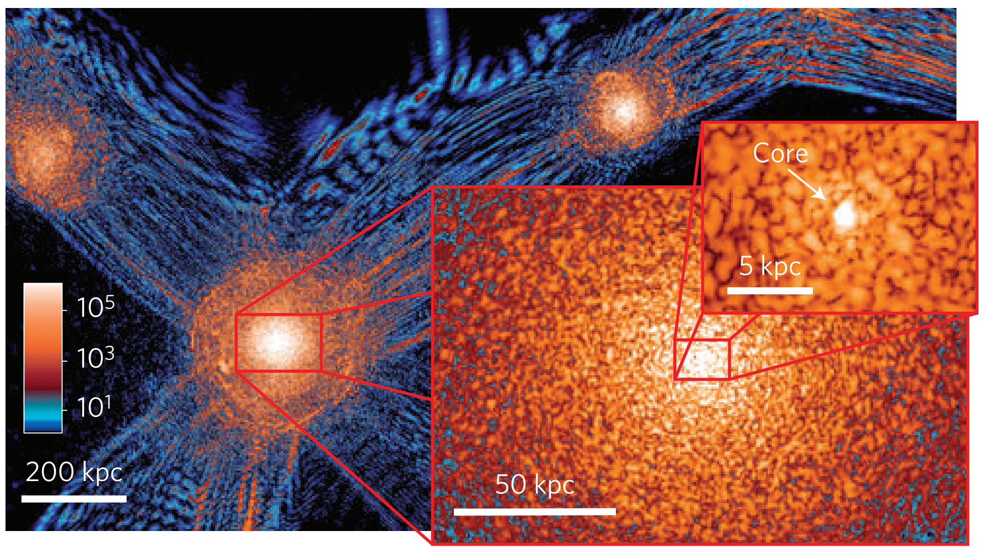


for example:

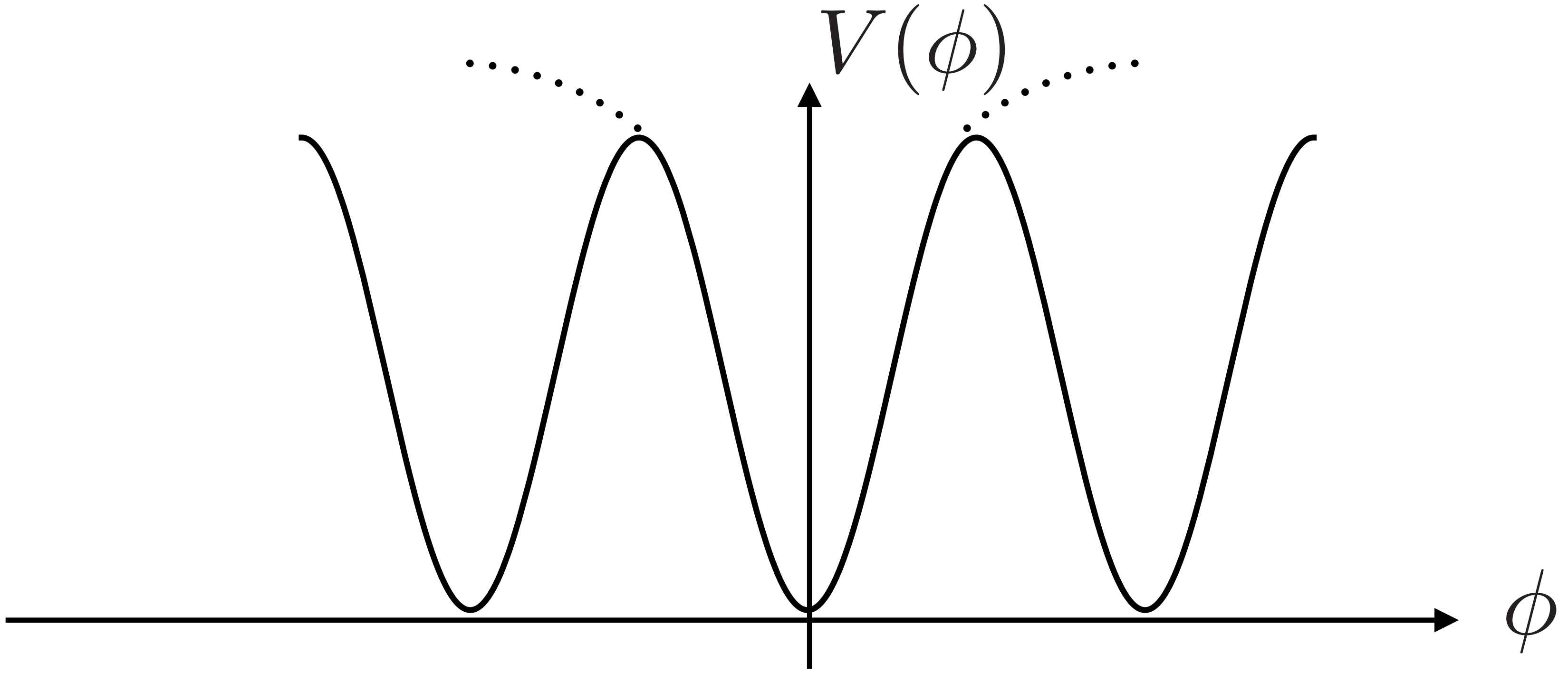
Starobinsky Inflation (1979)
Silverstein & Westhpal (2008)
Kallosh & Linde (2013)

dark matter: axion-like fields





Schive et. al (2014)



for example:

Peccei & Quinn (1977)
Hu, Barkana & Gruzinov (2000)
Arvanitaki et. al (2009)
Hui et. al (2016)

implications

- eq. of state after inflation ?
- stochastic gravitational wave-generation ?
  - constrained by $N_{\rm eff}$ or direct detection
- primordial black hole formation ?

....

- distinguishability from WIMPS ?
- compact objects
  - eg. sources of gravitational waves ?

eq. of motion for cosmological fields

$$\Box\phi + V'(\phi) = 0$$

nonlinear Klein-Gordon eq.

$$G_{\mu\nu} = \frac{1}{m_{\rm pl}^2} T_{\mu\nu}$$

Einstein Eq.

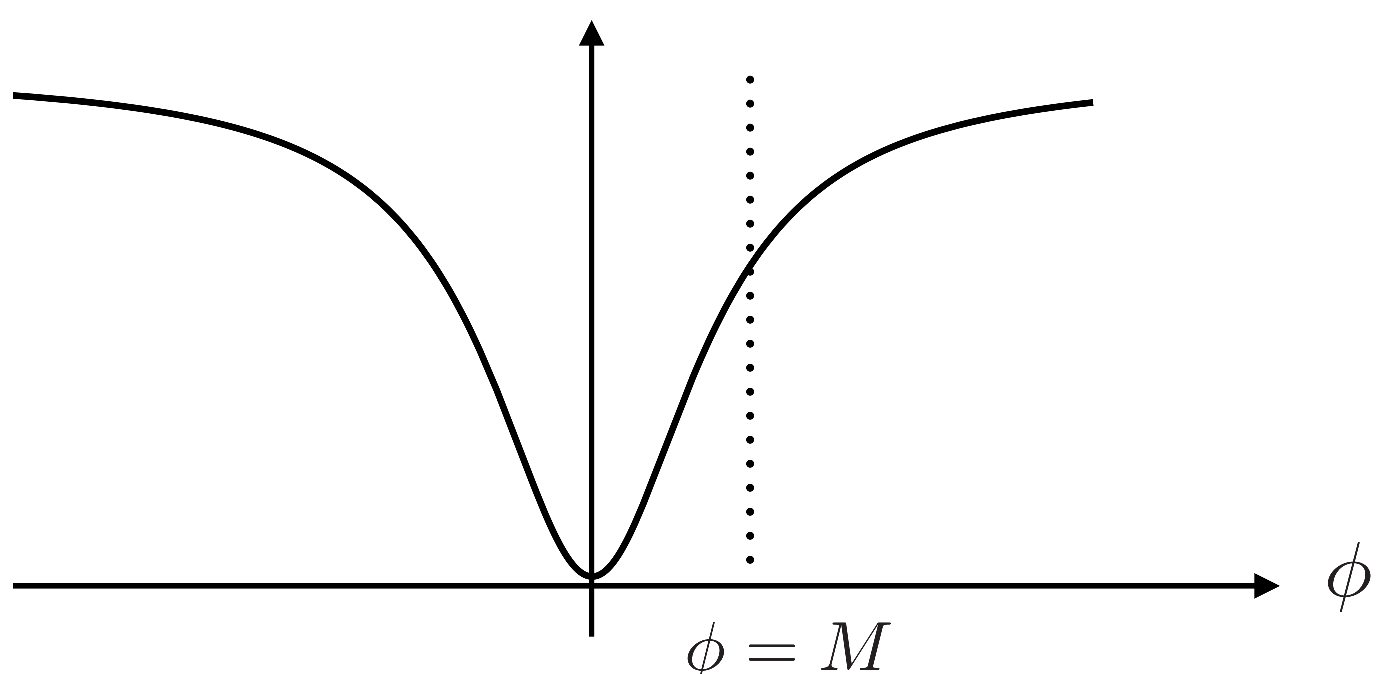
examples:

$$V(\phi) = \frac{m^2 M^2}{2} \tanh^2\left(\frac{\phi}{M}\right)$$

$$V(\phi) = m^2 M^2 \left[\sqrt{1 + \frac{\phi^2}{M^2}} - 1\right]$$

$$V(\phi) = m^2 M^2 \left[1 - \cos\frac{\phi}{M}\right]$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + V_{\rm nl}(\phi)$$



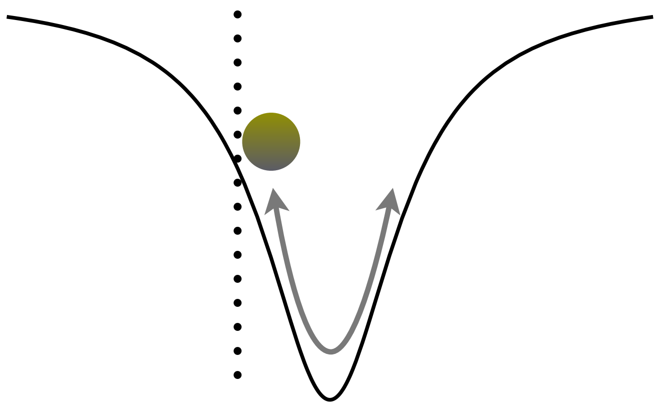
dynamics of oscillating fields

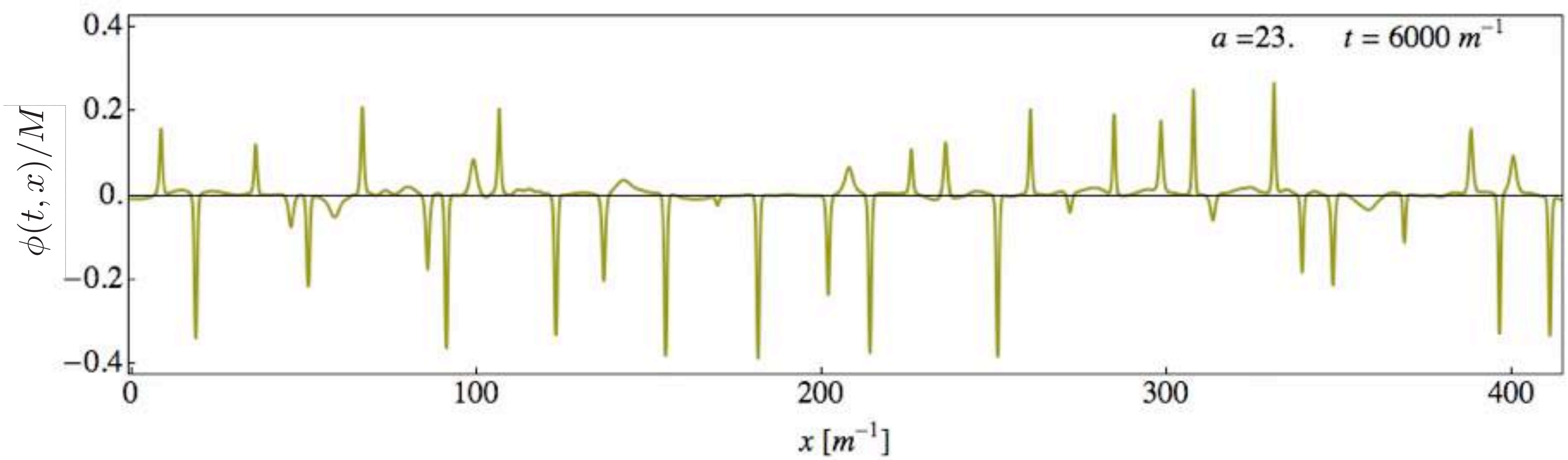
expansion ✓

self-interactions ✓

gravitational int. ✗

$$\Box\phi = V'(\phi)$$

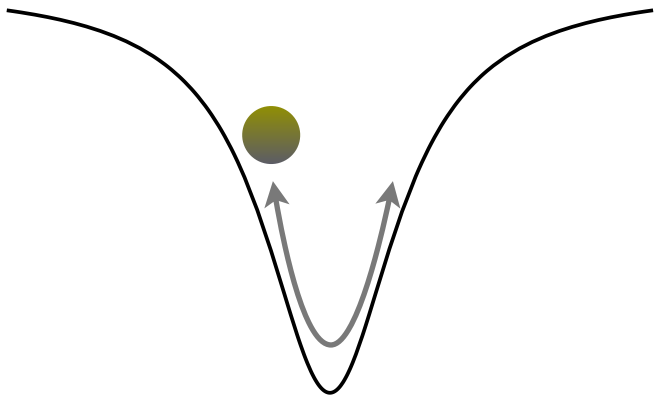


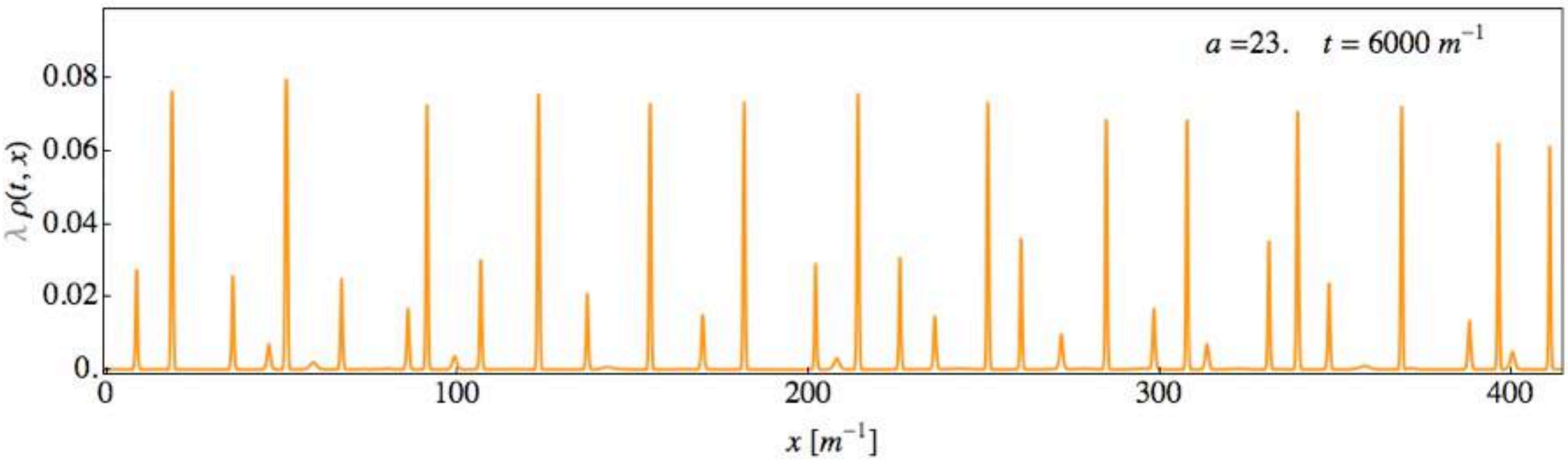


MA (2010)
Khlopov, Malomed & Zeldovich (1985)

dynamics of oscillating fields

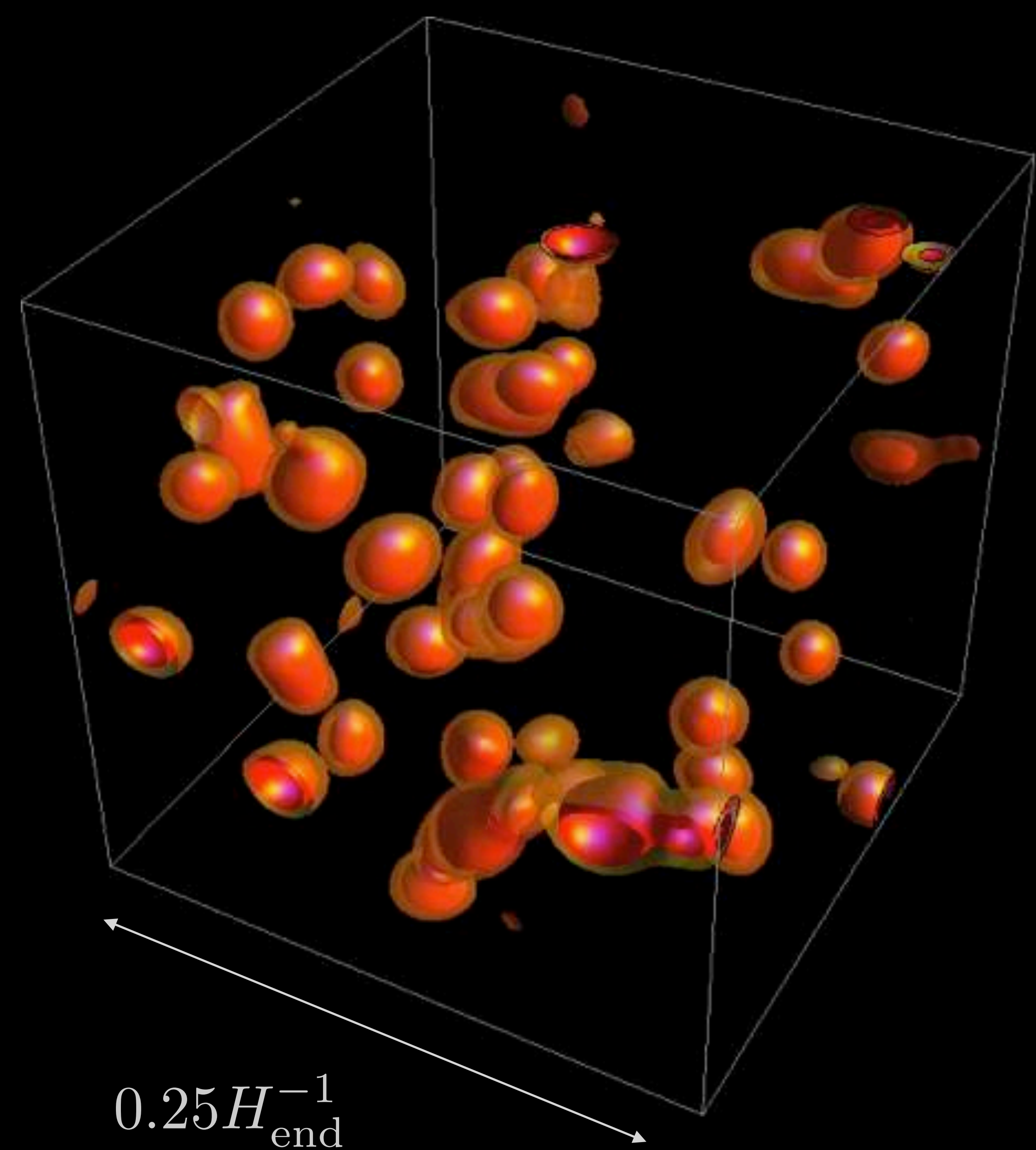
$$\Box\phi = V'(\phi)$$





soliton formation in relativistic fields

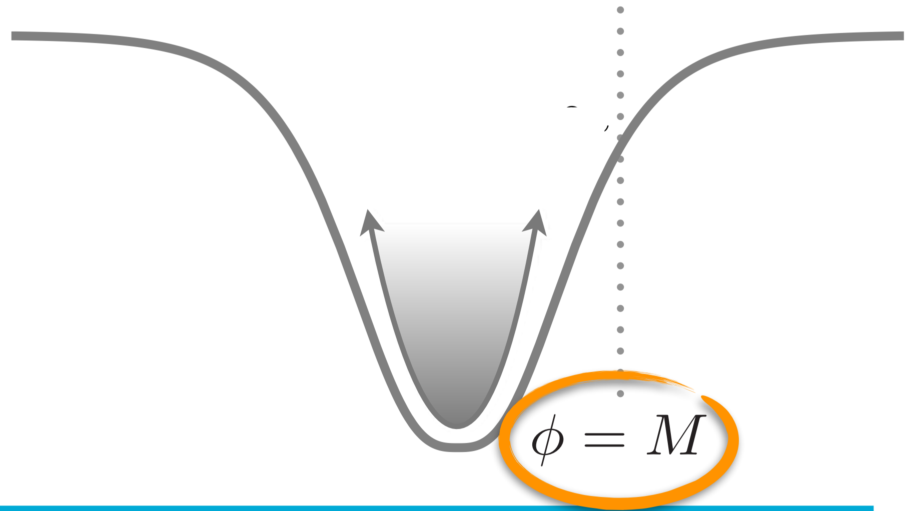
- expansion ✓
- self-interactions ✓
- gravitational int. ✗

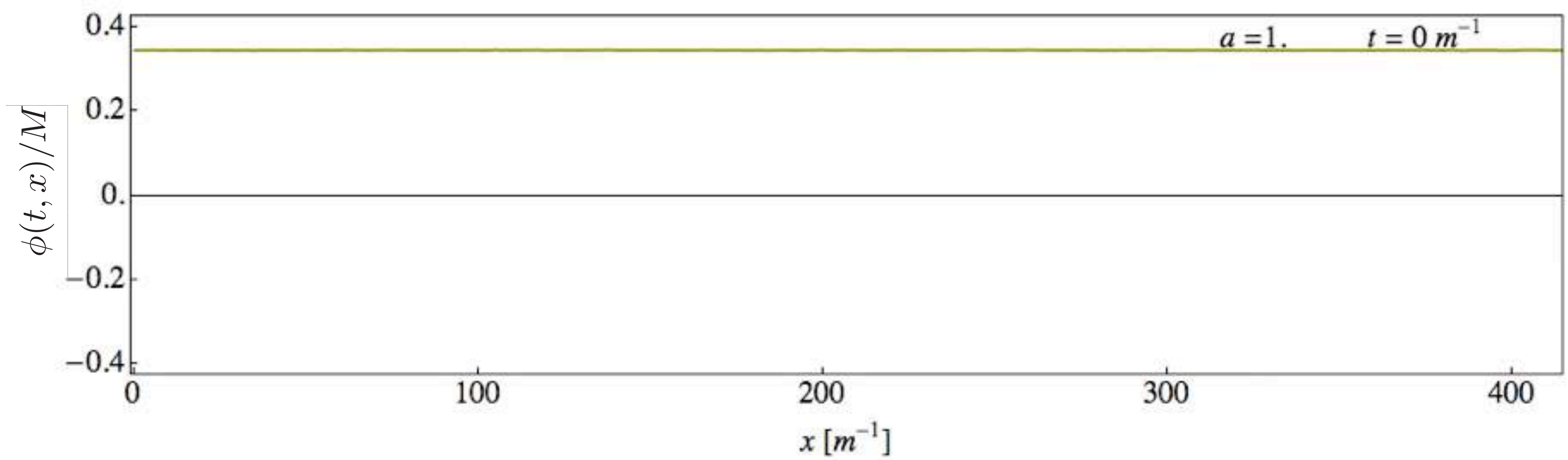


MA, Easther, Finkel, Flaugher & Hertzberg (2011)

condition for rapid fragmentation ?

