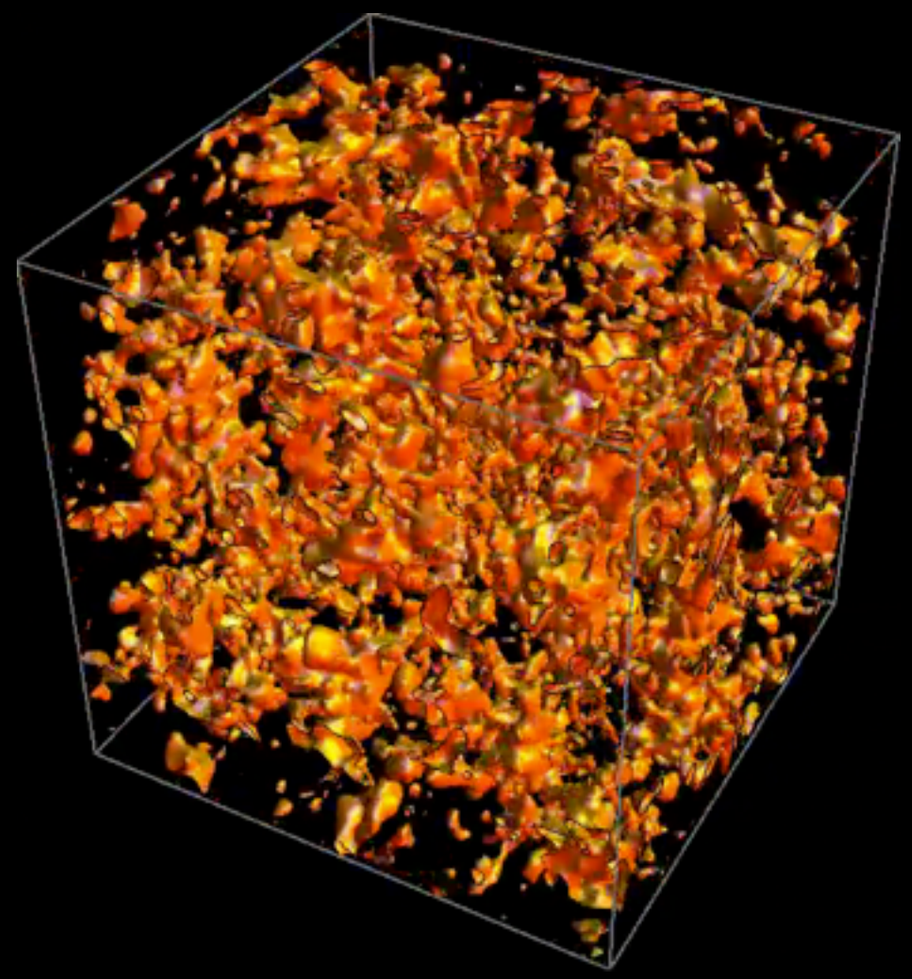
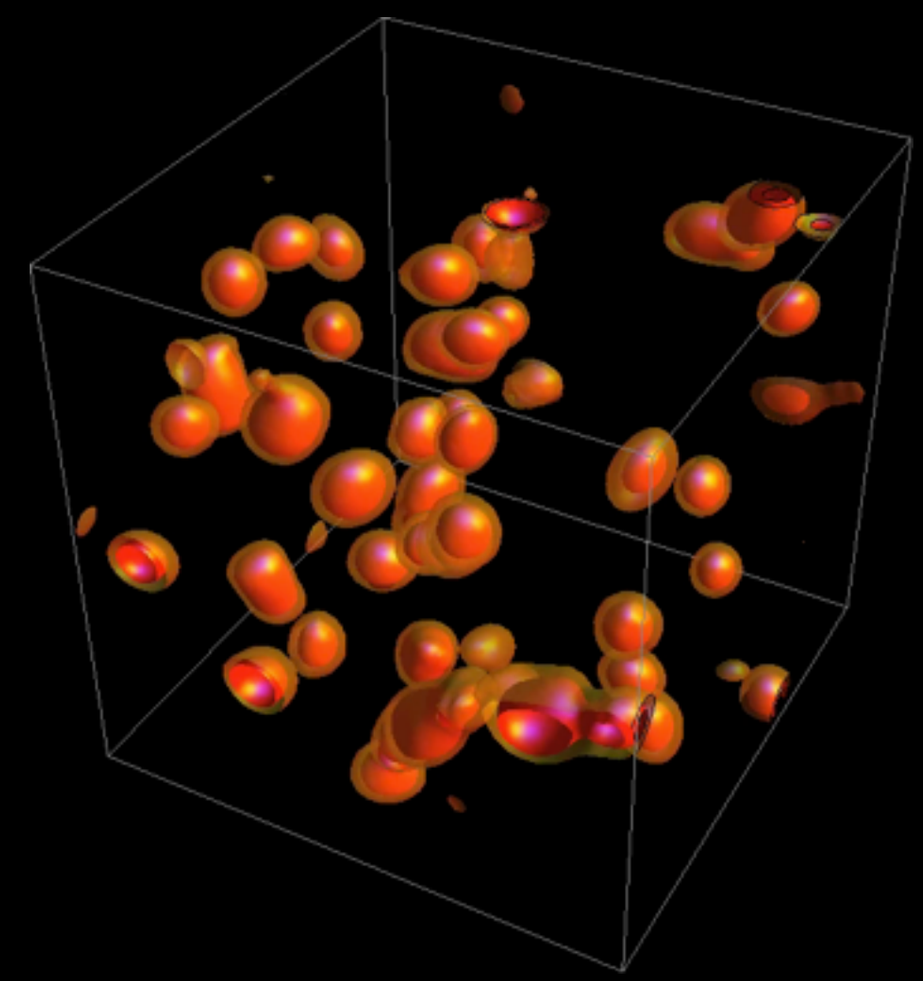
Nonperturbative Dynamics of Cosmological Scalar Fields



