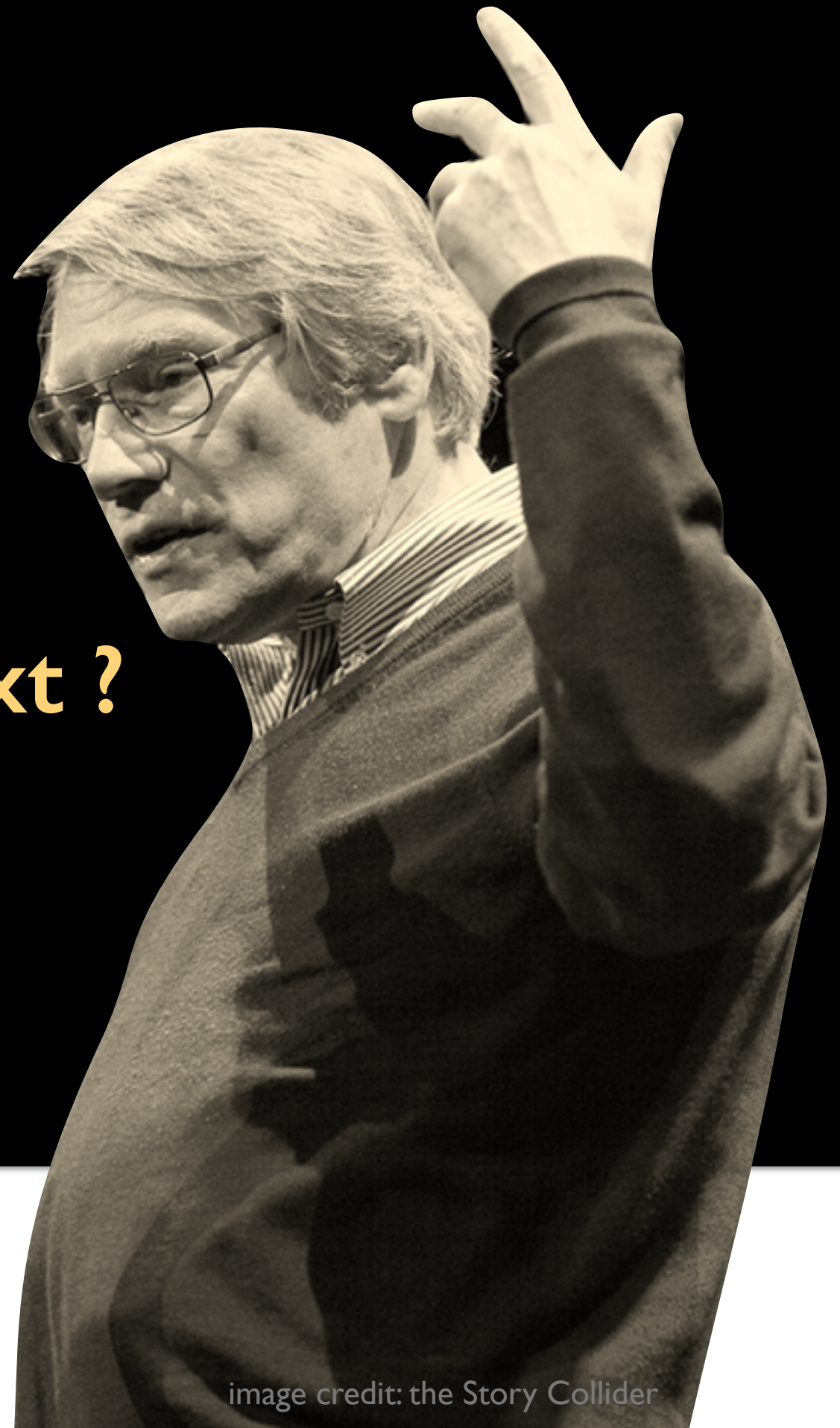


Inflation Ends, What's Next ?

Mustafa A. Amin

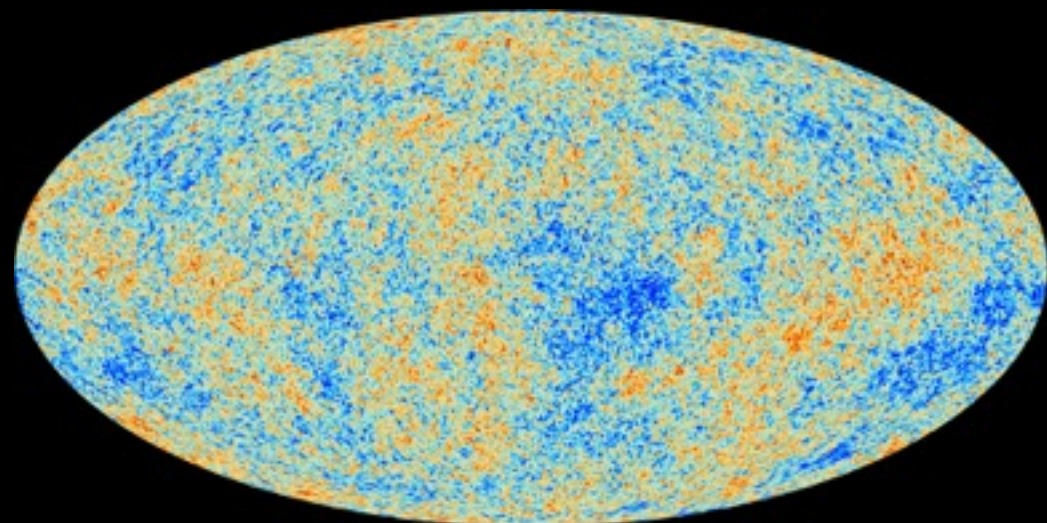


image credit: the Story Collider

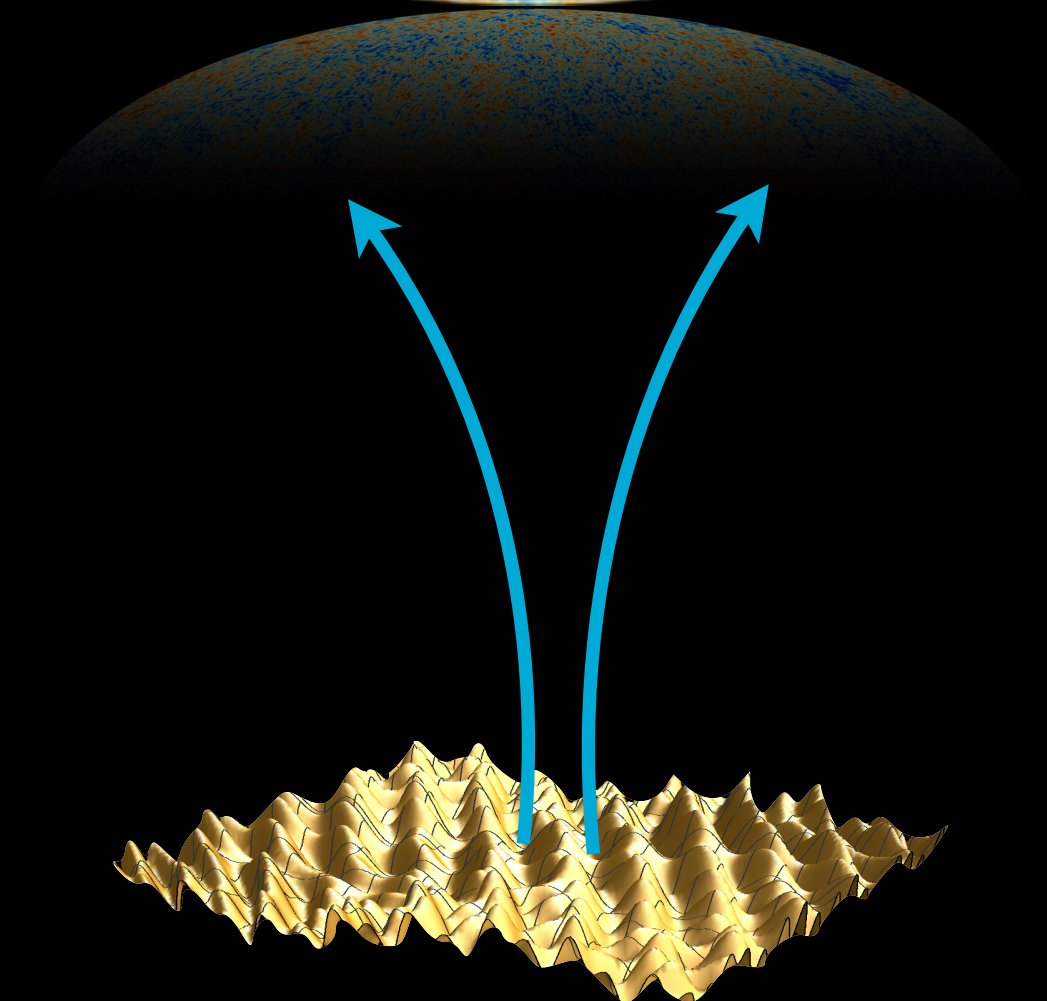
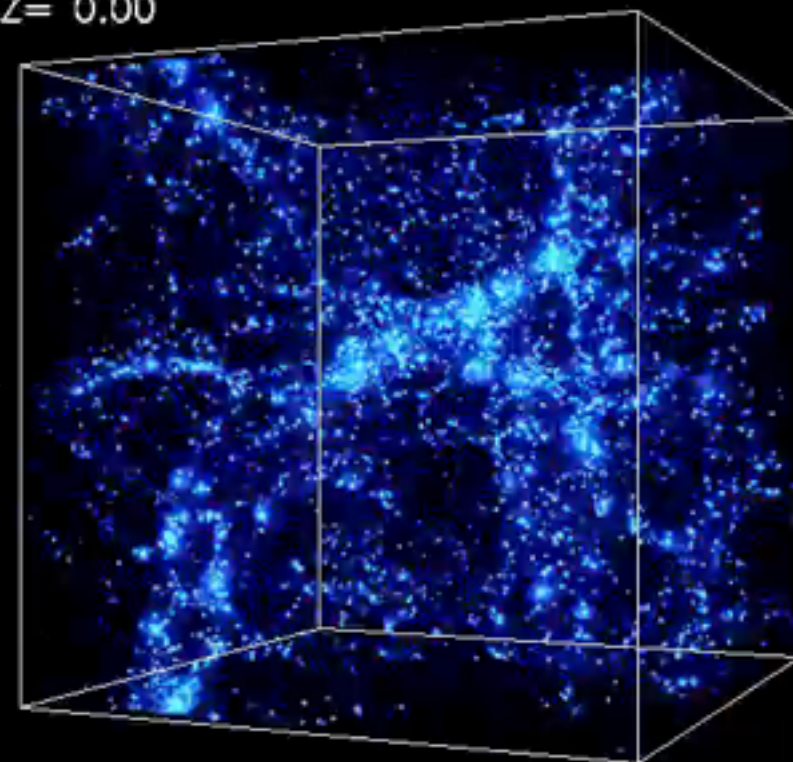




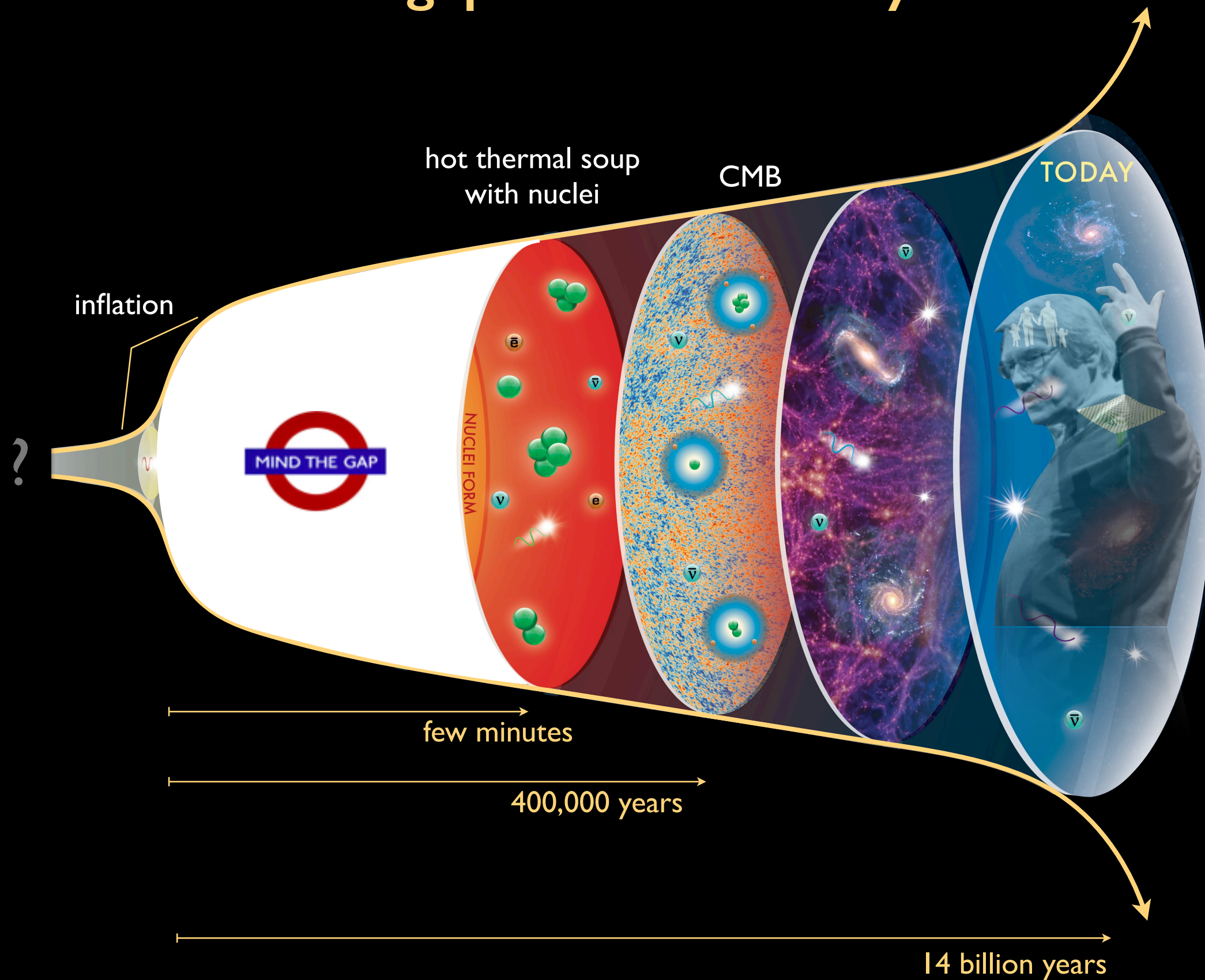
Alan's cosmic history

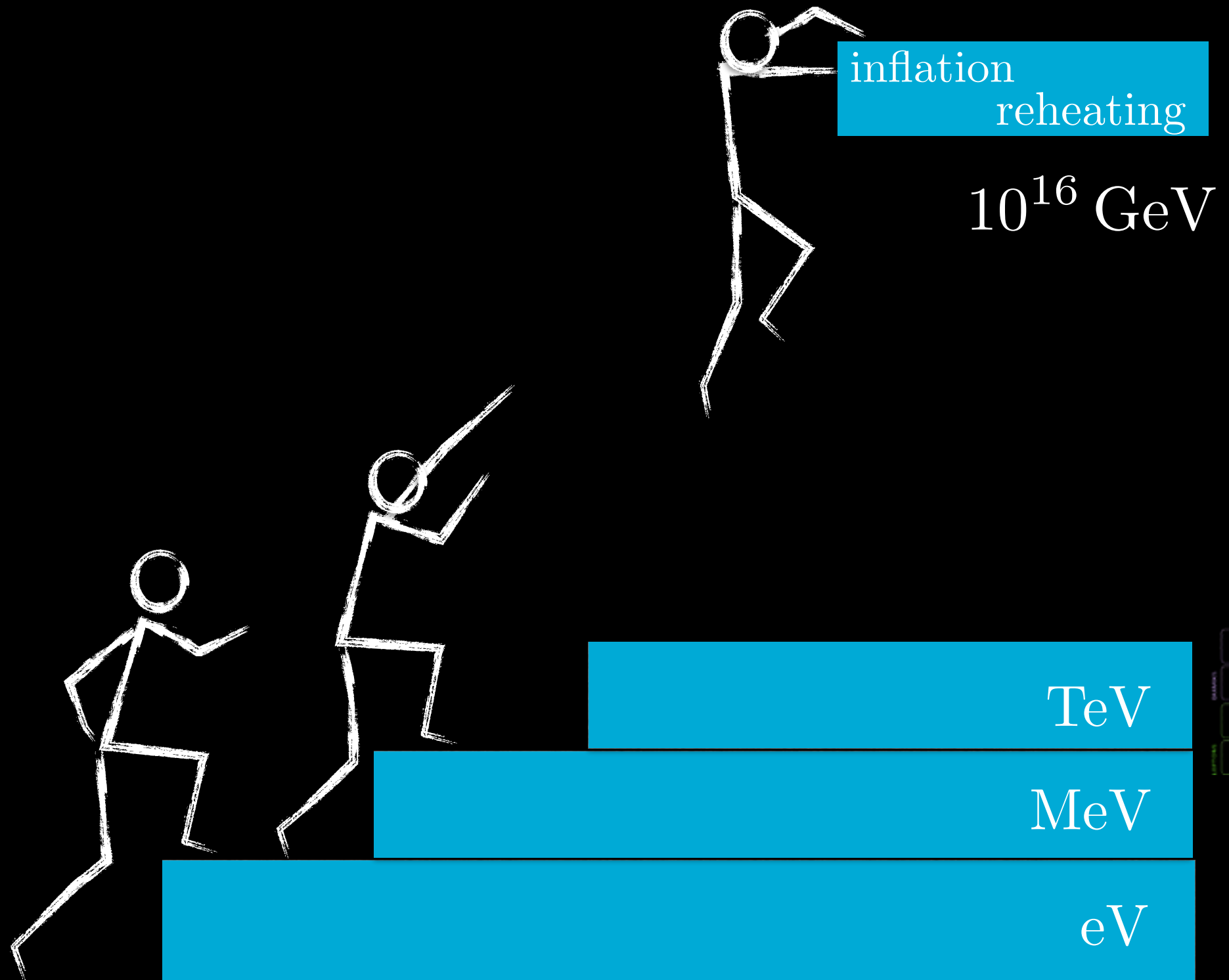


$Z = 0.00$

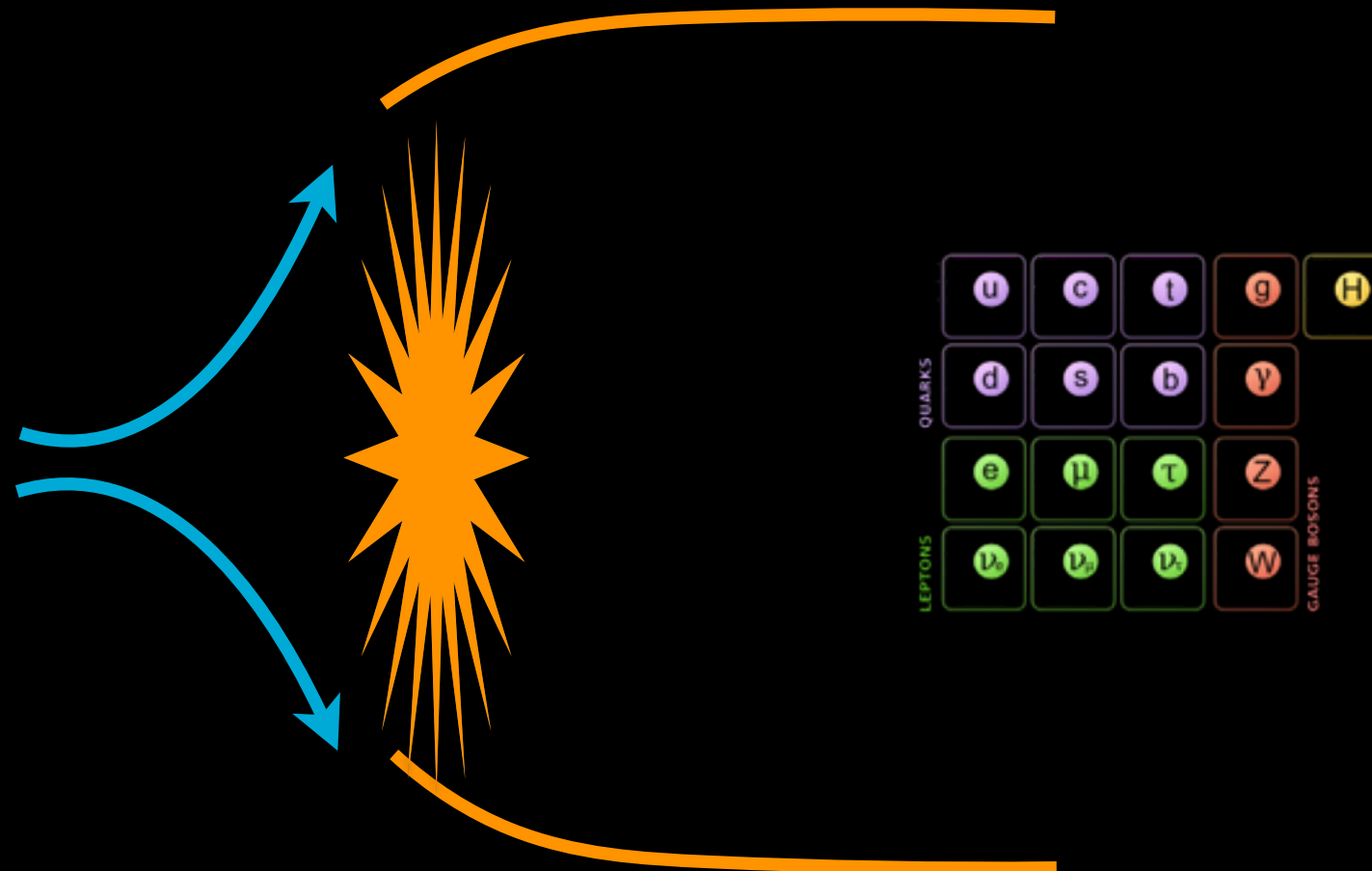


a gap in Alan's history





what happens at the end of inflation ?