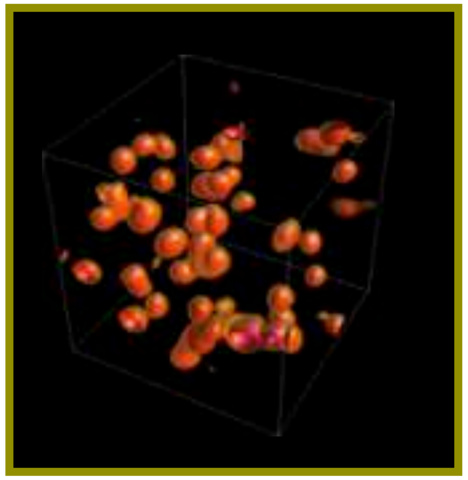
$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\rm pl}}{M} \gg 1$$



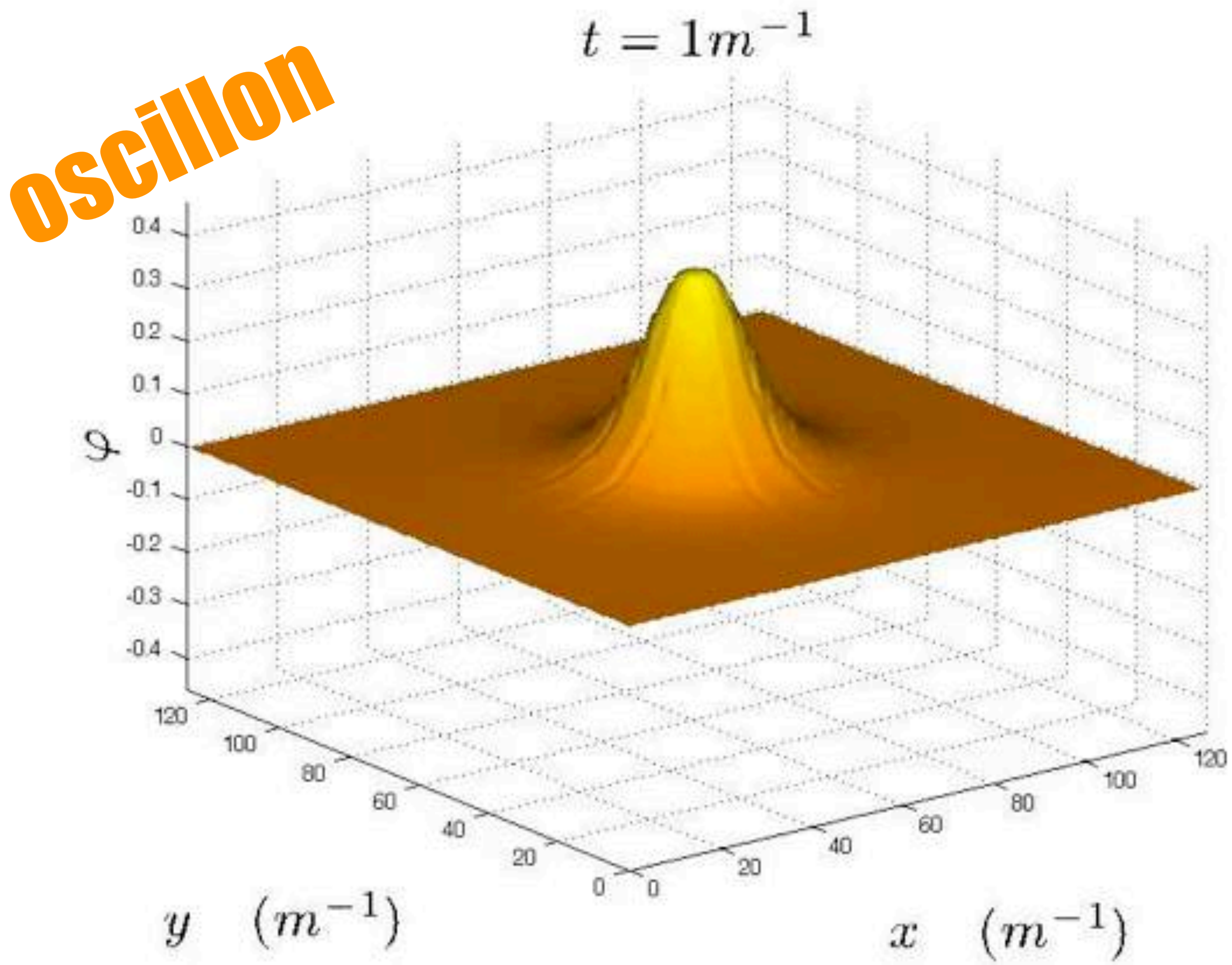


MA (2010)

solitons?



(1) oscillatory (2) spatially localized (3) **very long lived**



existence and stability:
MA (2013)
MA & Shirokoff (2010)

Segur & Kruskal (1987)
Hertzberg (2011)
Mukaido et. al (2016,17)

Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland et. al (1995)

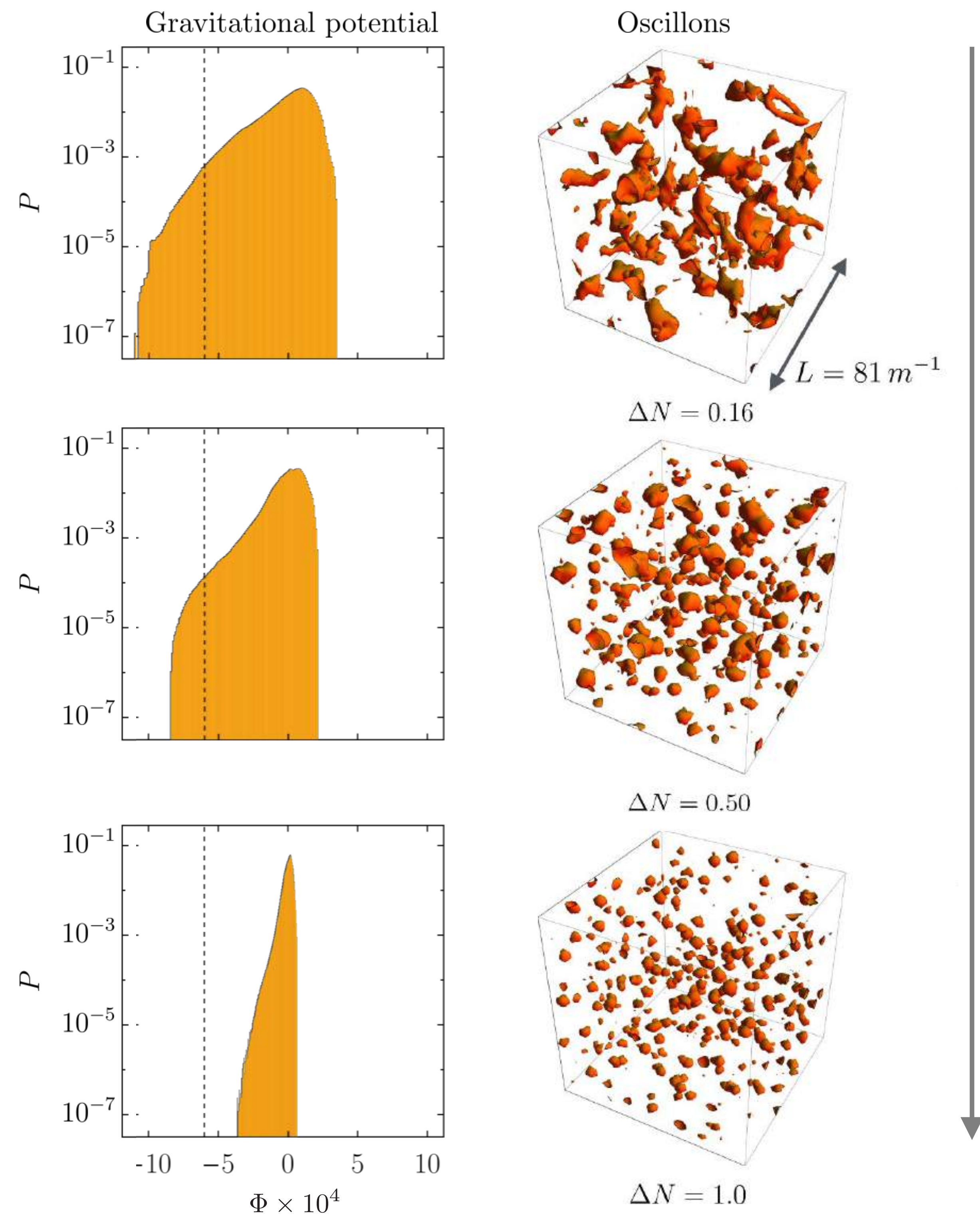
expansion ✓

self-interactions ✓

gravitational int. ✗

passively calculated gravitational potential

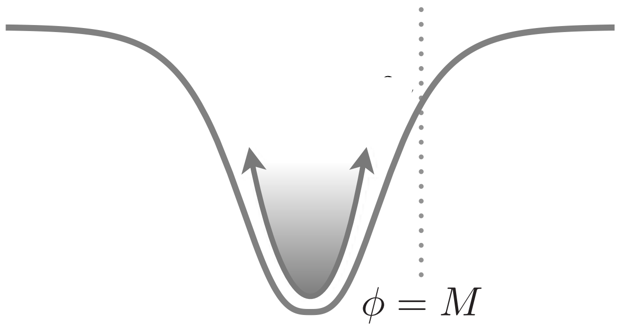
$$\Phi \lesssim \text{few} \times 10^{-3}$$



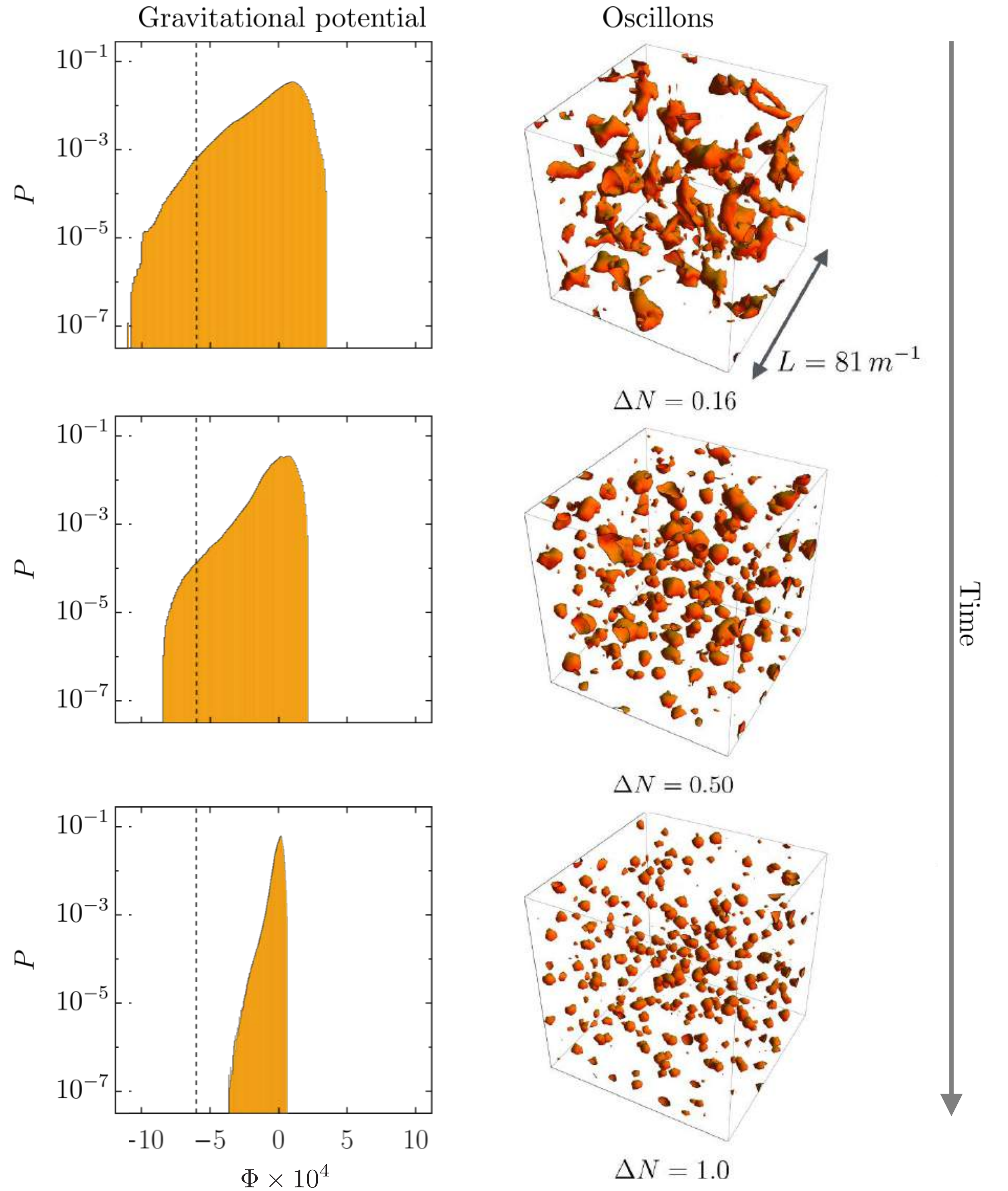
w/ K. Lozanov (2019)

expansion ✓
self-interactions ✓
gravitational int. ✗

passively calculated gravitational potential



$$|\Phi|_{\rm sol} \lesssim 10 \times \left(\frac{M}{m_{\rm pl}}\right)^2$$



$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\rm pl}}{M} \gg 1$$

expansion ✓
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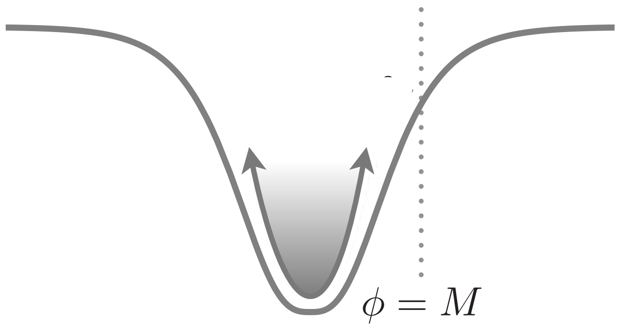
passively calculated gravitational potential



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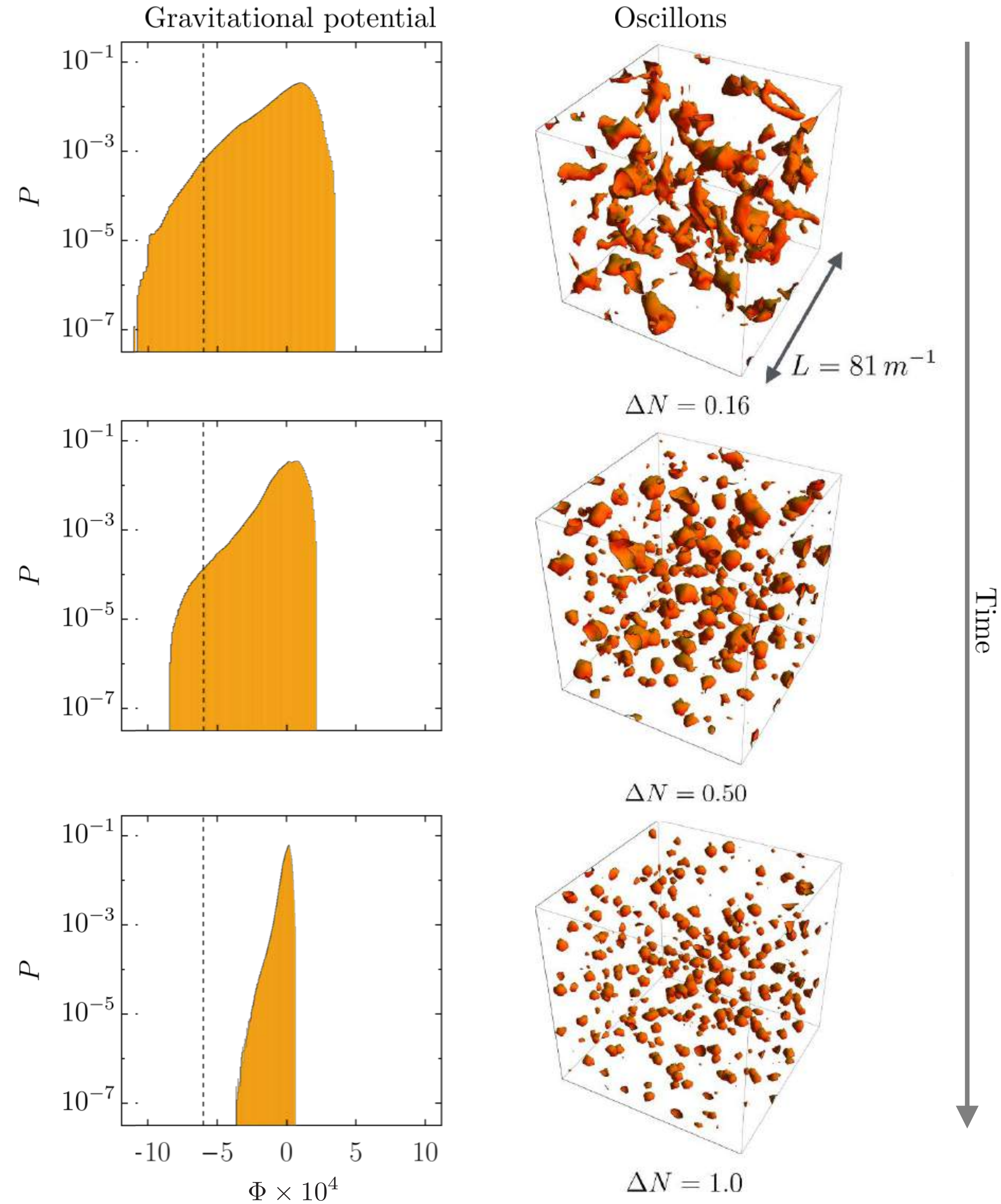
not easy to form black holes from individual solitons*