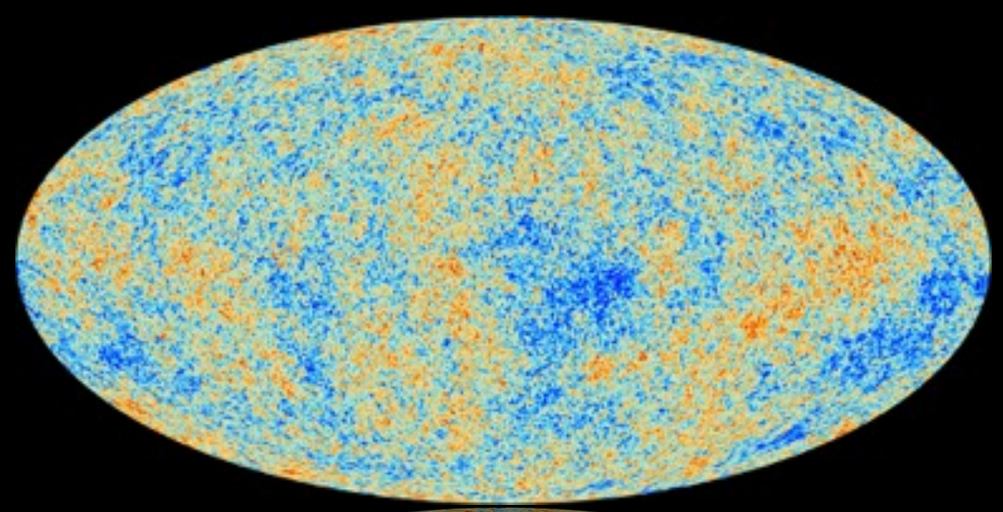


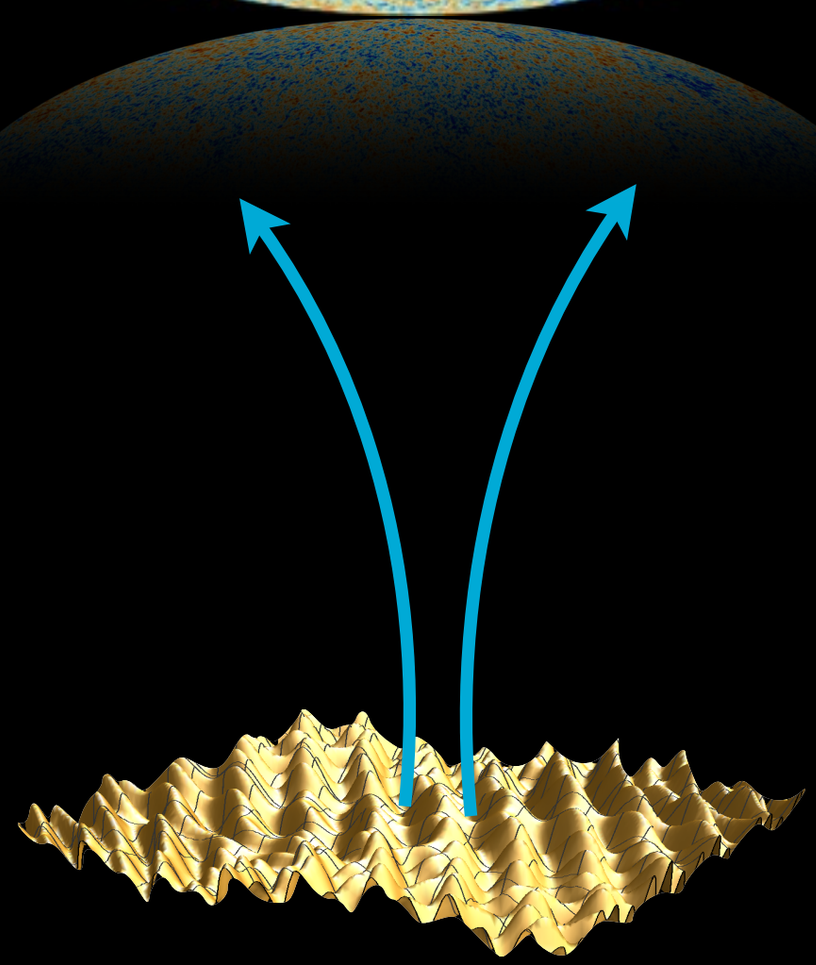
Mustafa A. Amin

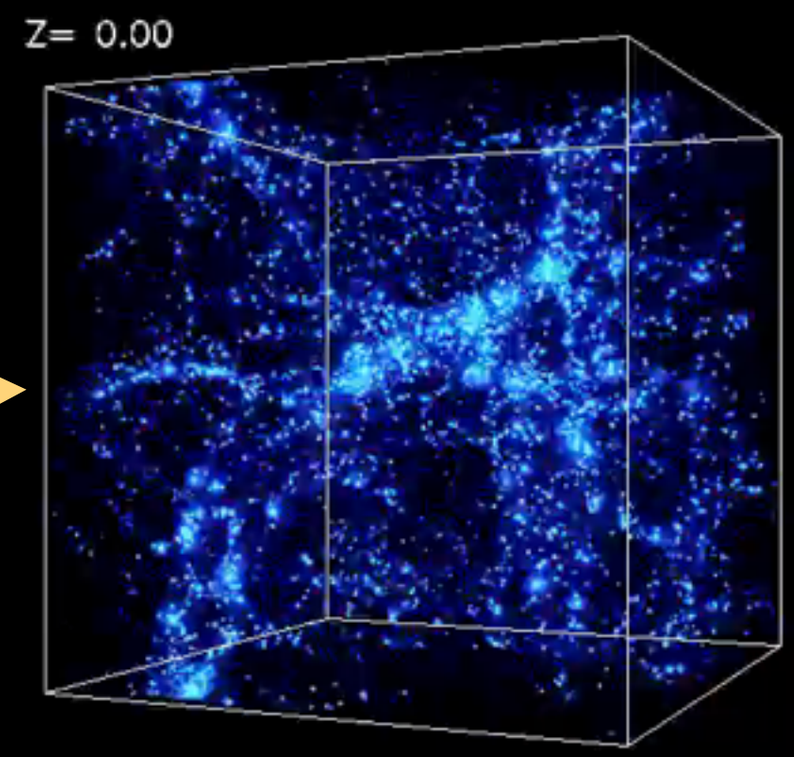
RICE

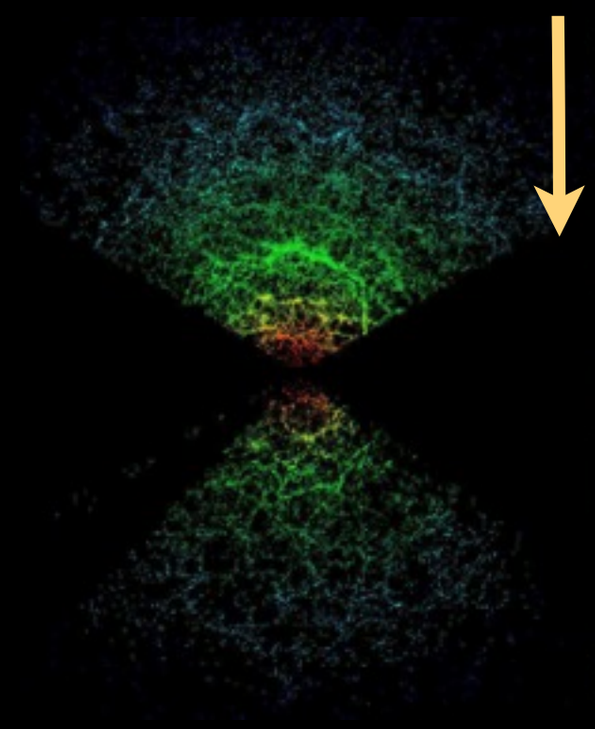
Post Inflationary String Cosmology, Bologna, Italy Sept 2017

inflationary cosmology: a calculable framework of initial perturbations*



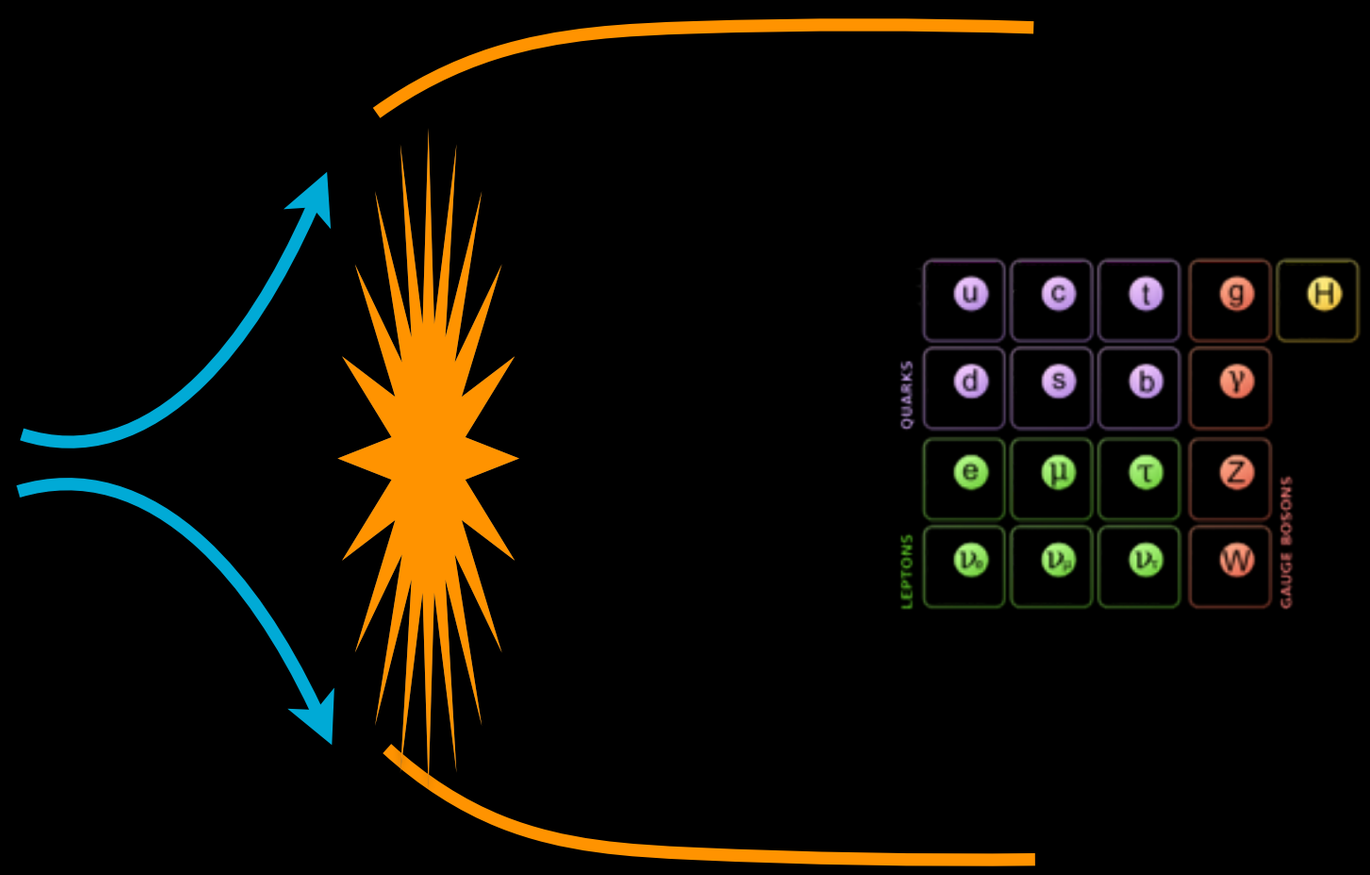


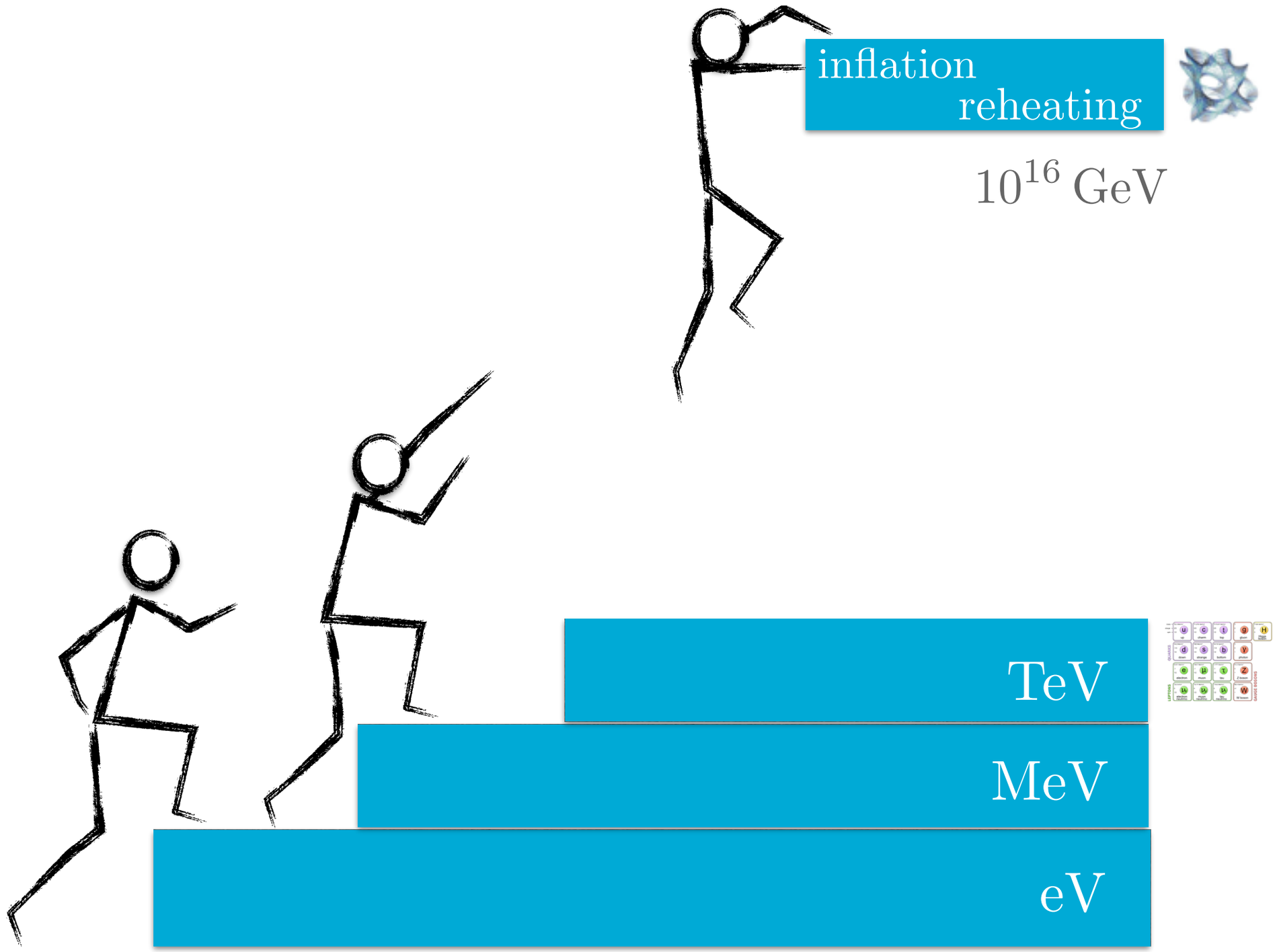




us

- how did inflation end ? (reheating)
- Standard Model?




inflation
reheating
10^{16} GeV
TeV
MeV
eV

general results possible ?

SIMPLE enough

COMPLEX enough

general results possible ?