COULD THE UNIVERSE HAVE RECOVERED FROM A SLOW FIRST-ORDER PHASE TRANSITION?*

Alan H. GUTH

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Received 3 March 1982
(Revised 6 September 1982)

We investigate the cosmological consequences of a phase transition which is driven primarily by slow nucleation of bubbles of the new phase via the effectively zero temperature quantum tunneling process of Coleman and Callan. These bubbles will asymptotically fill an arbitrarily large fraction of the space, yet they never percolate. Instead they form finite clusters, with each cluster dominated by a single largest bubble. The large scale thermalization required by the original “inflationary universe” scenario does not take place. The Coleman-De Luccia formalism for bubble formation in curved space is reviewed, with minor extensions. We argue that a single uncollided bubble would contain much less total entropy than the observed universe, unless the Higgs field potential involves widely disparate mass scales, as in the new inflationary universe scenario. We also argue that finite clusters are unlikely to yield a homogeneous and isotropic region containing sufficient entropy. Thus, unless the Higgs potential has the special form required by the new inflationary scenario, it appears quite implausible that there was such a phase transition in our past.



at the
White House

end of inflation

SIMPLE

complicated

COMPLEX

end of inflation

SIMPLE

complicated

COMPLEX

end of inflation

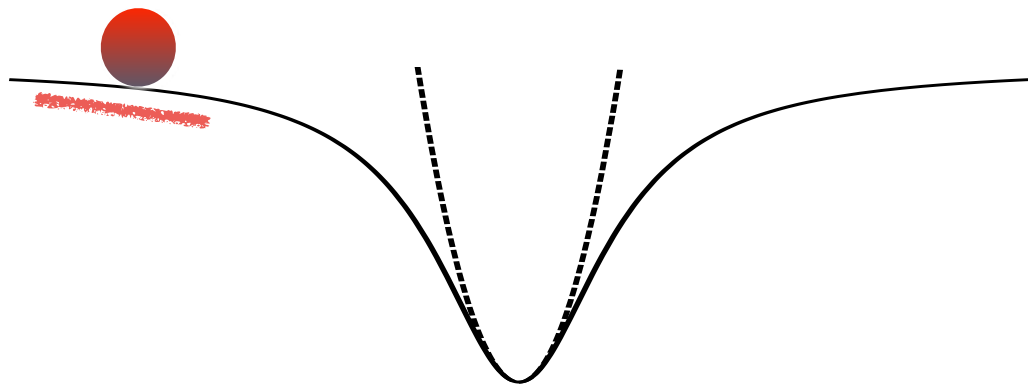
SIMPLE

complicated

COMPLEX

constraints from observations

$$V(\phi) \propto \phi^p$$

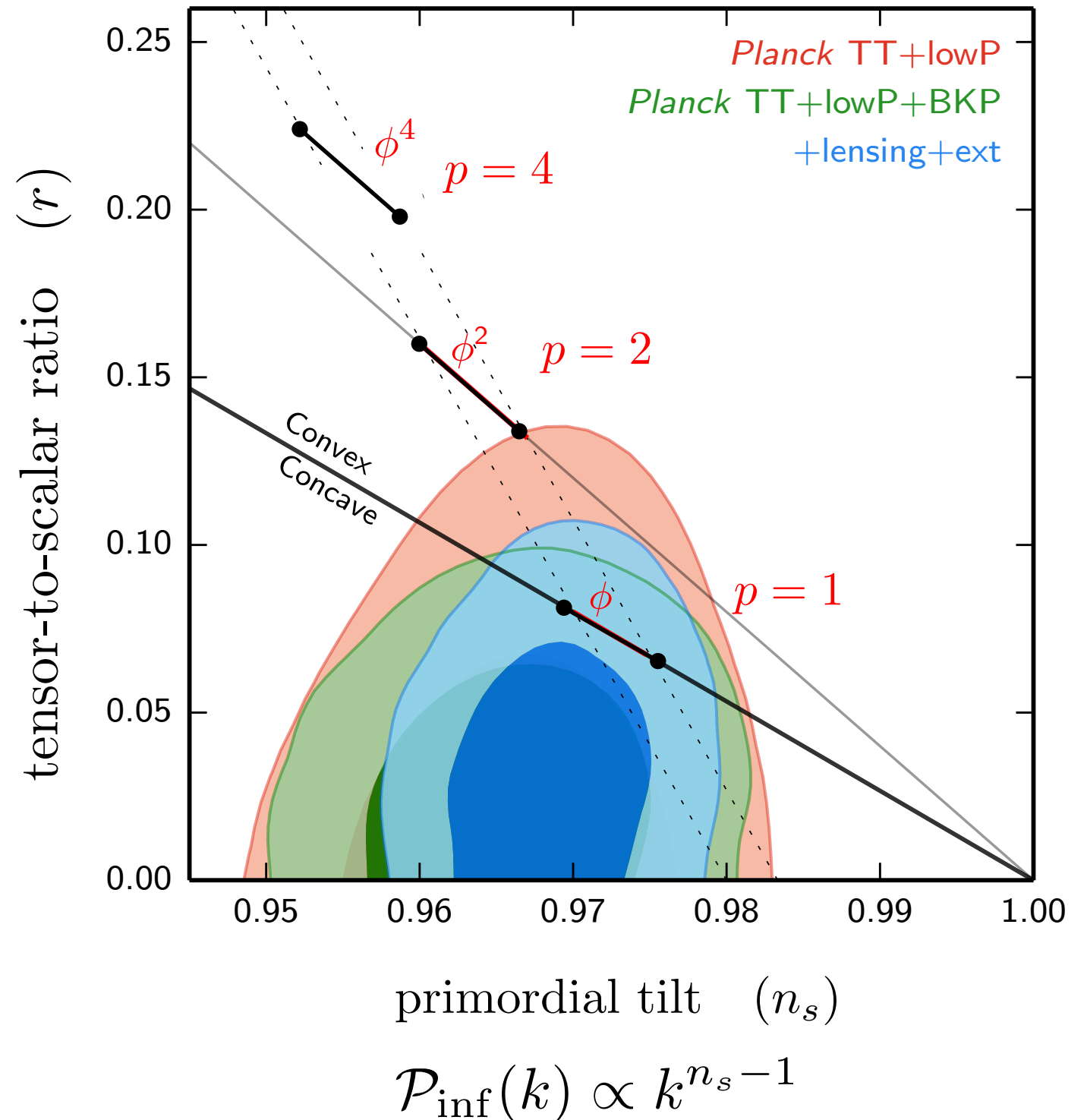


for example:

Silverstein & Westphal (2008)

McAllister et. al (2014)

Kalosh & Linde (2014)



inflation ends, details depend on:

- shape of the potential (self couplings)
- couplings to other fields

χ, ψ



mass	charge	spin	particle
$< 0.3 \text{ MeV}/c^2$	$2/3$	$1/2$	u (up)
$< 1.275 \text{ GeV}/c^2$	$2/3$	$1/2$	c (charm)
$< 173.1 \text{ GeV}/c^2$	$2/3$	$1/2$	t (top)
0	0	1	g (gluon)
$< 125 \text{ GeV}/c^2$	0	0	H (Higgs boson)
$< 4.8 \text{ MeV}/c^2$	$-1/3$	$1/2$	d (down)
$< 95 \text{ MeV}/c^2$	$-1/3$	$1/2$	s (strange)
$< 4.2 \text{ GeV}/c^2$	$-1/3$	$1/2$	b (bottom)
0	0	1	γ (photon)
$< 0.511 \text{ MeV}/c^2$	-1	$1/2$	e (electron)
$< 105.7 \text{ MeV}/c^2$	-1	$1/2$	μ (muon)
$< 1.777 \text{ GeV}/c^2$	-1	$1/2$	τ (tau)
$< 91.2 \text{ GeV}/c^2$	0	1	Z (Z boson)
$< 0.2 \text{ eV}/c^2$	0	$1/2$	ν_e (electron neutrino)
$< 0.17 \text{ MeV}/c^2$	0	$1/2$	ν_μ (muon neutrino)
$< 1.8 \text{ MeV}/c^2$	0	$1/2$	ν_τ (tau neutrino)
$< 80.4 \text{ GeV}/c^2$	± 1	1	W (W boson)

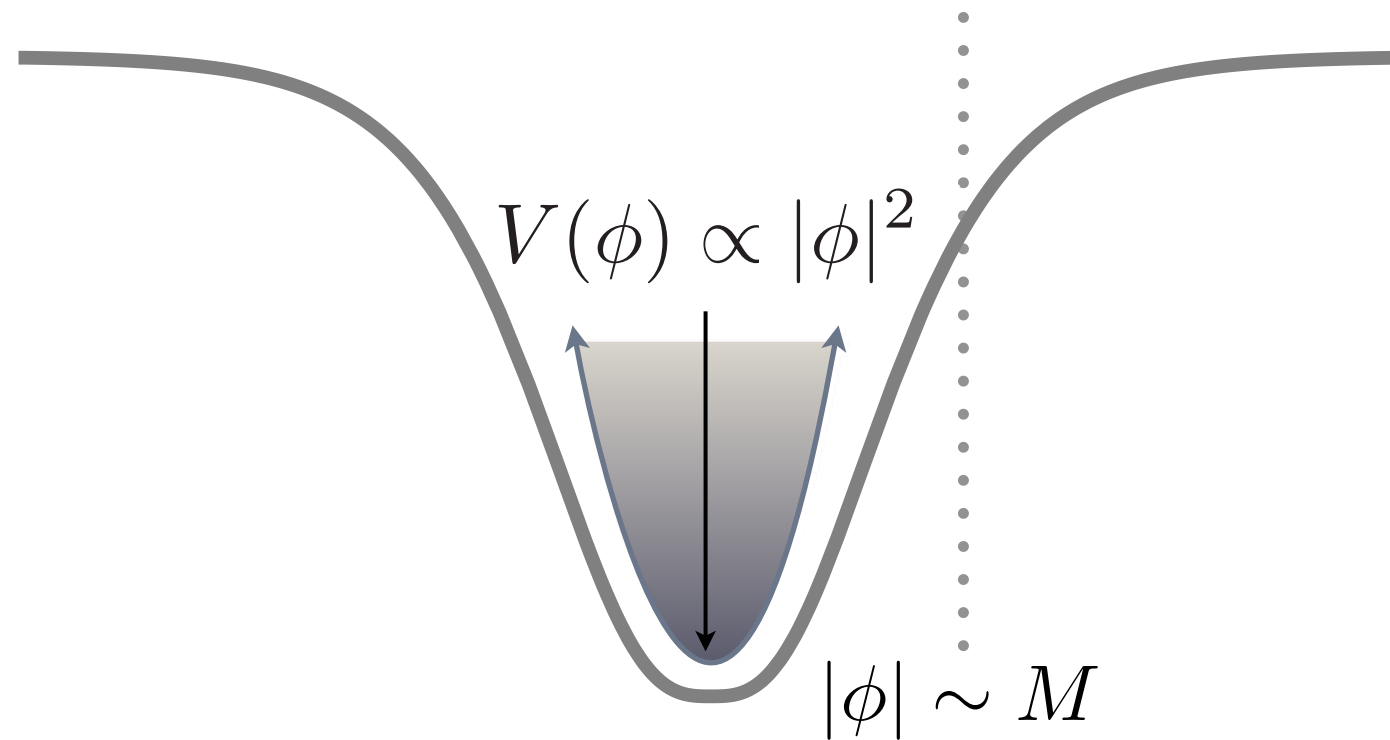
for example:

Kofman, Linde & Starobinsky (1994)

Shtanov, Traschen & Brandenberger (1995)

review: MA, Kaiser, Karouby & Hertzberg (2014)

end of inflation in “simple” models



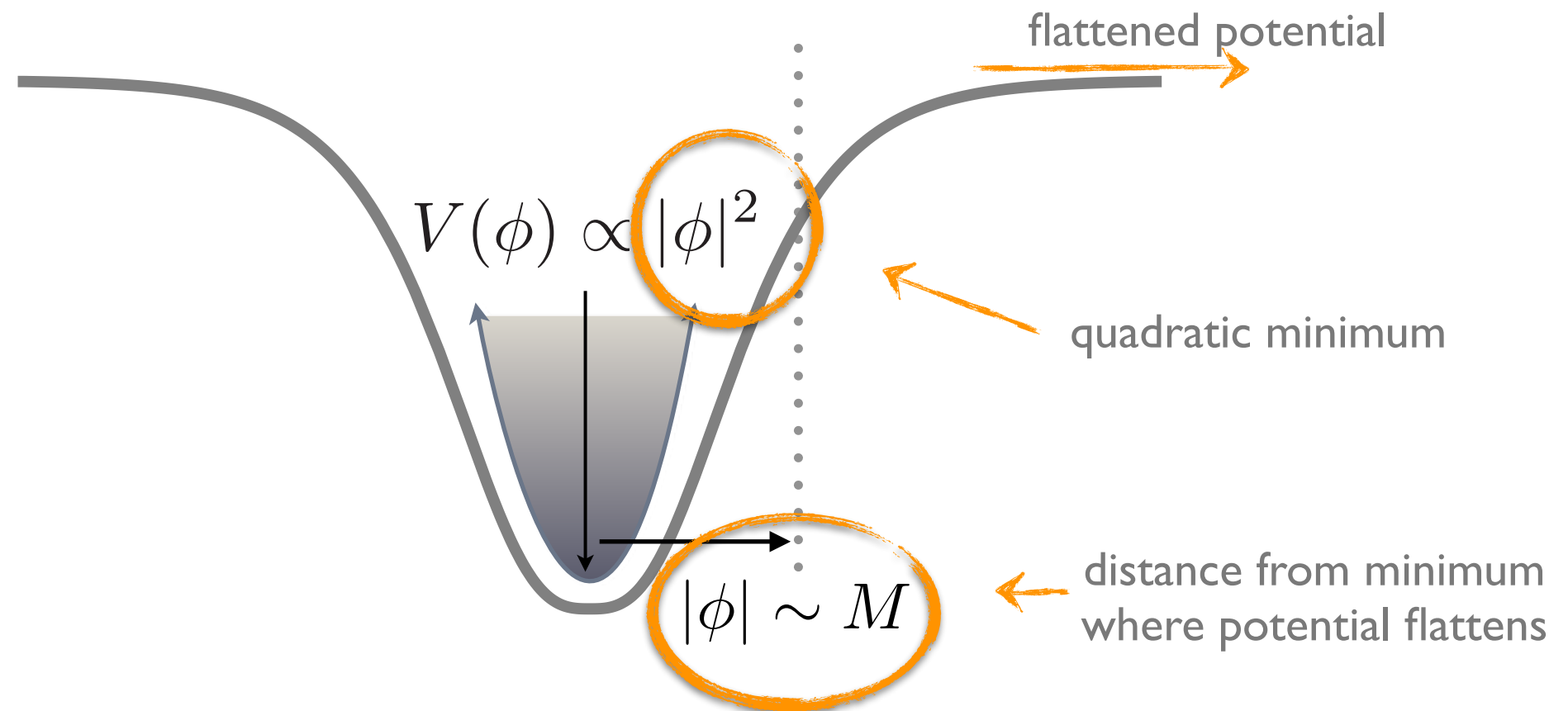
- shape of the potential (self couplings)

- ~~couplings to other fields~~



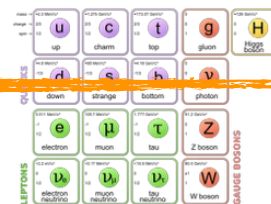
χ, ψ

end of inflation in “simple” models



- shape of the potential (self couplings)

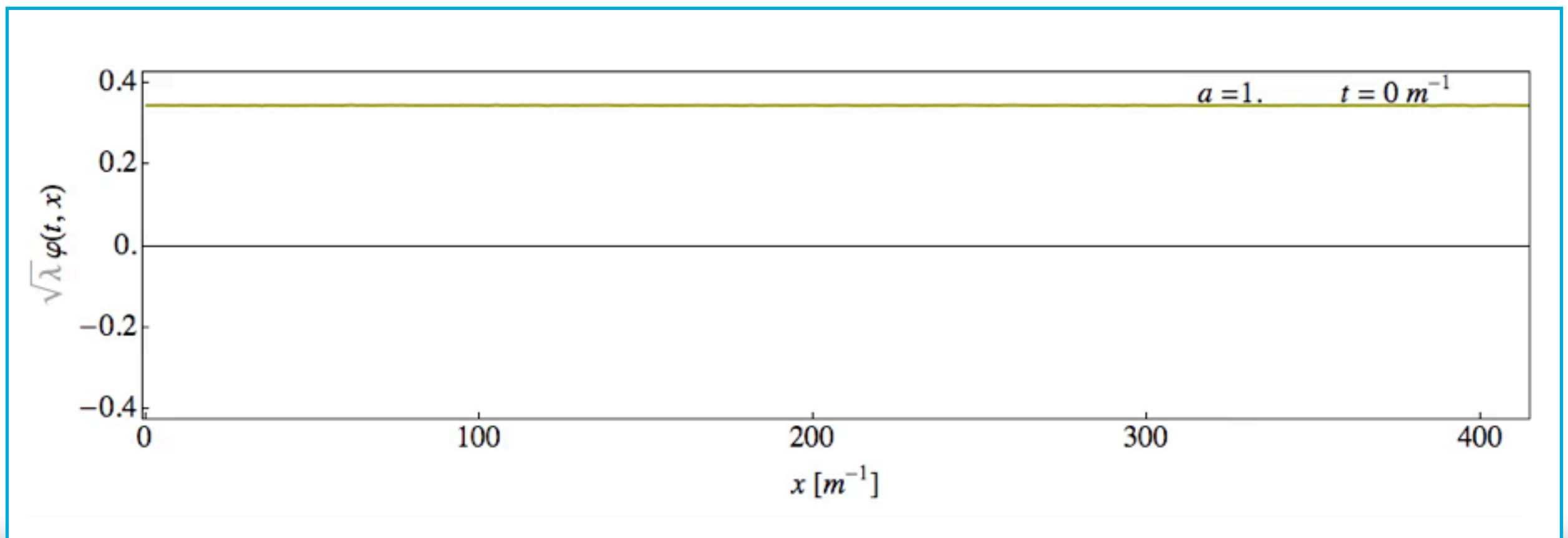
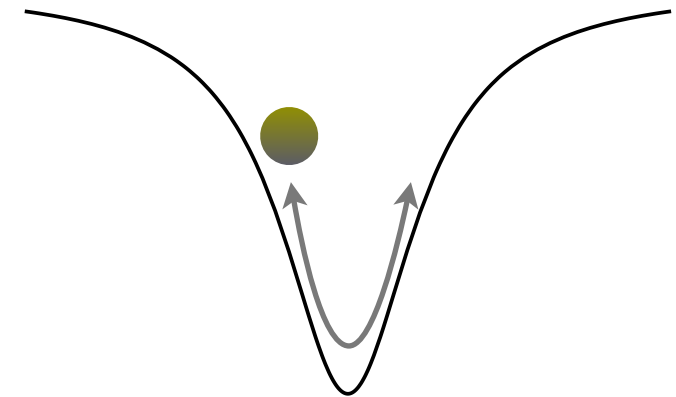
- ~~couplings to other fields~~



χ, ψ

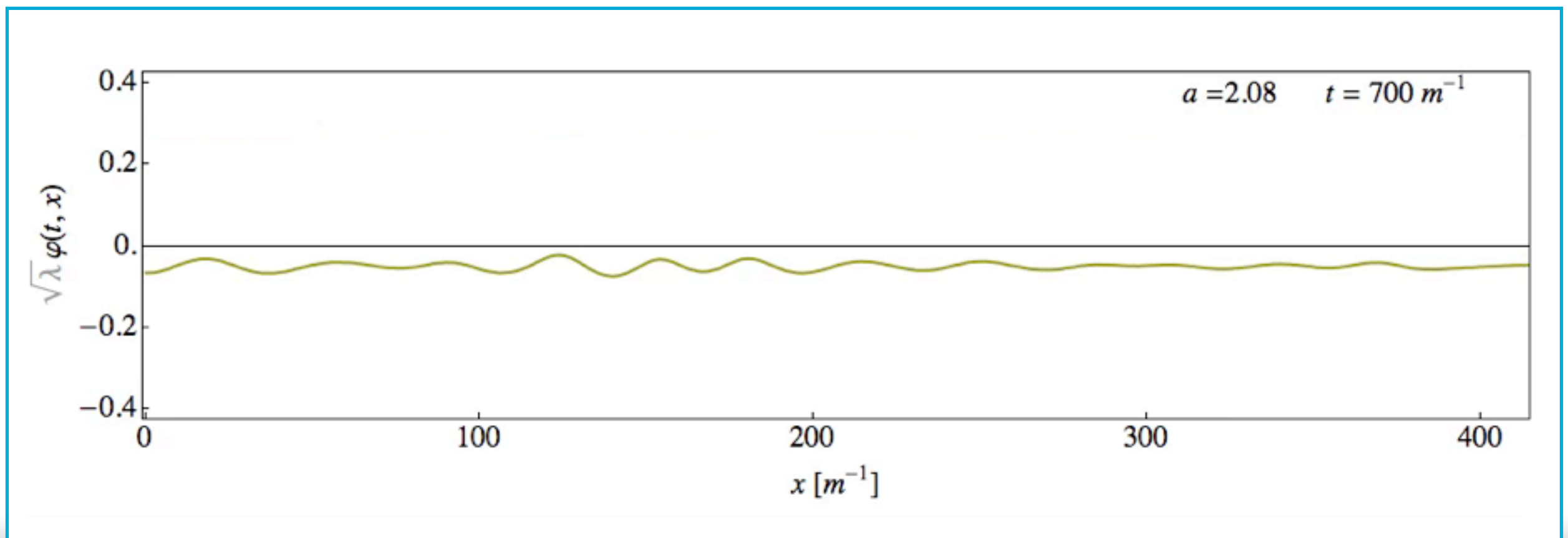
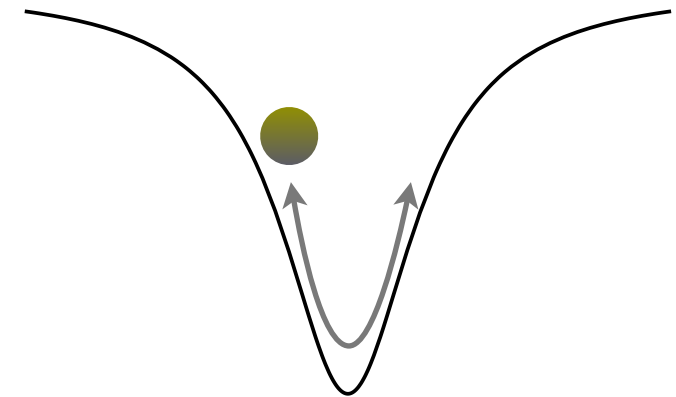
dynamics after inflation