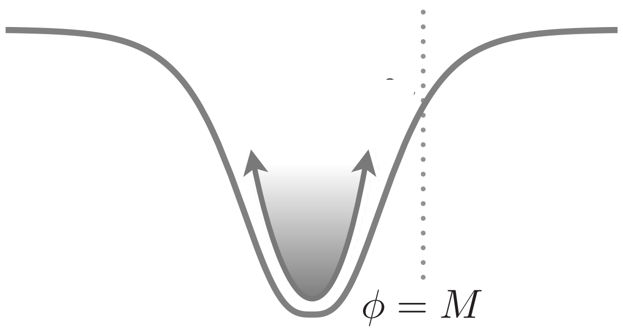
$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\rm pl}}{M} \gg 1$$

expansion ✓

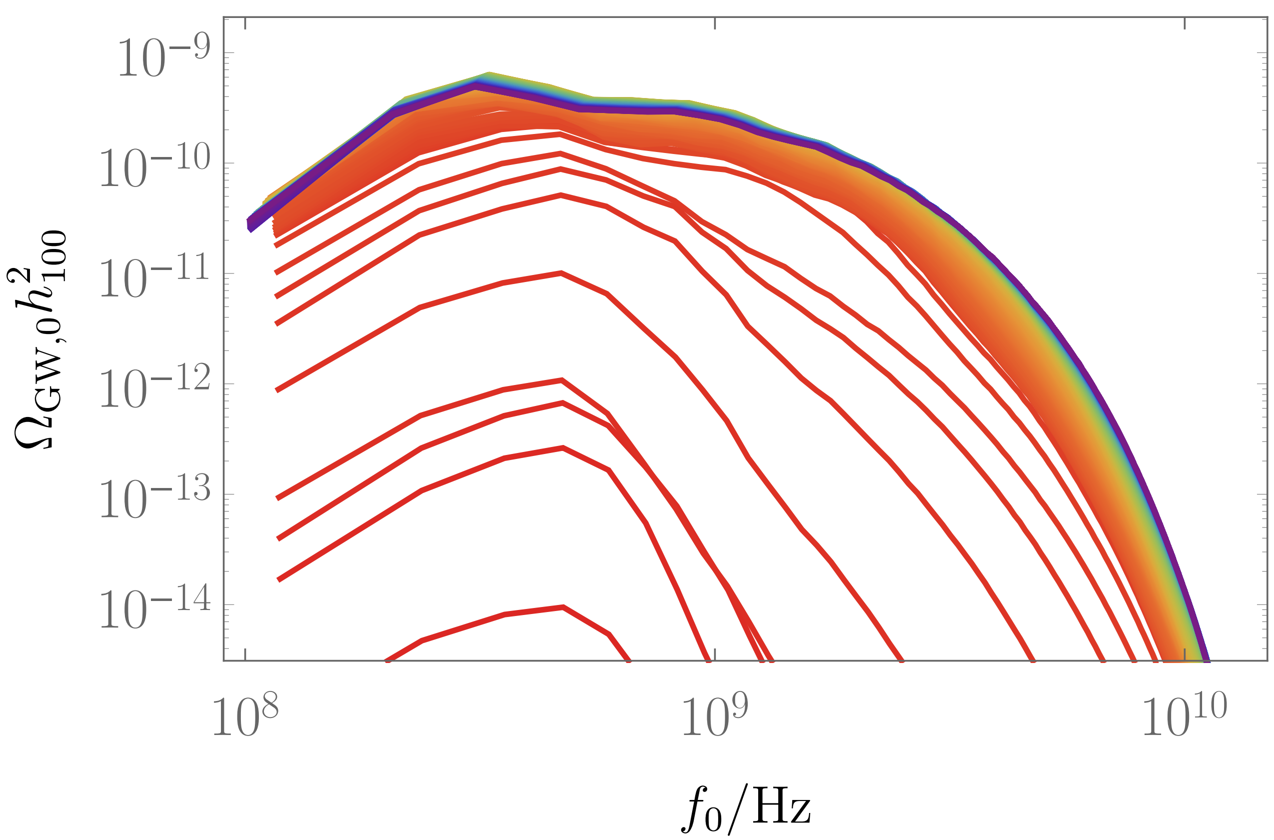
self-interactions ✓

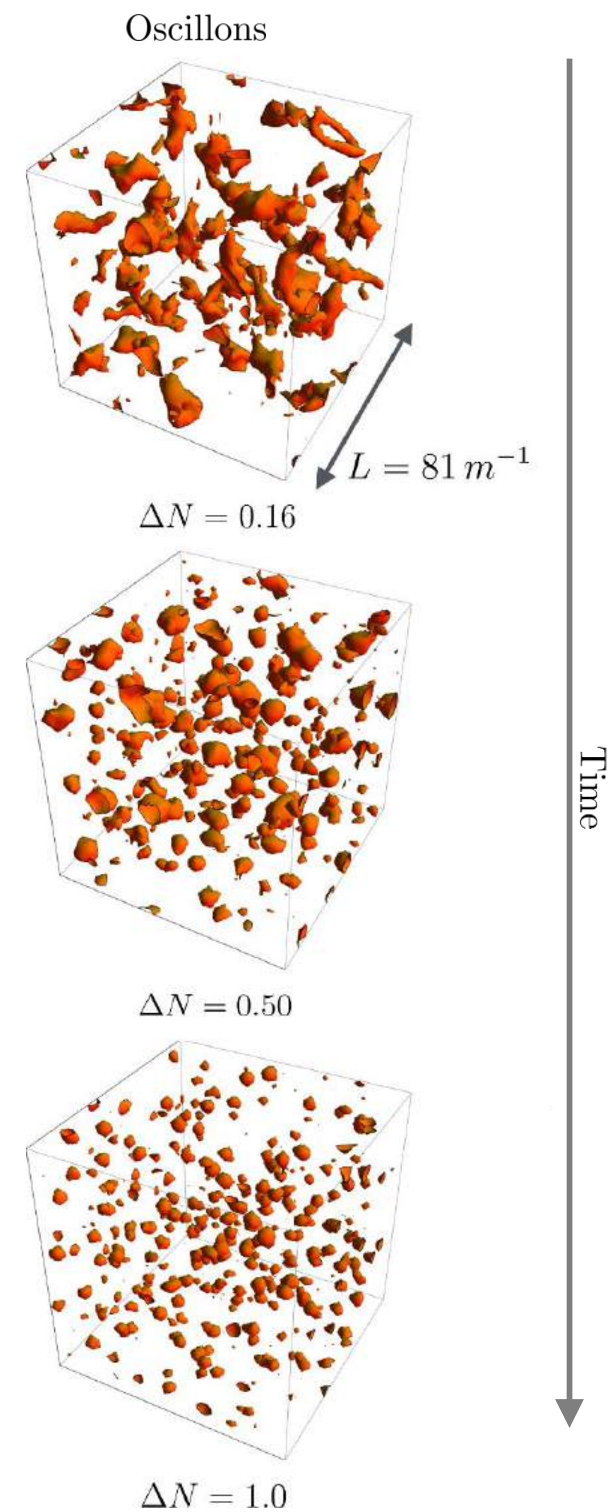
gravitational int. ✗

passively calculated gravitational waves



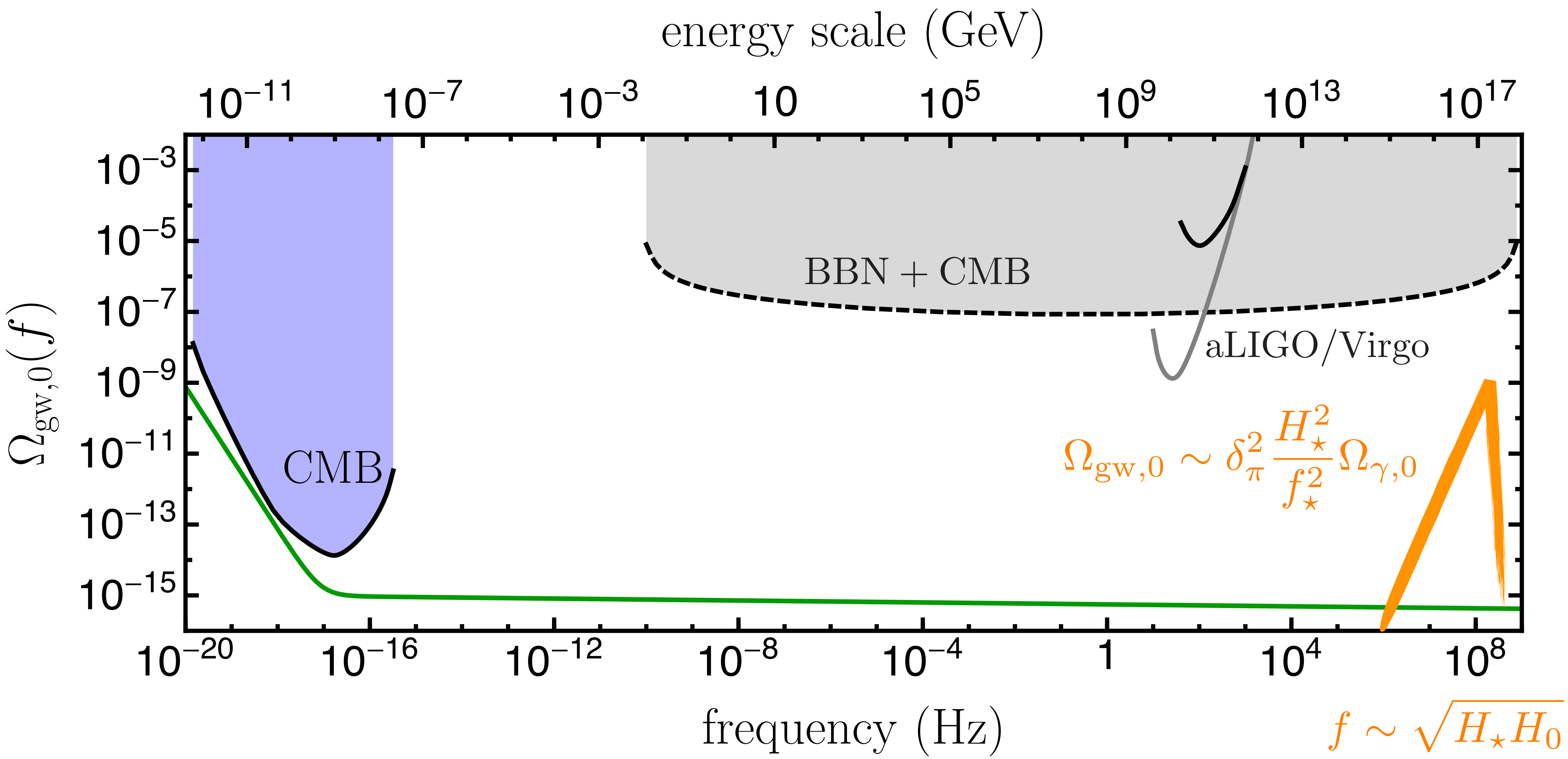
$$\Omega_{\mathrm{GW},0} h_{100}^2 \sim 10^{-6} \left(\frac{M}{m_{\mathrm{Pl}}} \right)^2 \lesssim \mathcal{O}[10^{-9}]$$





$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\mathrm{pl}}}{M} \gg 1$$

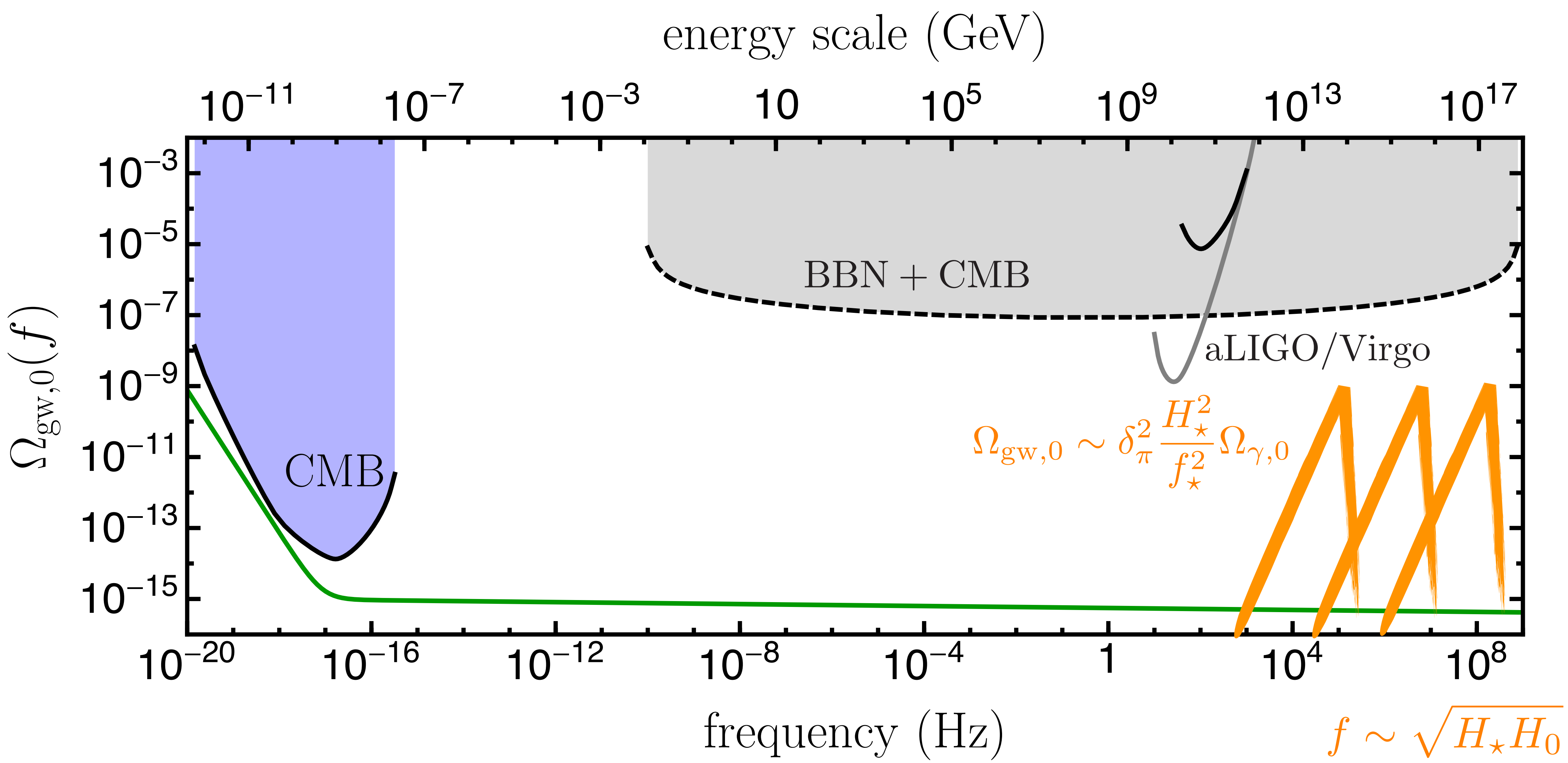
can be constrained ?



$$\Omega_{\rm gw}(f) \sim \left[e^{-N_{\rm rad}(1-3w)}]a_{\rm eq}\right] \delta_\pi^2 (H_*/f_*)^2$$

also see Lasky et. al (2016)

can be constrained ?



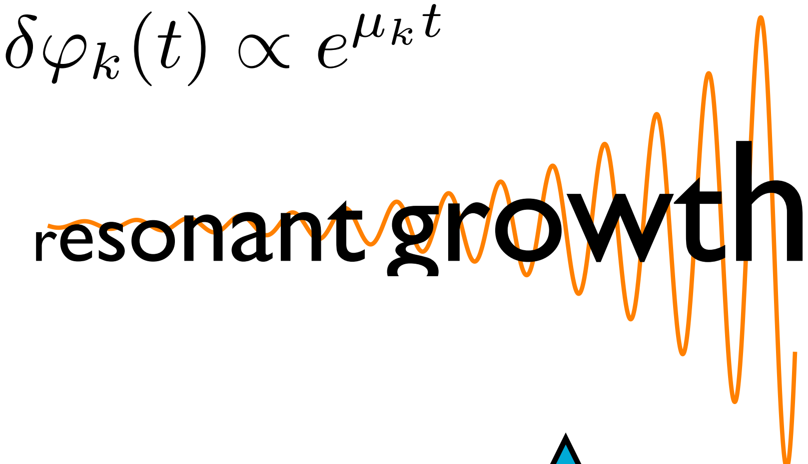
$$\Omega_{\mathrm{gw},0}h^2 \lesssim 1.12 - 1.68 \times 10^{-7} \qquad (\mathrm{CMB-S4})$$

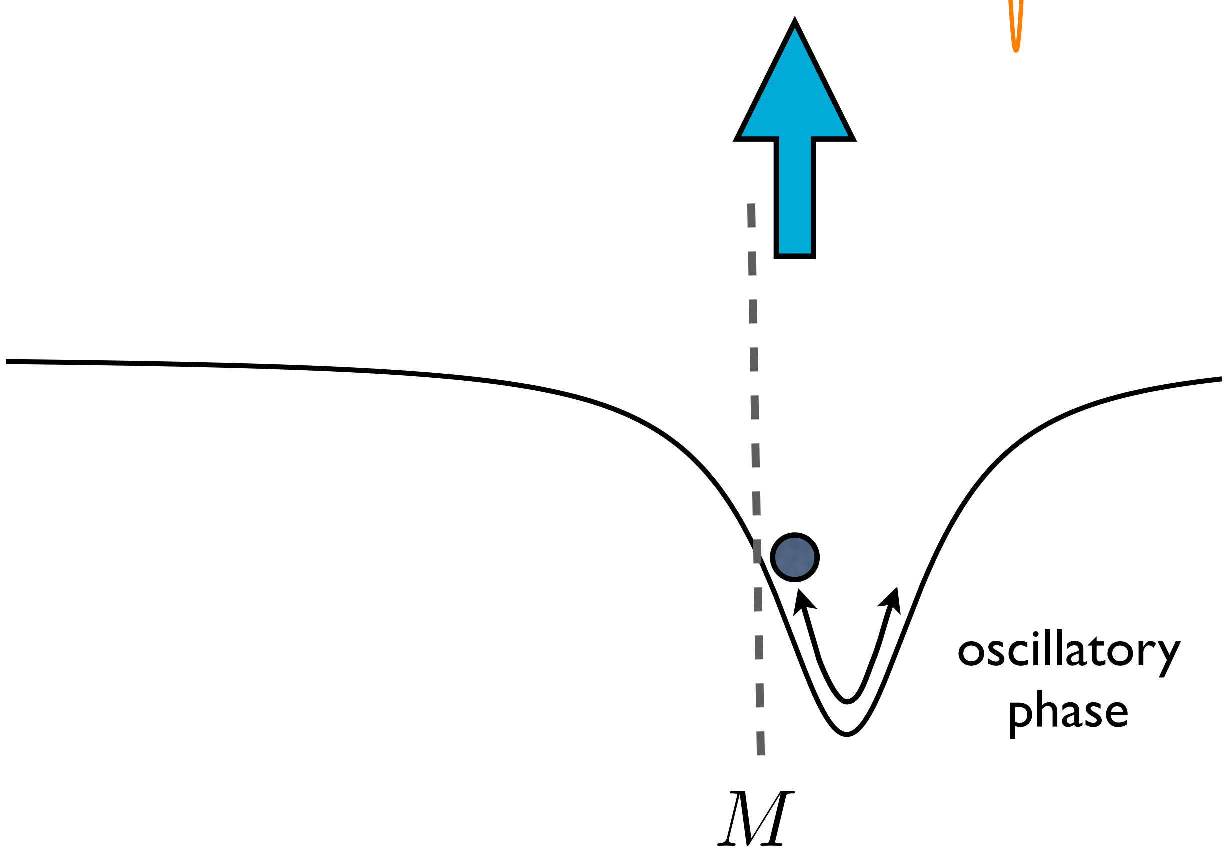
expansion ✓
self-interactions ✓
gravitational int. ✗

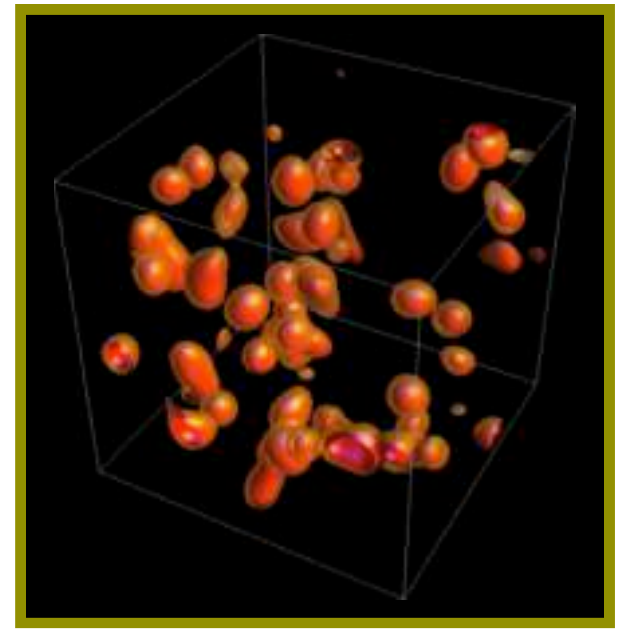
summary so far ...

$$\frac{\mu_k}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

$$\delta\varphi_k(t) \propto e^{\mu_k t}$$







$$|\Phi|_{\rm sol} \lesssim 10 \times \left(\frac{M}{m_{\rm pl}}\right)^2$$

$$\Phi \lesssim \text{few} \times 10^{-3}$$

not easy to form black holes from individual solitons*

$$\Omega_{{\rm GW},0} h_{100}^2 \sim 10^{-6} \left(\frac{M}{m_{\rm Pl}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$

expansion ✓

self-interactions ✓

gravitational interactions

active gravity ?