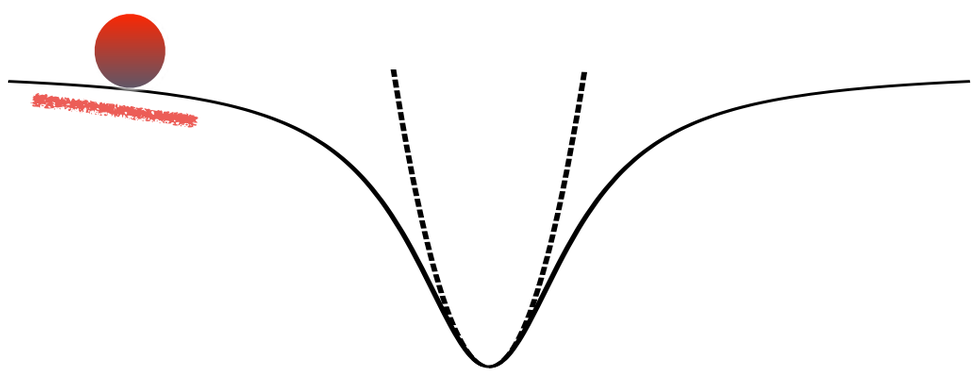
SIMPLE enough

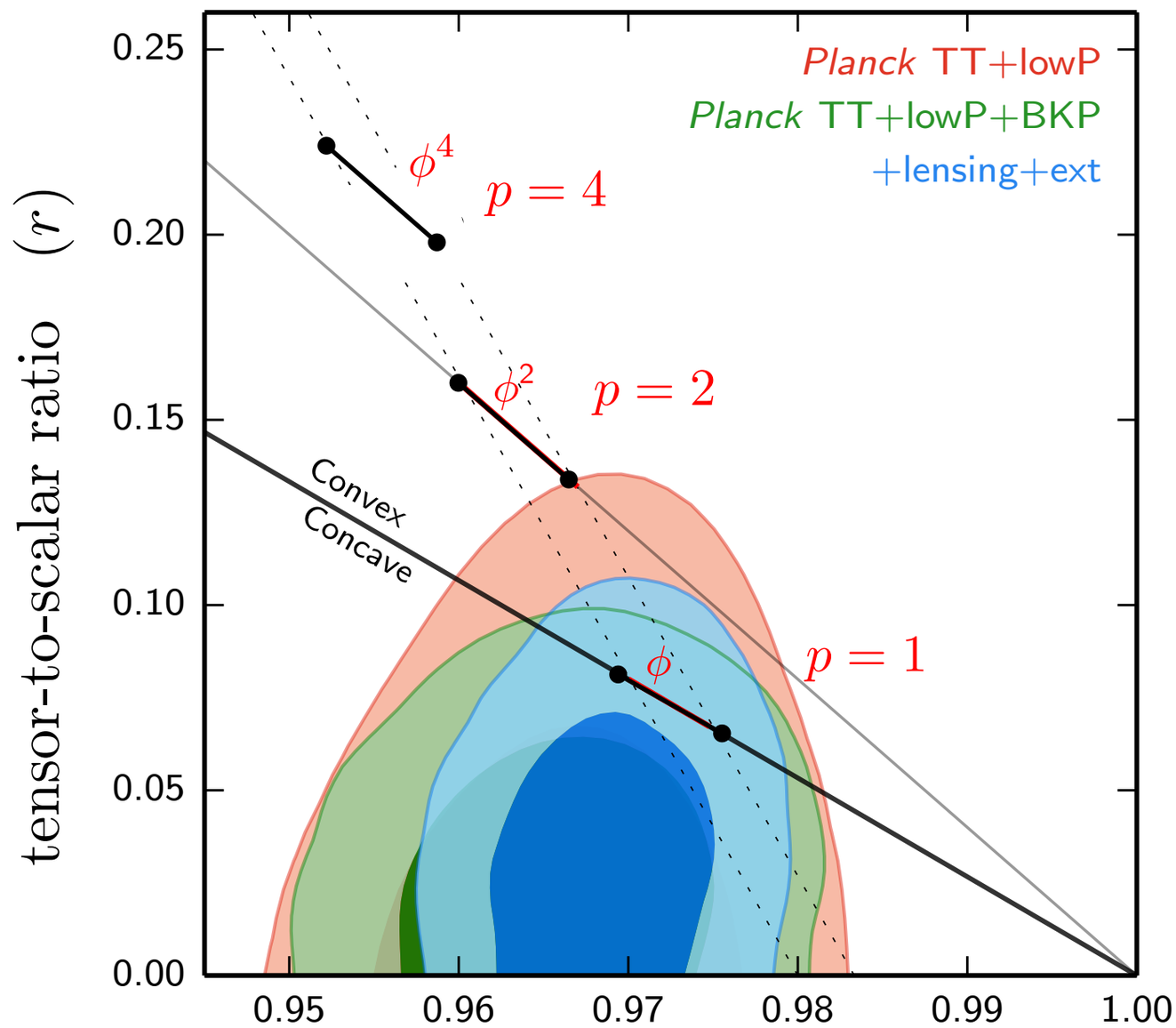
COMPLEX enough

constraints from observations

refer to F. Finelli's talk on Monday

$V(\phi) \propto \phi^p$





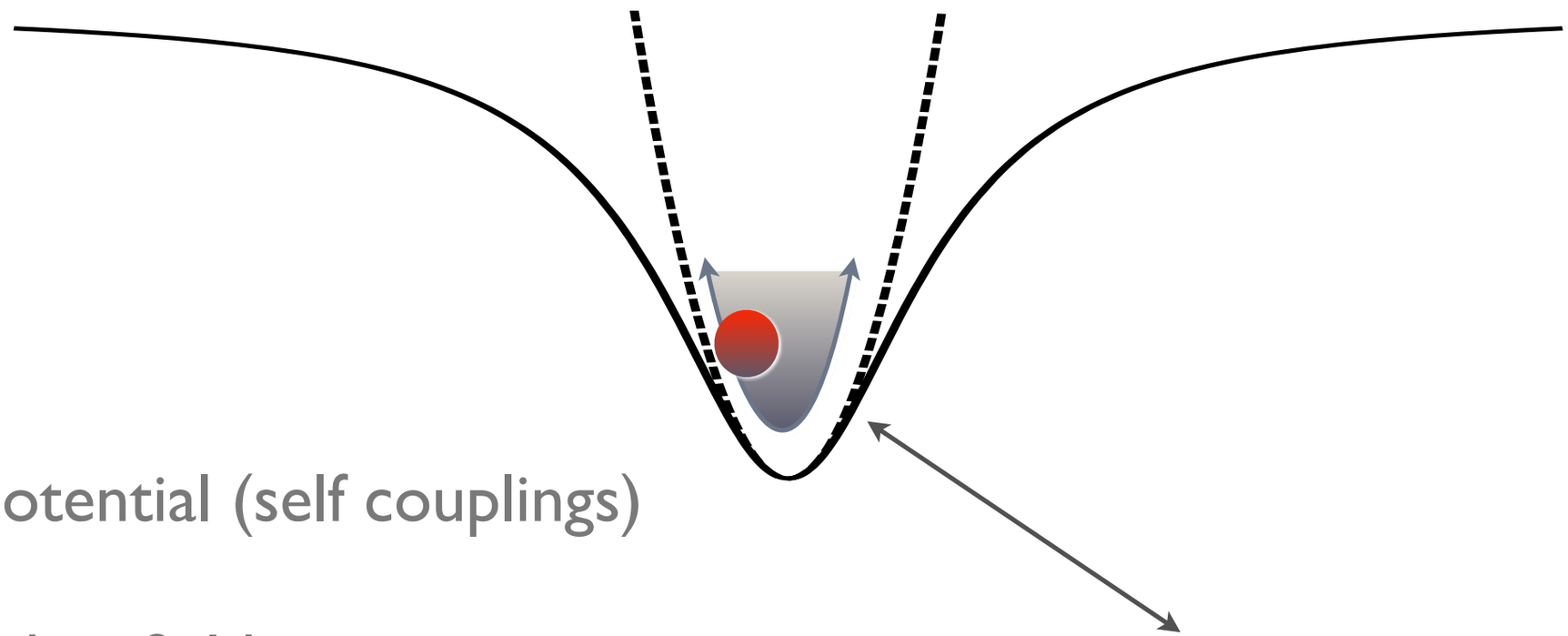
primordial tilt (n_s)

$\mathcal{P}_{\rm inf}(k) \propto k^{n_s - 1}$

for example:

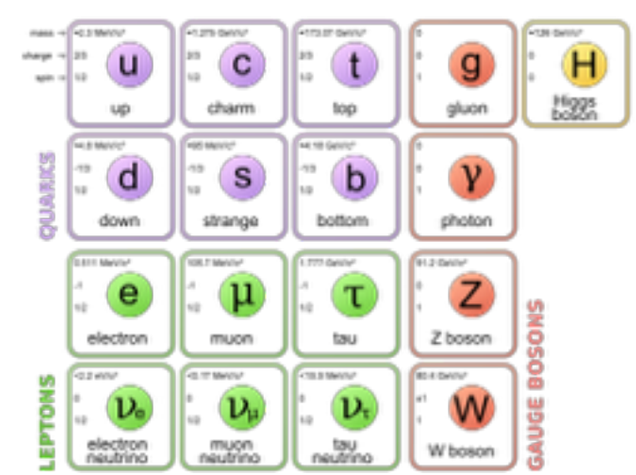
Silverstein & Westhpal (2008)
McAllister et. al (2014)
Kallosh & Linde (2014)

energy transfer: “reheating”



- shape of the potential (self couplings)
- couplings to other fields

χ, ψ

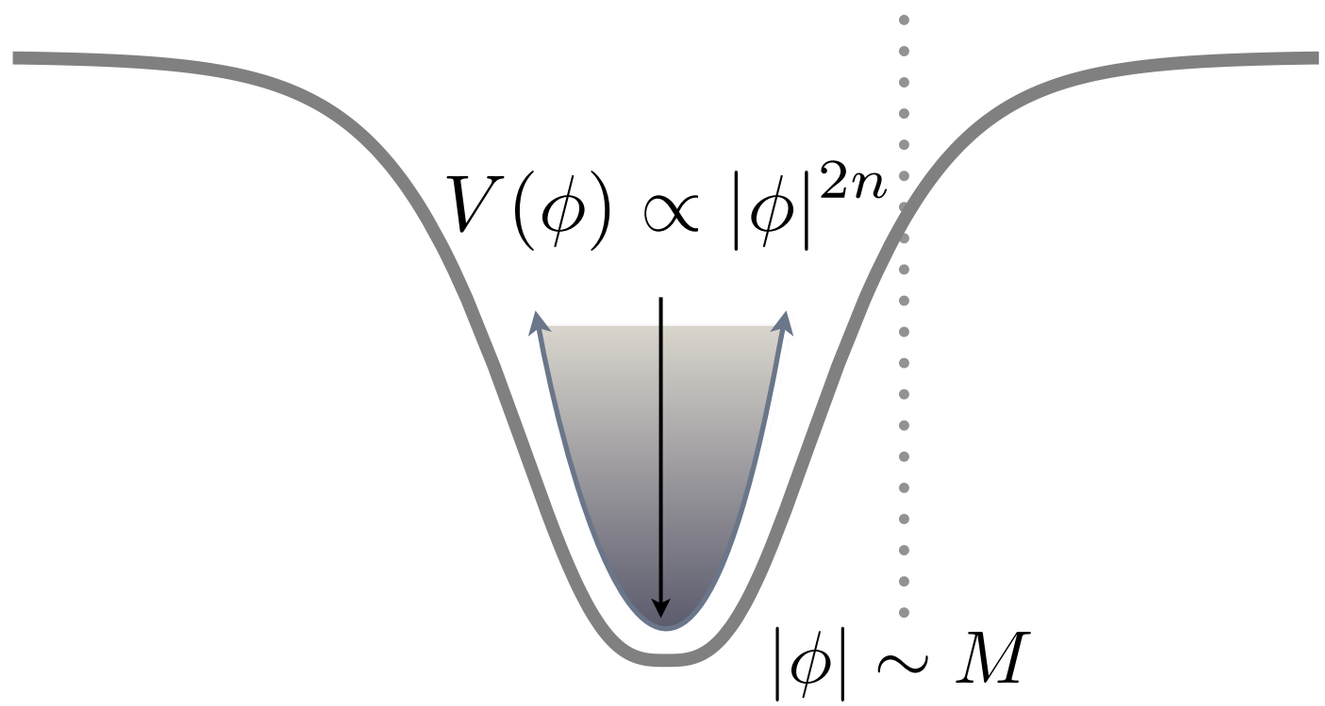


Traschen & Brandenburger (1990)
Kofman, Linde & Starobinsky (1994)
Shtanov, Traschen & Brandenberger (1995)
Kofman, Linde & Starobinsky (1997)
review: MA, Kaiser, Karouby & Hertzberg (2014)

end of inflation in “simple” models

for example:

Silverstein & Westhpal (2008)
McAllister et. al (2014)
Kallosh & Linde (2014)



- shape of the potential (self couplings)
- ~~couplings to other fields~~  $\chi\ ,\ \psi$

end of inflation in “simple” models

for example:

Silverstein & Westhpal (2008)
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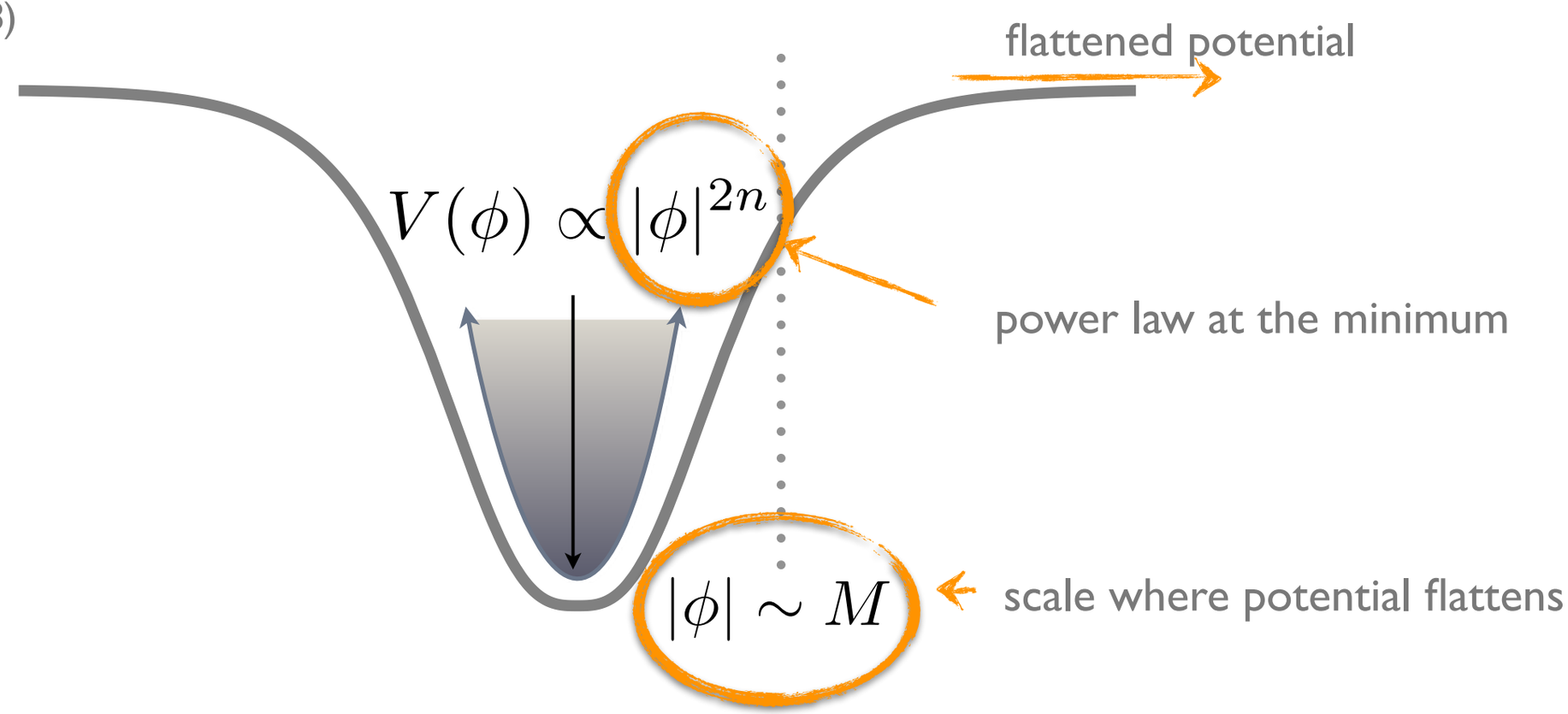
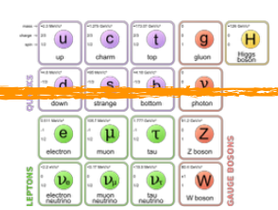
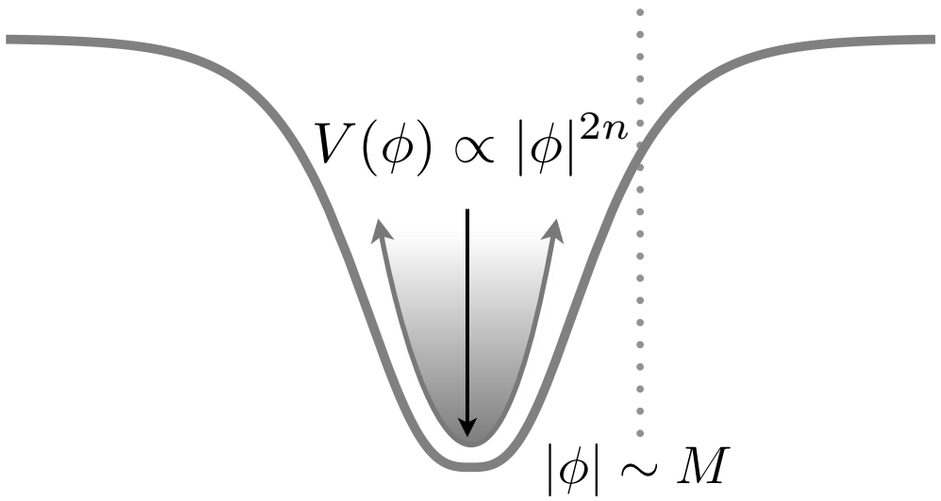


- shape of the potential (self couplings)
- ~~couplings to other fields~~  $\chi\,,\,\psi$

end of inflation in "simple" models



(i) what are the dynamics ?

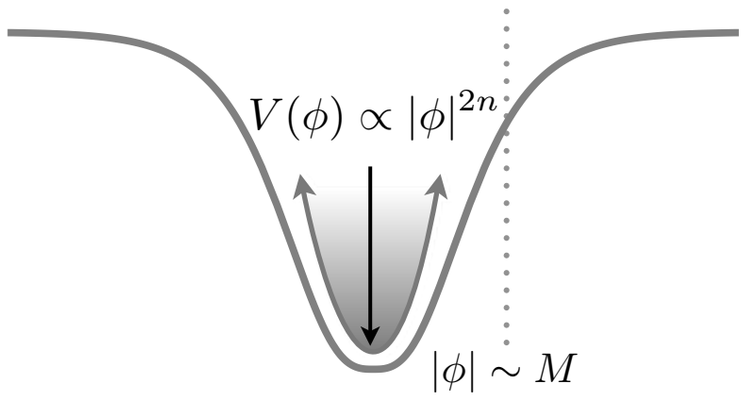
(ii) eq. of state & how long to radiation domination ?

(iii) obs. consequences ?

Lozanov & MA, *Phys Rev. Lett* (2017)

end of inflation in "simple" models

*can be applied to any cosmologically dominant scalar field



(i) what are the dynamics ?

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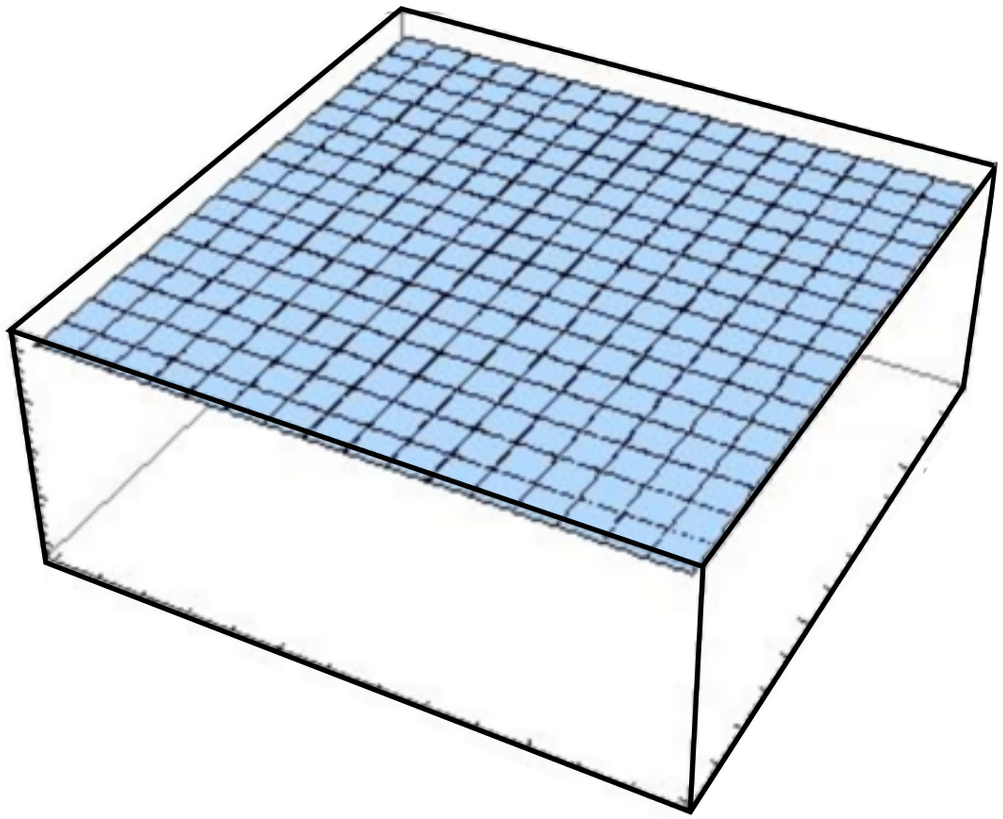
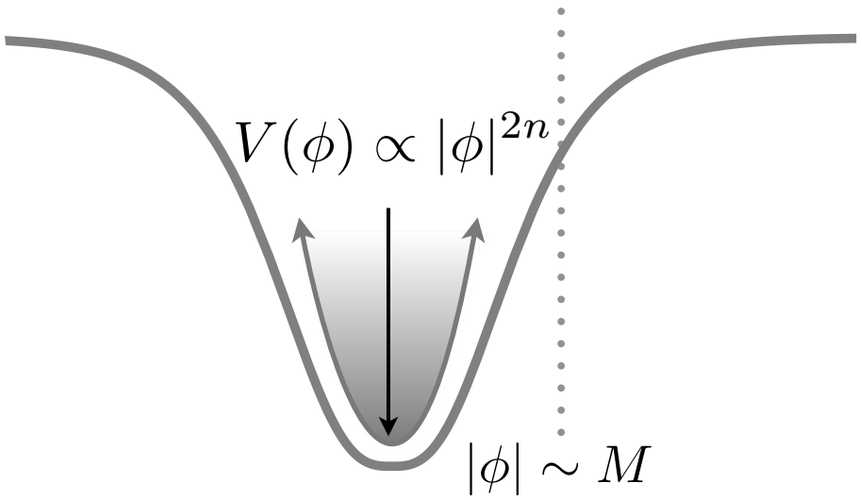
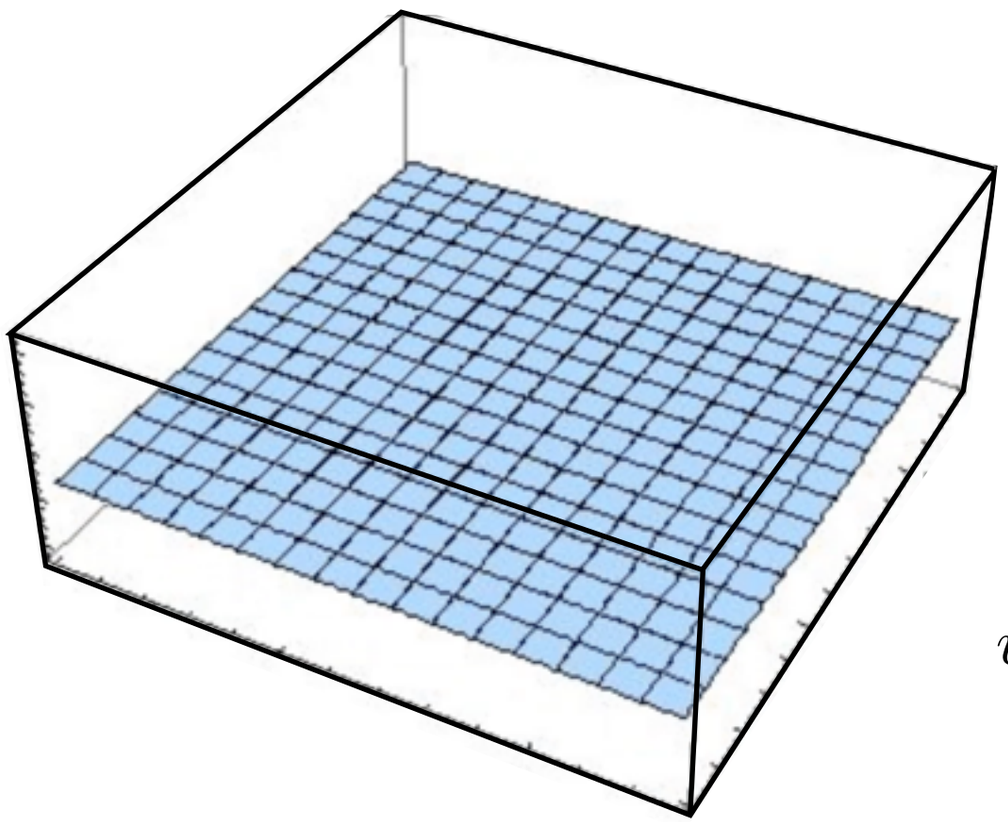
Lozanov & MA, *Phys Rev. Lett* (2016/17)