$$\square \varphi = V'(\varphi)$$



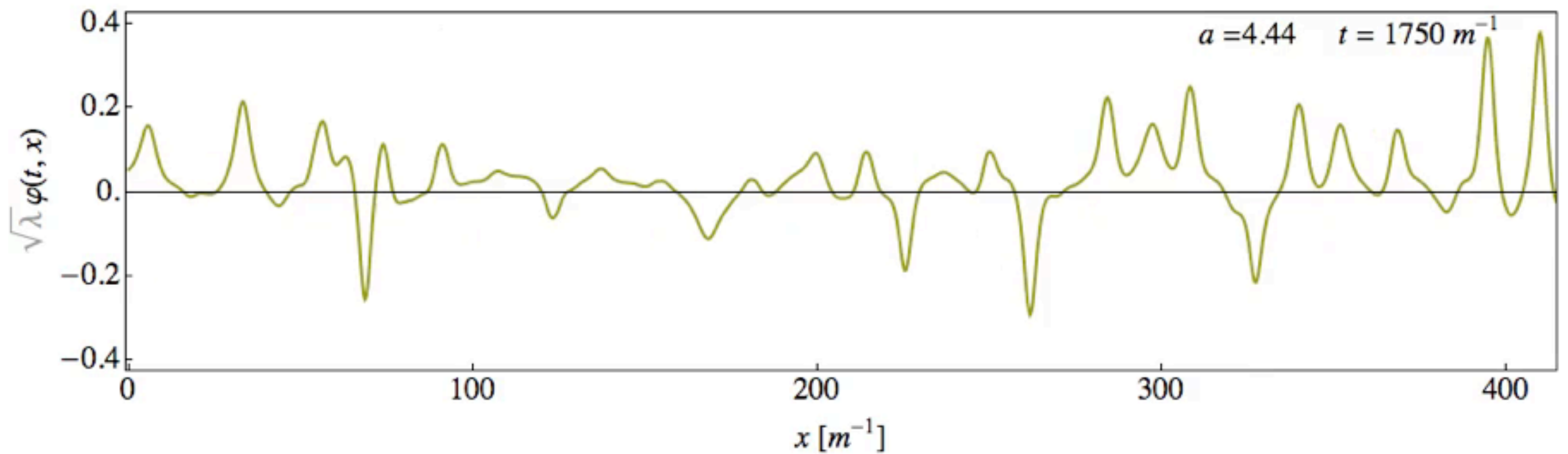
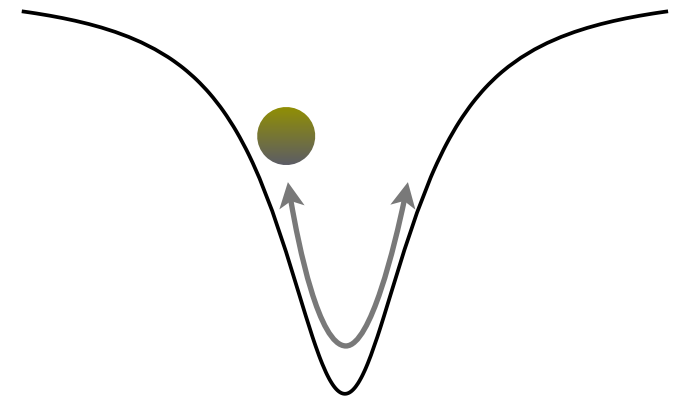
dynamics after inflation

$$\square \varphi = V'(\varphi)$$



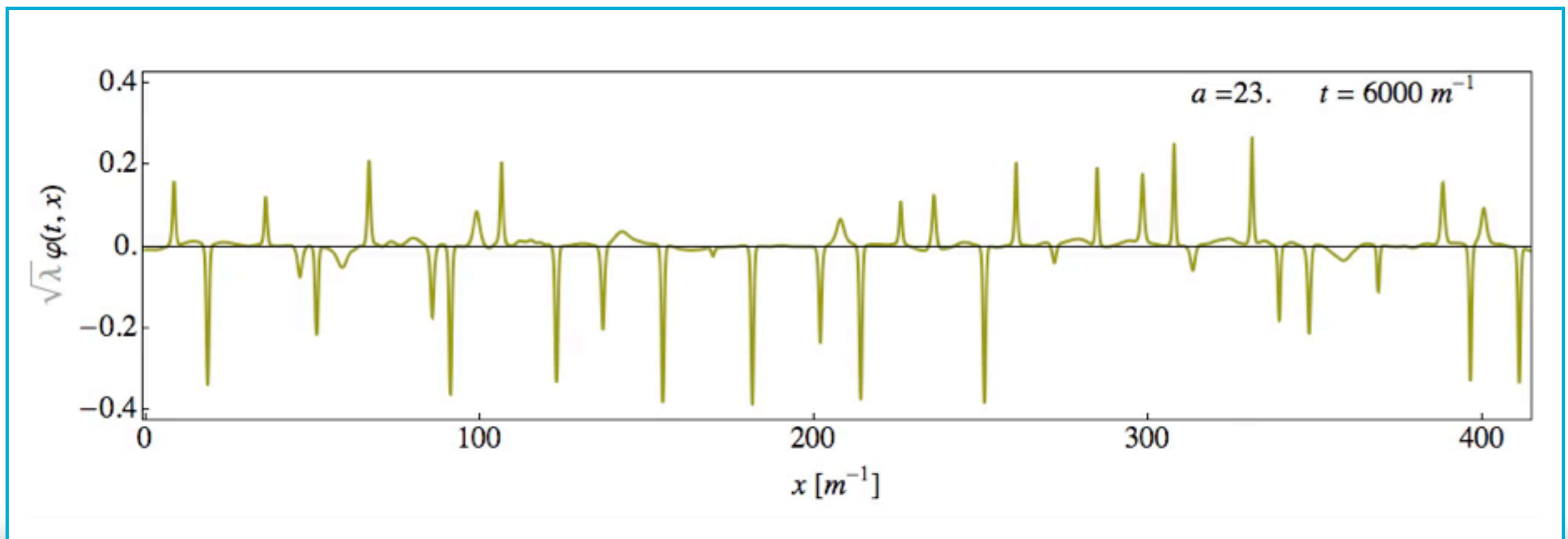
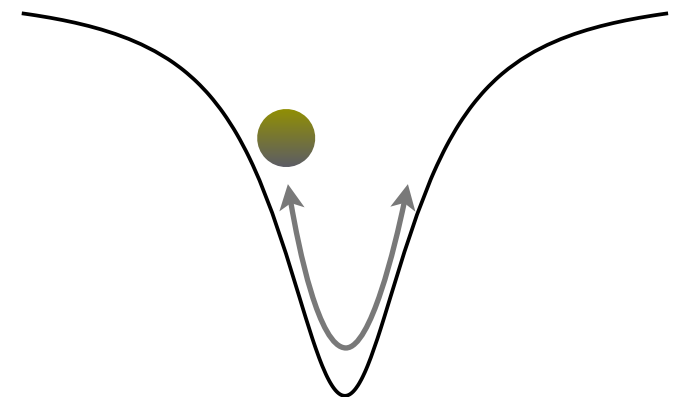
dynamics after inflation

$$\square \varphi = V'(\varphi)$$



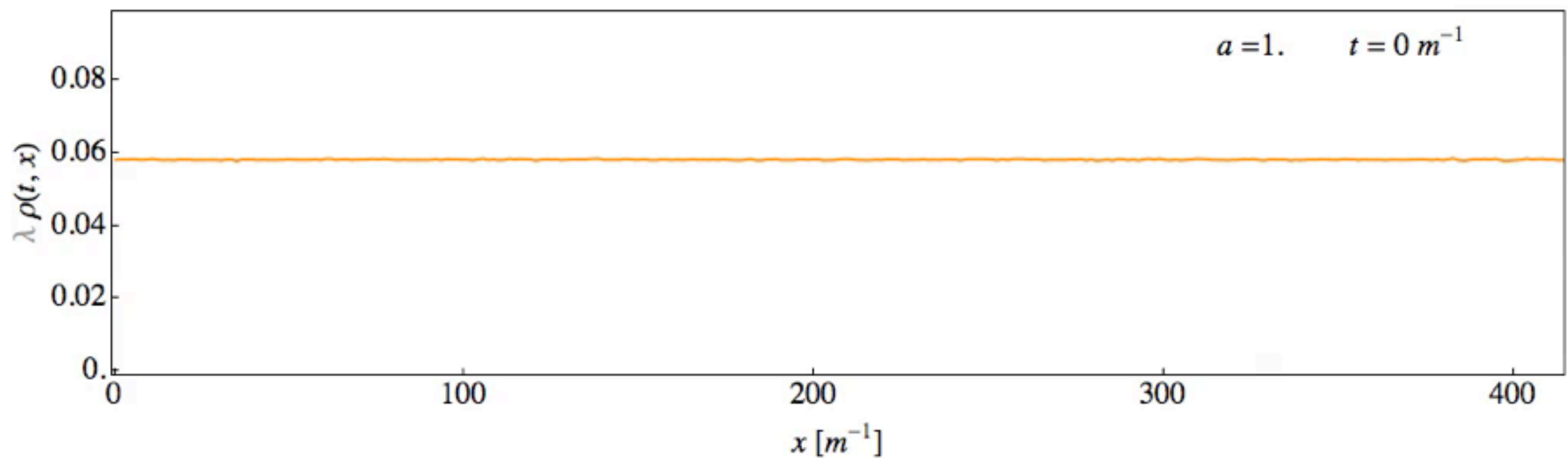
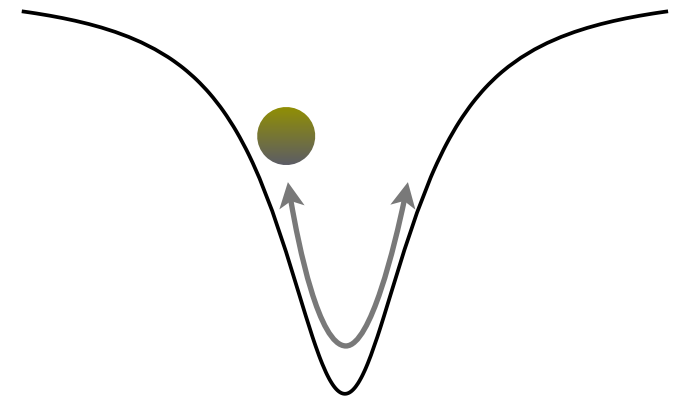
dynamics after inflation

$$\square \varphi = V'(\varphi)$$



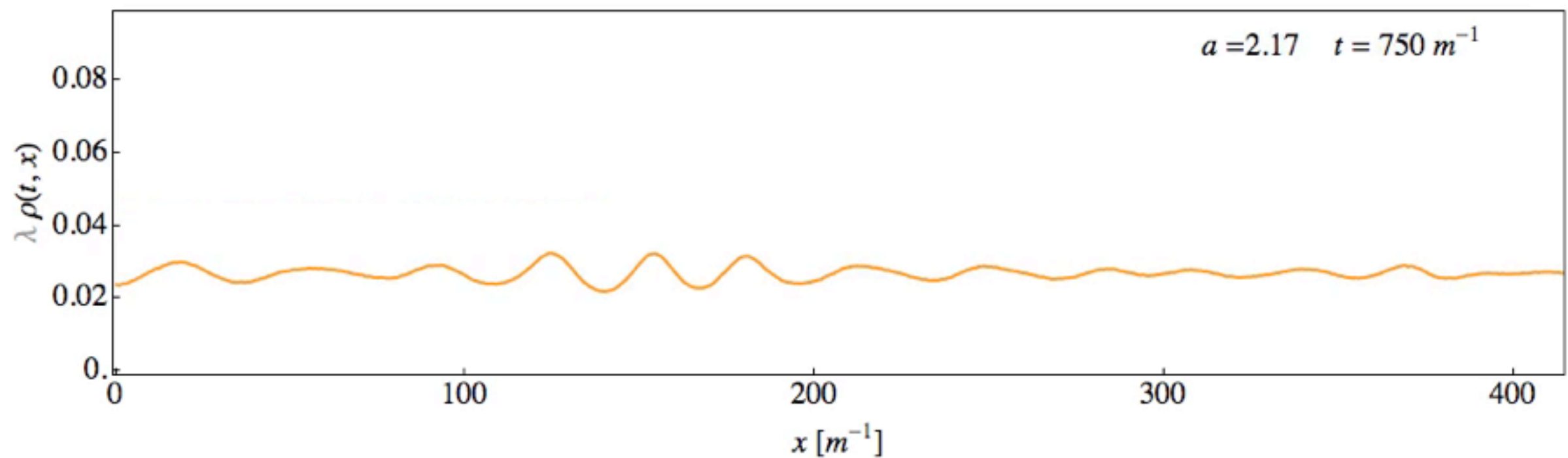
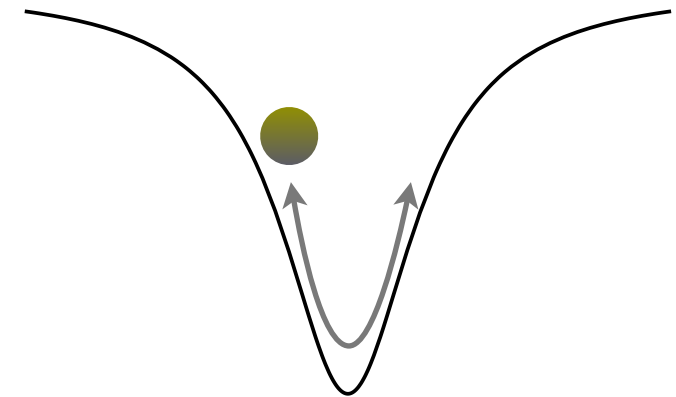
dynamics after inflation

$$\square \varphi = V'(\varphi)$$



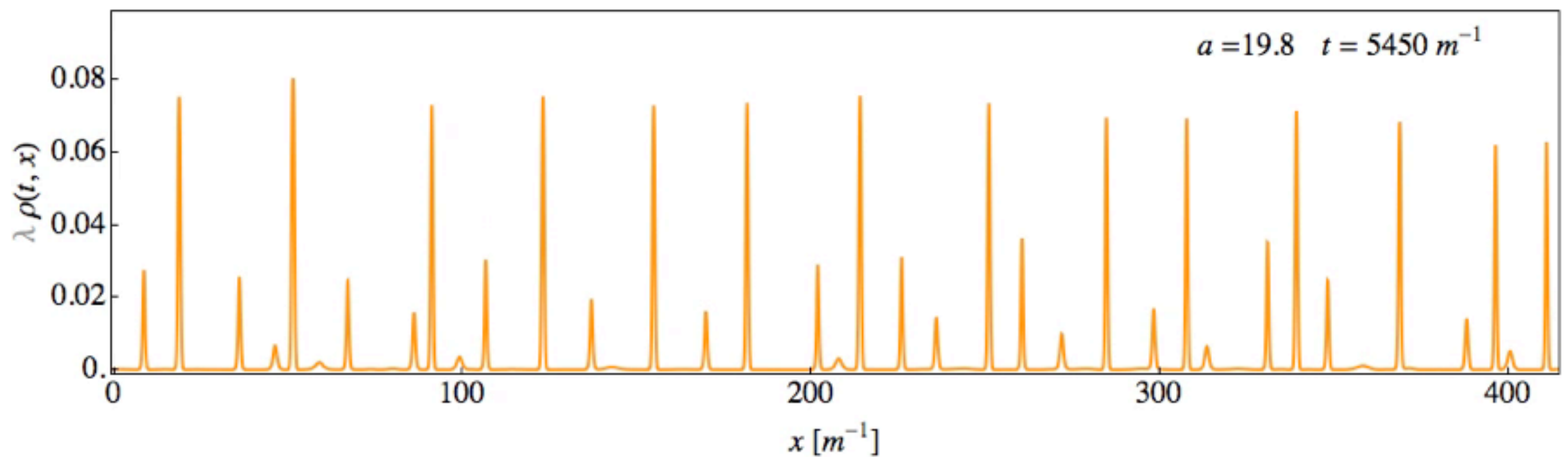
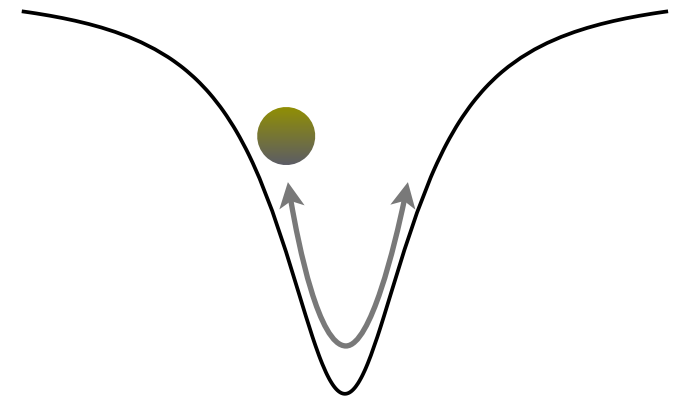
dynamics after inflation

$$\square \varphi = V'(\varphi)$$

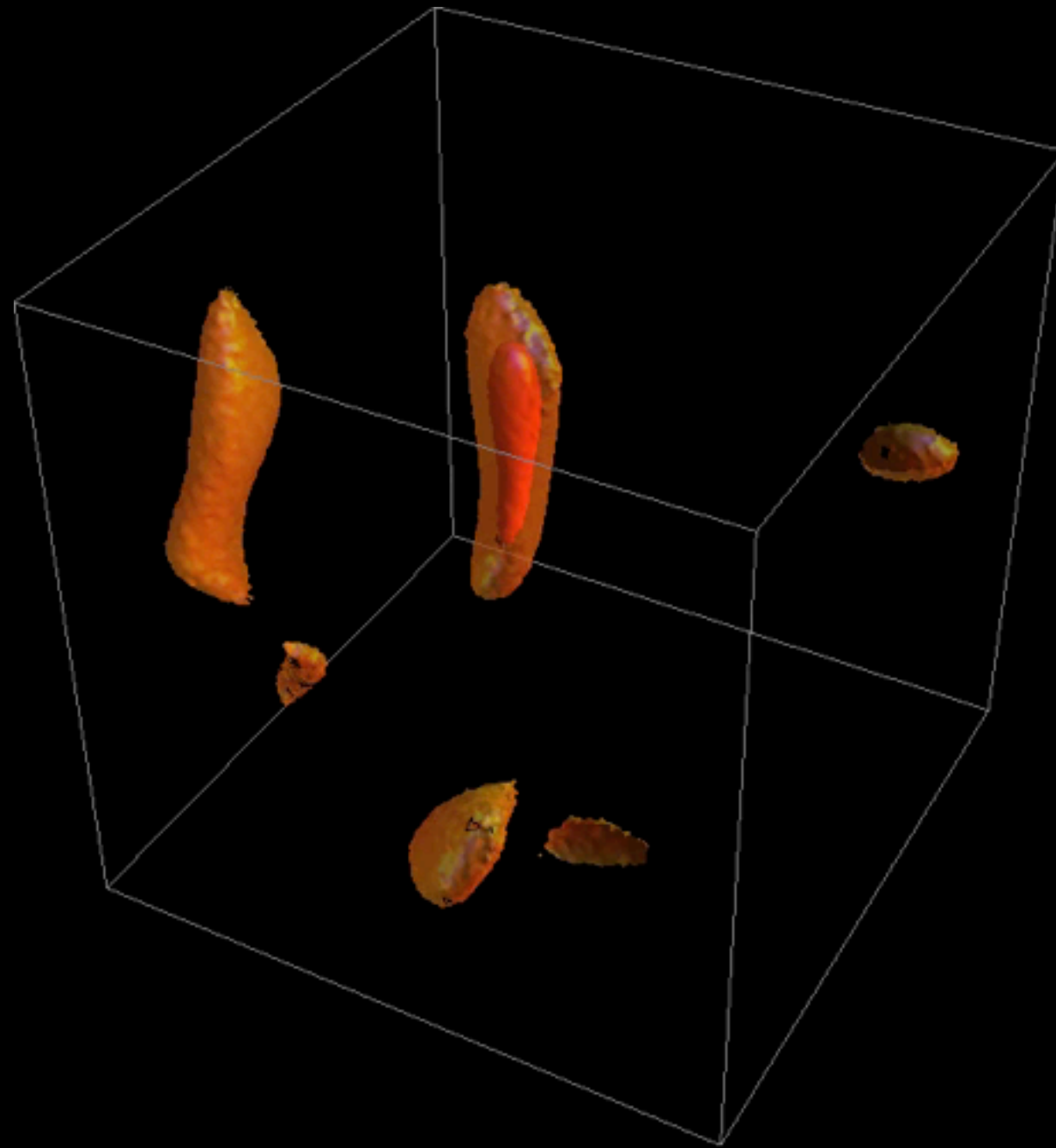


dynamics after inflation

$$\square \varphi = V'(\varphi)$$

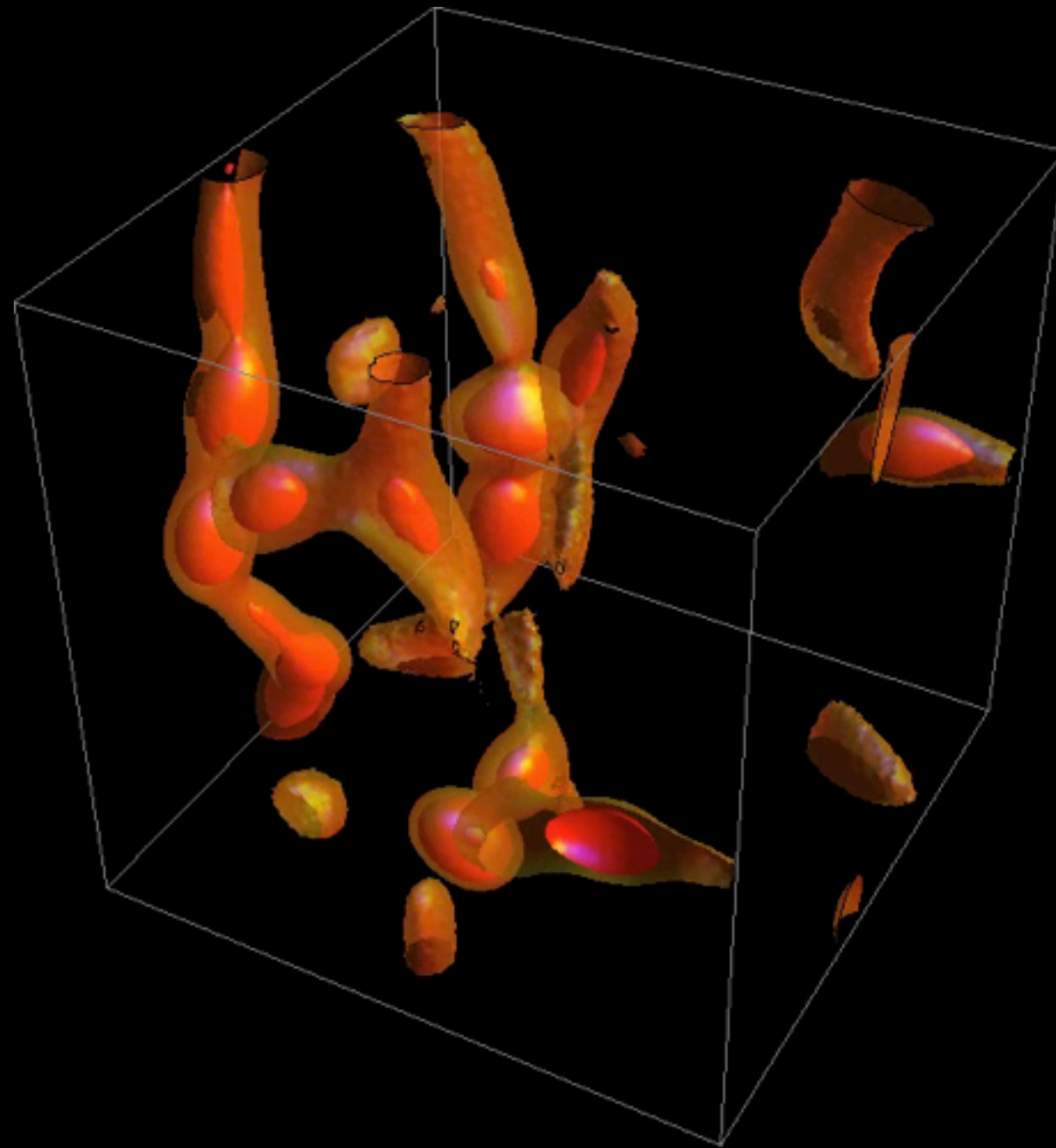


now in 3D:
(iso-density surfaces)