- gravitational clustering takes time …
- long time makes it difficult to resolve very fast oscillatory time scale

a way forward …

- rapid oscillatory behavior of fields (integrate out ?)
- size of solitons and instability length scales $\gtrsim m^{-1}$
- gravity is weak
- non-relativistic simulations including local gravitational interactions ?

“non-relativistic” limit

$$\Box\phi + V'(\phi) = 0$$

$$G_{\mu\nu} = \frac{1}{m_{\rm pl}^2} T_{\mu\nu}$$

$$\phi(t, \mathbf{x}) = \frac{\sqrt{2}}{m} \Re[e^{-imt} \psi(t, \mathbf{x})]$$

$$\frac{|\nabla|}{m}, \frac{\partial_t}{m} \ll 1$$

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Phi)d\mathbf{x}^2$$

$$|\Phi| \ll 1$$

↓ non-linear Schrodinger eq.

↓ Poisson eq. + Friedmann eq.

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)$$

non-relativistic case

$$\left[i\left(\partial_t + \frac{3}{2}H\right) + \frac{1}{2a^2}\nabla^2 - U'_{\rm nl}(|\psi|^2) - \Phi\right]\psi = 0\,,$$

nonlinear Schrodinger eq.

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nonlinear Schrodinger eq.

$$\frac{\nabla^2}{a^2}\Phi = \frac{\beta^2}{2}\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right] - \frac{3}{2}H^2\,,$$

Poisson eq.

$$H^2 = \frac{\beta^2}{3}\overline{\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right]}\,,$$

Friedmann eq.

$$mx^\mu \to x^\mu$$

length/time units

$$\frac{\psi}{mM} \to \psi$$

non-linearity

$$\beta \equiv \frac{M}{m_{\rm pl}}$$

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

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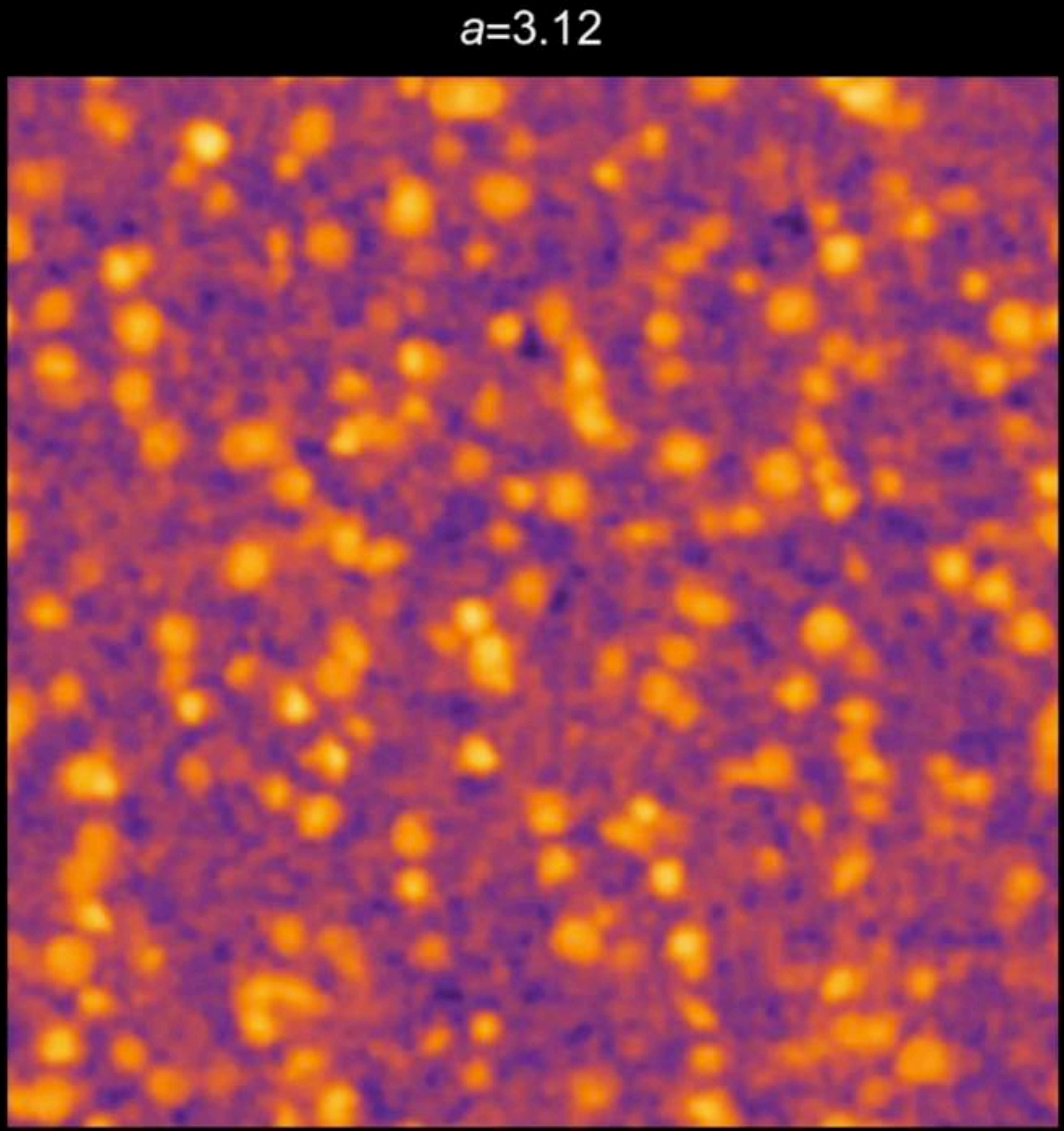
$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

relativistic* ✓

self-interactions ✓

gravitational int. ✗



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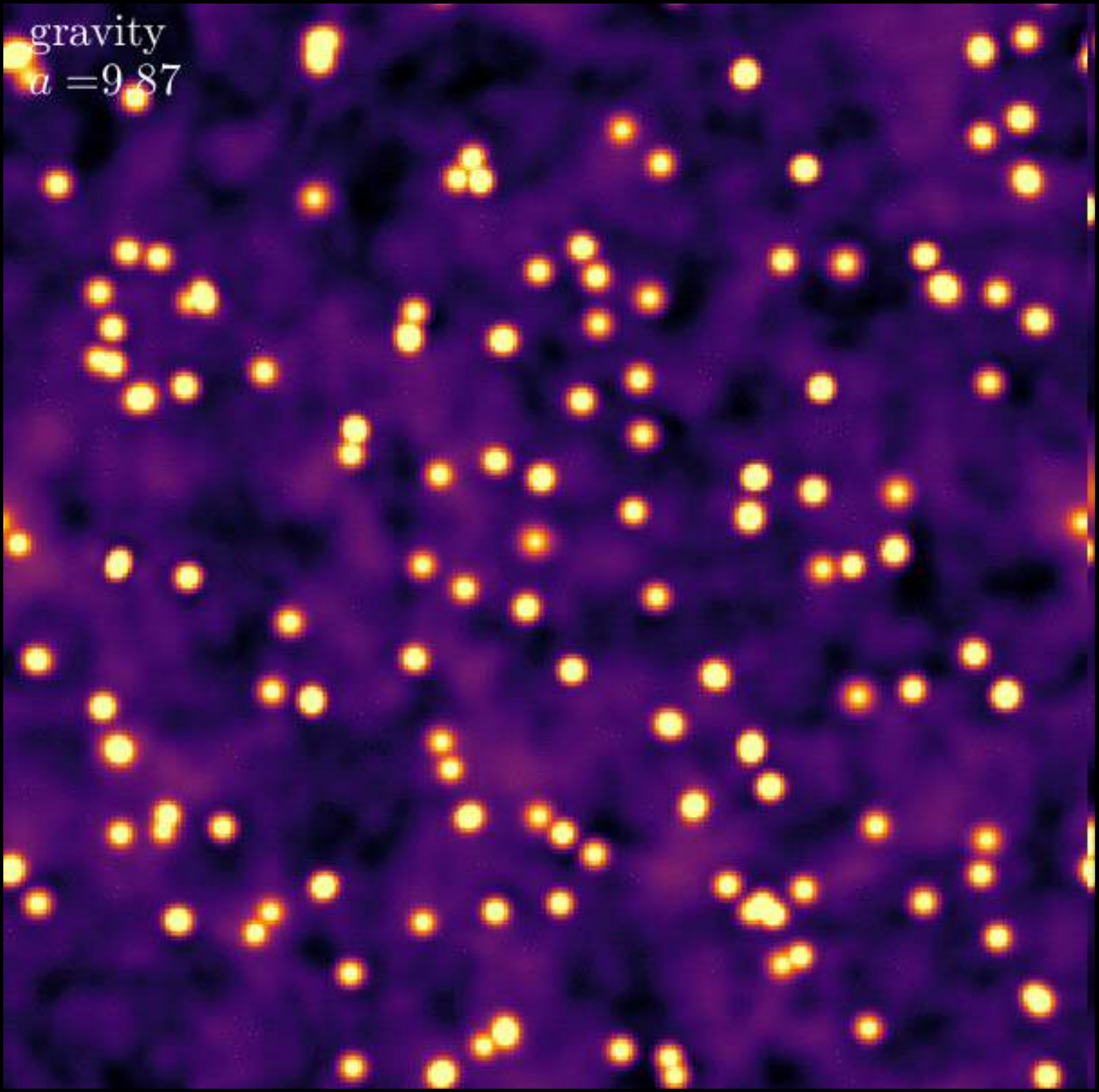


two linear instabilities

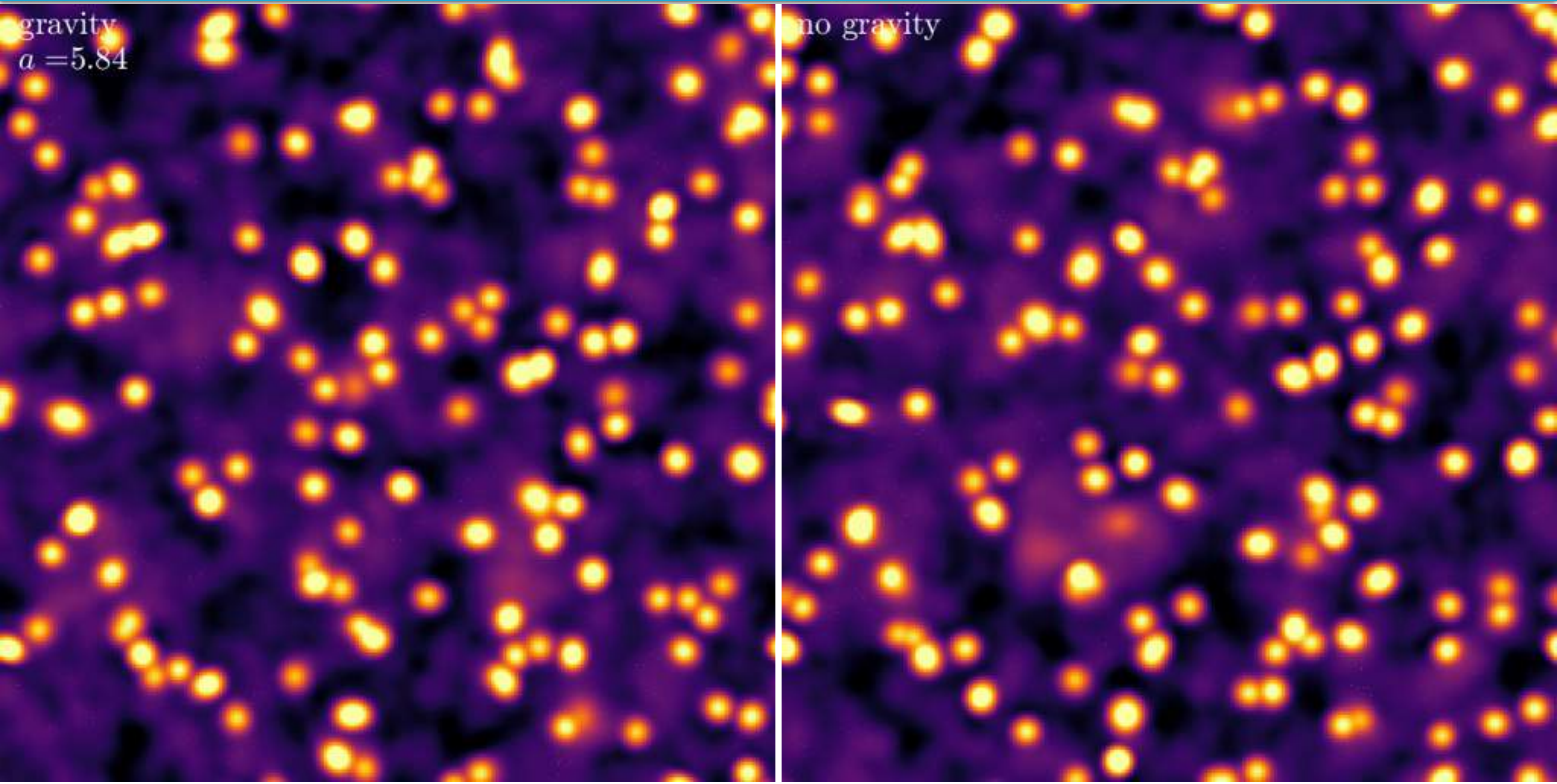
$$\psi(t,\mathbf{x}) = \bar{\psi}(t)\left[1+\varepsilon\frac{\delta\psi_{\mathbf{k}}(t)}{\bar{\psi}(t)}e^{i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$|\delta\psi_{\mathbf{k}}/\bar{\psi}| \sim e^{\mu_k t}$$

self-interactions	gravitational interactions
$k^2 < -4\lvert\bar{\psi}\rvert^2 U''_{\rm nl}(\lvert\bar{\psi}\rvert^2)\,,$	$k < k_J \approx \sqrt{\sqrt{2}\beta\lvert\bar{\psi}\rvert}$
$\mu_k = \left\lvert i\frac{k}{2}\sqrt{k^2+4\lvert\bar{\psi}\rvert^2 U''_{\rm nl}(\lvert\bar{\psi}\rvert^2)}\right\rvert$	$\mu_k = \sqrt{\frac{1}{2}\beta^2\lvert\bar{\psi}\rvert^2 - \frac{k^4}{4}}$
$\frac{\mu_k}{H} \sim \frac{1}{\beta}$	$\frac{\mu_k}{H} \sim 1$

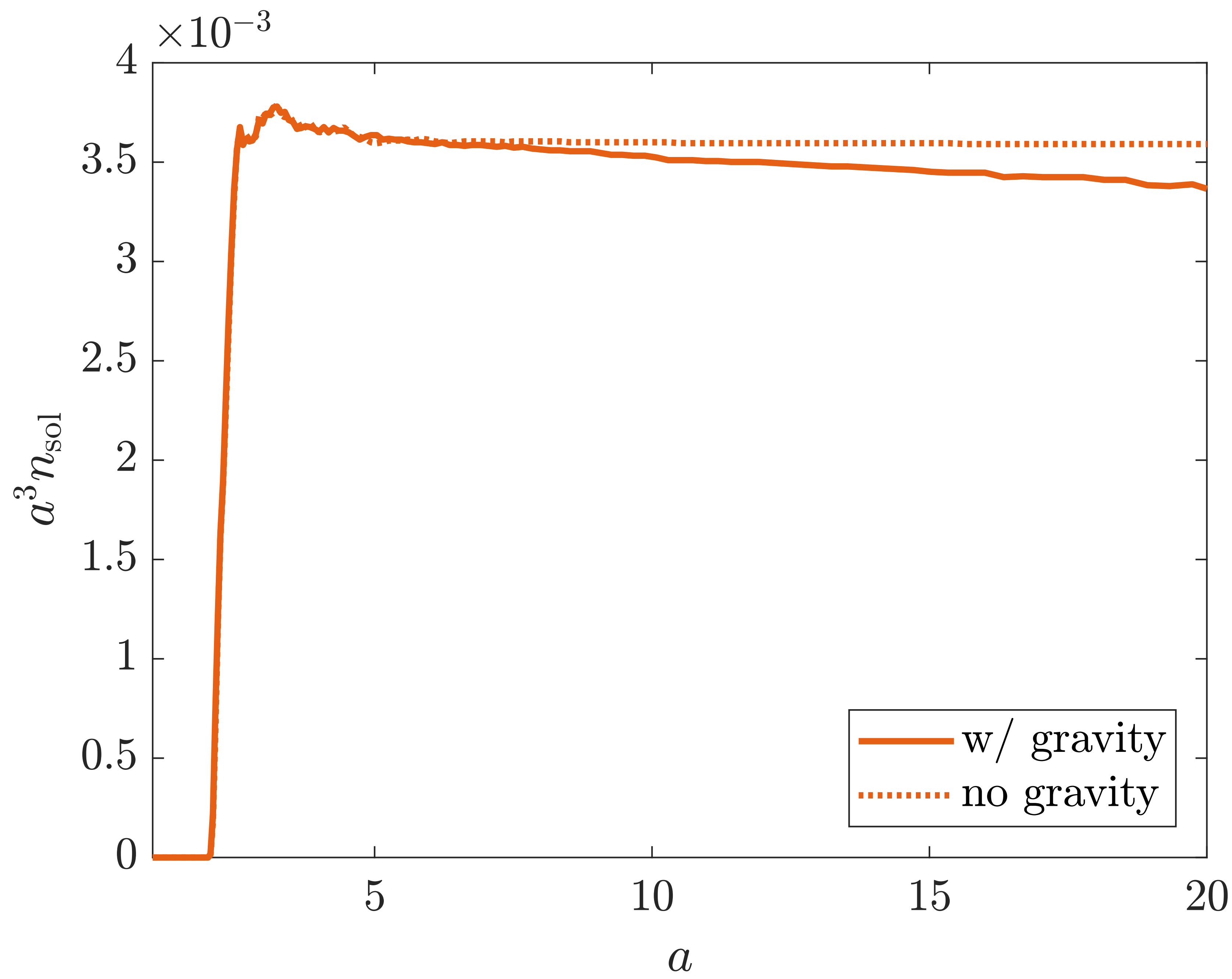
For $\beta \equiv \frac{M}{m_{\rm pl}} \ll 1$ self-interaction instability dominates