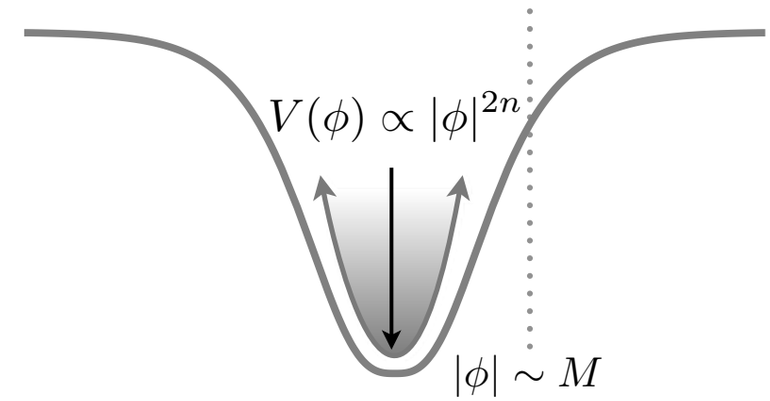
homogeneous dynamics





homogeneous eq. of state



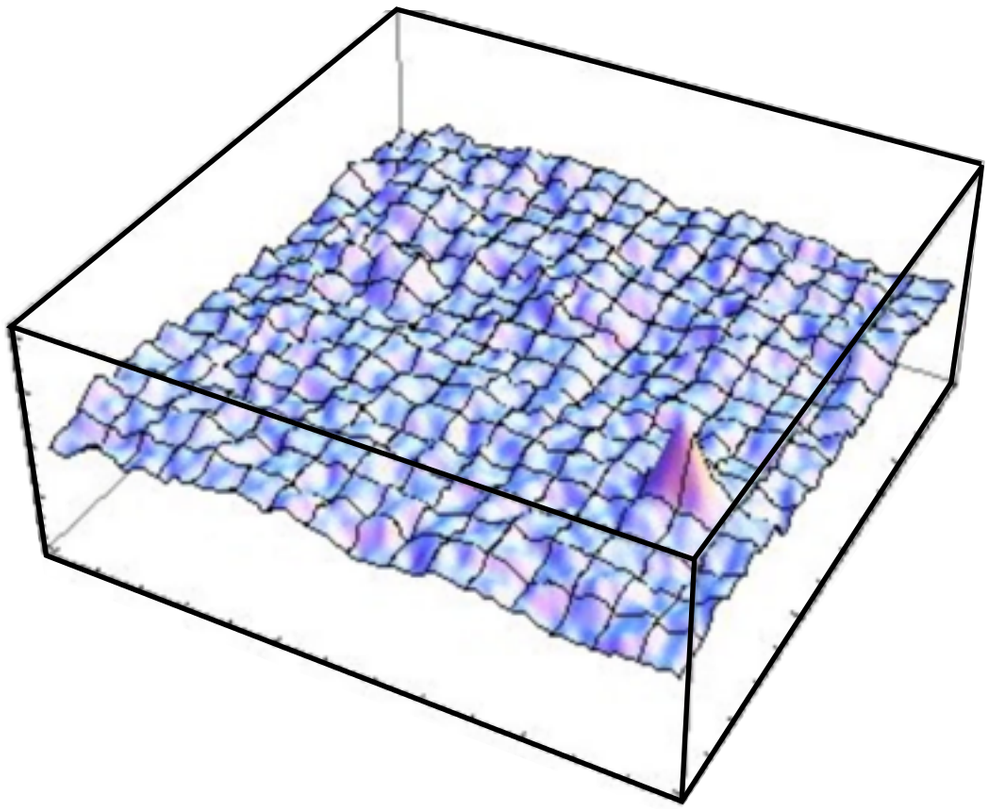


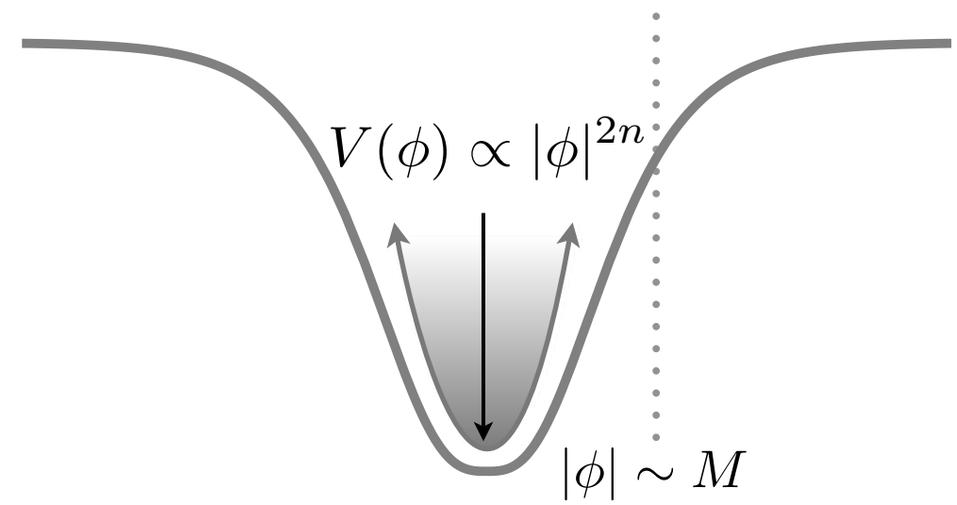
$$\text{eq. of state } w = \frac{\text{pressure}}{\text{density}}$$

$$w \equiv \frac{\langle p \rangle_{\rm s}}{\langle \rho \rangle_{\rm s}} = \frac{\langle \dot{\phi}^2/2 - (\nabla\phi)^2/6a^2 - V \rangle_{\rm s}}{\langle \dot{\phi}^2/2 + (\nabla\phi)^2/2a^2 + V \rangle_{\rm s}} \approx \frac{n-1}{n+1}$$

Turner (1983)
* can be obtained from a viral theorem

fragmentation is (almost) inevitable





(i) existence of wings (self-couplings) $M \lesssim m_{\rm pl}$

and/ or

(ii) non-quadratic minimum $n > 1$

* directly related to competition between growth rate and expansion, but duration depends on parameters