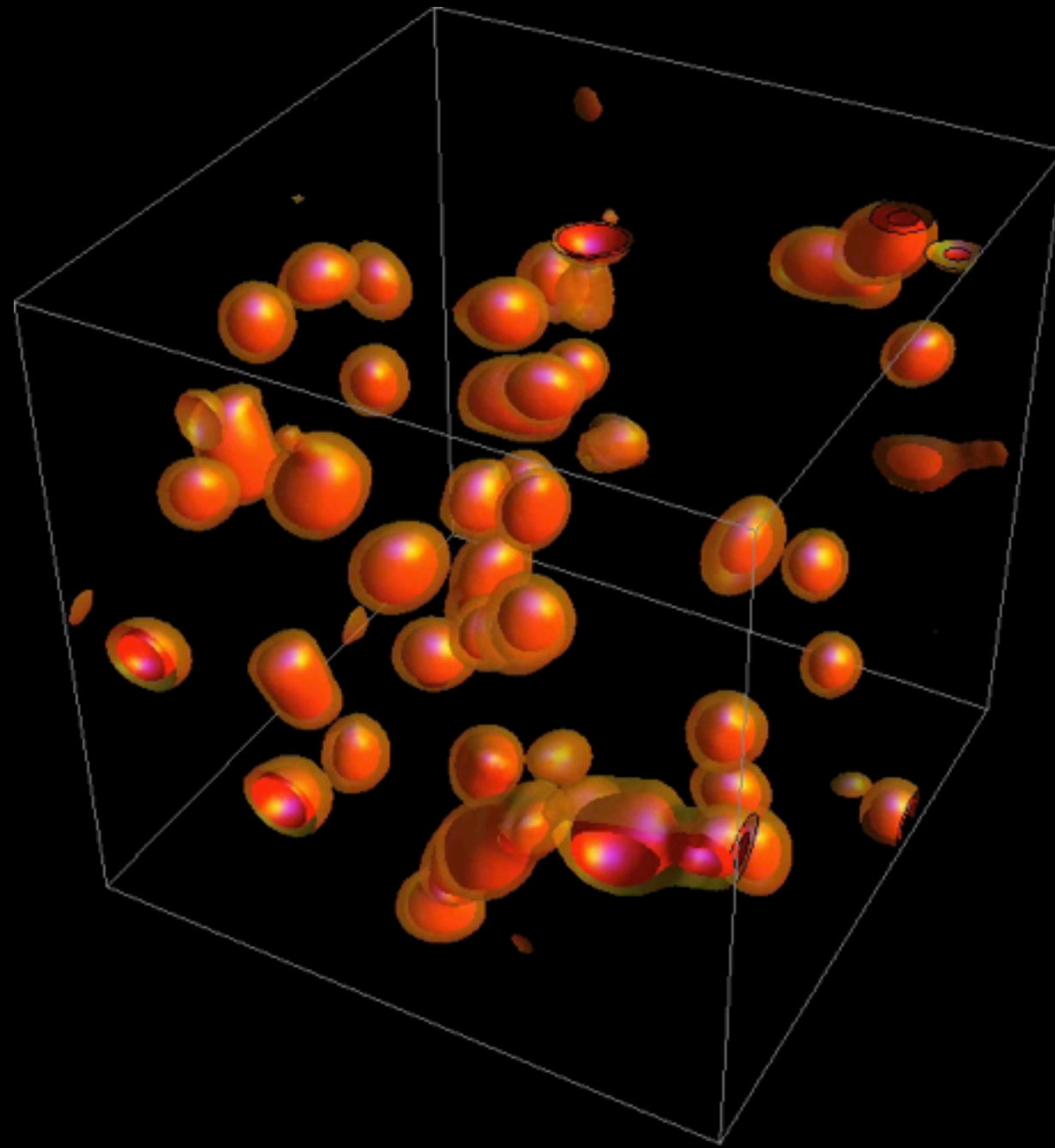
MA, Easter, Finkel, Flaughner & Hertzberg (2011)

now in 3D:
(iso-density surfaces)



MA, Easter, Finkel, Flaugher & Hertzberg (2011)

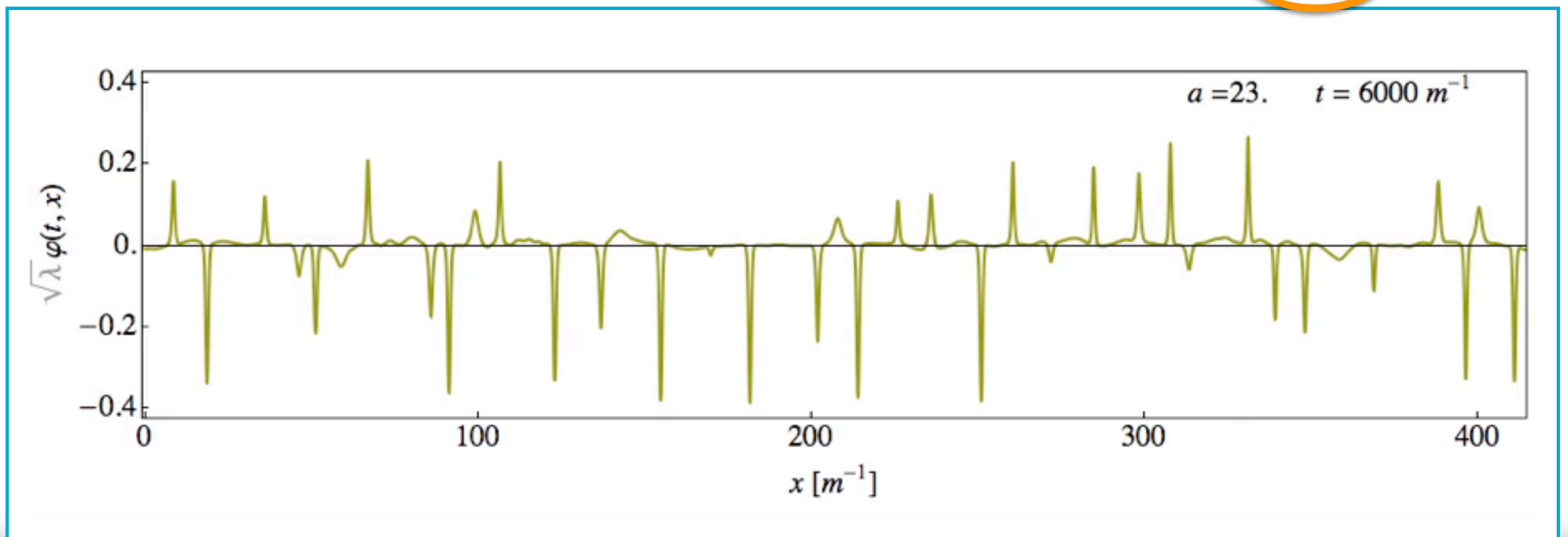
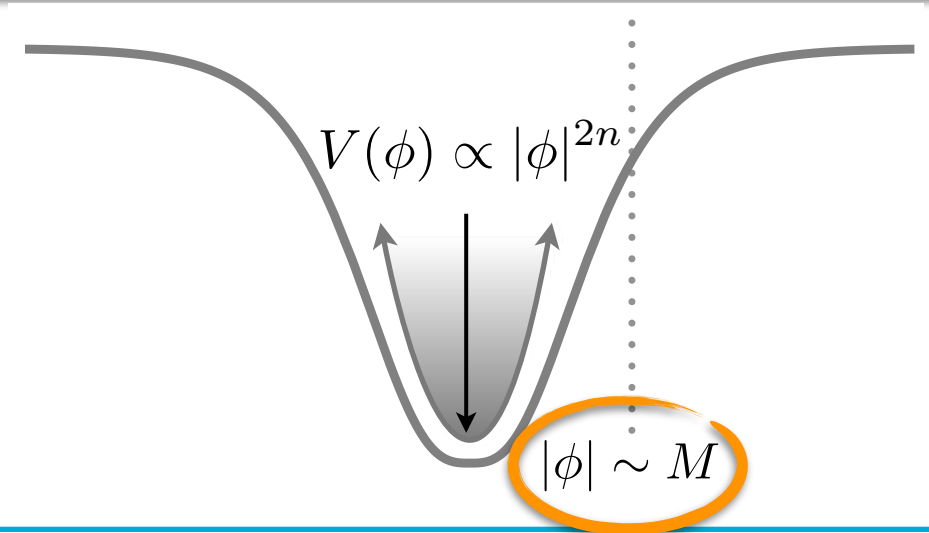
now in 3D: (iso-density surfaces)



MA, Easter, Finkel, Flaugher & Hertzberg (2011)

condition for emergence of oscillons after inflation

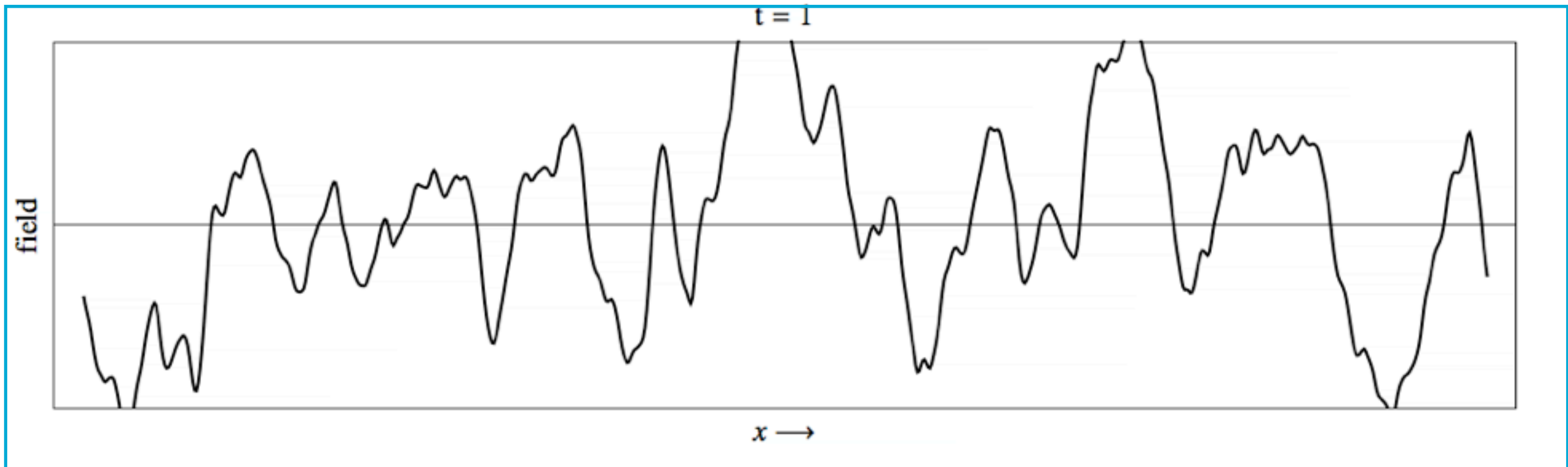
$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\text{pl}}}{M} \gg 1$$



has Alan thought about this ?

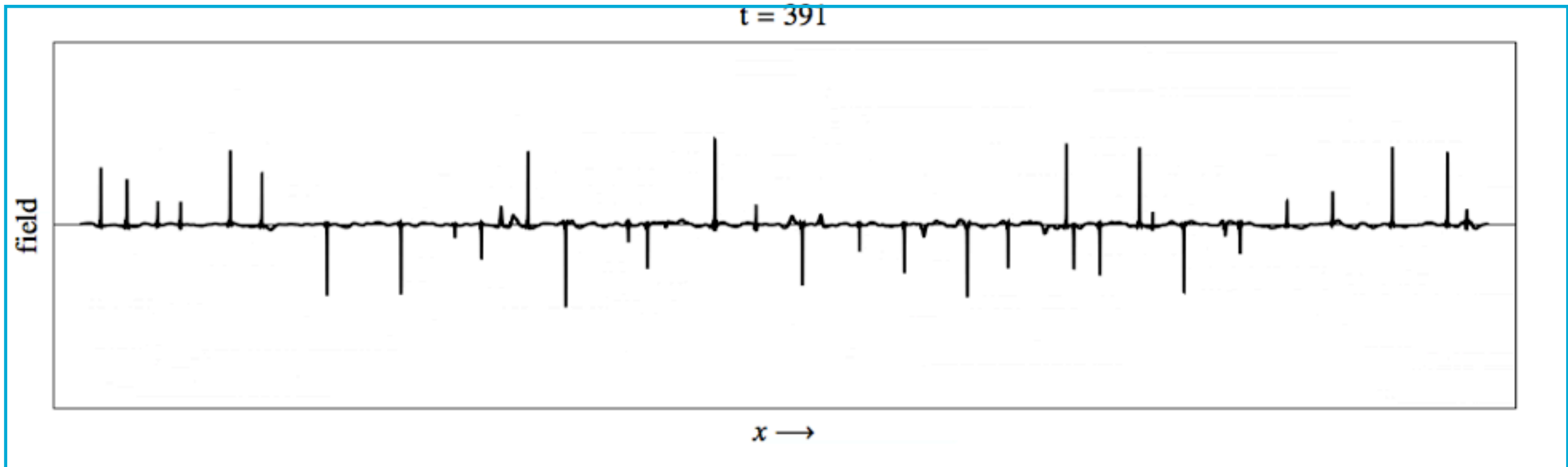


“quasi-thermal” field configuration



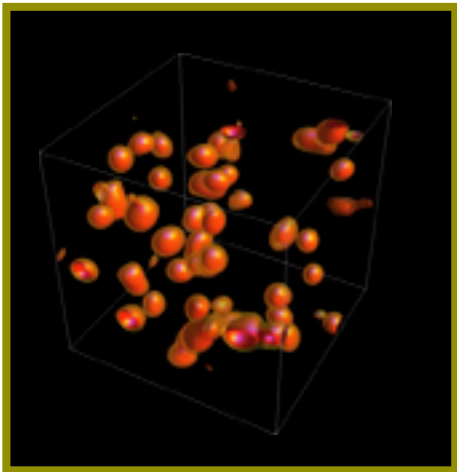
simulation of “quasi-thermal” example in
Farhi, Graham, **Guth**, Iqbal, Rosales, Stamatopoulos 2008

“quasi-thermal” field configuration



simulation of “quasi-thermal” example in
Farhi, Graham, **Guth**, Iqbal, Rosales, Stamatopoulos 2008

lumps ?



(1) oscillatory (2) spatially localized (3) **very long lived**

$$\mathcal{L} = T(X, \varphi) - V(\varphi)$$

$$T(X, \varphi) = X + \xi_2 X^2 + \xi_3 \varphi X^2 + \dots$$

$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{\lambda_3}{3}\varphi^3 + \frac{\lambda_4}{4}\varphi^4 + \frac{\lambda_5}{5}\varphi^5 + \dots$$

$$\Delta = \xi_2 - \lambda_4 + \frac{10}{9}\lambda_3^2 > 0.$$

existence and stability:

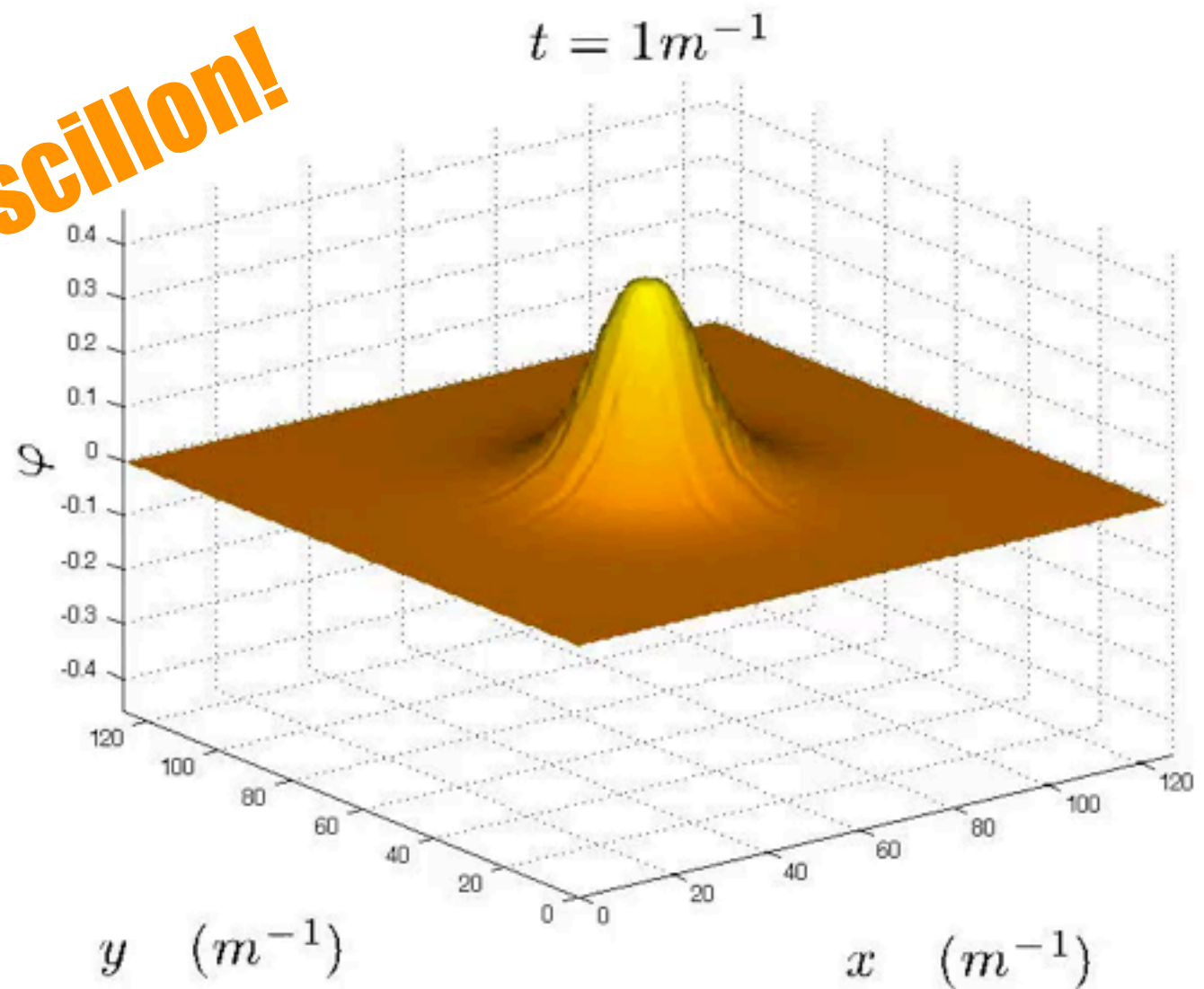
MA (2013)

MA & Shirokoff (2010)

Hertzberg (2011)

Sfakianakis (2015)

oscillon!