perturbations become nonlinear & form solitons



$$a^3 n_{\rm sol} \sim (k_{\rm nl}/2\pi)^3$$

co-moving number density of solitons

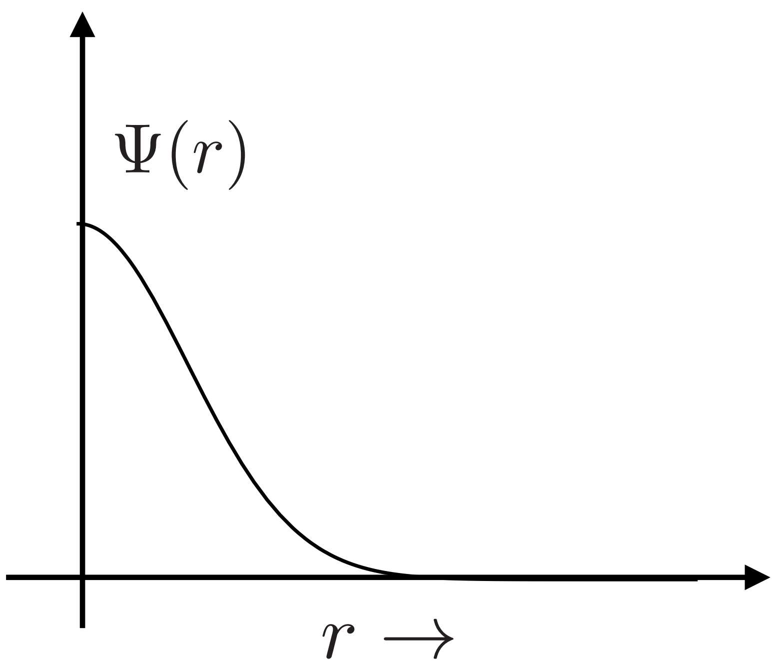


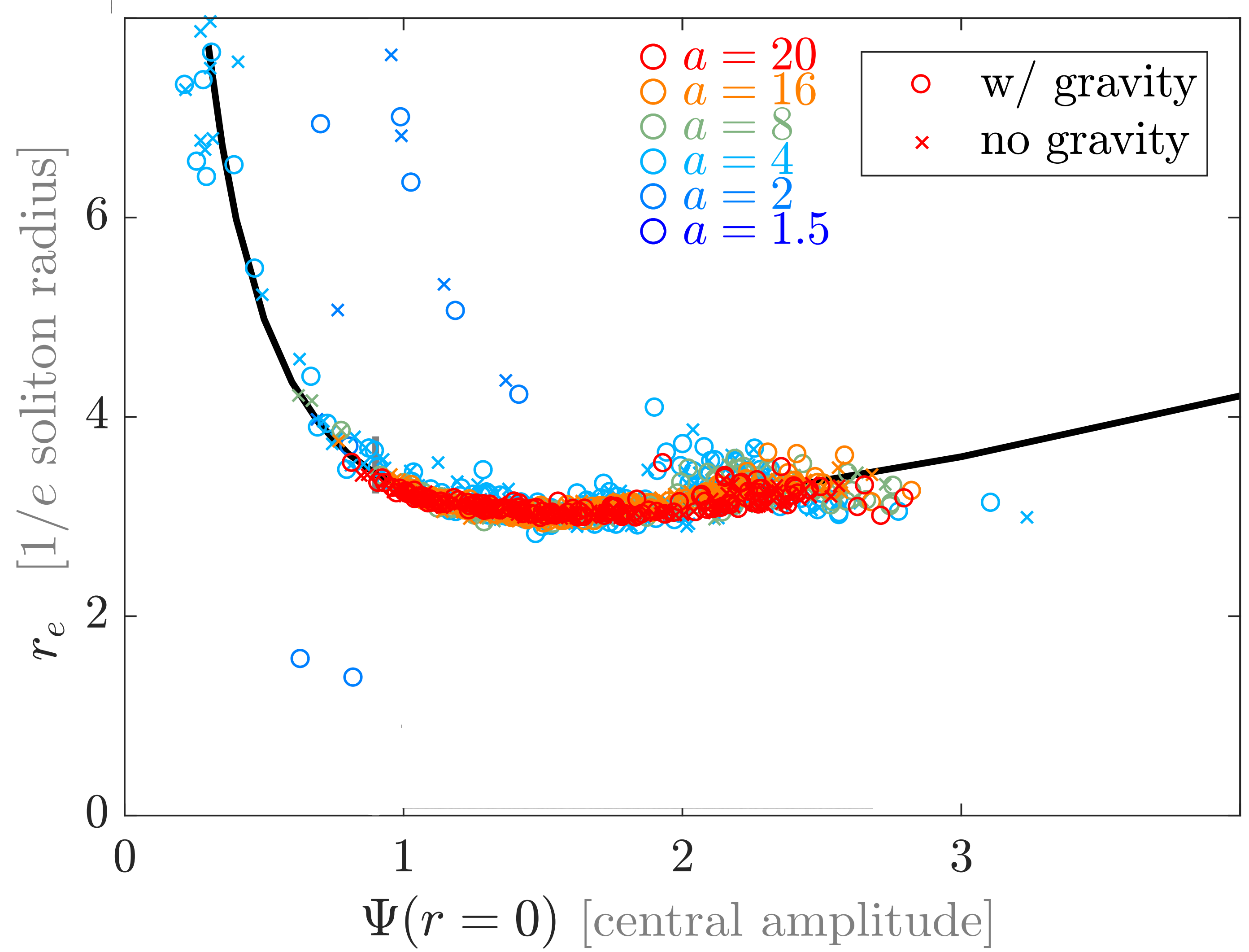
$$\frac{d\ln(a^3 n_{\rm sol})}{d\ln a} \simeq 0.1$$

few x 10^3 /Hubble volume at time of formation

individual solitons

$$\psi(t,r) = e^{-i\nu t}\Psi(r)$$





$$\phi(t,r) = \sqrt{2}\Re[e^{-it}\psi(t,r)] = \sqrt{2}\Psi(r)\cos[(1+\nu)t]$$

$$\nu < 0, \qquad |\nu| \ll 1$$

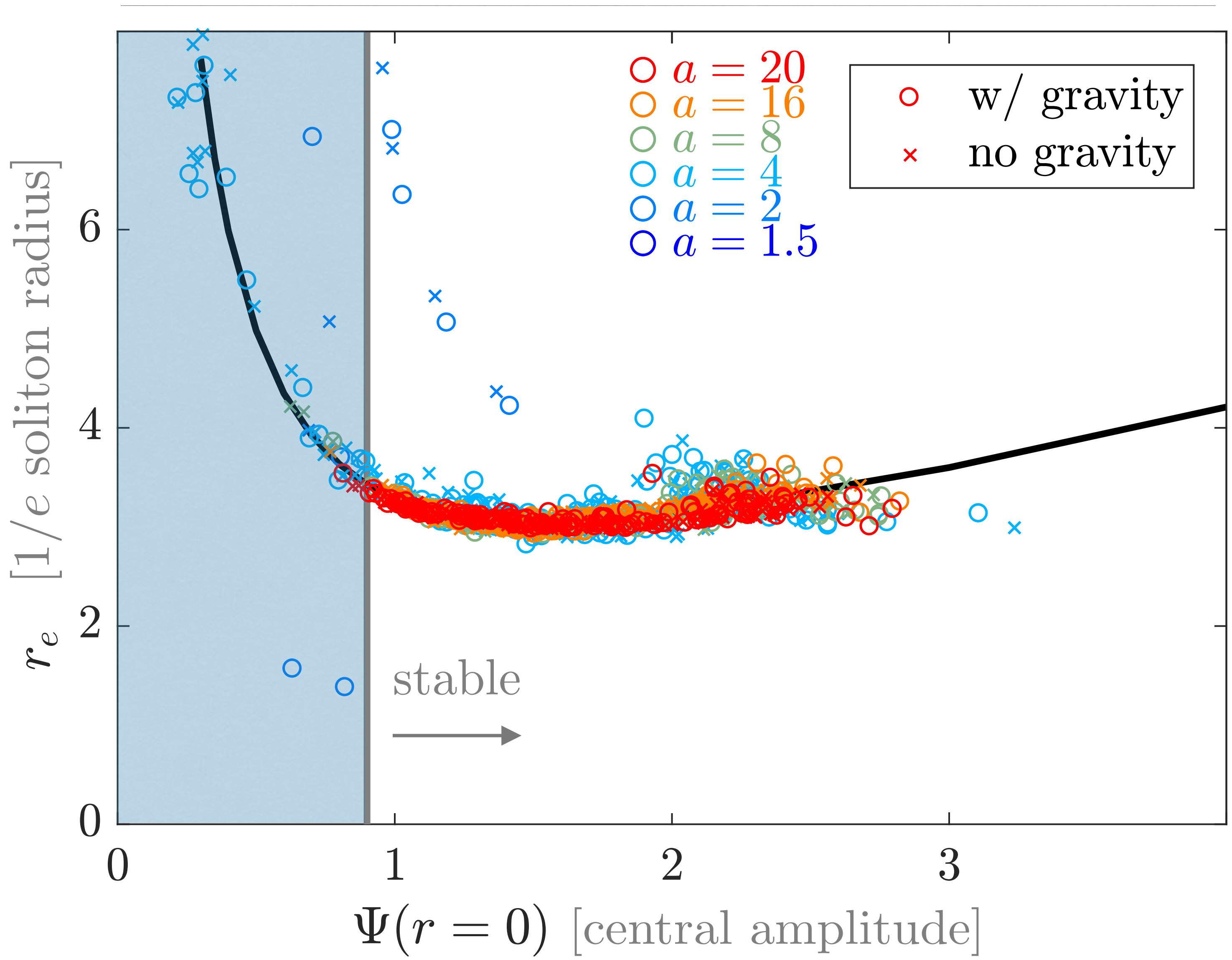
individual solitons stability

$$\psi(t,r) = e^{-i\nu t}\Psi(r)$$

$$\mathcal{N} \equiv \int d^3 r \Psi^2(r)$$

stable iff:
Vakhitov Kolokolov (1973)

$$\frac{d\mathcal{N}}{d(-\nu)} > 0$$

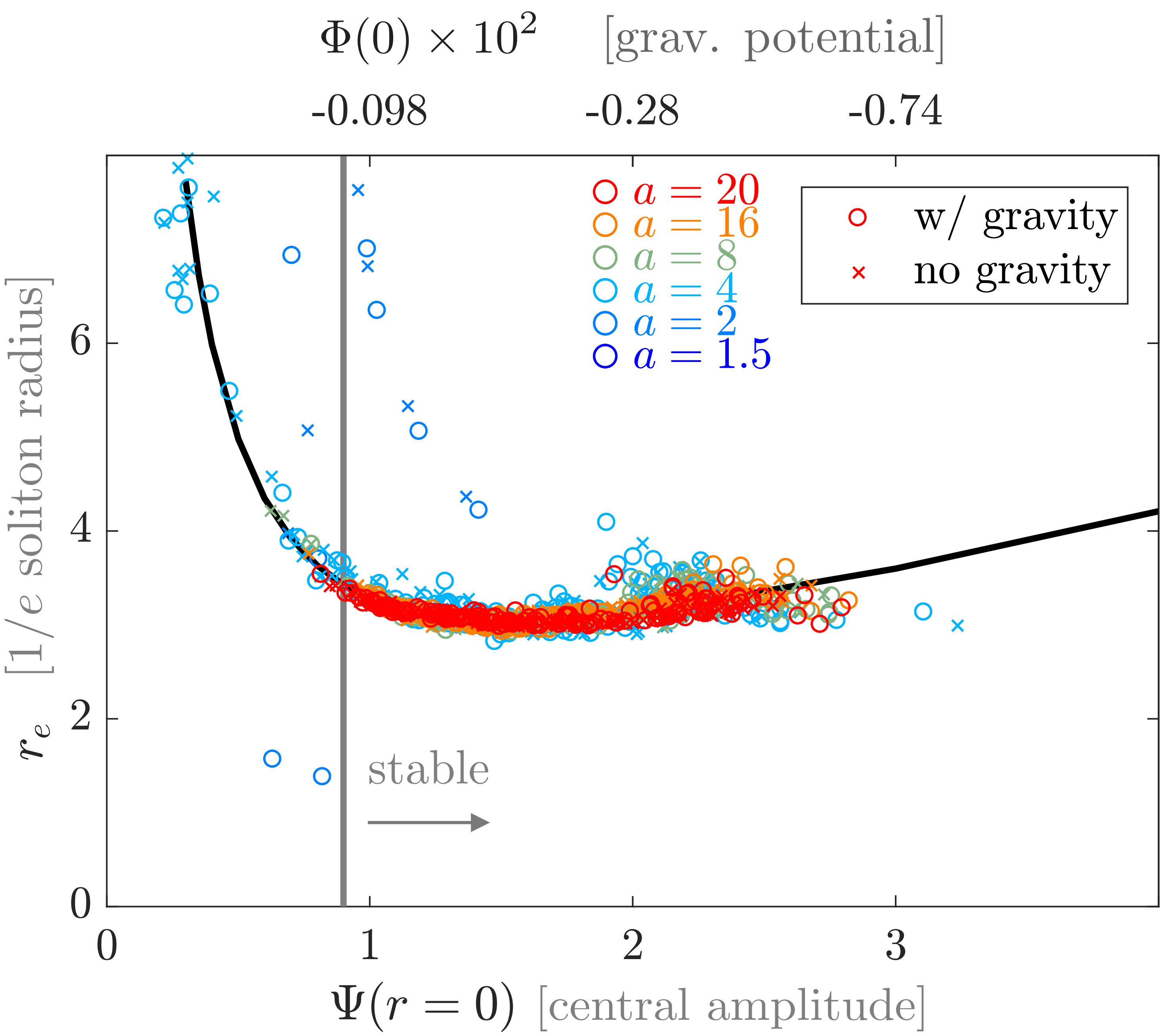


— stability with gravitational interactions needs to be investigated

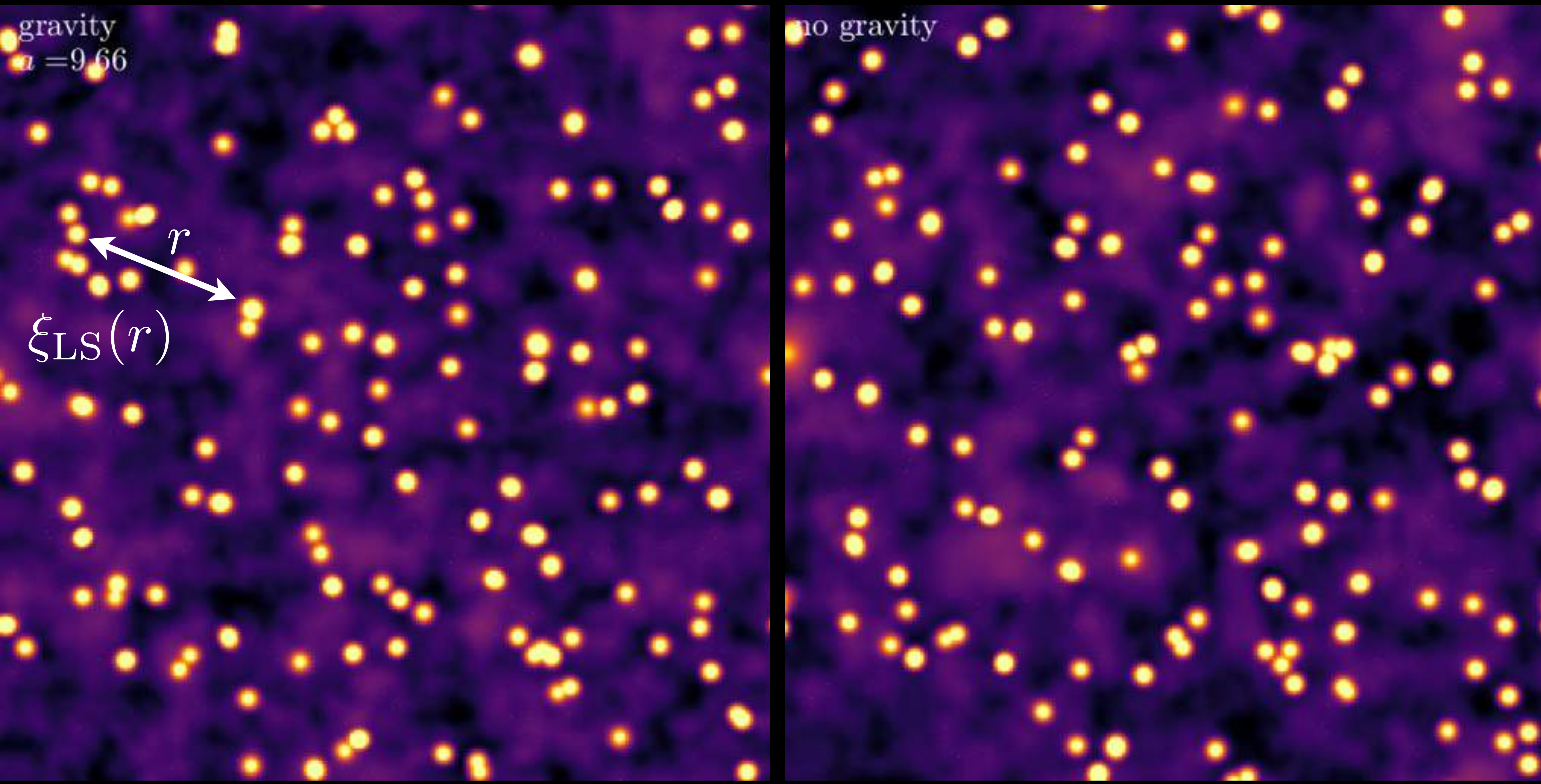
gravity remains weak

$$|\Phi| \sim \mathcal{O}[10^{-3}]$$

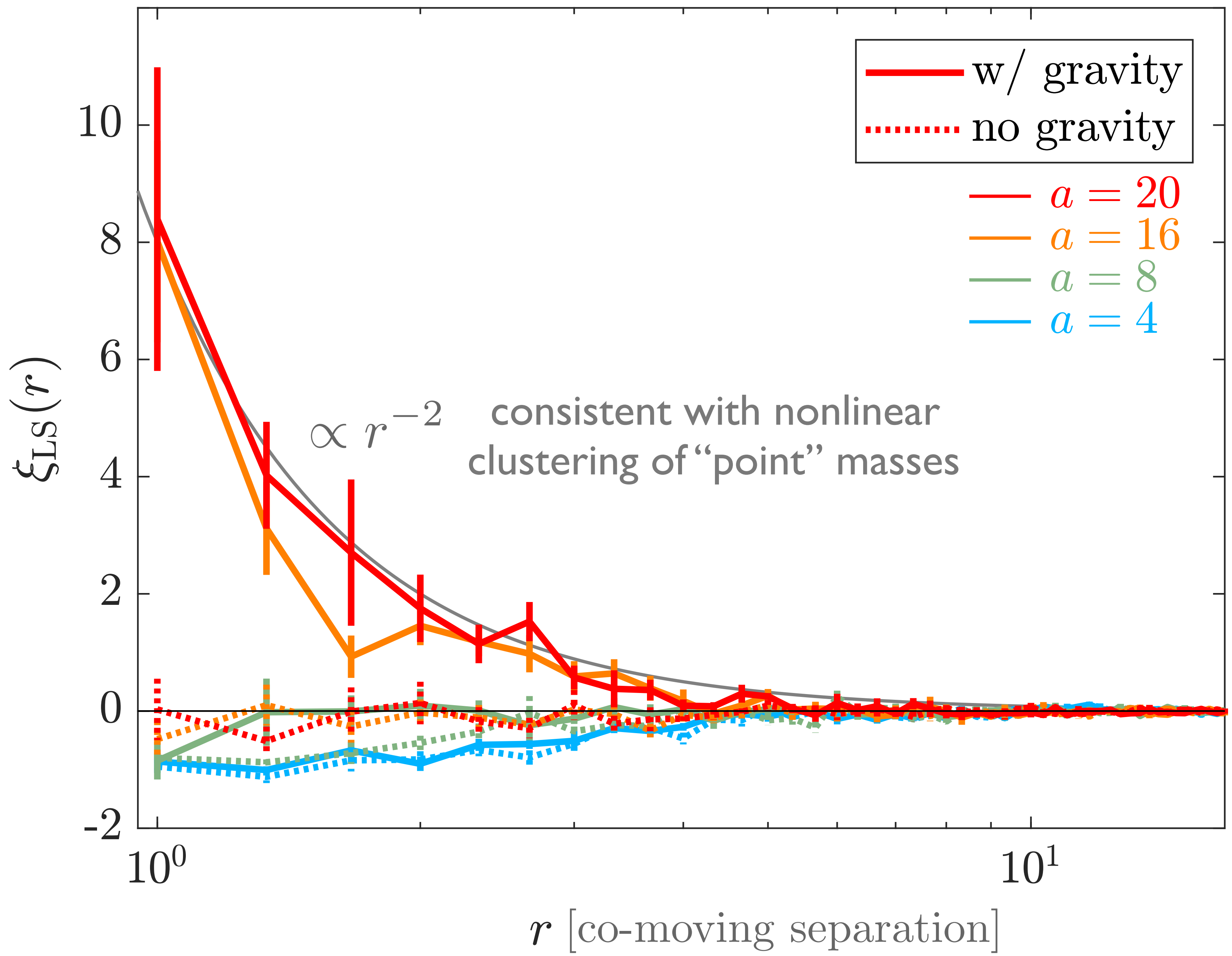
not easy to form black holes from individual solitons still