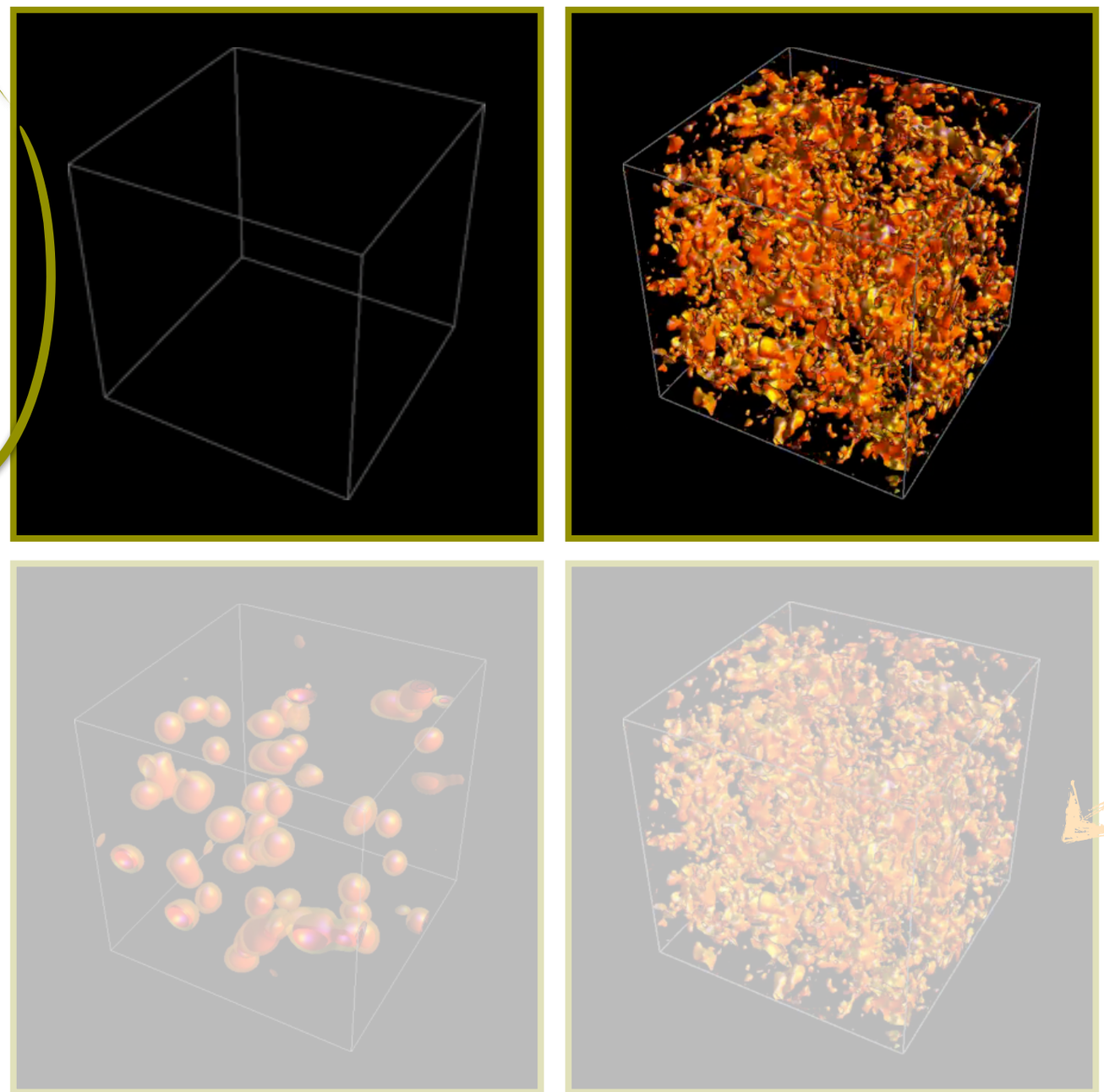
result of fragmented dynamics

* after sufficient time

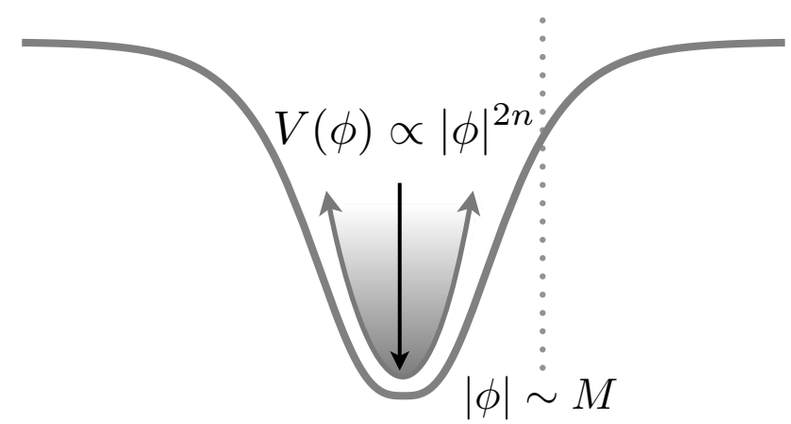
$n = 1$ $n > 1$

$M \sim m_{\rm pl}$

$M \ll m_{\rm pl}$



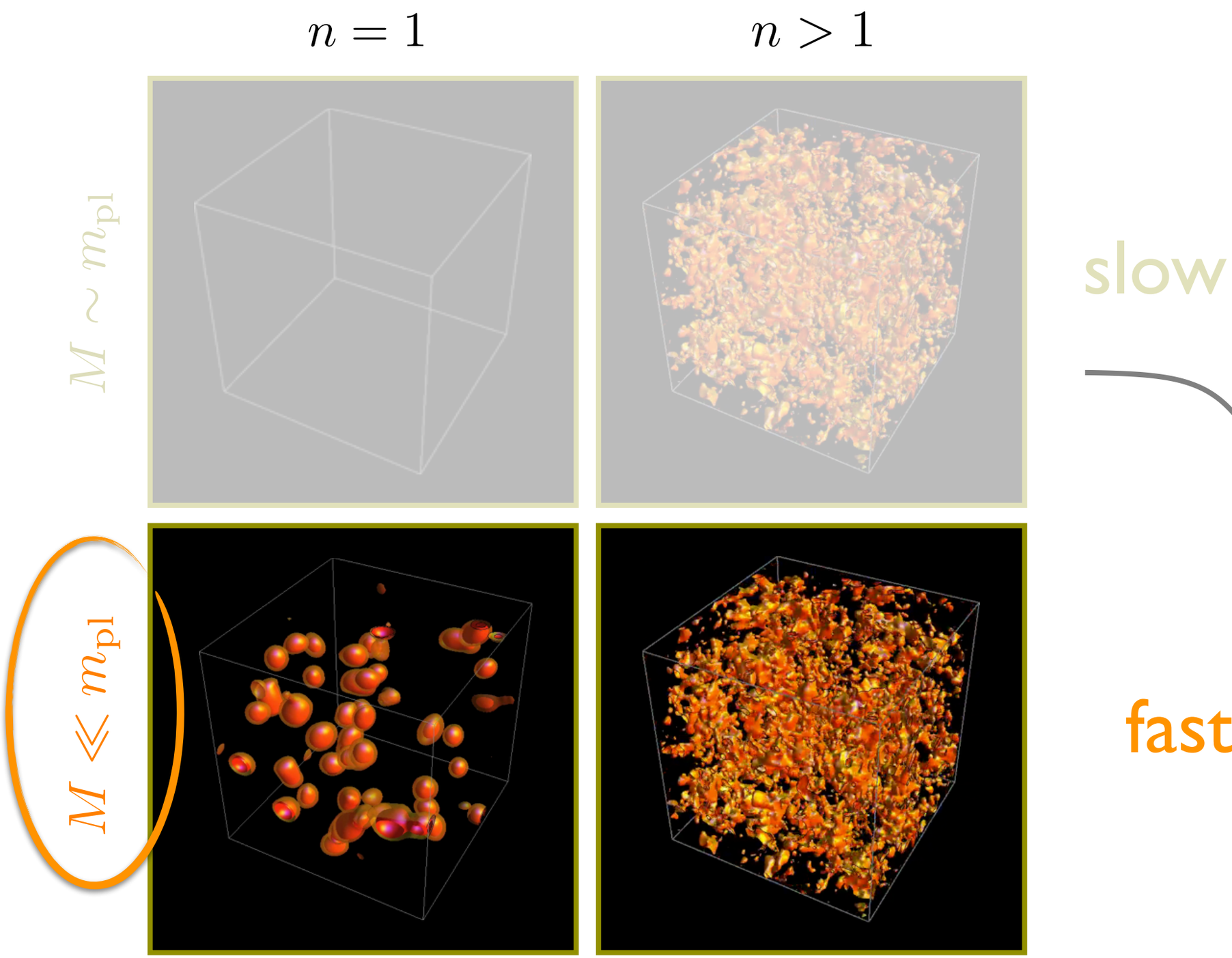
slow

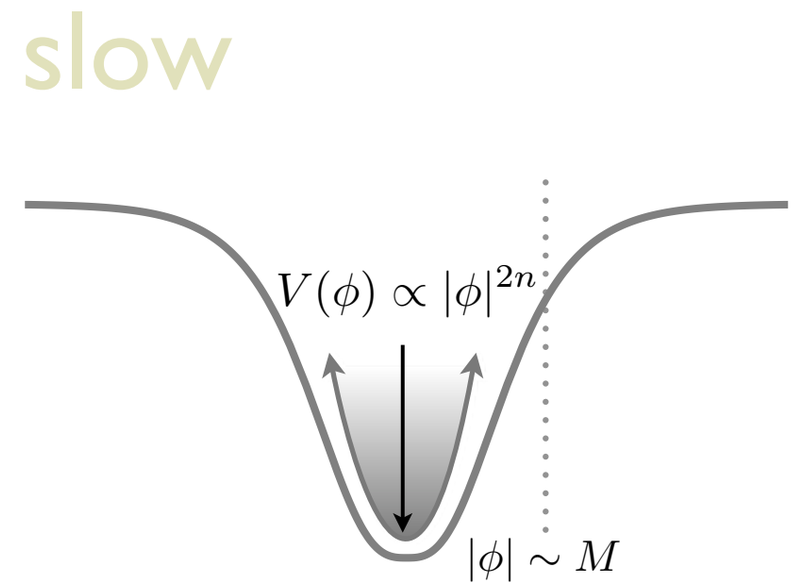


quickly

result of fragmented dynamics

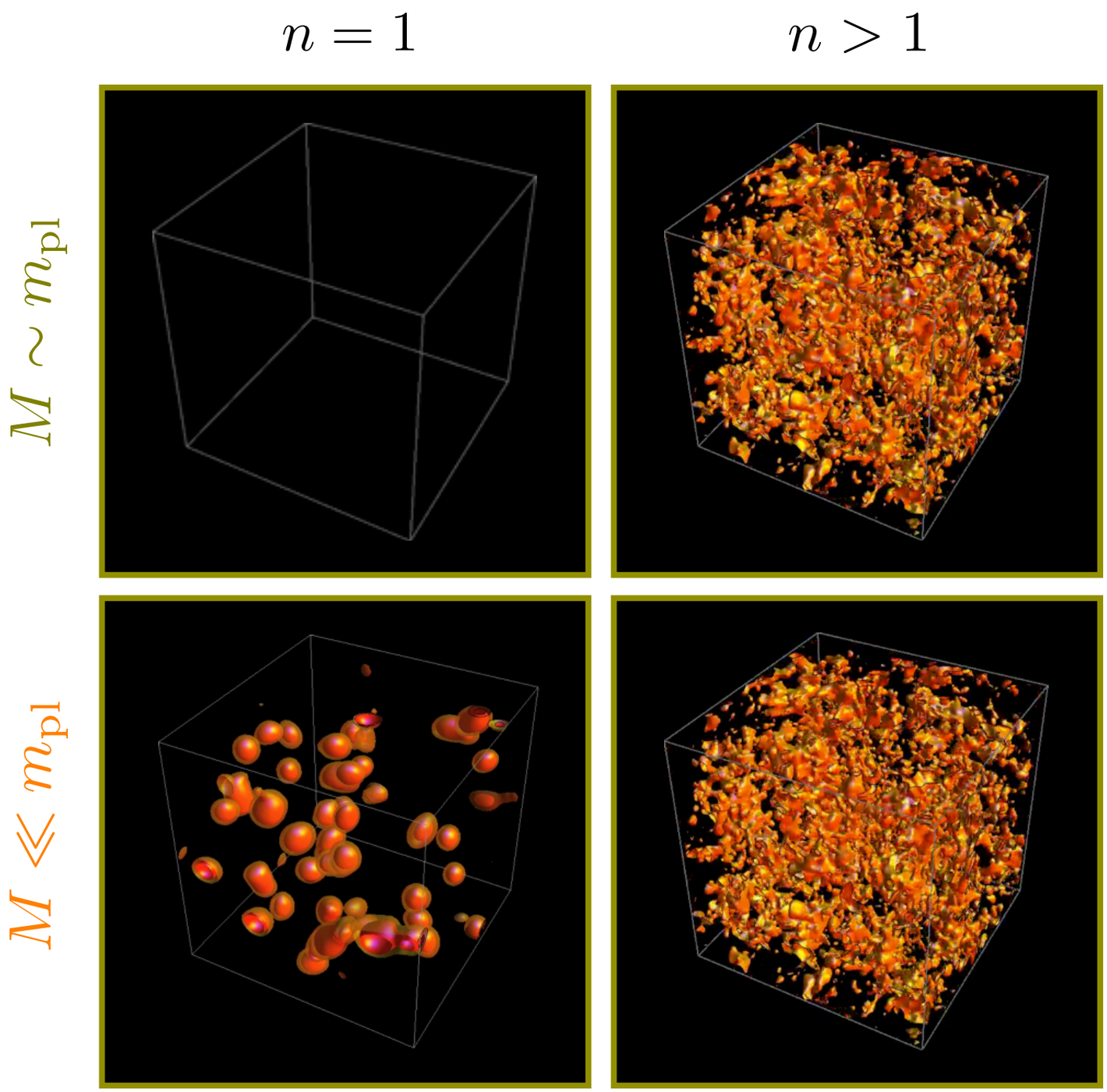
* after sufficient time

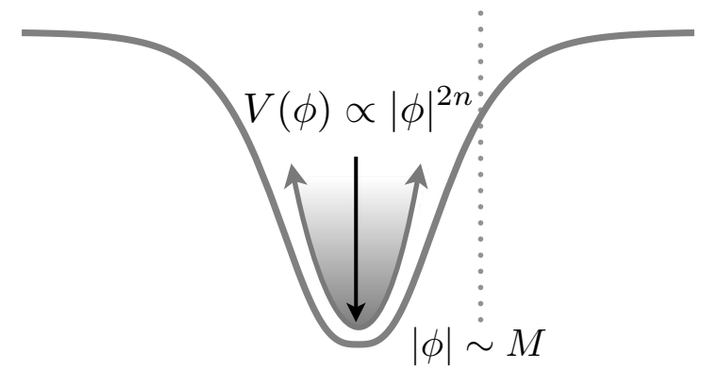




eq. of state

* after sufficient time



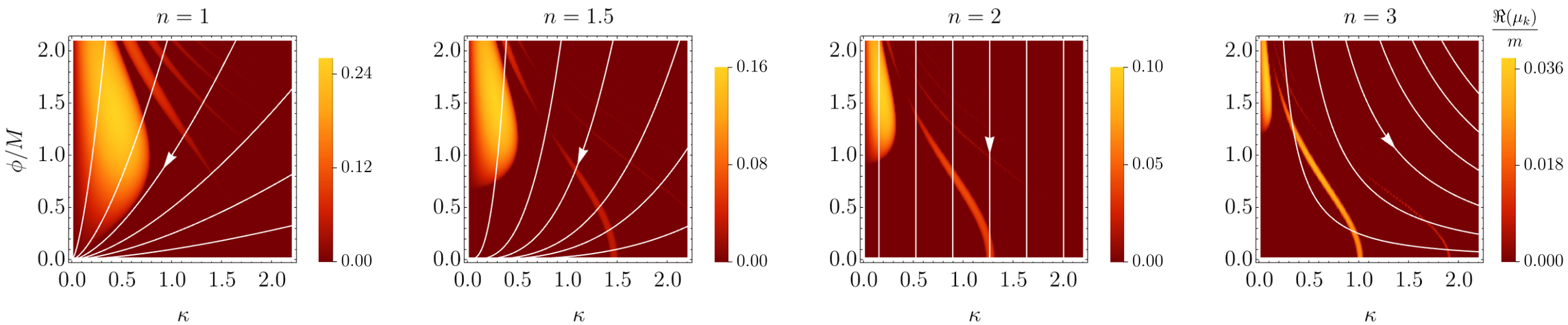


$$w \to \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

independent of M

$$w \neq \frac{n-1}{n+1}$$

4 ingredients for understanding the results



(i) rapid fragmentation due to broad band $\quad [|\Re(\mu_k)|/H]^0_{\max} = f(n)(m_{\rm Pl}/M) \qquad M \ll m_{\rm pl}$