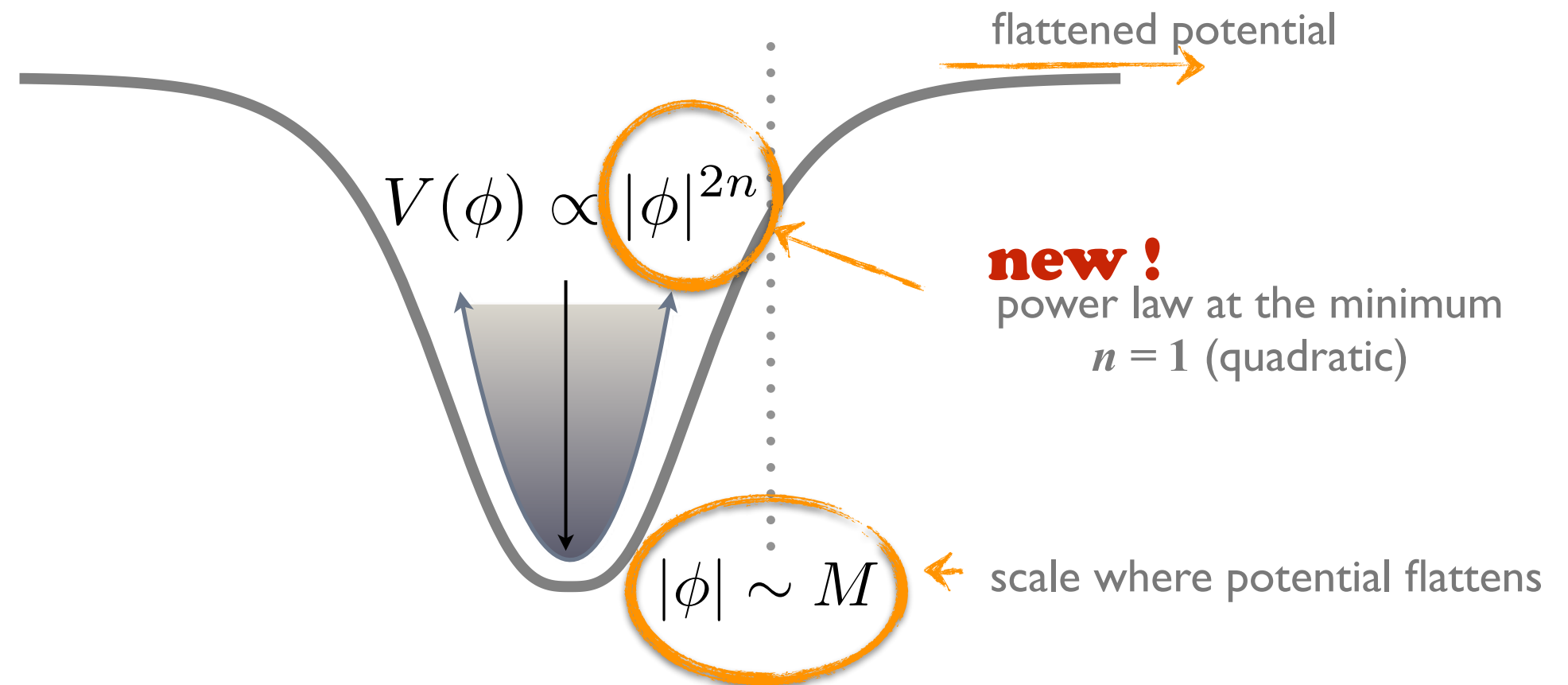


Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland, Gleiser and Mueller et al. (1995) ...

consequences ?

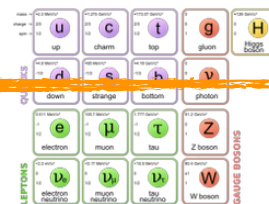
- equation of state/duration to radiation domination ? (if coupled to other fields)
- black holes ?
- gravitational waves ?
Zhou, Copeland, Easther, Finkel, Mou & Saffin (2013)
Antusch, Cefala, Krippendorf, Muia, Orani, Quevedo (2017)

end of inflation in “simple” models



- shape of the potential (self couplings)

- ~~couplings to other fields~~



χ, ψ

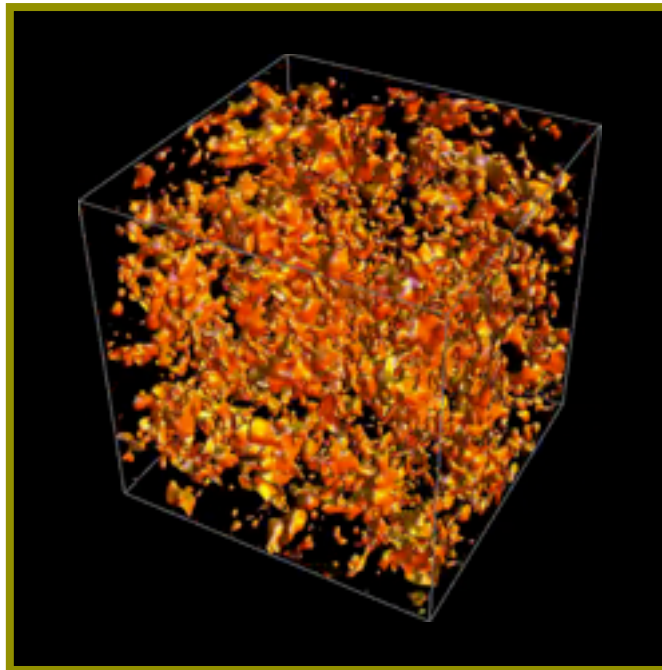
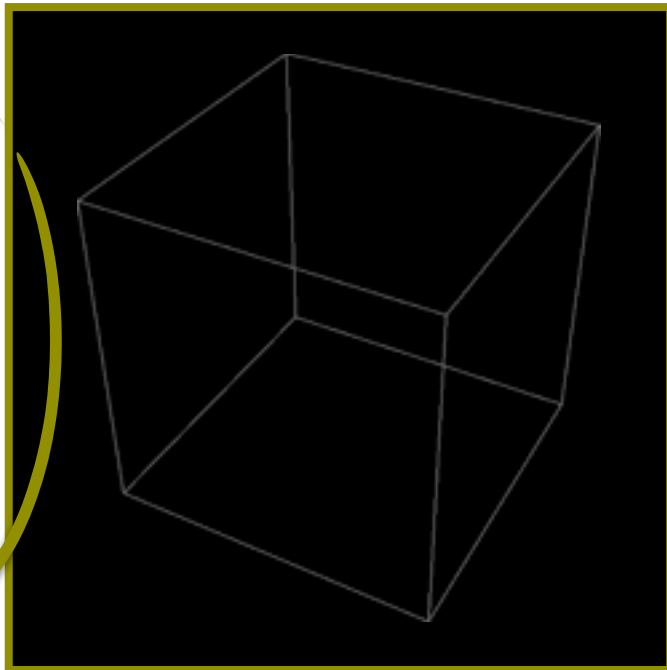
result of fragmented dynamics

* after sufficient time

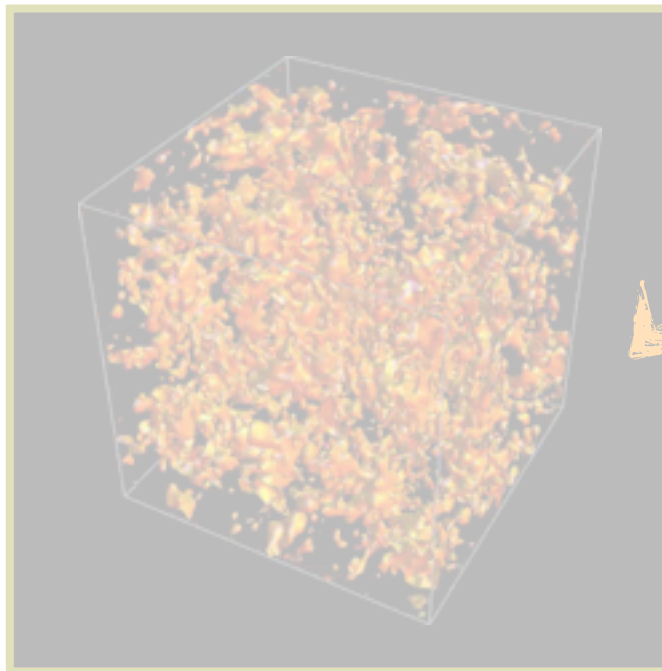
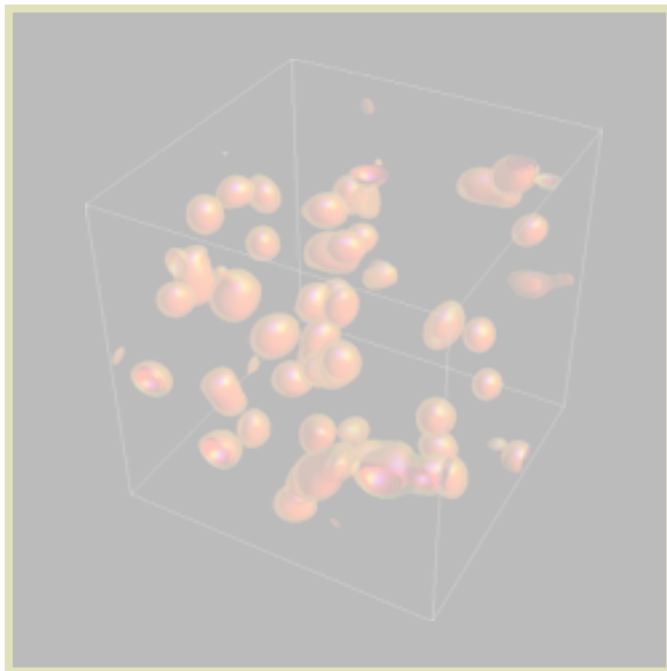
$n = 1$

$n > 1$

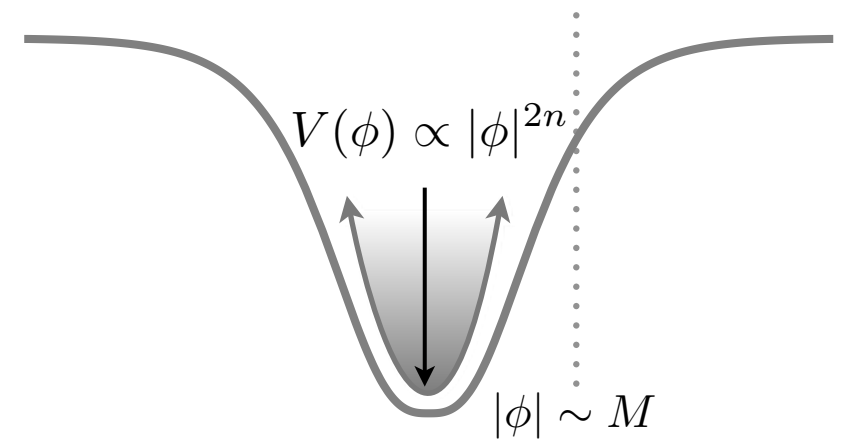
$M \sim m_{\text{pl}}$



$M \ll m_{\text{pl}}$



slow



quickly

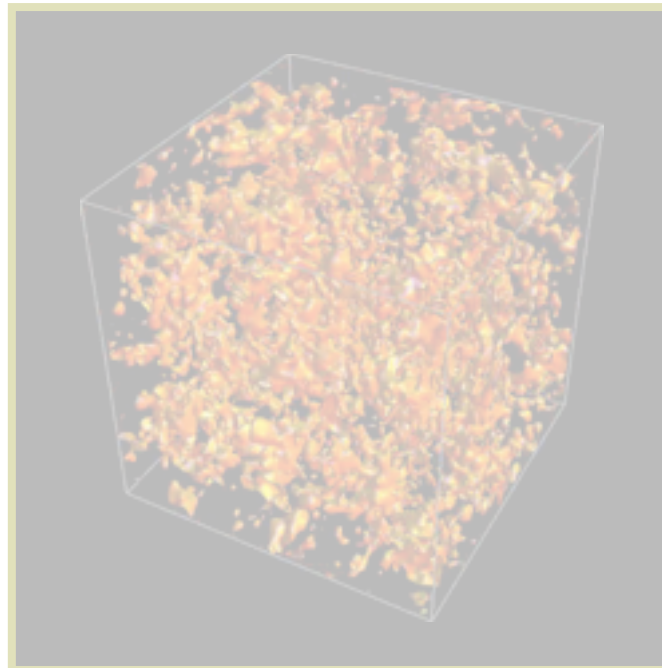
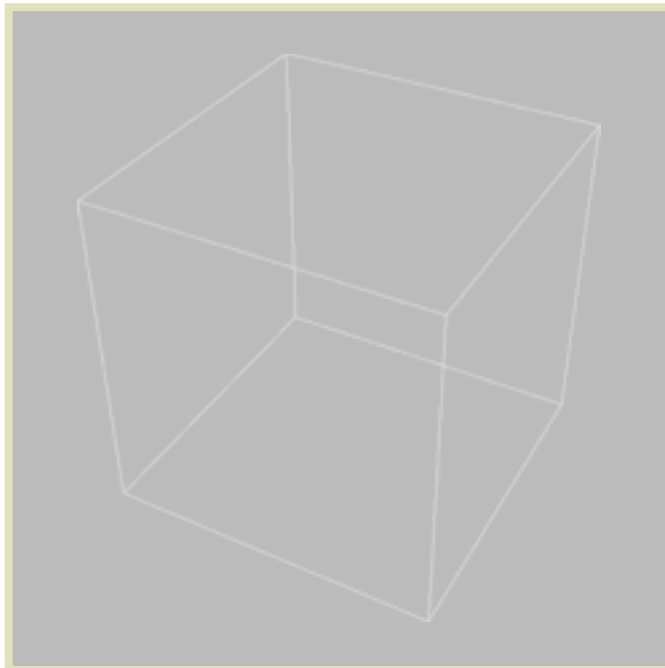
result of fragmented dynamics

* after sufficient time

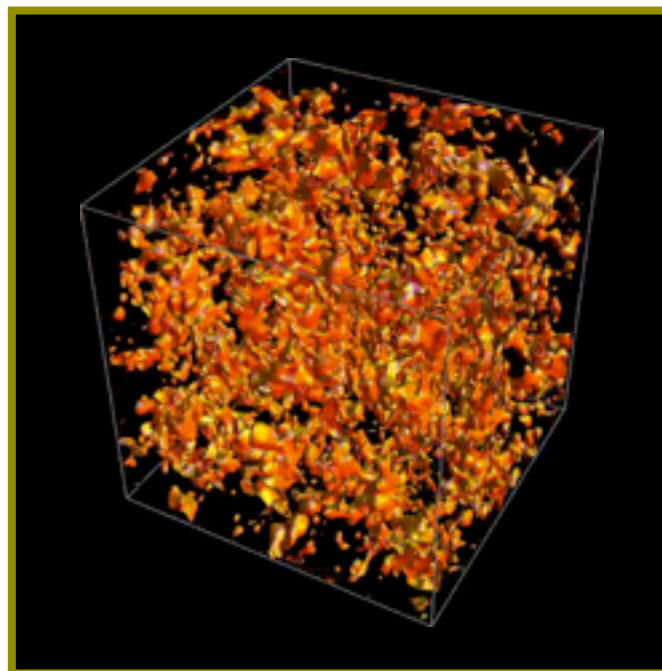
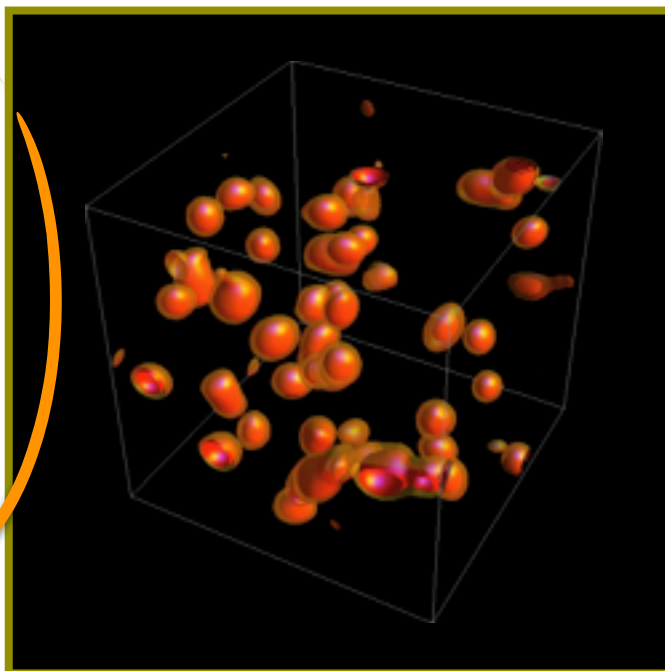
$n = 1$

$n > 1$

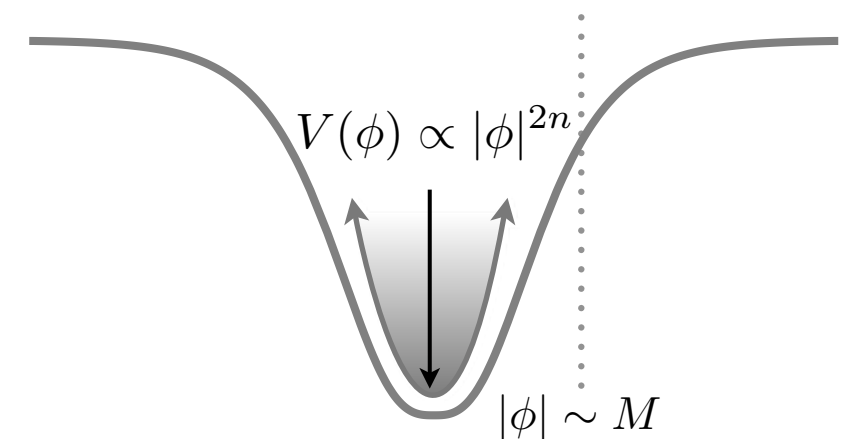
$M \sim m_{\text{pl}}$



$M \ll m_{\text{pl}}$



slow



fast

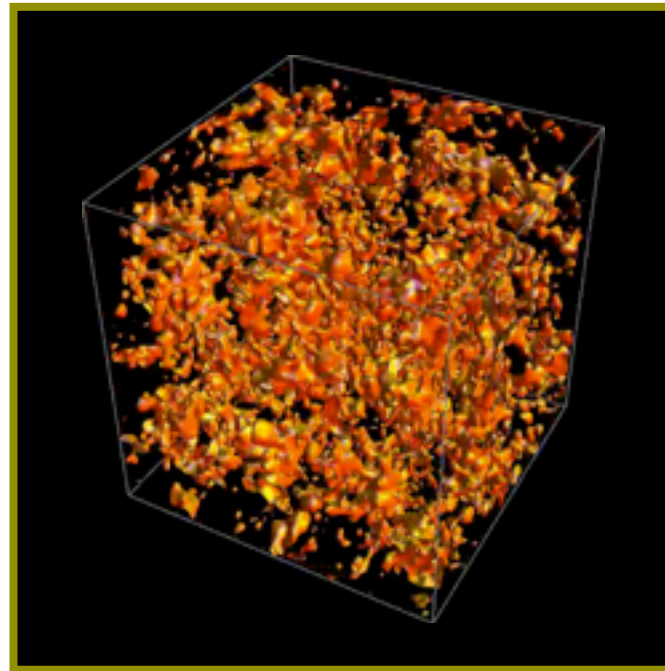
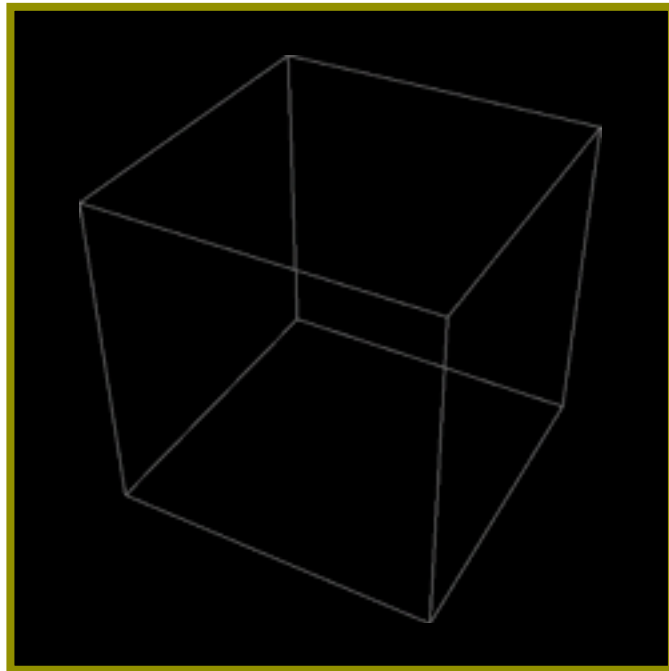
eq. of state

* after sufficient time

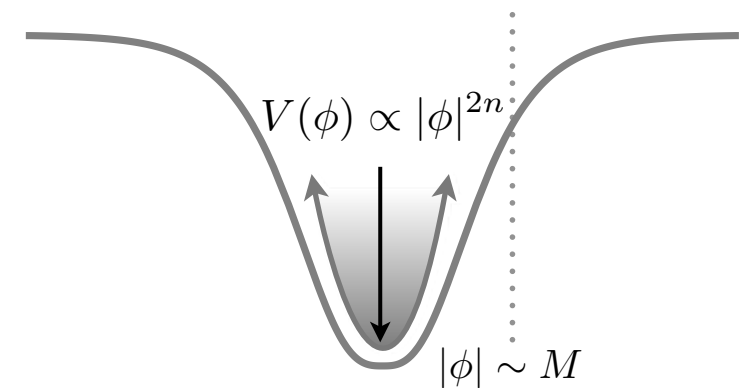
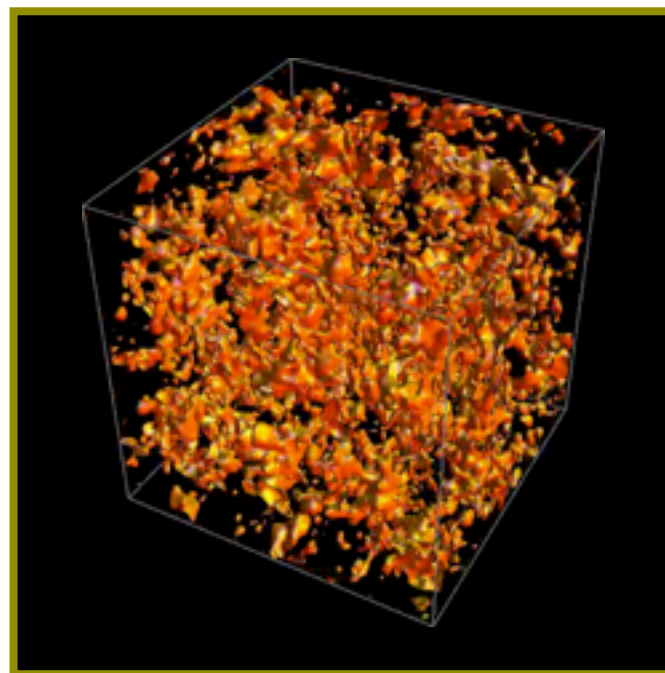
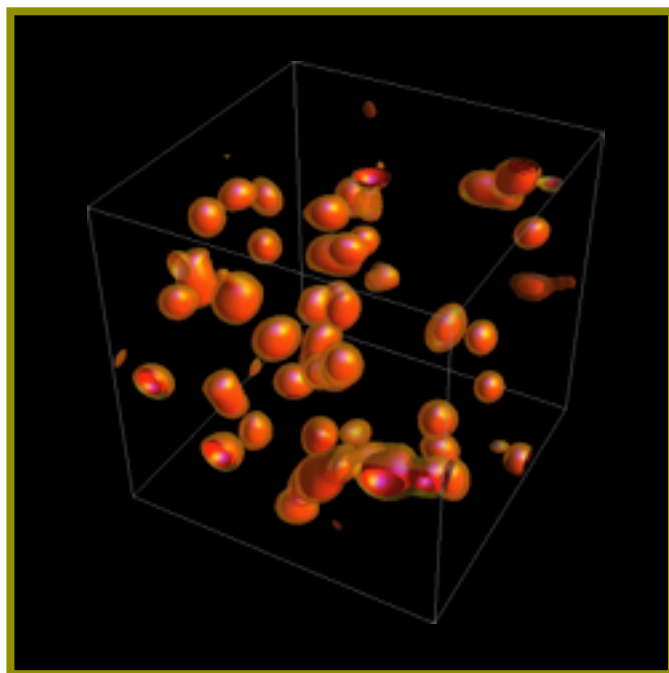
$n = 1$