gravitational clustering of solitons

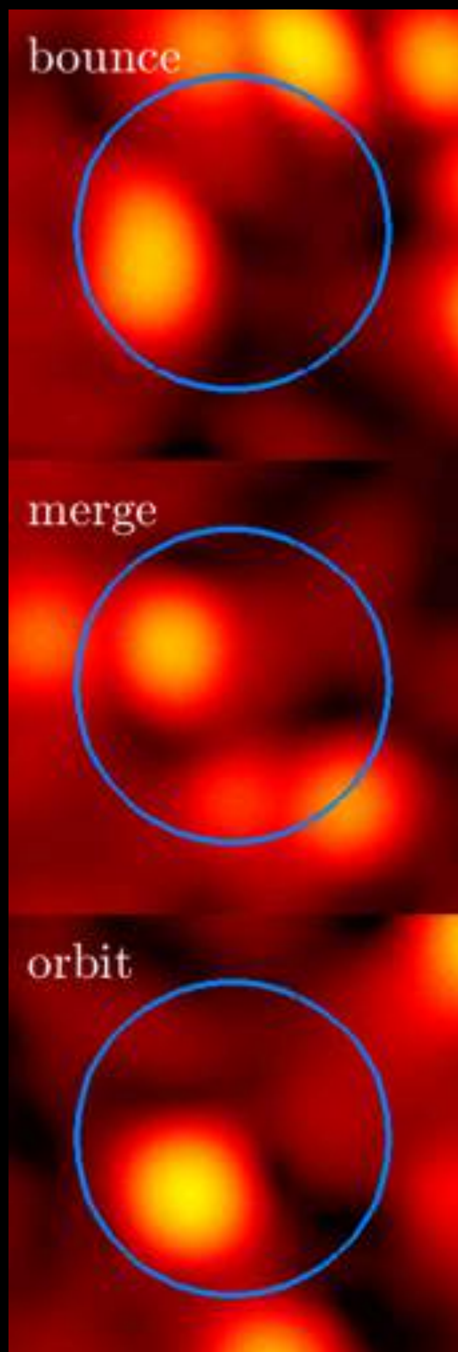


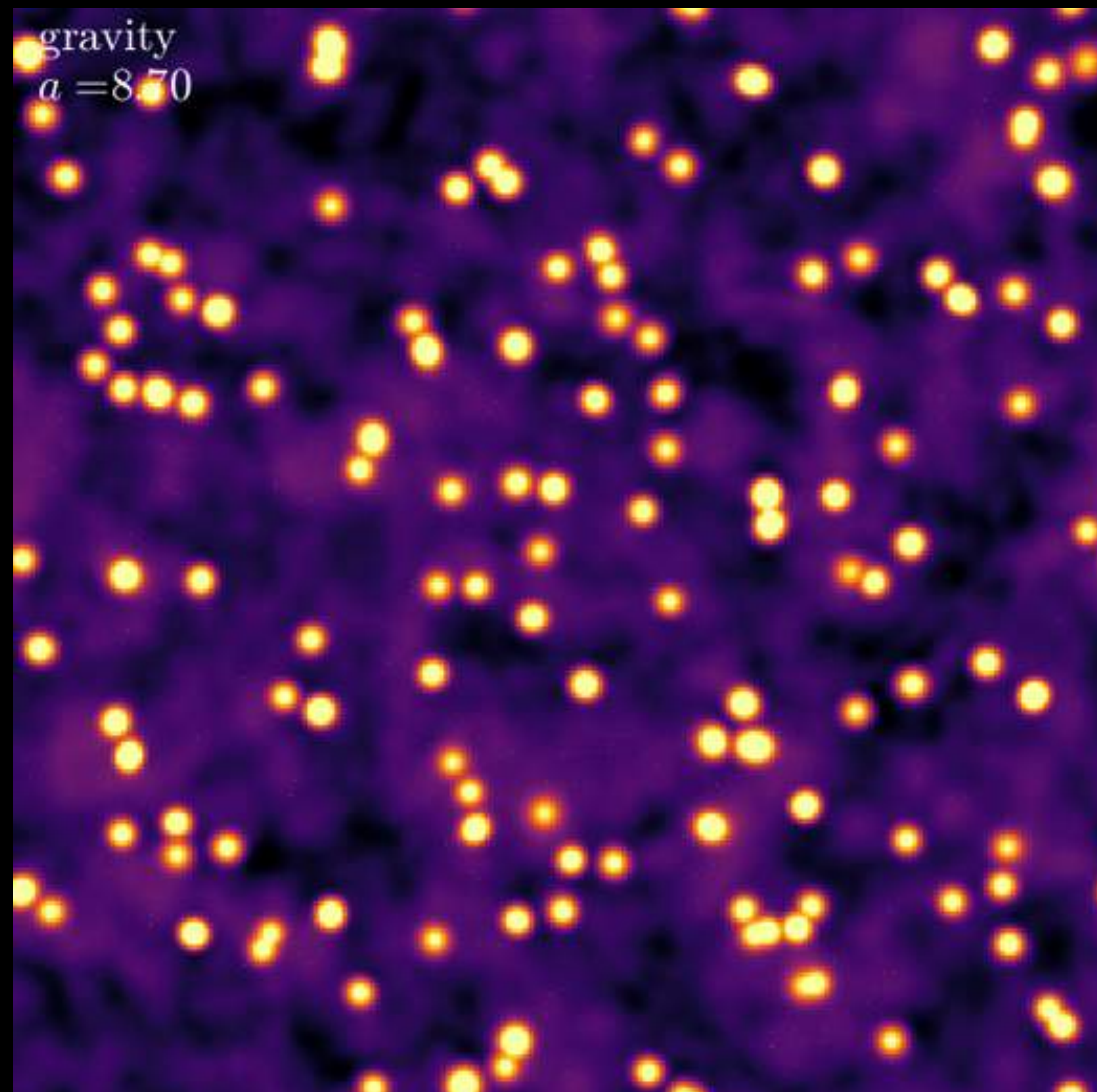
gravitational clustering of solitons



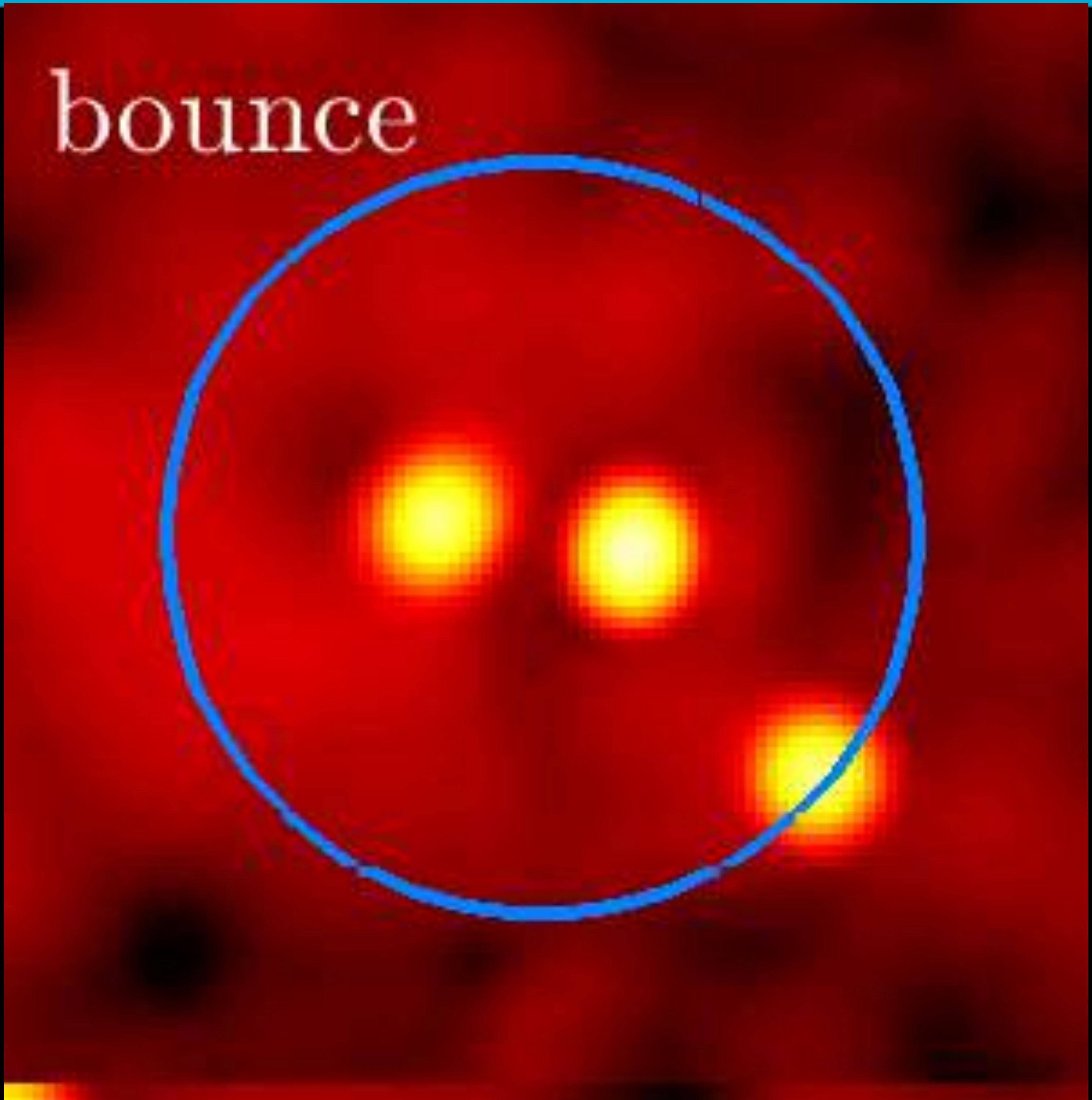
*theoretical arguments for r^{-2} in Saslaw 1980