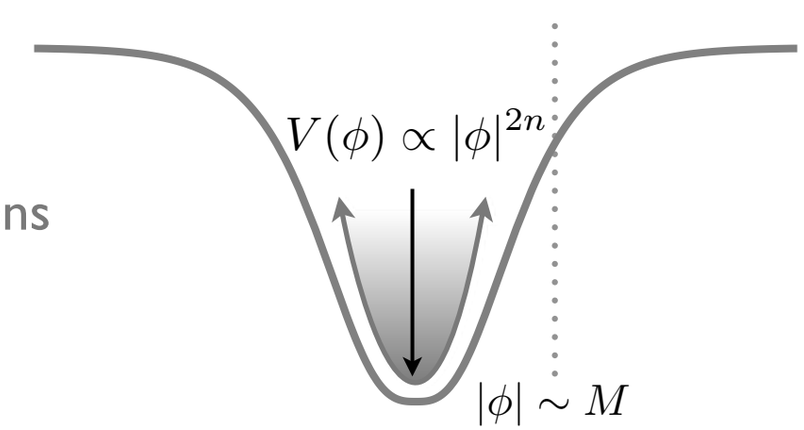
(ii) importance of the narrow band $\quad [\Re(\mu_k)/H]^1 \propto m_{\rm Pl}/|\bar{\phi}| \qquad |\bar{\phi}| \ll M$

(iii) getting stuck in the instability band $\quad |\dot{\kappa}| \sim H\kappa$

(iv) how gradients redshift compared to the potential energy

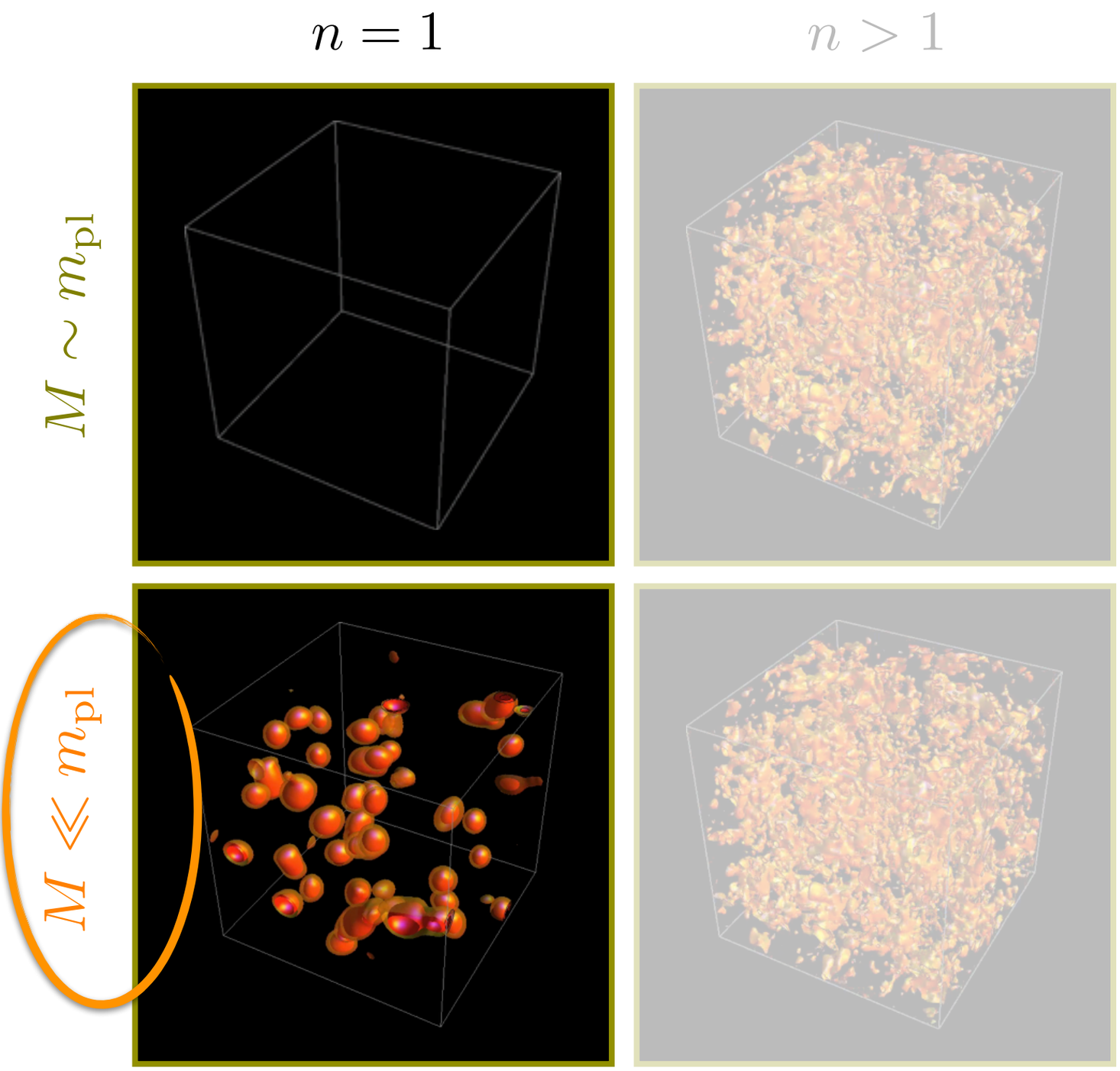
$$w \to \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

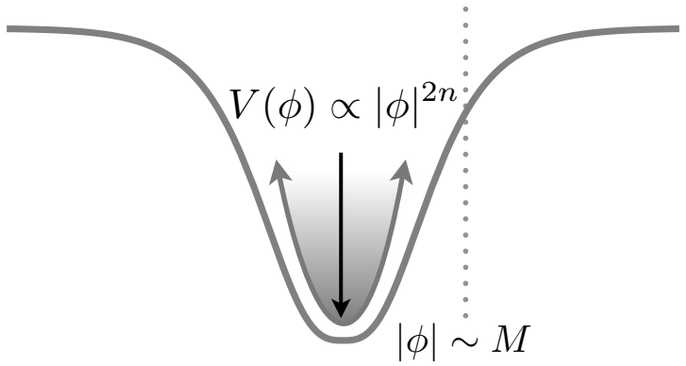
* formation of solitons



focus on $n = 1$

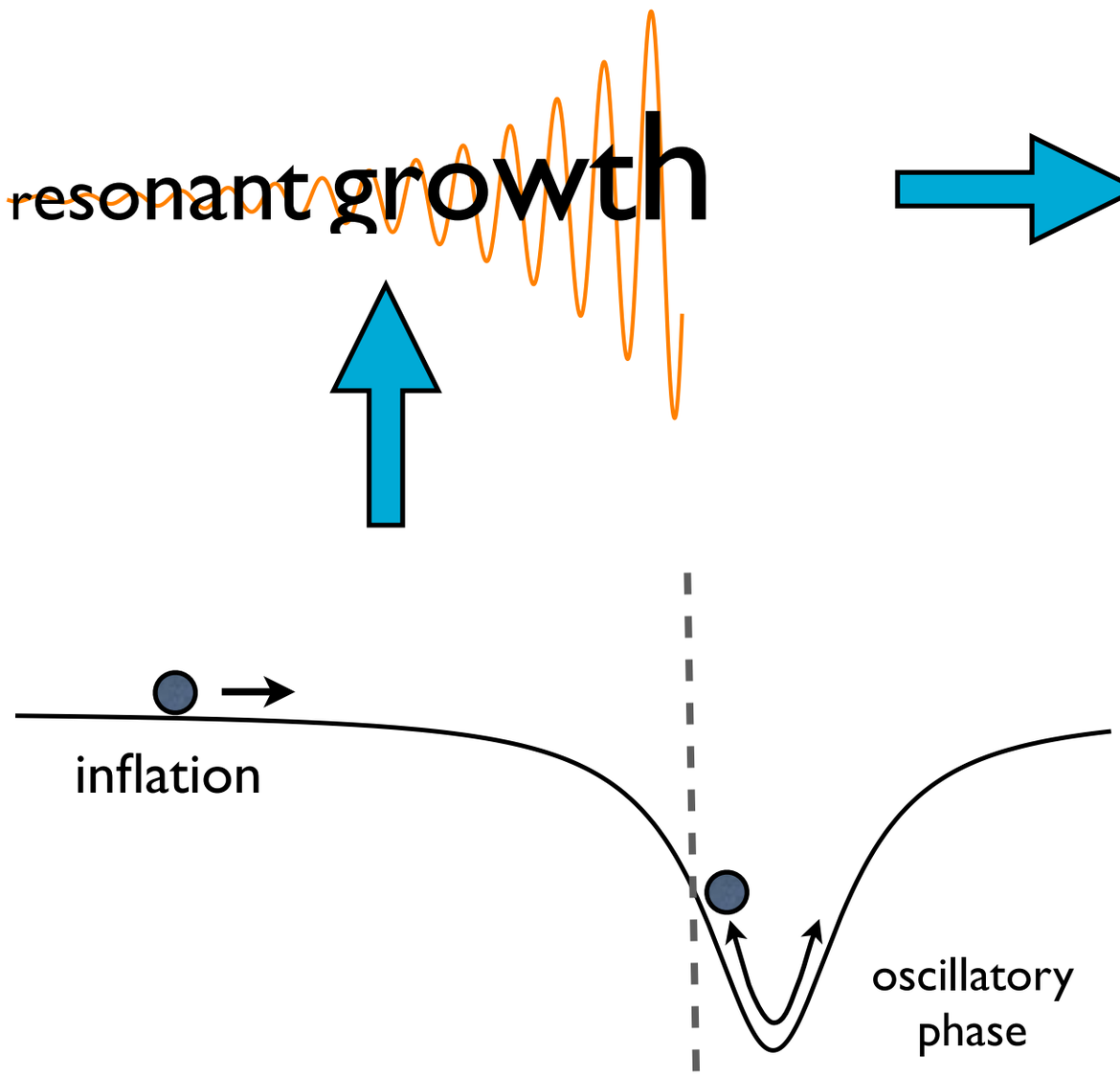
(*quadratic minimum)