$n > 1$

$M \sim m_{\text{pl}}$



$M \ll m_{\text{pl}}$



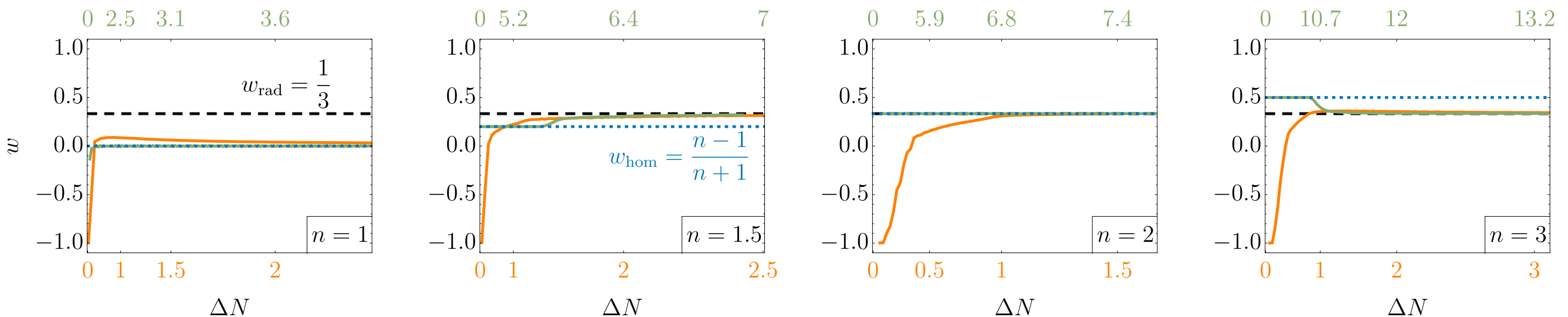
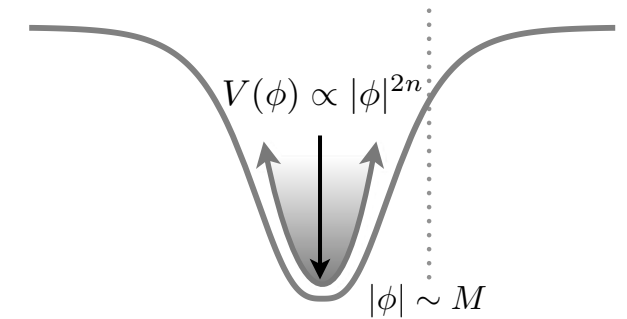
$$w \rightarrow \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

independent of M

$$w \neq \frac{n-1}{n+1}$$

duration to radiation domination

green = inefficient initial resonance
orange = efficient initial resonance



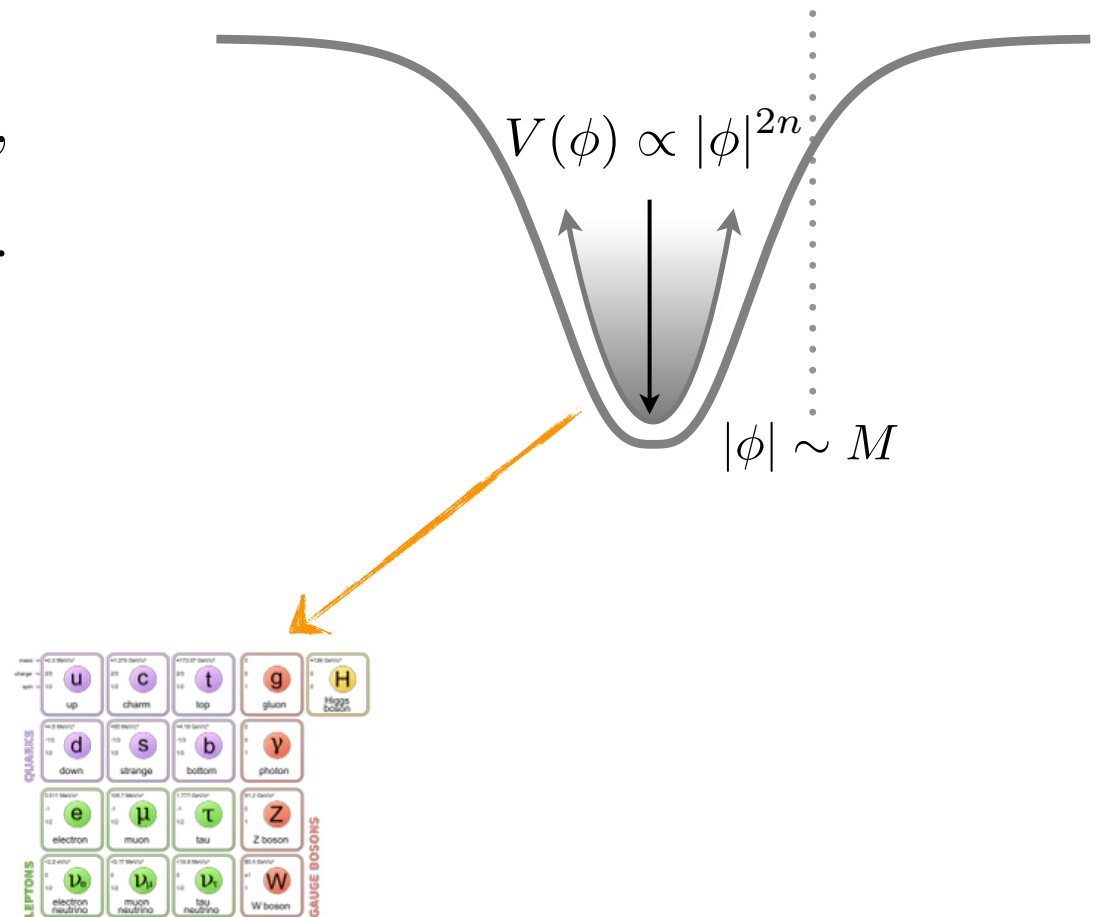
from analytic considerations

$$\Delta N_{\text{rad}} \sim \begin{cases} 1 \\ \frac{n+1}{3} \ln \left(\frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) \end{cases} \quad \begin{aligned} &M \lesssim 10^{-2} m_{\text{Pl}}, \\ &M \gtrsim 10^{-2} m_{\text{Pl}}. \end{aligned}$$

an upper bound on duration to radiation domination

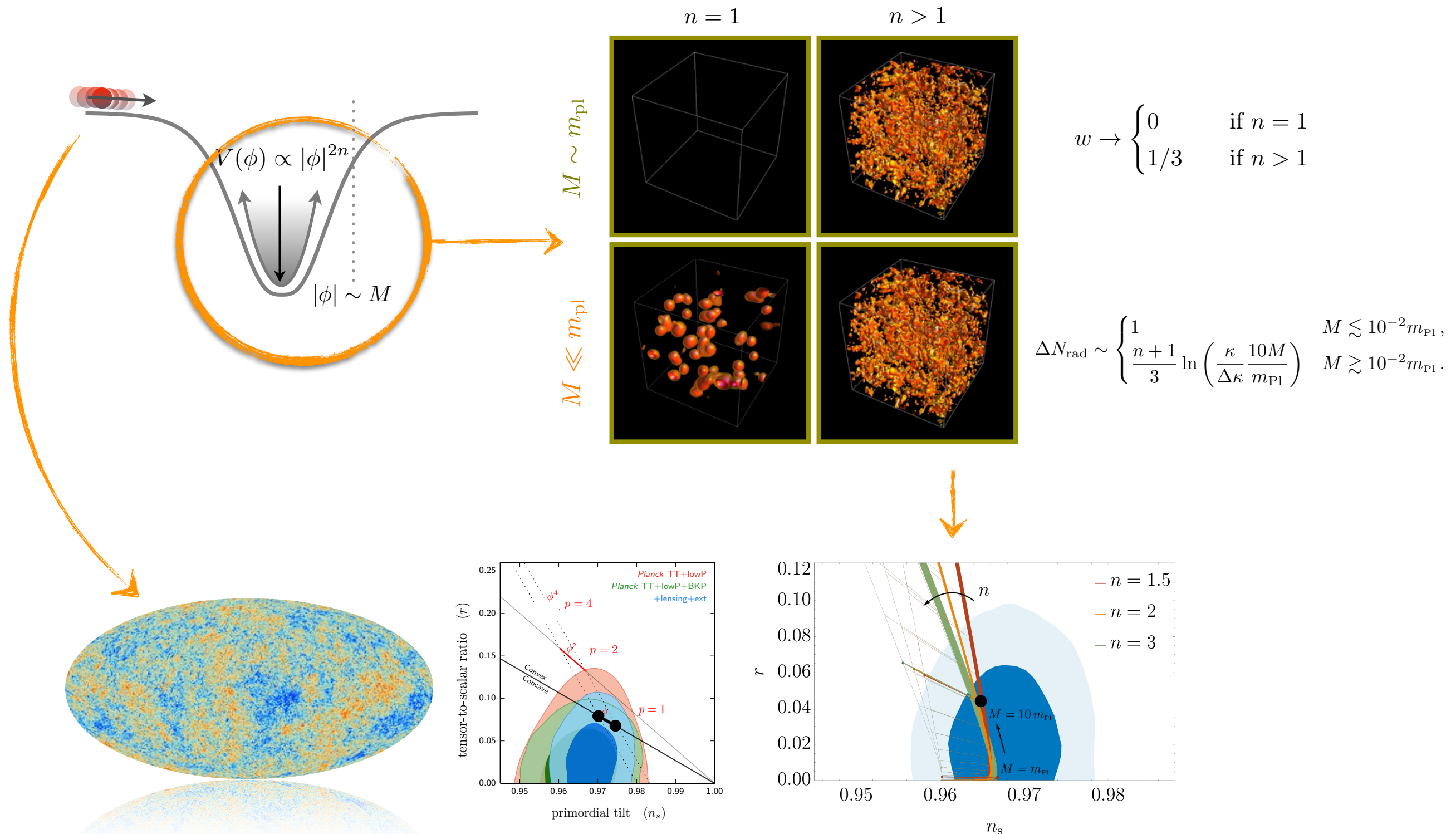
$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left(\frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

additional *light (massless) fields* can
only decrease the duration!



* decay to significantly massive fields can change this conclusion

summary: “simple” models of cosmological scalar field dynamics



end of inflation



SIMPLE

complicated

COMPLEX

end of inflation

SIMPLE

$$\phi \bar{\psi} \psi$$

$$\phi F^{\mu\nu} F_{\mu\nu}$$

$$\phi^2 \chi^2$$

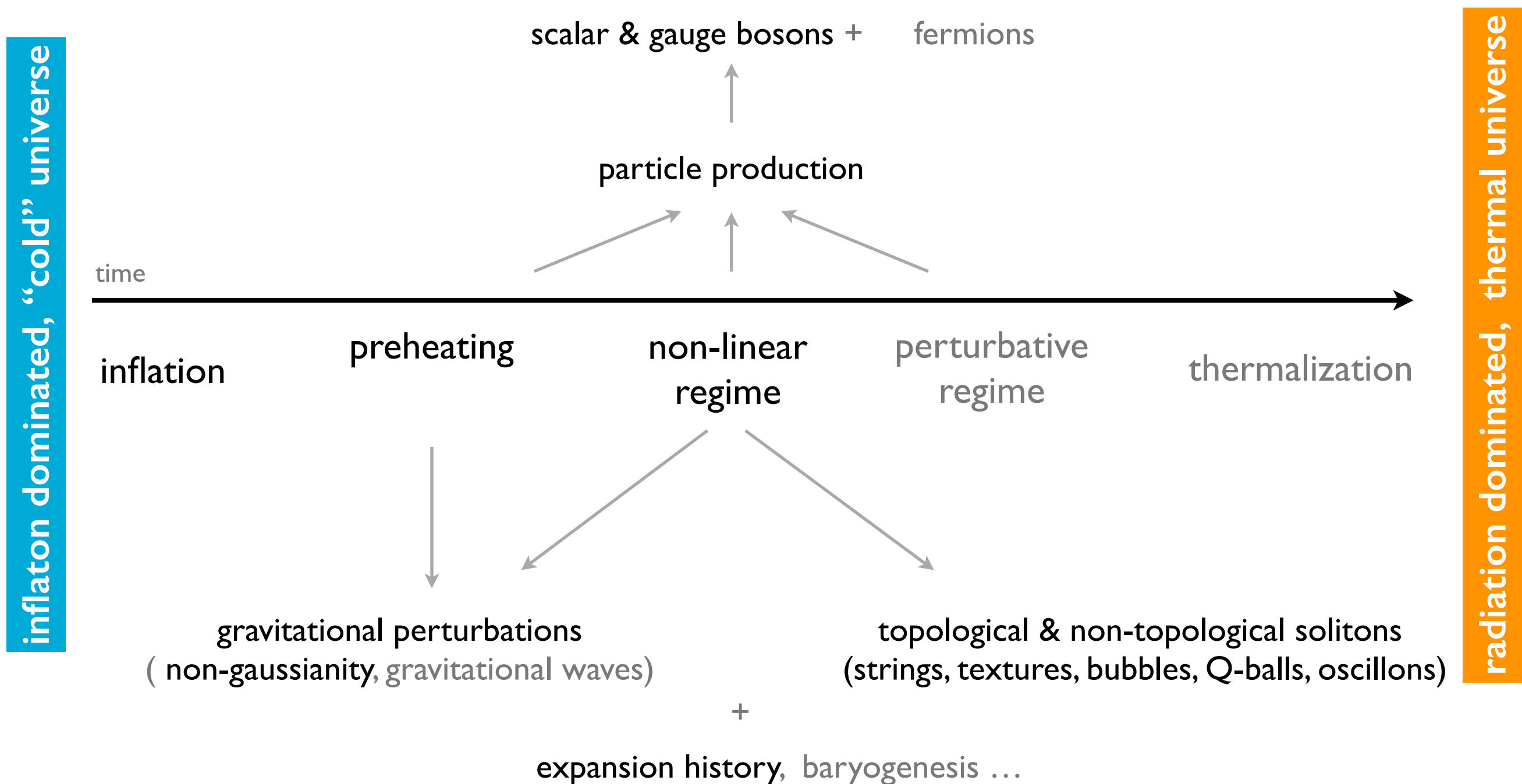
complicated

$$\phi \chi^2$$

$$J^\mu(\phi) A_\mu$$

COMPLEX

things that can happen



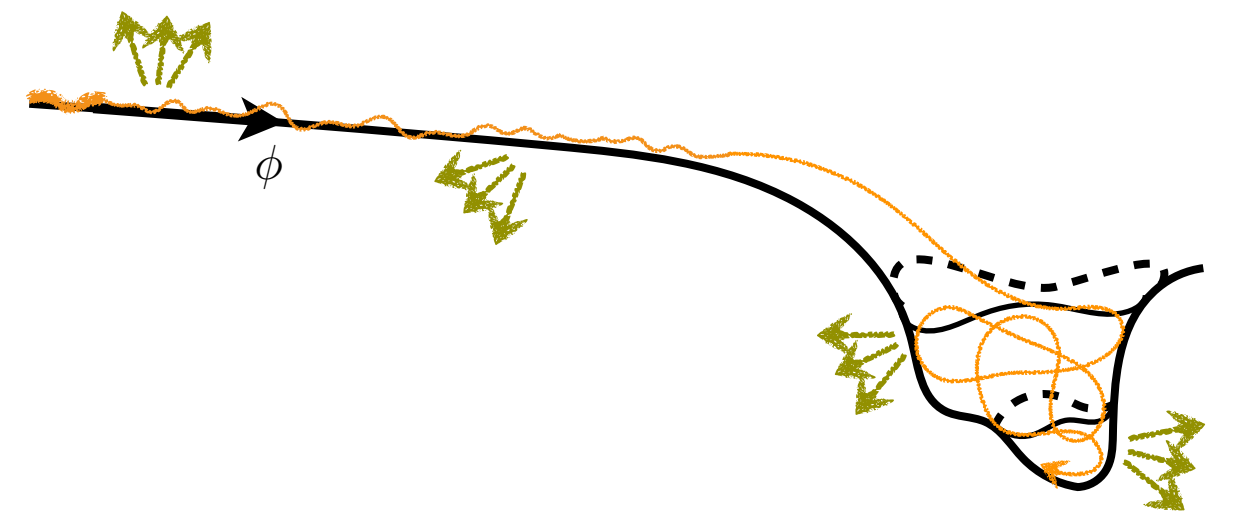
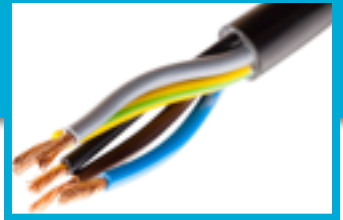
end of inflation

SIMPLE

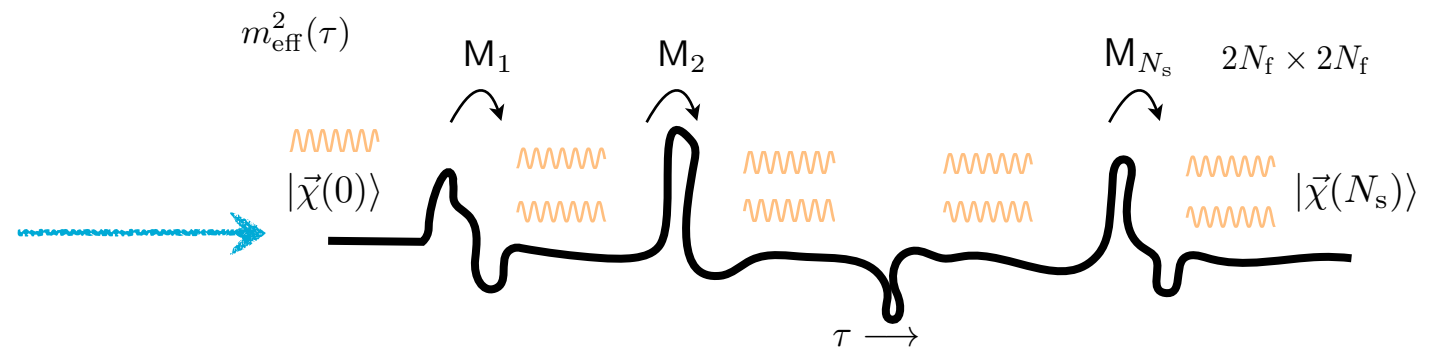
complicated

COMPLEX

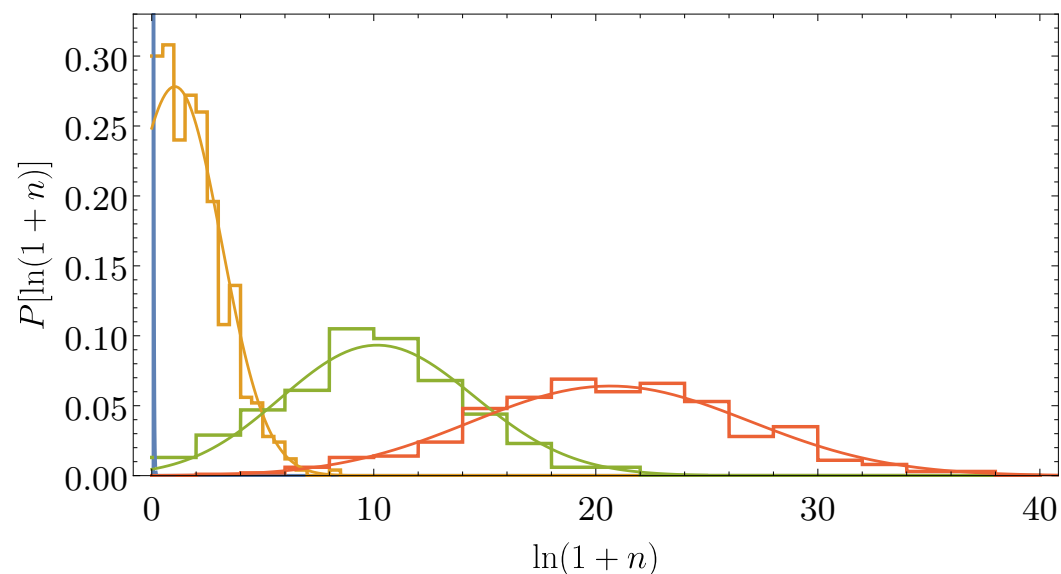
complex enough: “universal” results



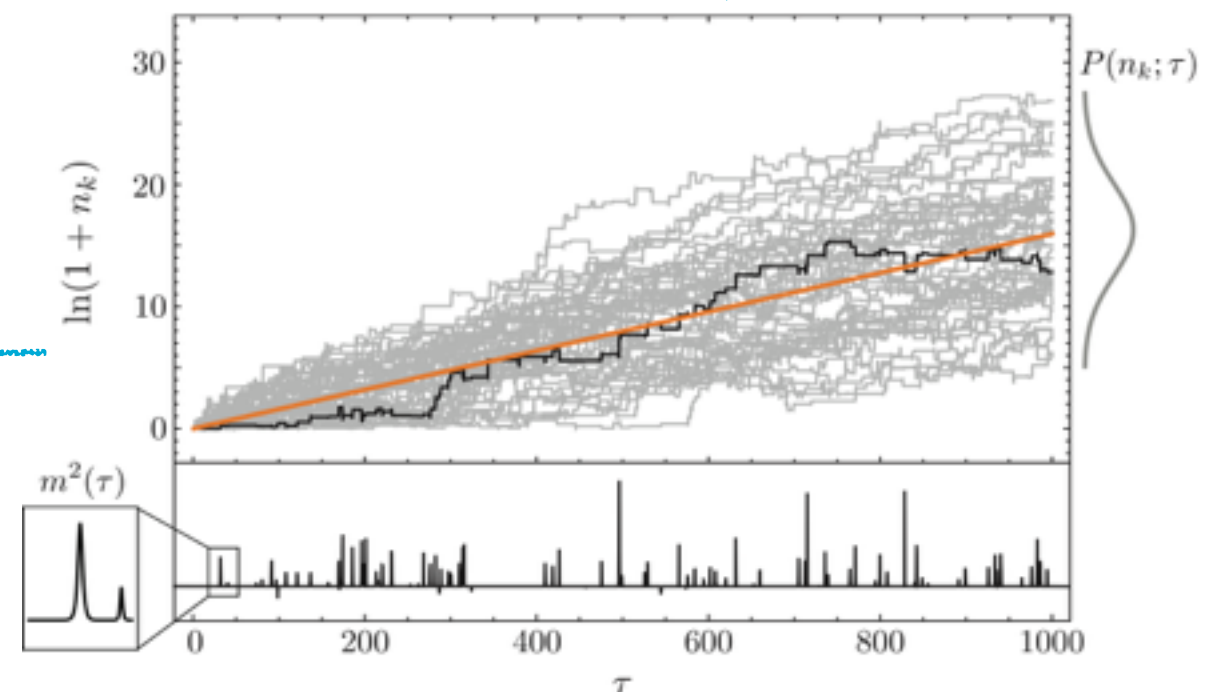
complex non-perturbative particle production



treat as scattering problems
(similar to that when dealing with impurities in wires)



universally log-normal distributions



occupation numbers grow exponentially
(universality similar to Anderson localization)