strong interactions

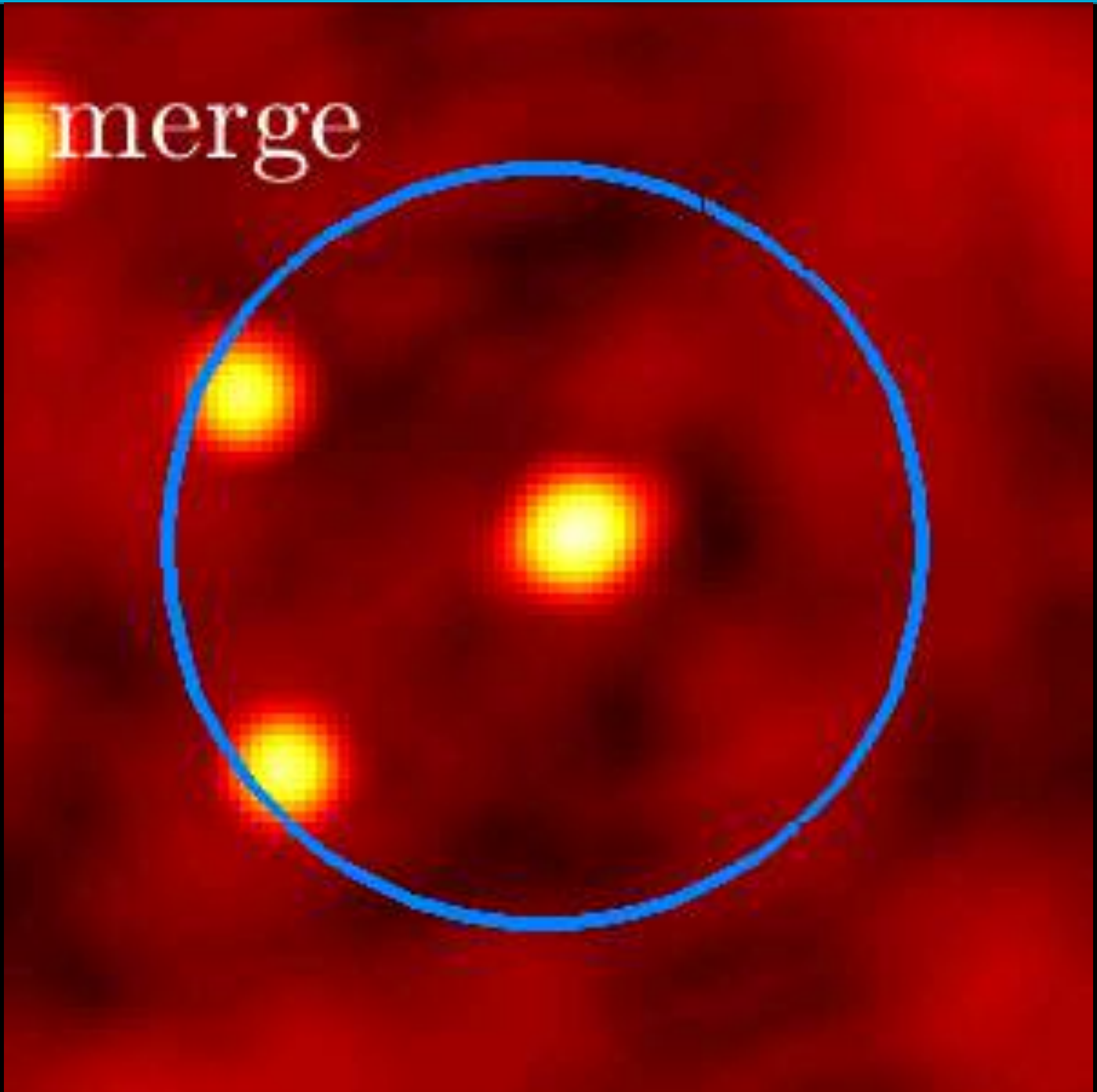




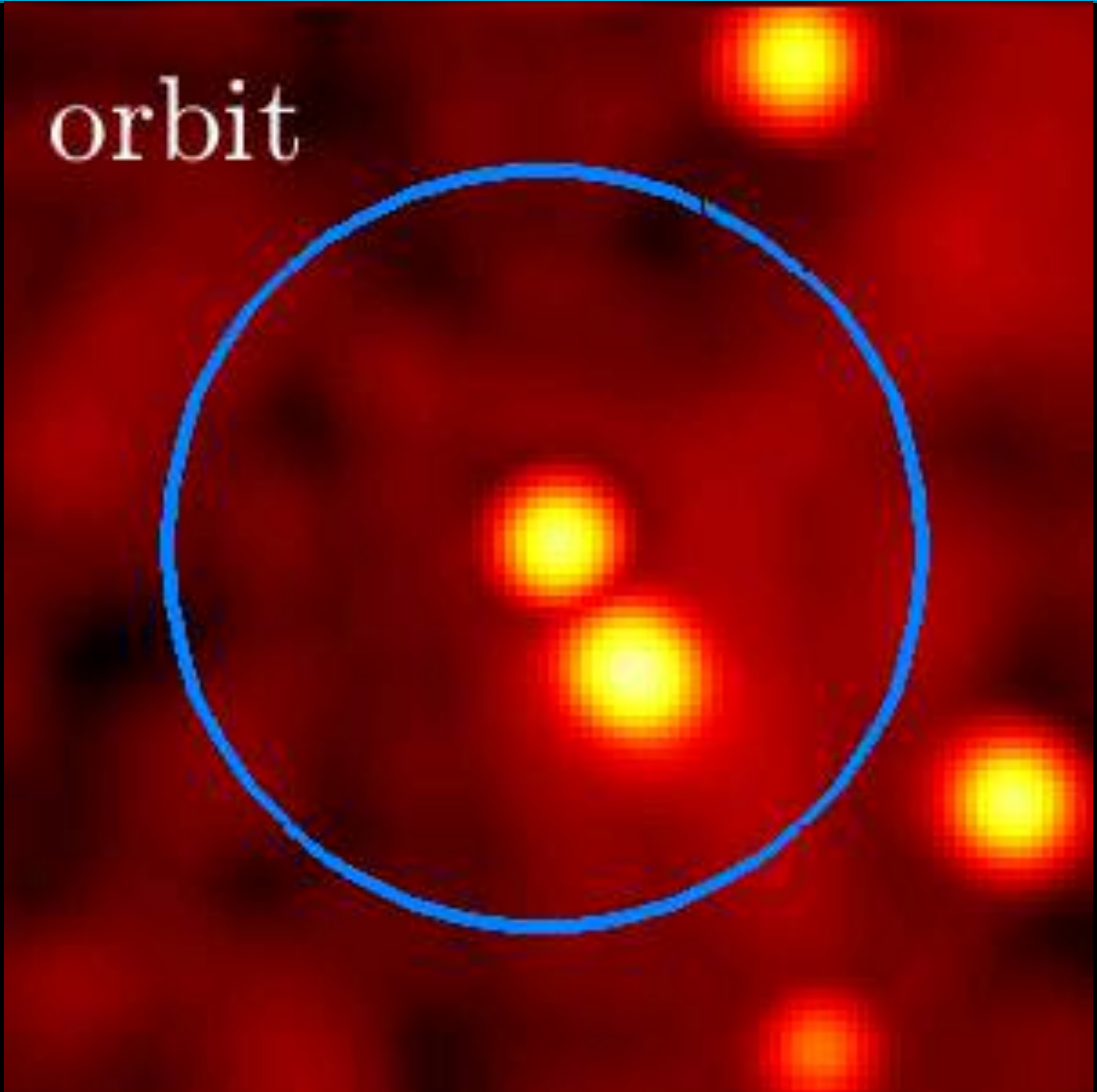
strong interactions: bounce



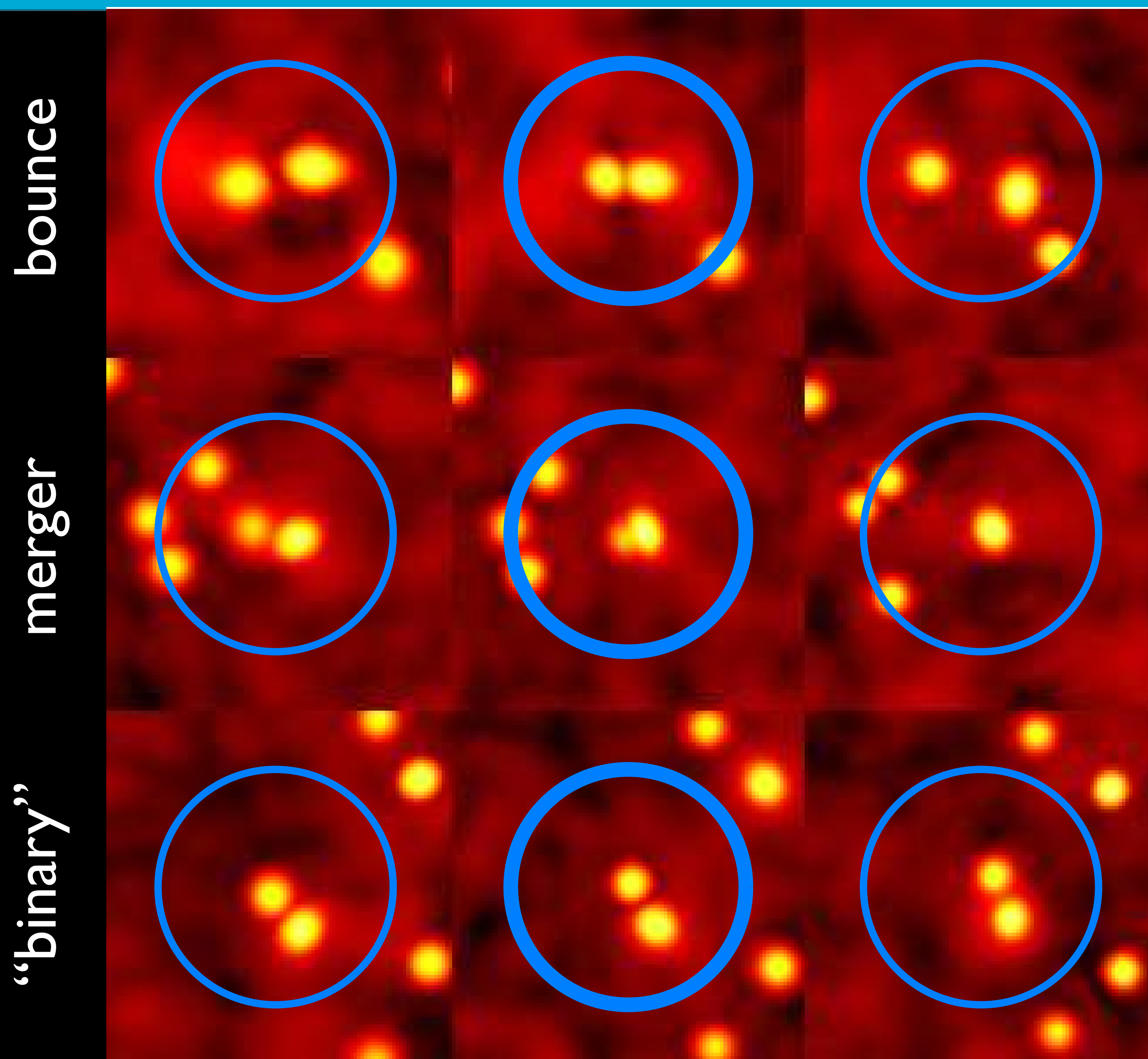
strong interactions: mergers



strong interactions: orbit



strong interactions



$$\psi_a(t,\mathbf{x}) = \Psi_a(\mathbf{x})e^{-i\nu_a t+\theta_a}$$

$$|\theta_1 - \theta_2| \simeq \pi$$

$$|\theta_1 - \theta_2| \simeq 0$$

* need better understanding
* & simulations

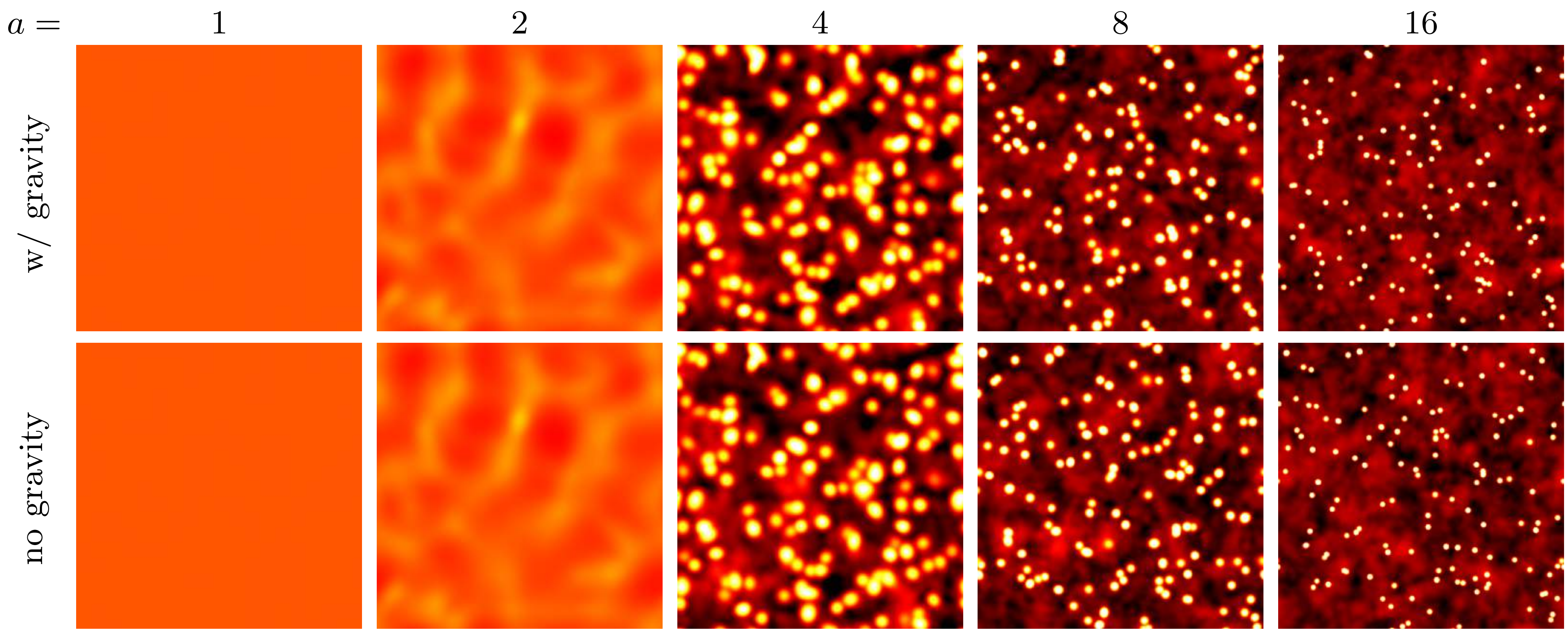
summary with gravitational interactions

self-interaction instability

soliton formation

gravitational clustering

strong interactions



things that need more work

- relativistic - non-relativistic connection
- long term state of the strongly interacting soliton gas — how does probability of PBH formation increases ?
- better understanding of the strong interactions

exploring individual interactions further with full numerical GR

$$S = \int d^4x \sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

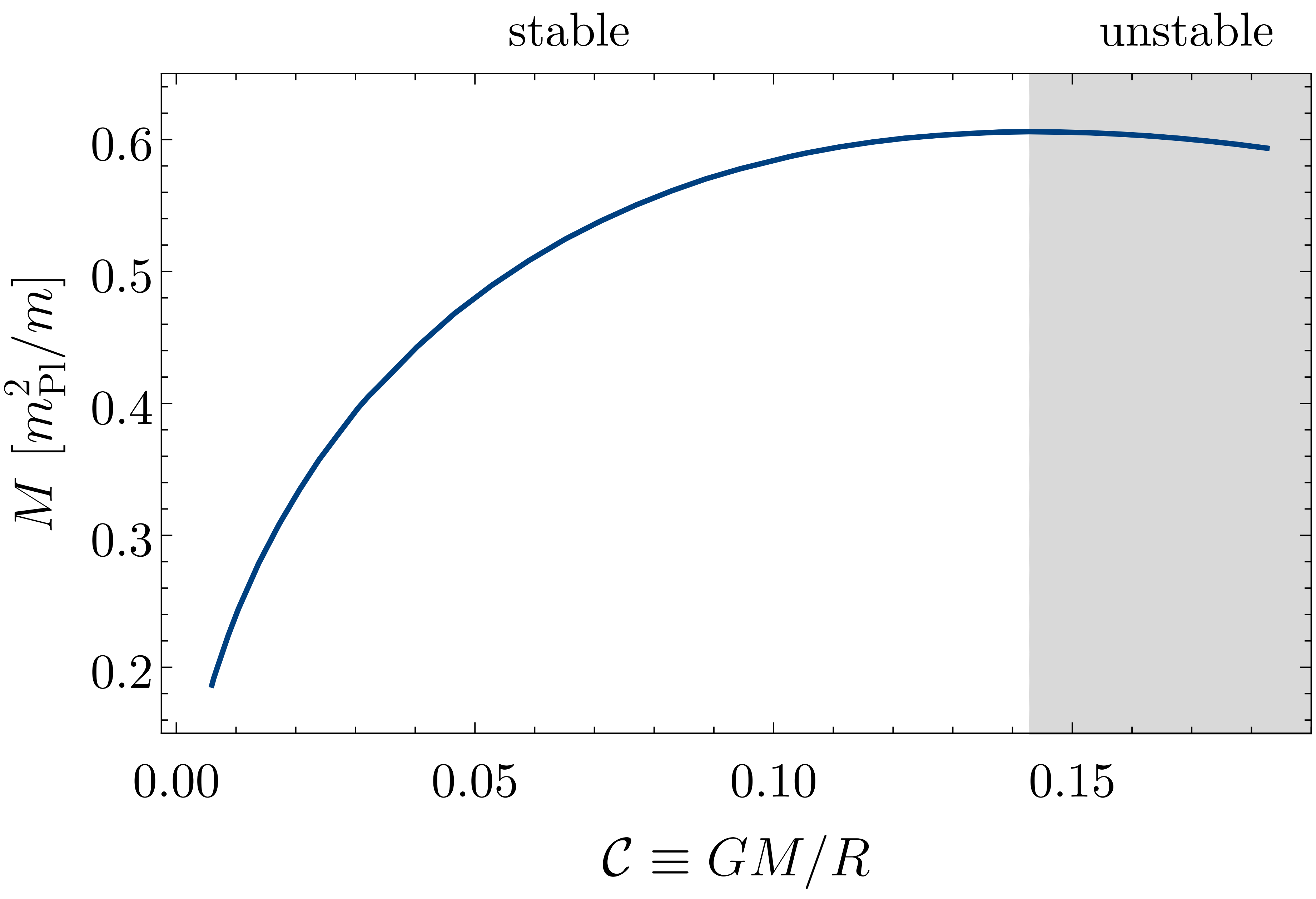
$$V(\phi) = \frac{1}{2}m^2\phi^2 + \cancel{V_{\rm nl}(\phi)}$$

ignore self-interactions

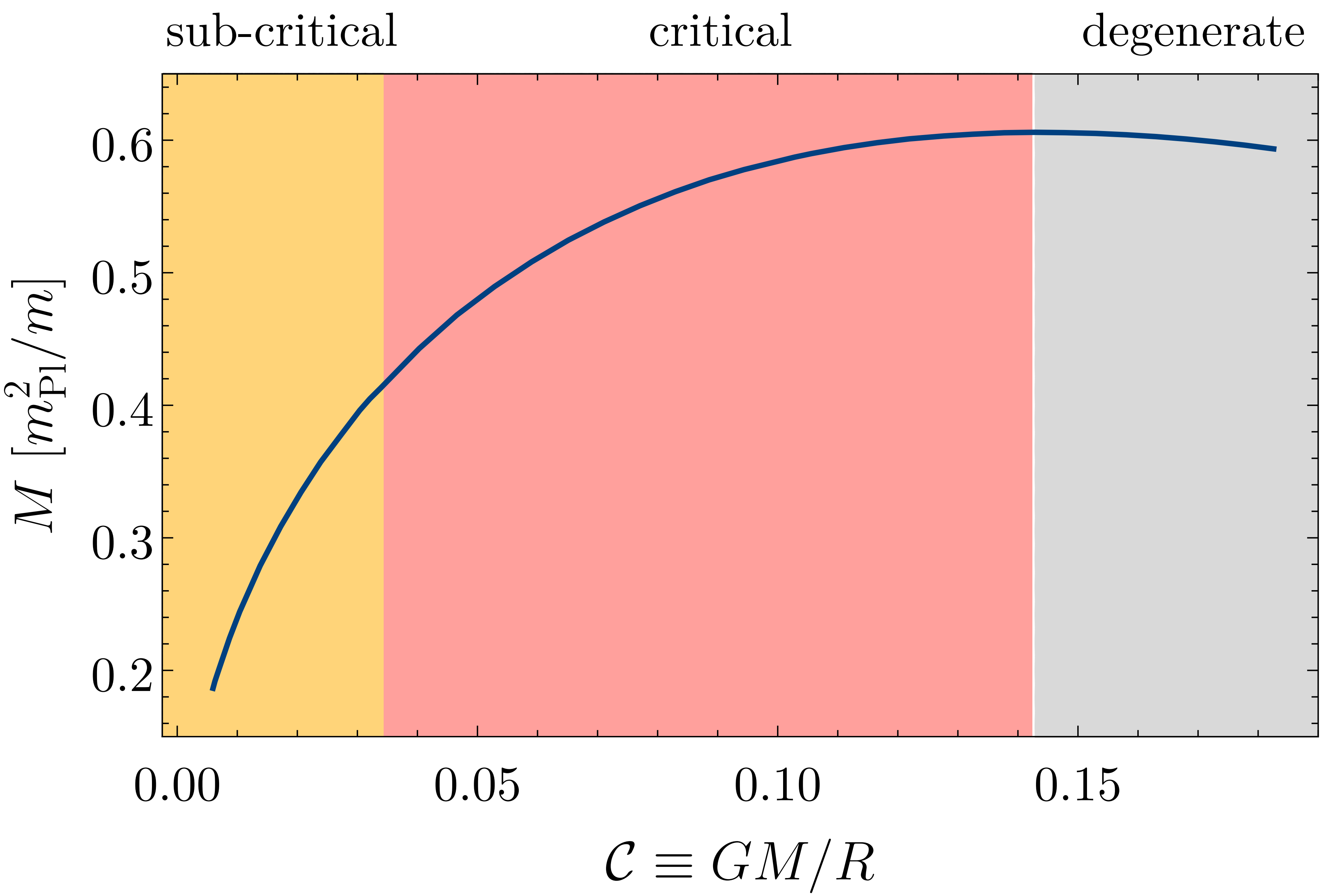
interested in gravitational wave emission from ultra-compact solitons

Helfer, Lim, Garcia & MA (2018)

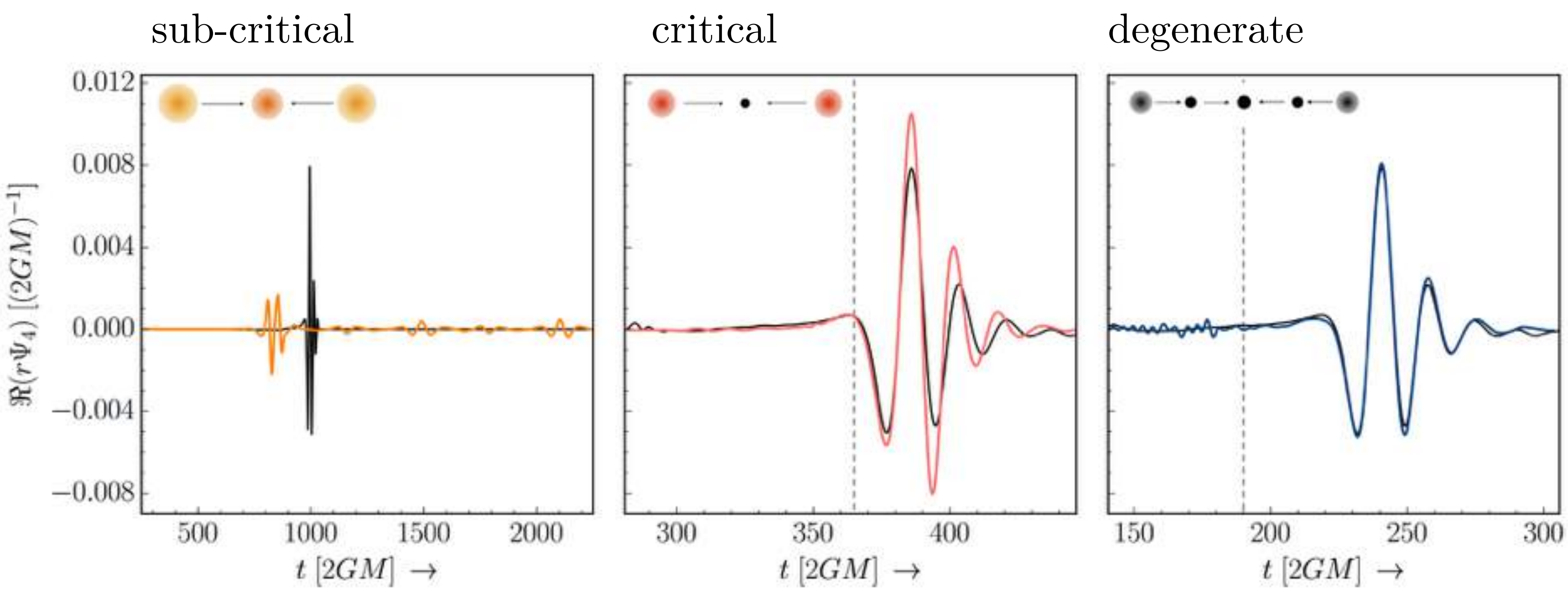
ultra-compact soliton



ultra-compact soliton

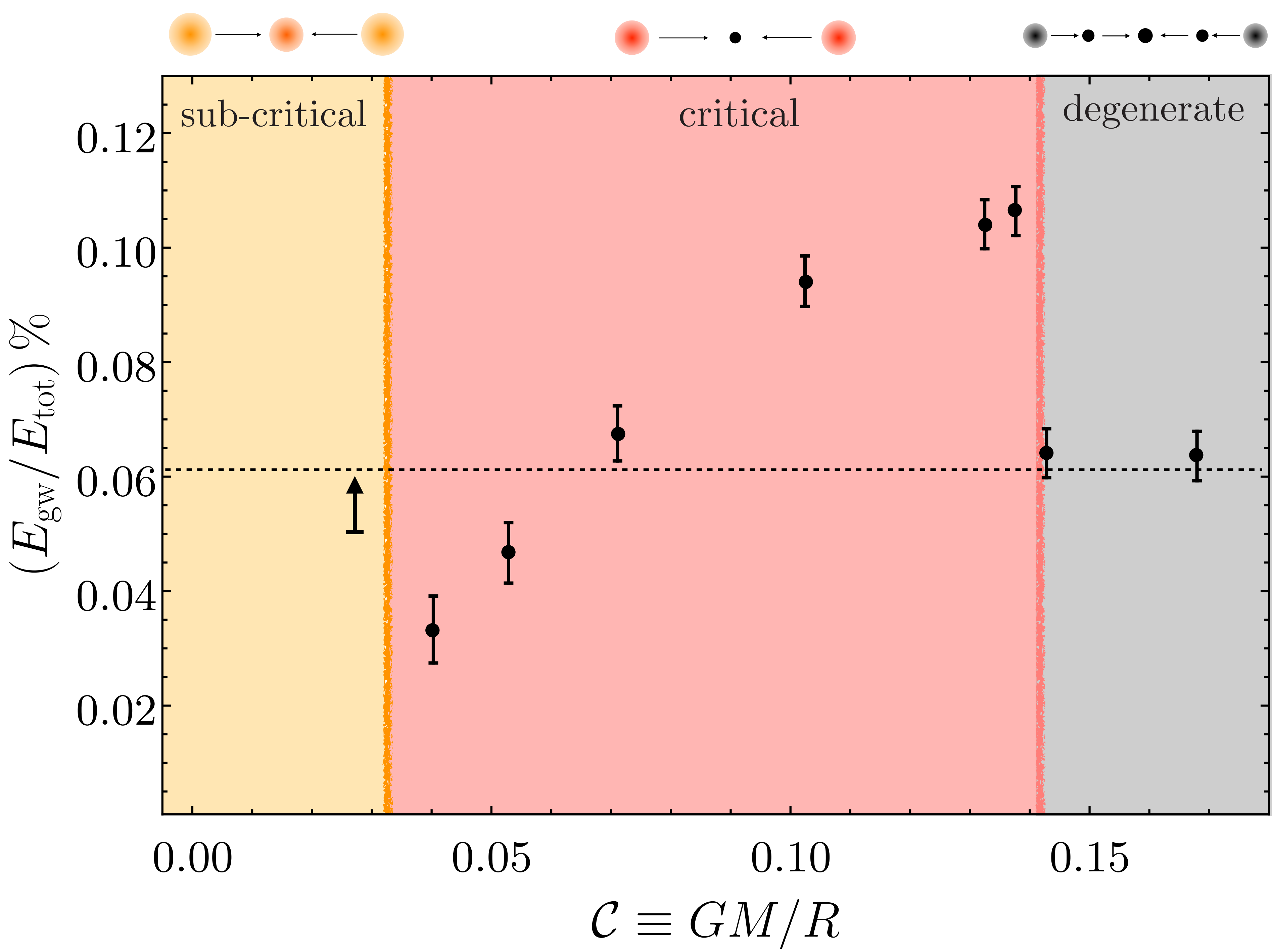


gravitational waves from ultra-compact soliton collisions



black = corresponding mass black hole g-wave signal

more energy in g-waves than corresponding mass BHs



important caveats

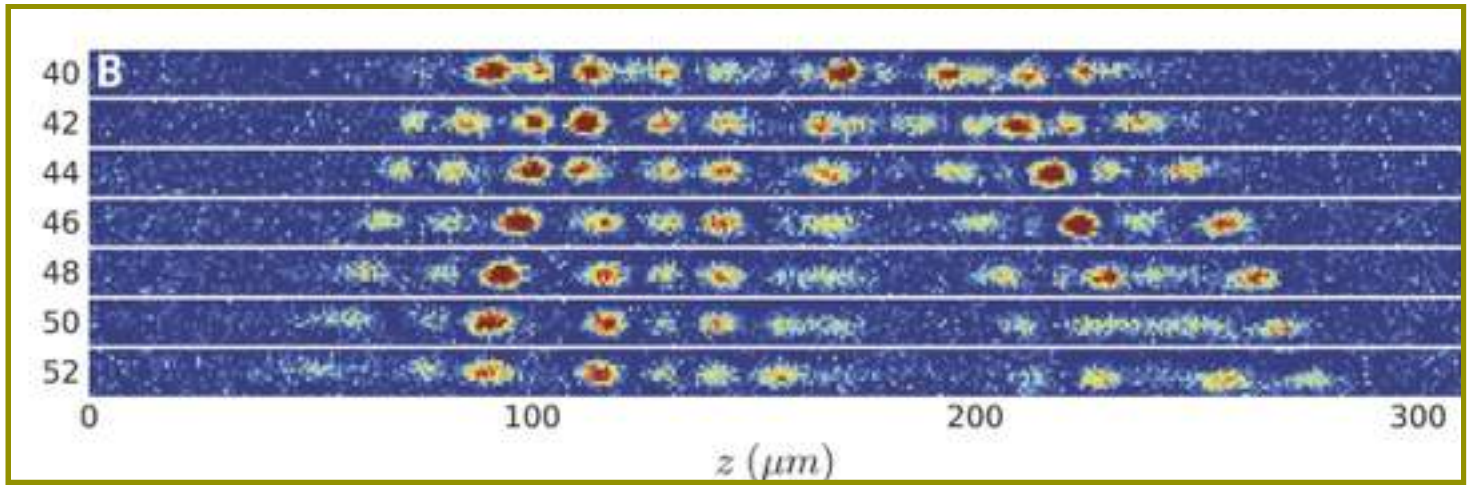
- solitons are in phase (out of phase ones can bounce)
- how likely are these to form ?
- head-on collisions: inspirals might change the answers (working on it — Helfer et. al)

things I did not discuss

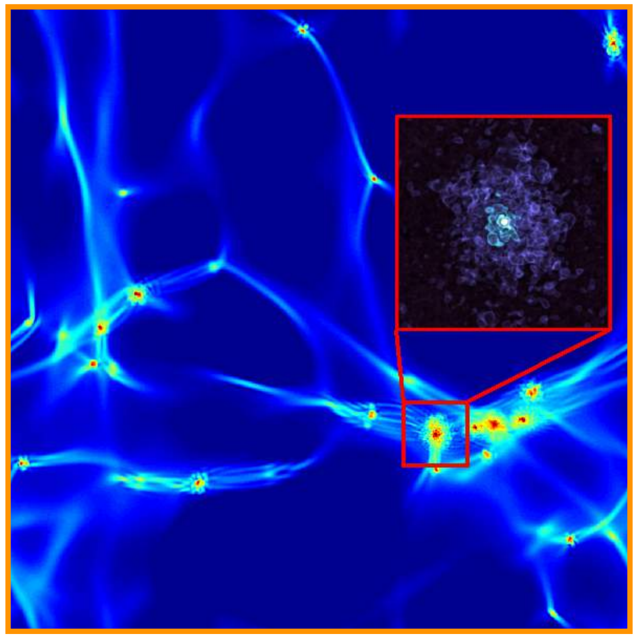
- axions in the early universe [there are differences with case discussed]
  - axitons — axion mini clusters …