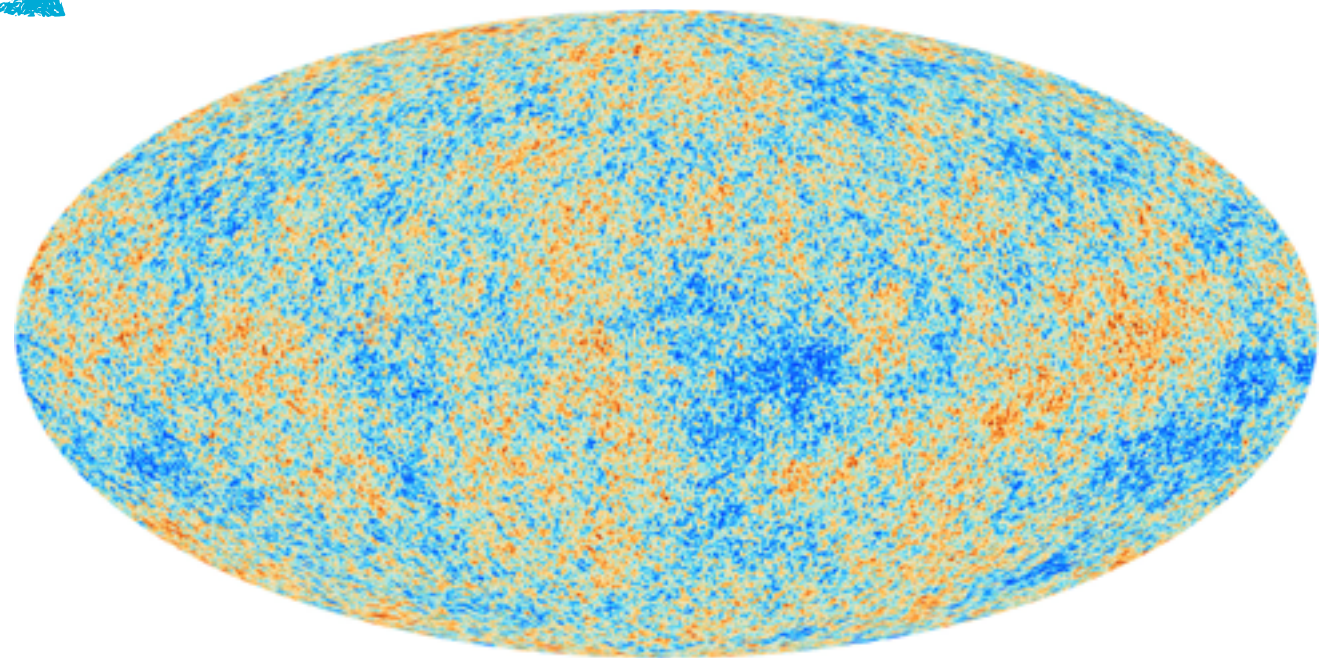
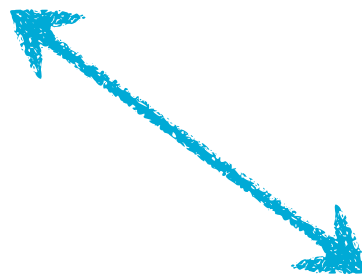
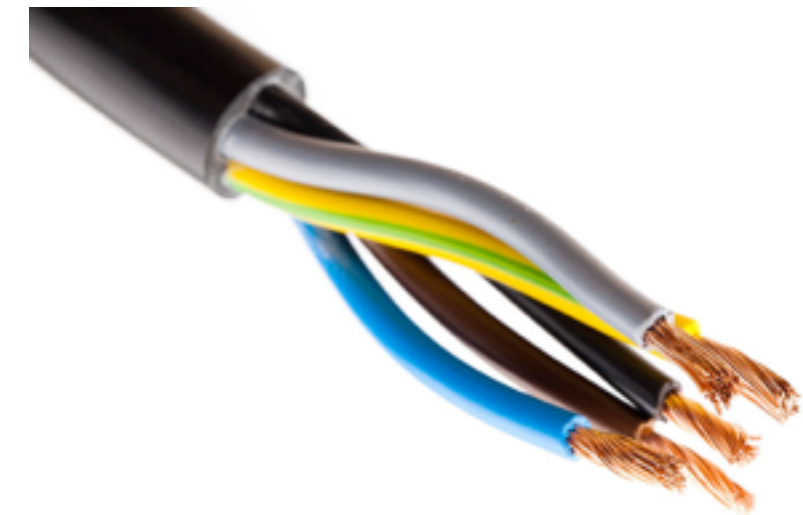
theory : its complicated (probably)

- inflation
- reheating after inflation



mass ~ 12 MeV/c ²	mass ~ 1.275 GeV/c ²	mass ~ 173.2 GeV/c ²	mass ~ 0	mass ~ 125 GeV/c ²
charge + 2/3	charge + 2/3	charge + 2/3	charge 0	charge 0
spin 1/2	spin 1/2	spin 1/2	spin 1	spin 0
u up	c charm	t top	g gluon	H Higgs boson
mass ~ 4.2 MeV/c ²	mass ~ 1.275 GeV/c ²	mass ~ 4.18 GeV/c ²	mass 0	
charge - 1/3	charge - 1/3	charge - 1/3	charge 0	
spin 1/2	spin 1/2	spin 1/2	spin 1	
d down	s strange	b bottom	γ photon	
mass ~ 0.511 MeV/c ²	mass ~ 105.7 MeV/c ²	mass ~ 1.777 GeV/c ²	mass 0	
charge - 1	charge - 1	charge - 1	charge 0	
spin 1/2	spin 1/2	spin 1/2	spin 1	
e electron	μ muon	τ tau	Z Z boson	
mass ~ 0	mass ~ 0.106 GeV/c ²	mass ~ 1.777 GeV/c ²	mass 80.4 GeV/c ²	
charge 0	charge 0	charge 0	charge 0	
spin 1/2	spin 1/2	spin 1/2	spin 1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
mass ~ 0	mass ~ 0	mass ~ 0	mass 80.4 GeV/c ²	
charge 0	charge 0	charge 0	charge ±1	
spin 1/2	spin 1/2	spin 1/2	spin 1	
				GAUGE BOSONS

inspiration from disordered wires



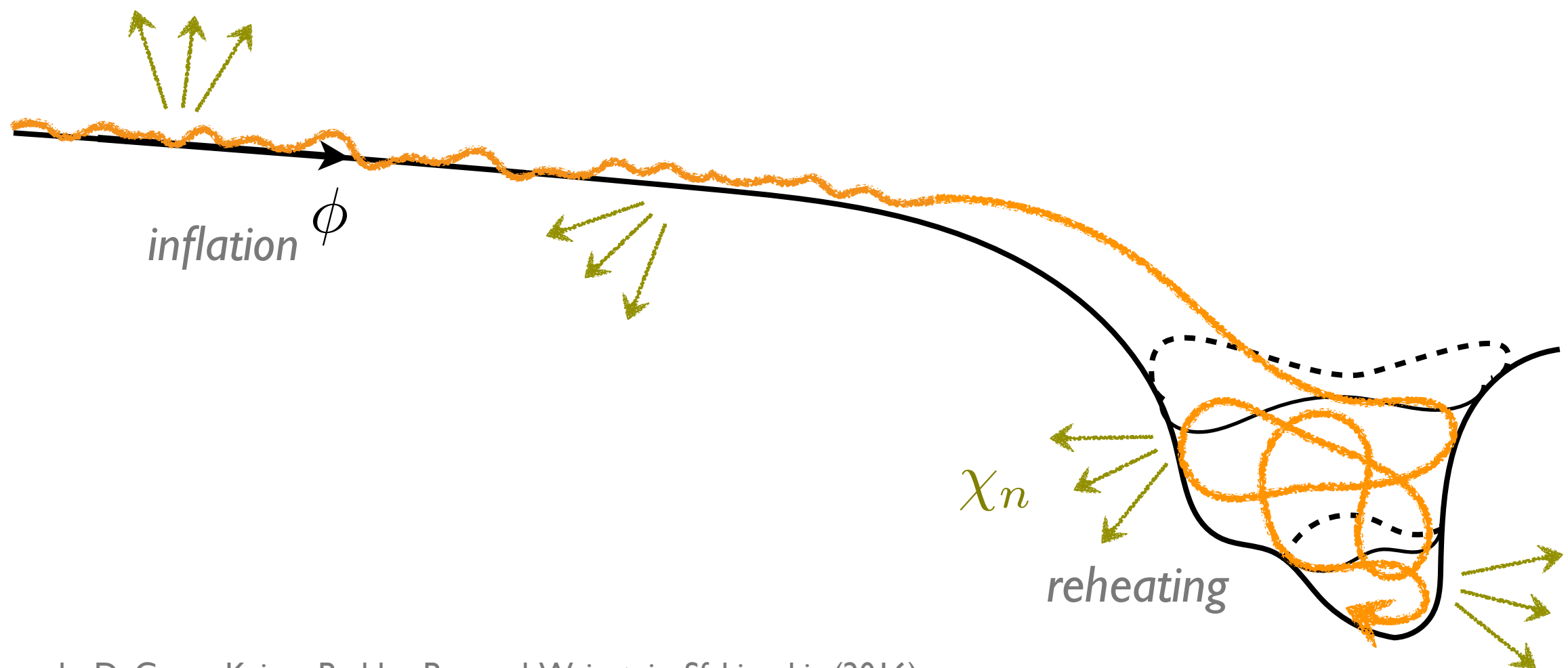


the framework



multifield inflation/reheating

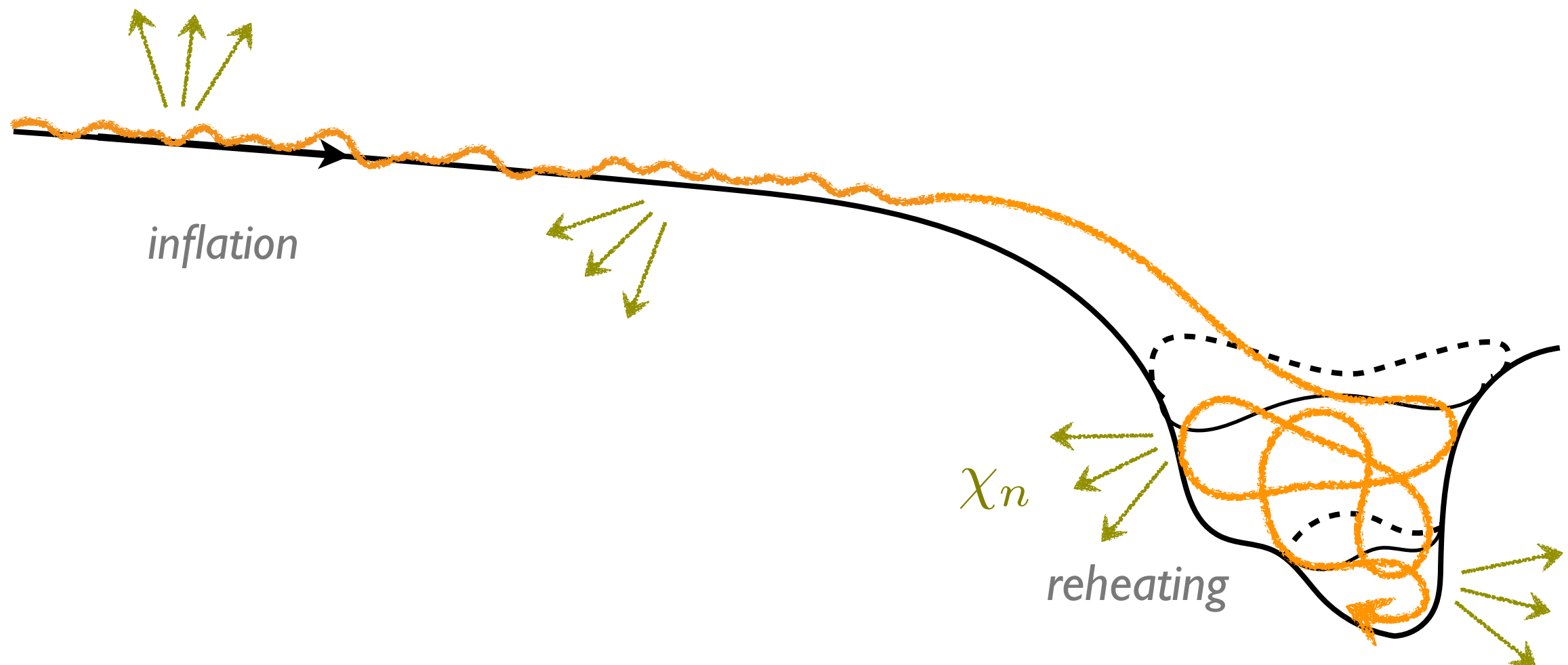
$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} G_{ab}(\phi^c) \partial^\mu \phi^a \partial_\mu \phi^b - V(\phi^c) + \dots \right]$$



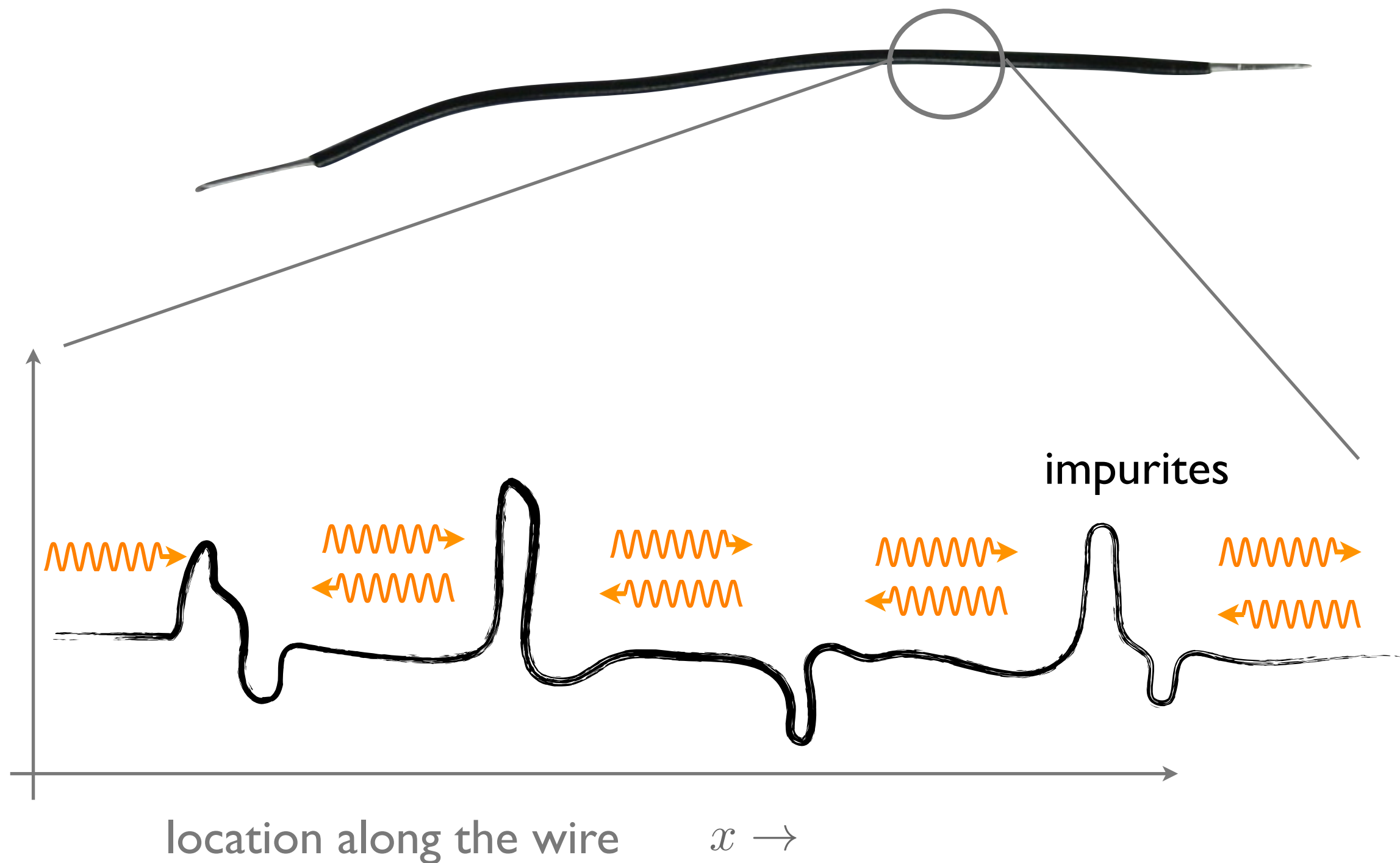
focus on perturbations

mode functions in Fourier space

$$\left(\frac{d^2}{d\tau^2} + \omega_I^2 \right) \chi_k^I(\tau) + \sum_{J=1}^{N_f} m_{IJ}^s(\tau) \chi_k^J(\tau) = 0,$$
$$\omega_I^2(k) = k^2 + m_I^2,$$

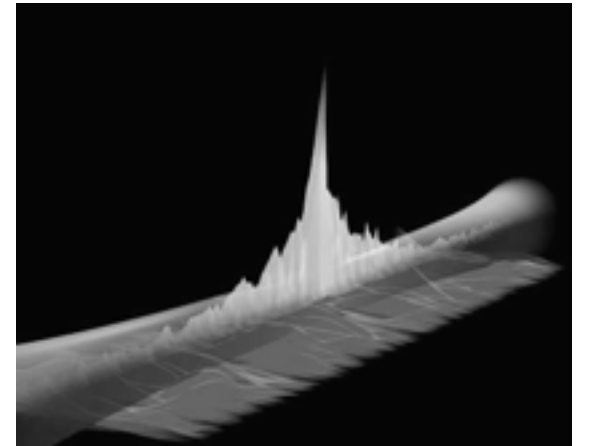


electron wave function: disordered wires

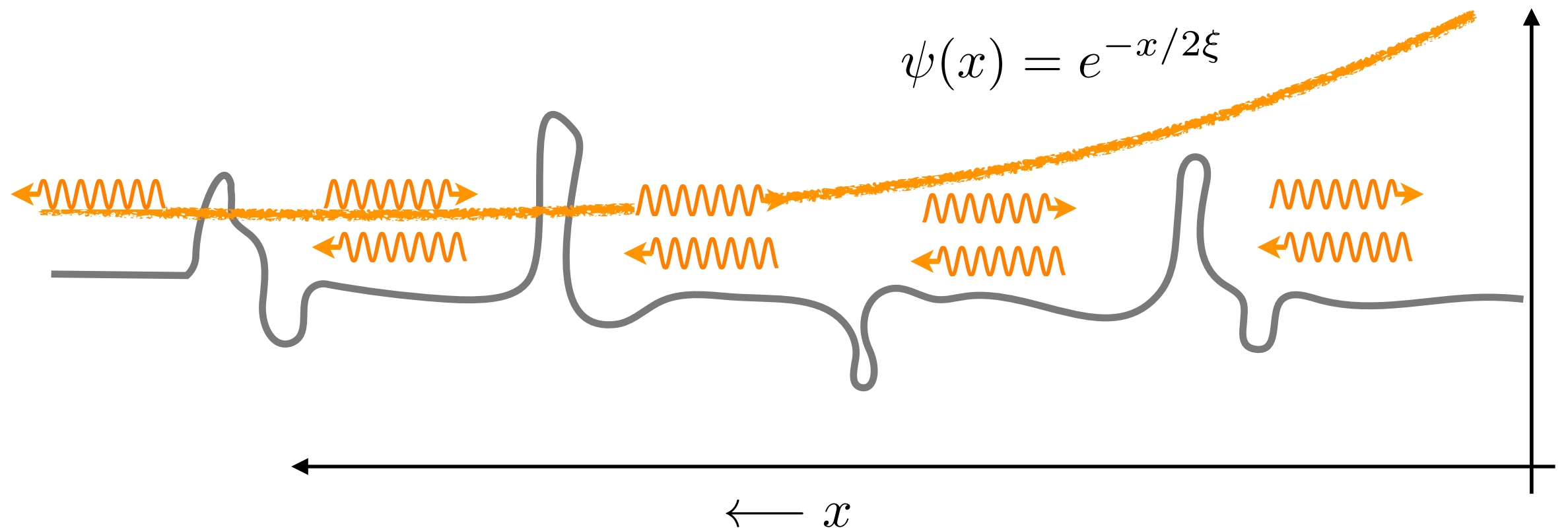


Anderson localization !