- dynamics in late-time ultra-light axions
  - solitons at centers of halos/galaxies
  - small-scale structure of CDM (include baryons — in progress)
  - dynamical friction

- solitons in Bose-Einstein condensates



Nguyen, Luo & Hulet (2017)



Mocz et. al …

relativistic* ✓

self-interactions ✓

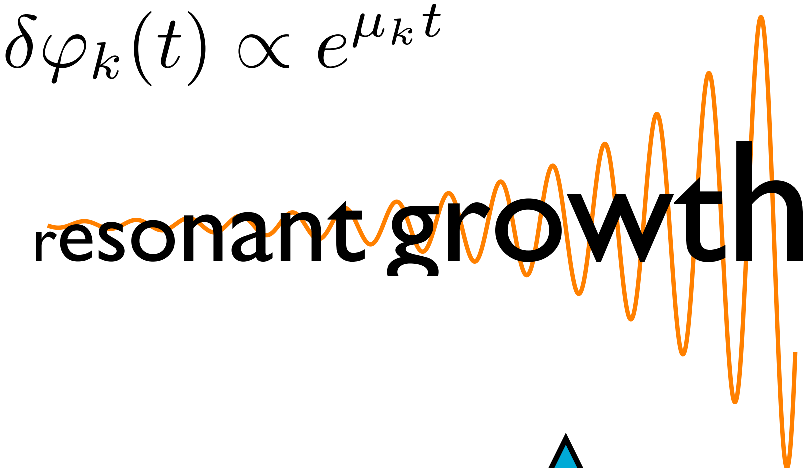
gravitational int. ✗

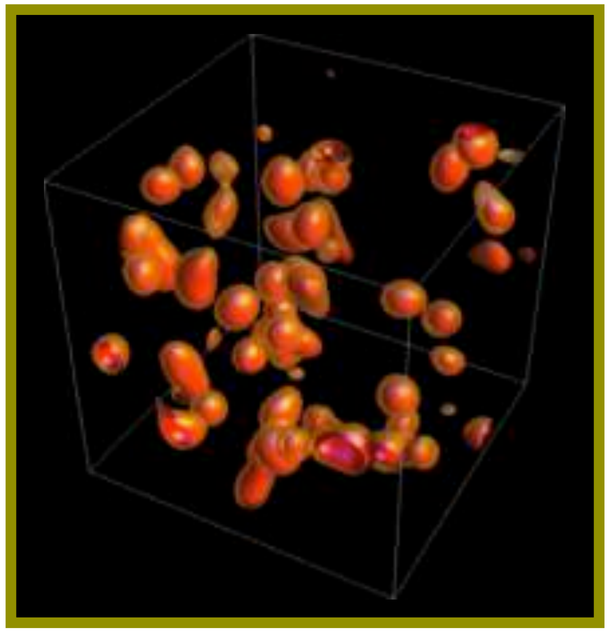
summary I

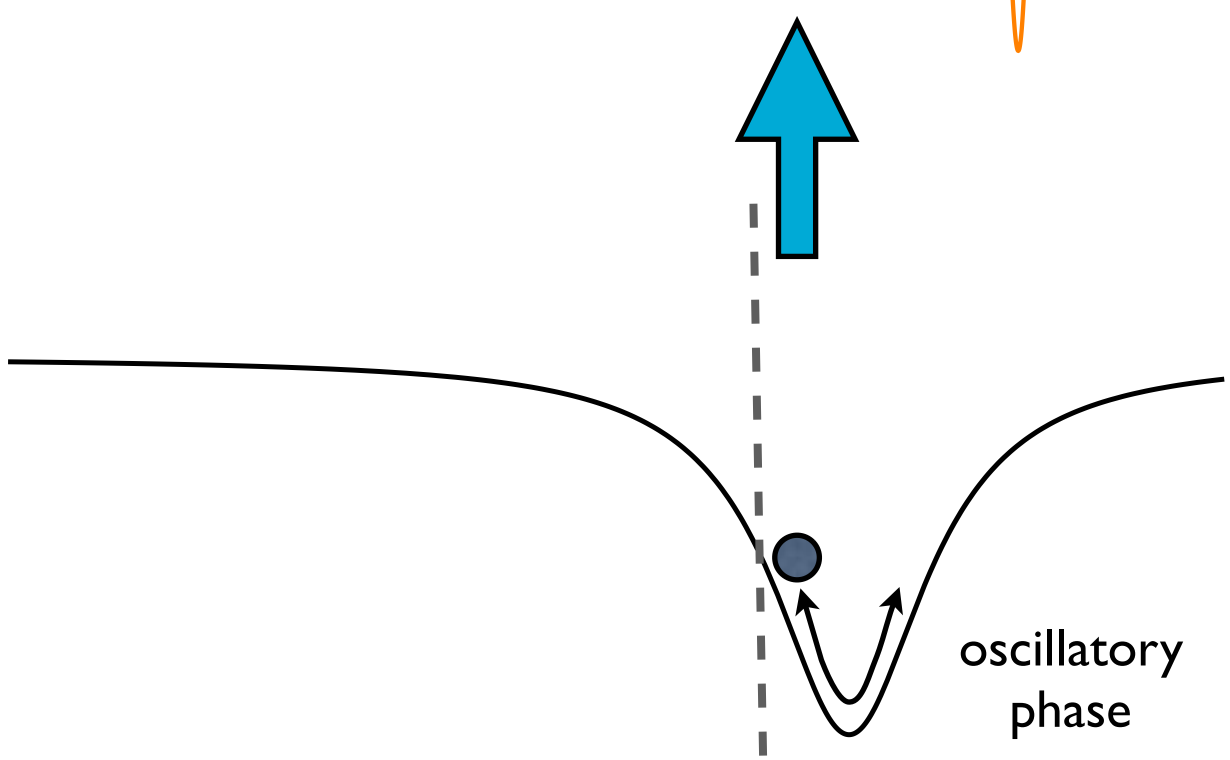
$$\frac{\mu_k}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

$$\delta\varphi_k(t) \propto e^{\mu_k t}$$

resonant growth







M

$$|\Phi|_{\rm sol} \lesssim 10 \times \left(\frac{M}{m_{\rm pl}}\right)^2$$

$$\Phi \lesssim \text{few} \times 10^{-3}$$

not easy to form black holes from individual solitons*

$$\Omega_{\rm GW,0} h_{100}^2 \sim 10^{-6} \left(\frac{M}{m_{\rm Pl}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$

relativistic ✗

self-interactions ✓

gravitational int. ✓

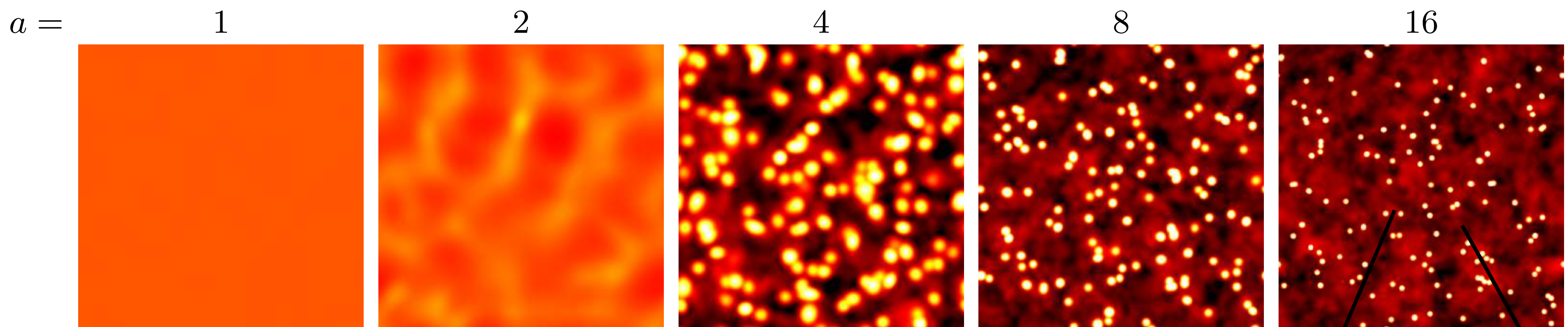
summary 2

self-interaction instability

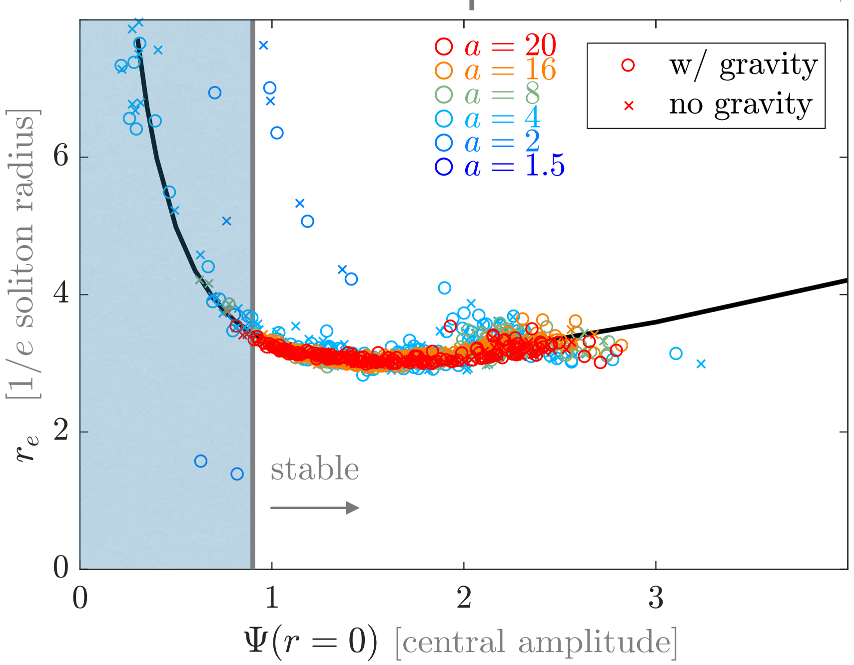
soliton formation

gravitational clustering

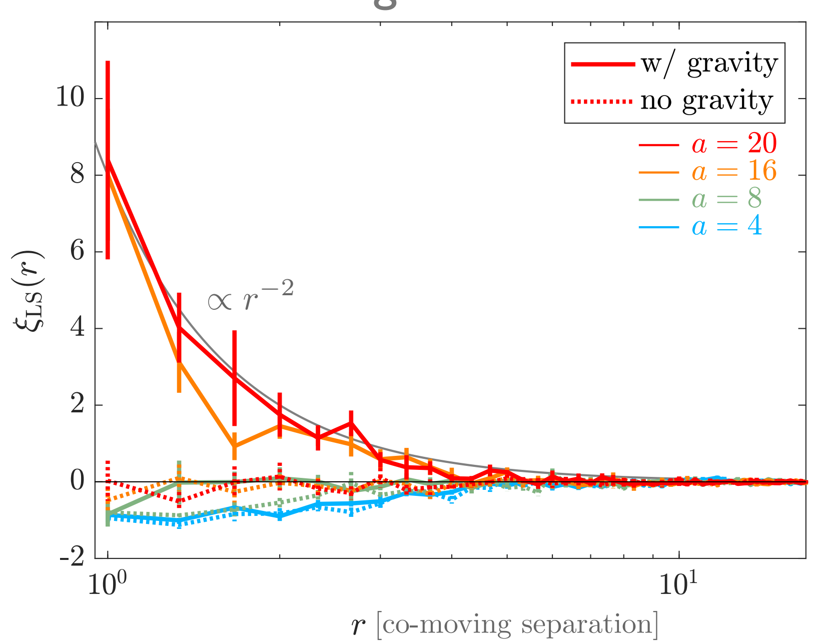
some late-time strong interactions

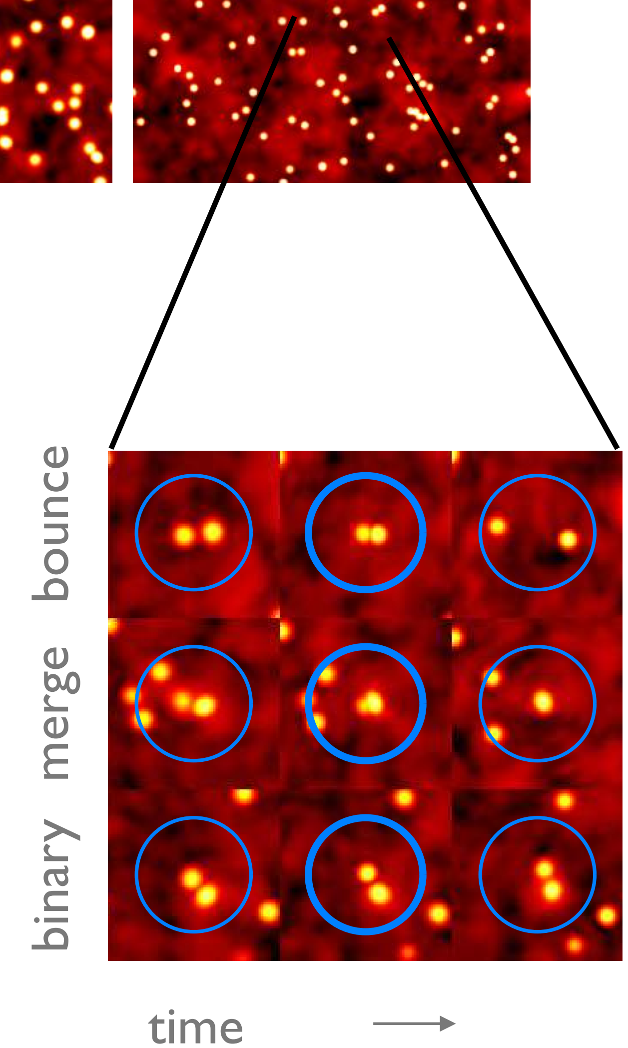


solitons match expectations



evidence for gravitational clustering



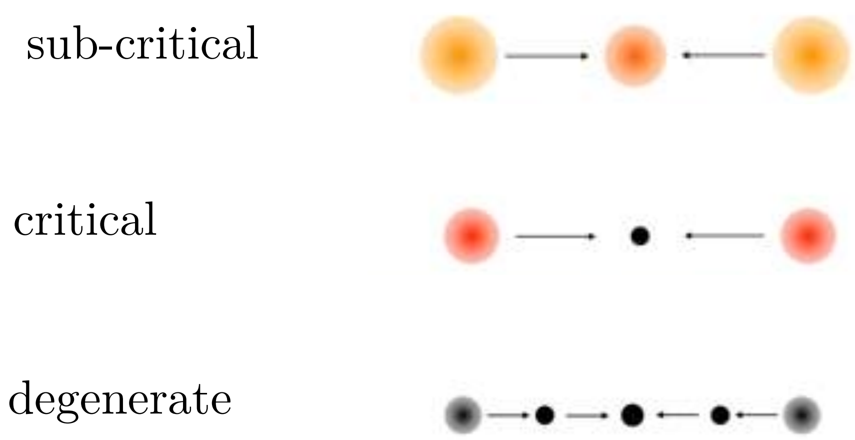


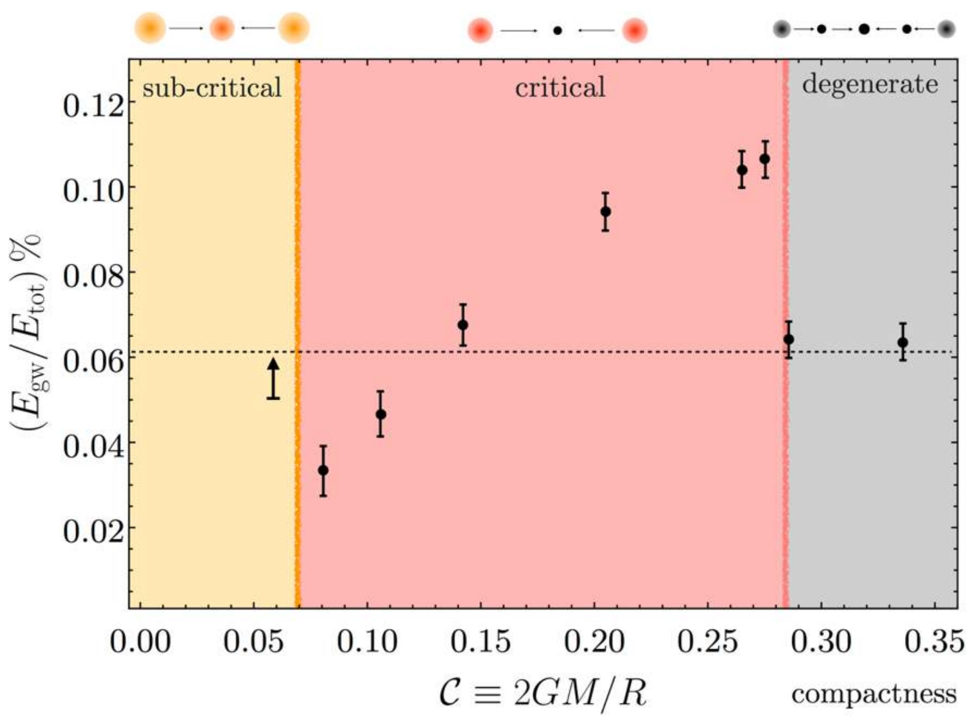
Numerical GR ✓

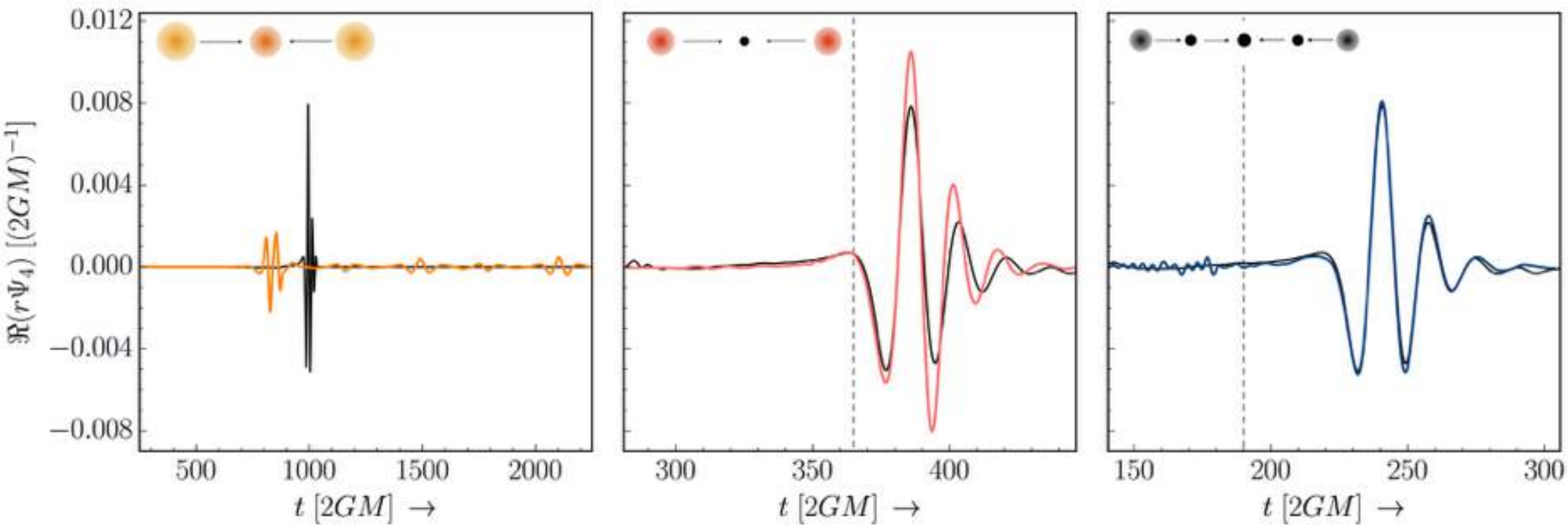
self-interactions ✗

formation ✗

summary 3







thanks