$$\psi''(x) + [k^2 - V(x)] \psi(x) = 0$$



Anderson 1957



complexity in time cosmology

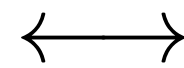


complexity in space wires

exponential growth in occupation number

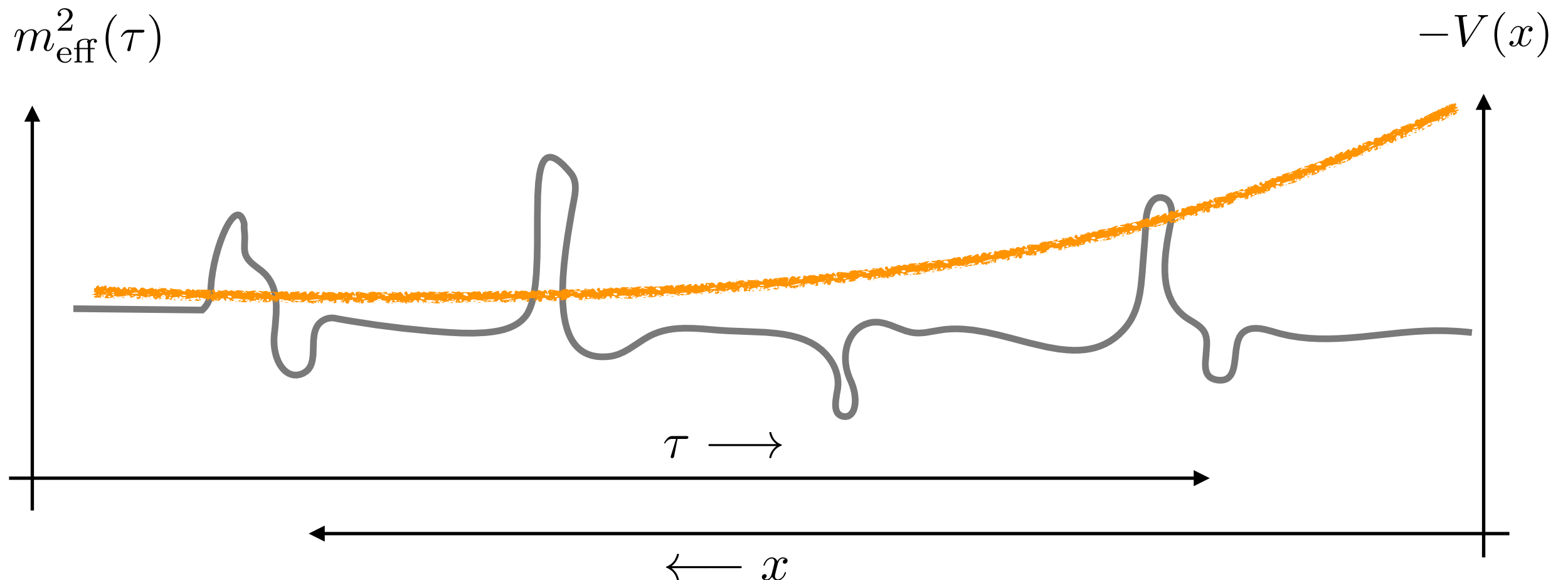
Anderson localization

$$\ddot{\chi}_k(\tau) + [k^2 + m_{\text{eff}}^2(\tau)] \chi_k(\tau) = 0$$



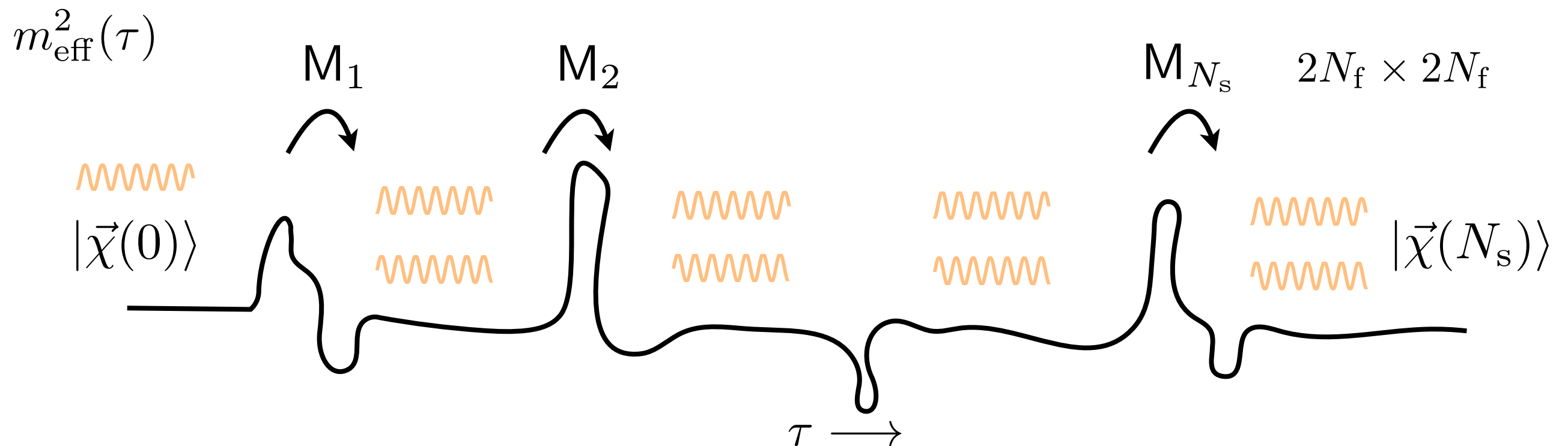
$$\psi''(x) + [k^2 - V(x)] \psi(x) = 0$$

simplified version!



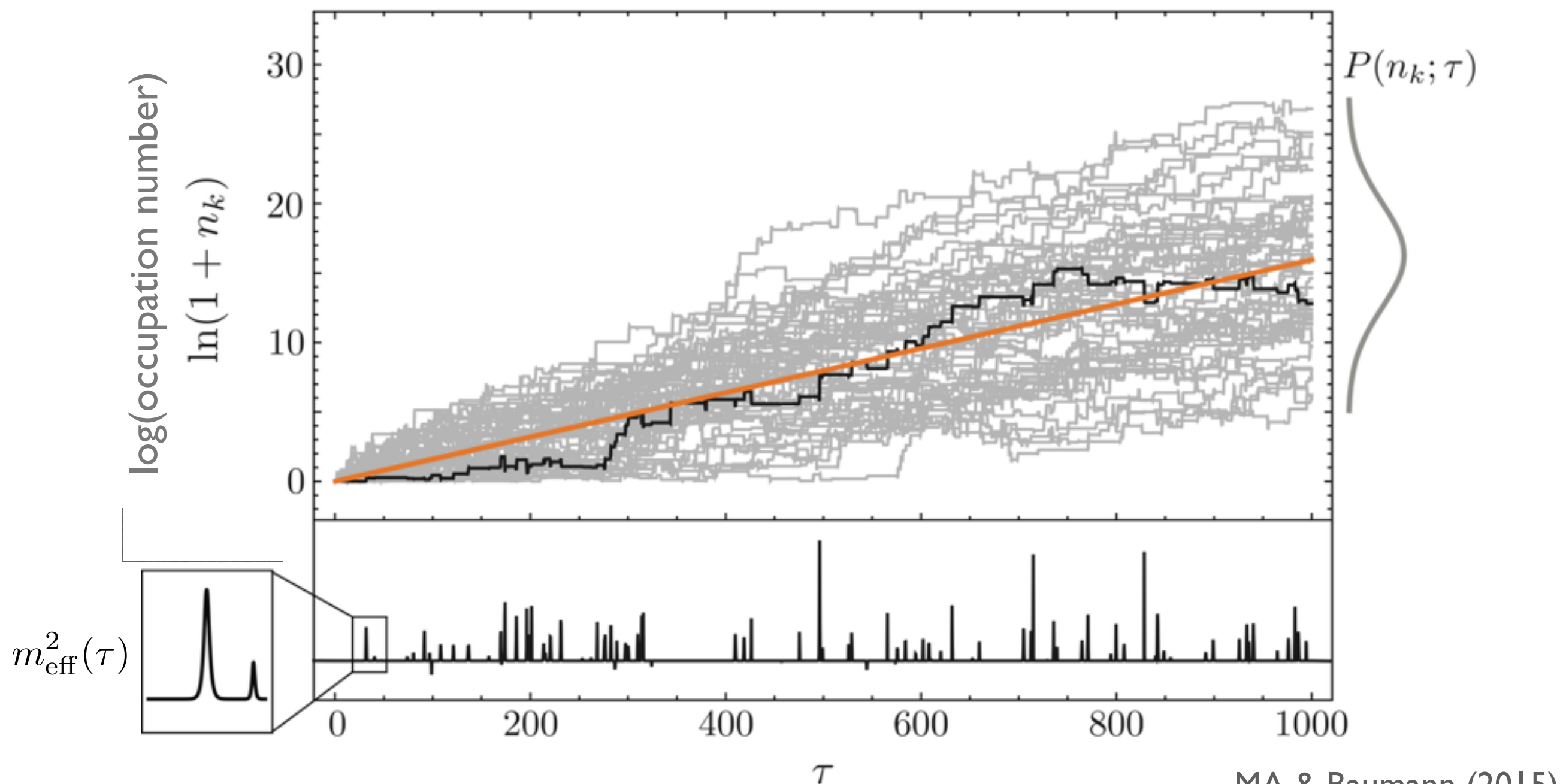
multifield particle production as scattering

$$|\vec{\chi}(N_s)\rangle = M |\vec{\chi}(0)\rangle \quad \text{where} \quad M \equiv M_{N_s} \cdots M_2 M_1$$



occupation number performs a drifted random walk

$$n(k, \tau) = \frac{1}{2\omega_k} (|\dot{\chi}_k|^2 + \omega_k^2 |\chi_k|^2)$$



multifield Fokker Planck equation

joint probability for occupation numbers satisfies the **a Fokker Planck-like** equation:

Dokhorov, Mello, Pereyra & Kumar = DMPK eq.

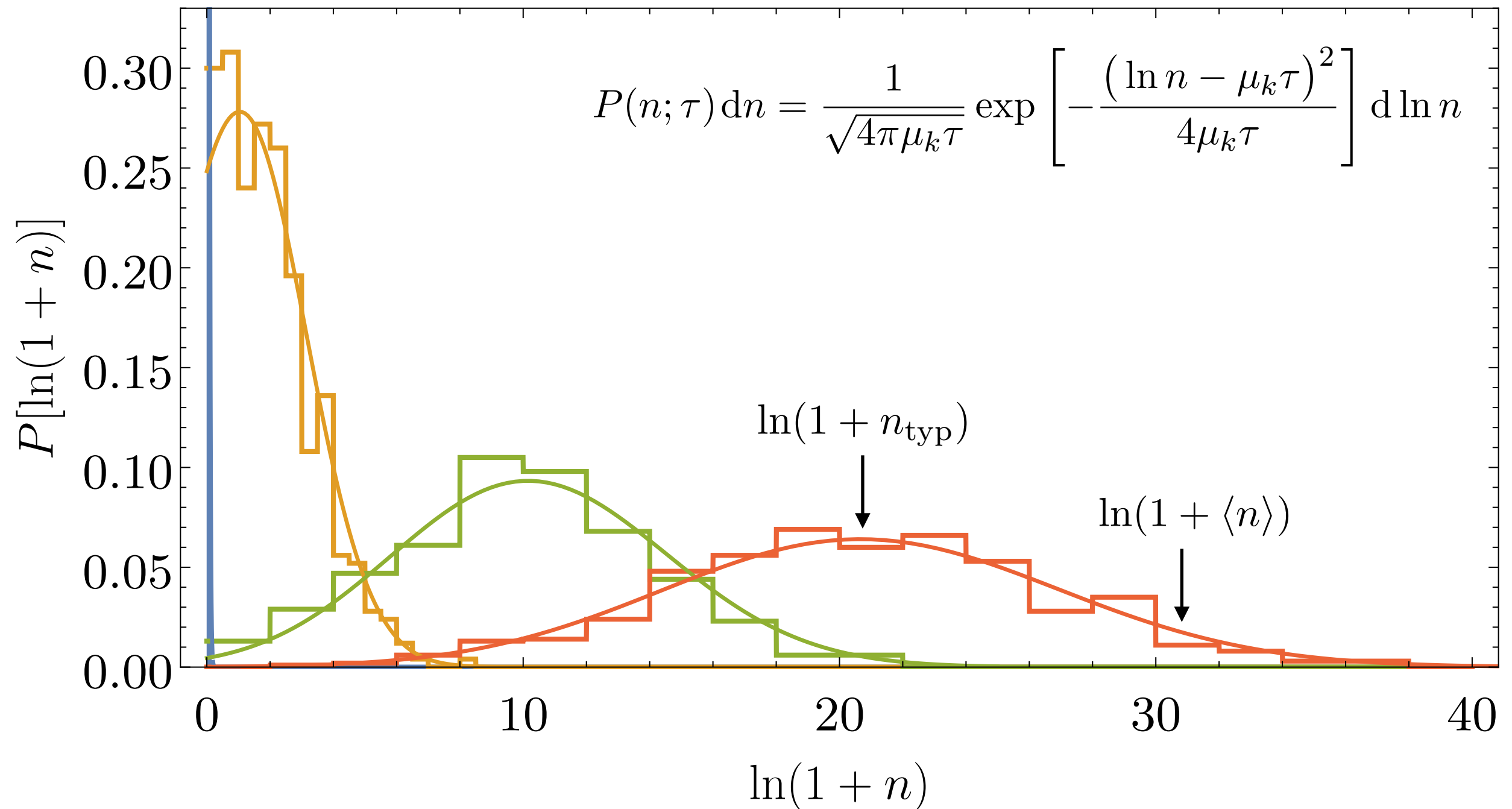
$$\begin{aligned} \frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n_a; \tau) = & \sum_{a=1}^{N_f} \left[(1 + 2n_a) + \frac{1}{N_f + 1} \sum_{b \neq a} \frac{n_a + n_b + 2n_a n_b}{n_a - n_b} \right] \frac{\partial P}{\partial n_a} \\ & + \frac{2}{N_f + 1} \sum_{a=1}^{N_f} n_a (1 + n_a) \frac{\partial^2 P}{\partial n_a^2} \end{aligned}$$

MA & Baumann 2015

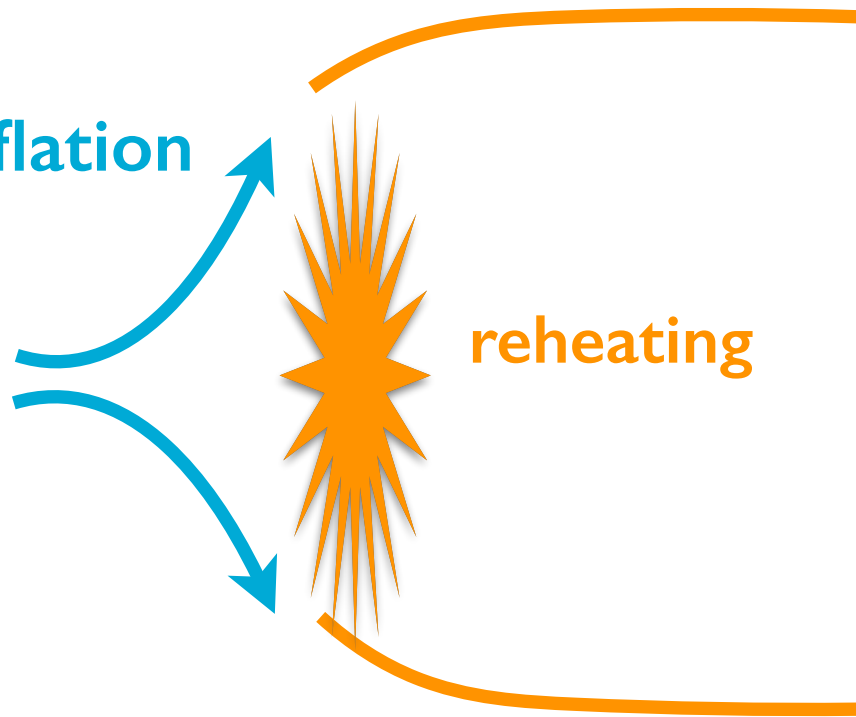
more general results in

MA, Garcia, Xie and Wen 2017

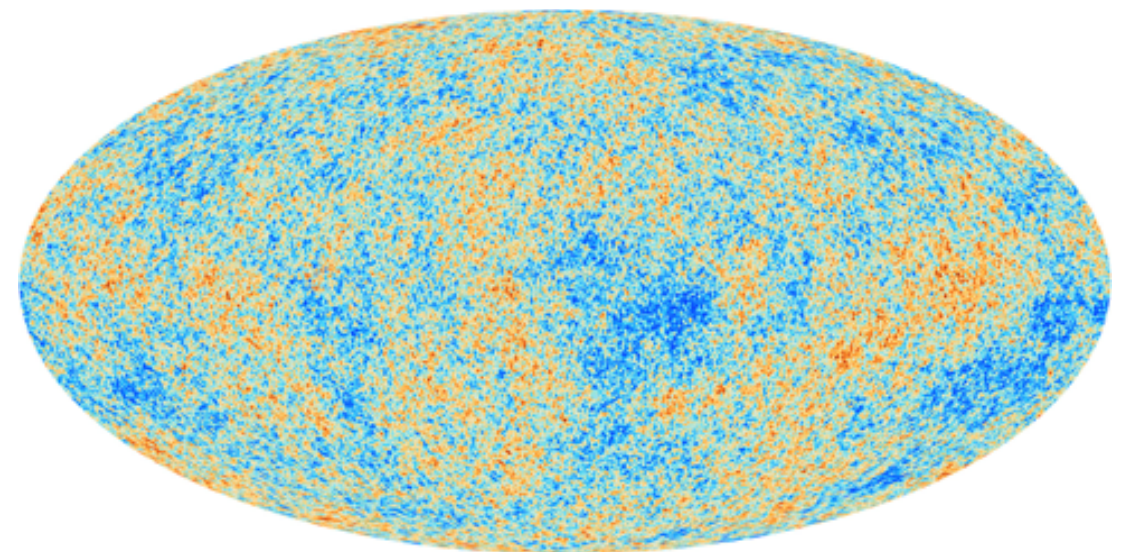
solution: “universal” distributions



inflation



applications



WORK IN
PROGRESS

combine particle production with driving and dissipation

background dynamics \rightarrow particle production \leftrightarrow curvature fluctuations

$$\langle n_{k_1} n_{k_2} \dots \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \dots \rangle$$

MA, Garcia, Baumann, Carlsten, Chia & Green

$$\ddot{\pi}_k + [3H + \mathcal{O}_d] \pi_k + \frac{k^2}{a^2} \pi_k = \mathcal{O}_s(\langle \chi \chi \dots \rangle_k)$$

$$\zeta_k = -H \pi_k$$

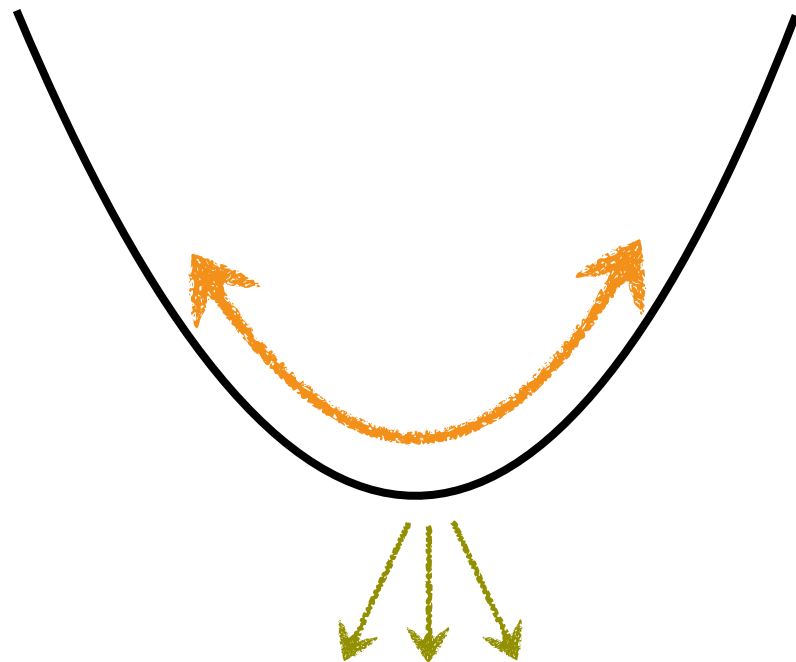
dissipation

driving

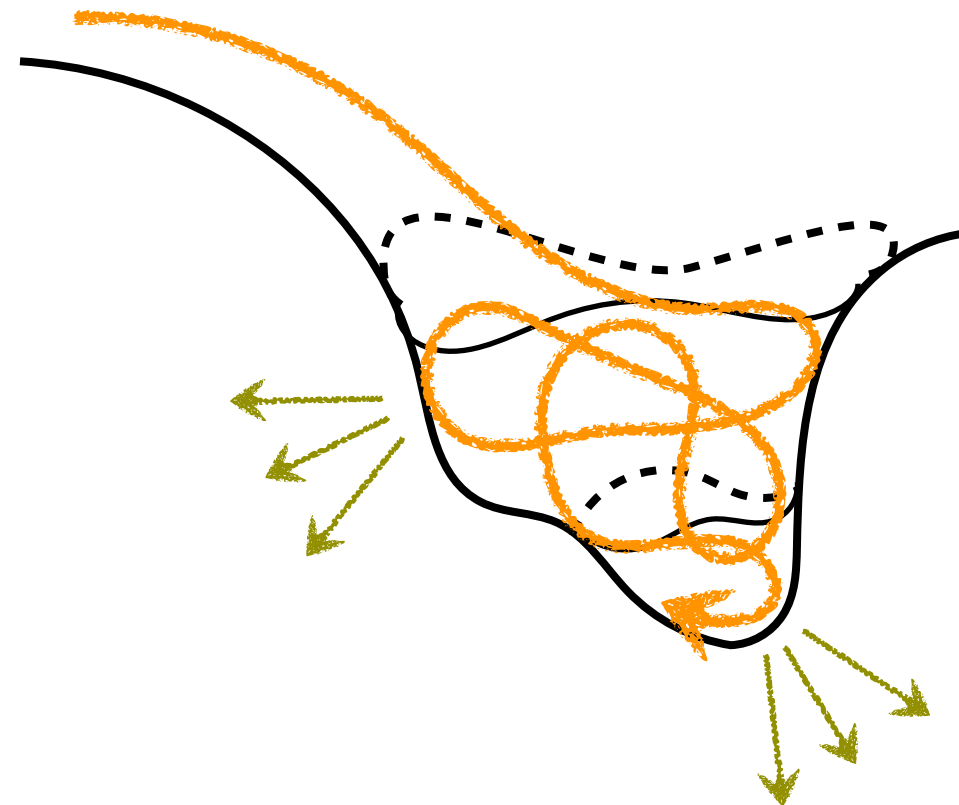


**WORK IN
PROGRESS**

applications : reheating



multichannel — multifield — statistical



model-insensitive description of a complicated reheating process.

Kofman, Linde & Starobinsky (1997)
Shtanov, Traschen & Brandenberger (1995)
Zanchin et. al (1998) & Bassett (1998) [with noise]
Barnaby, Kofman & Braden et. al 2010 [quasiperiodic]
Giblin, Nesbit, Ozsoy, Sengor & Watson (2017)

Inflation Ends, What's Next ?