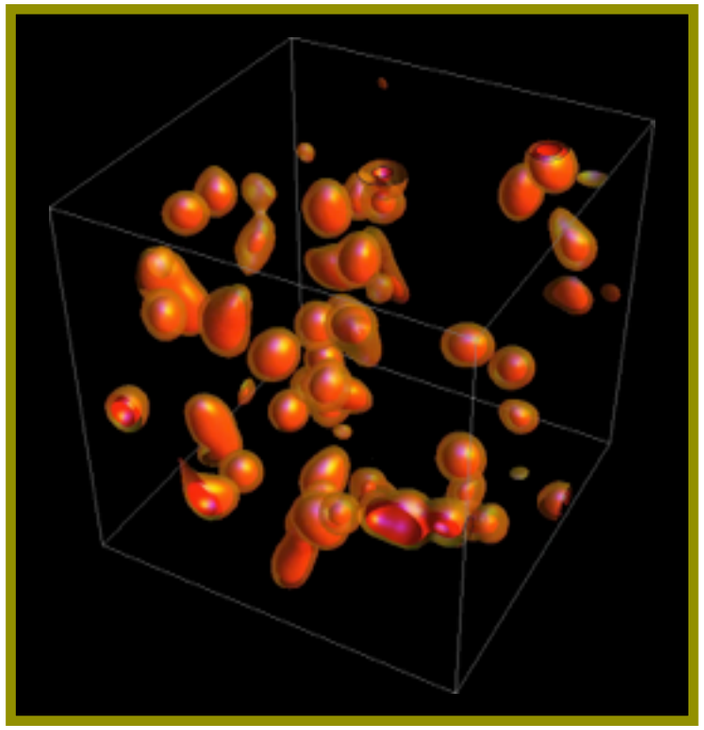




$$w \to \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

the process

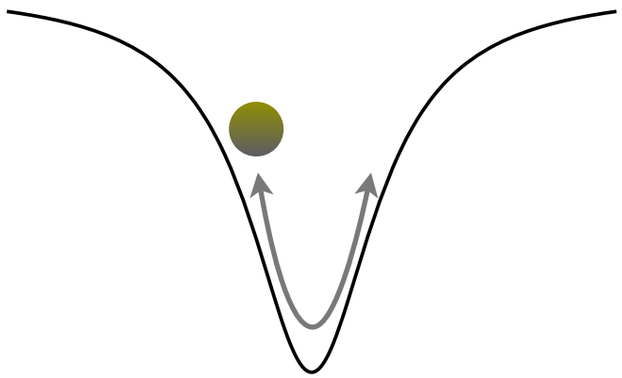


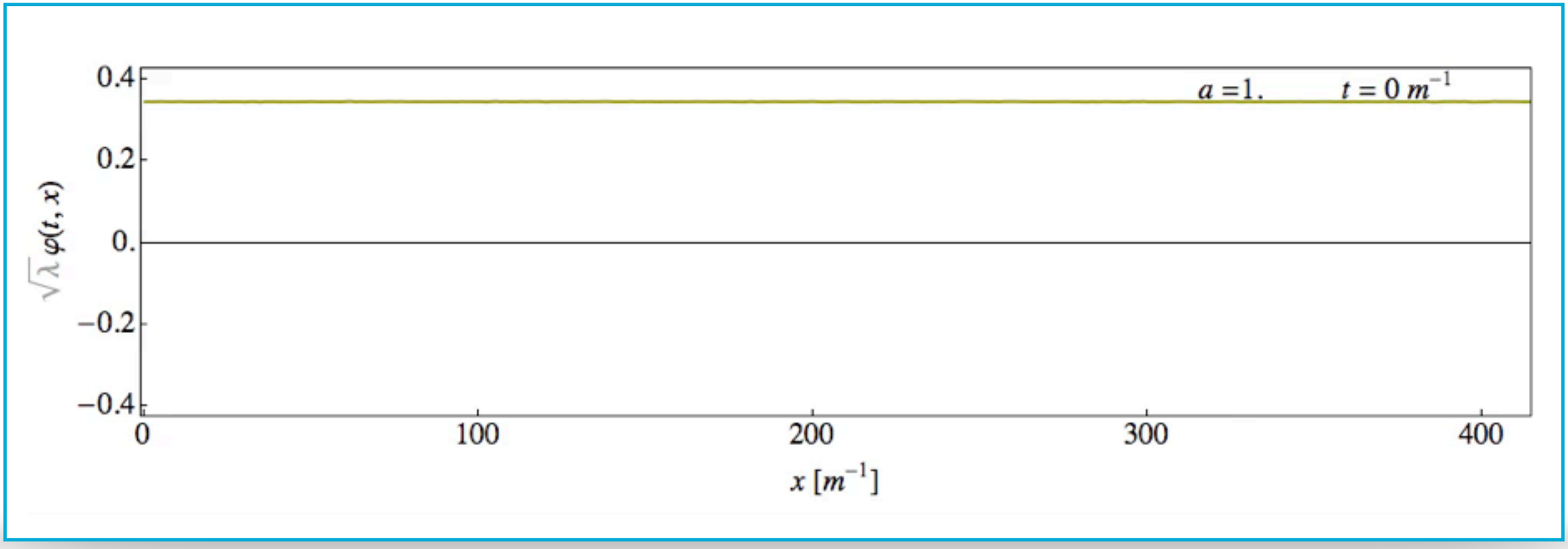


dynamics for $n = 1$

(*quadratic minimum)

$$\Box\varphi = V'(\varphi)$$





MA (2010)
Khlopov et. al (1985)

dynamics for $n = 1$

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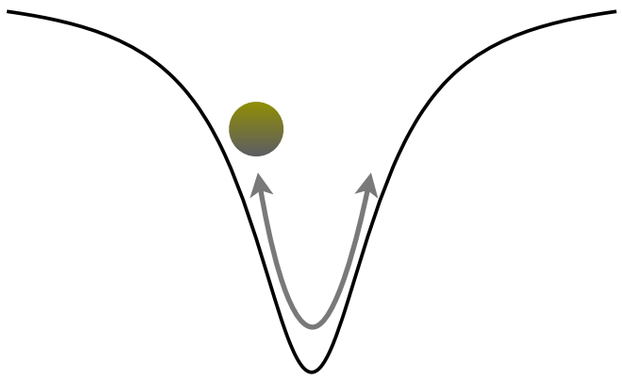

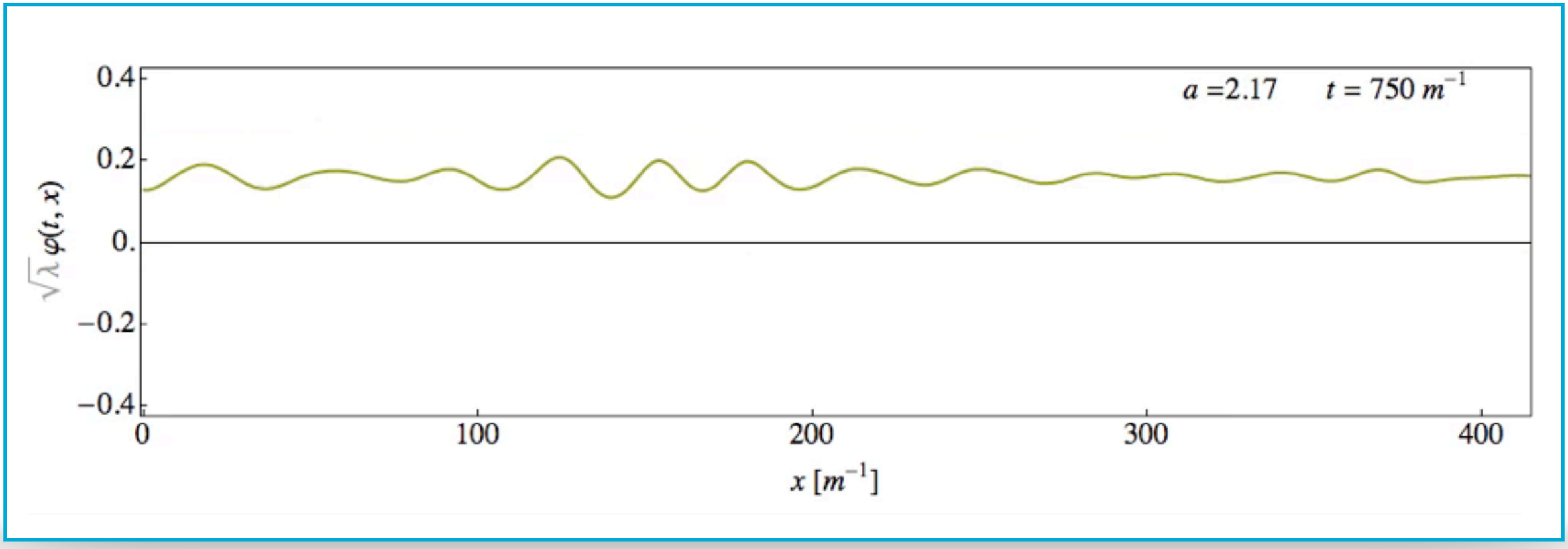

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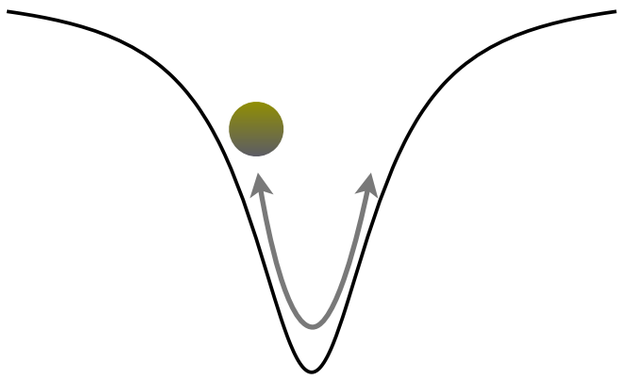

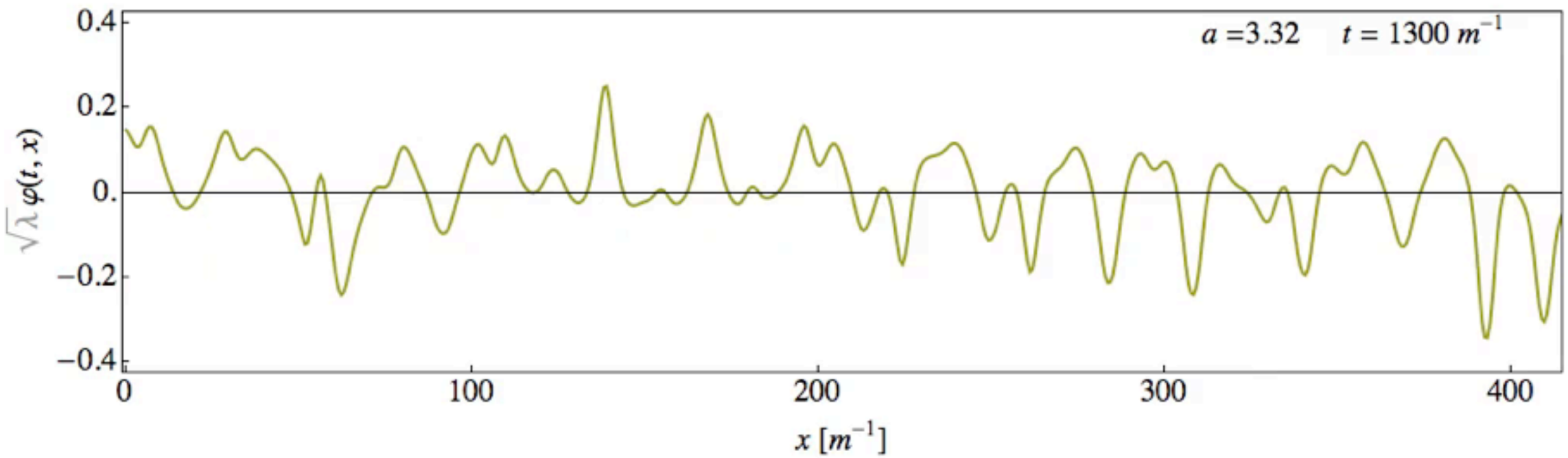

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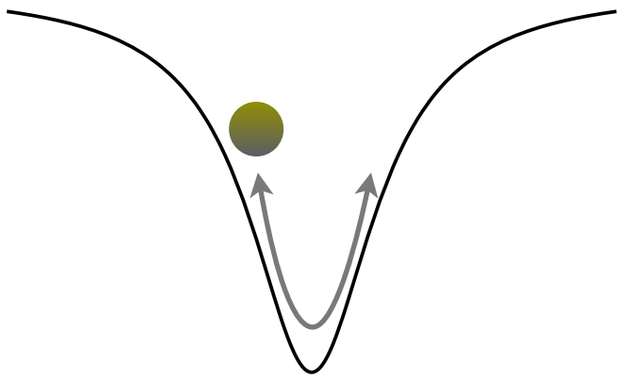

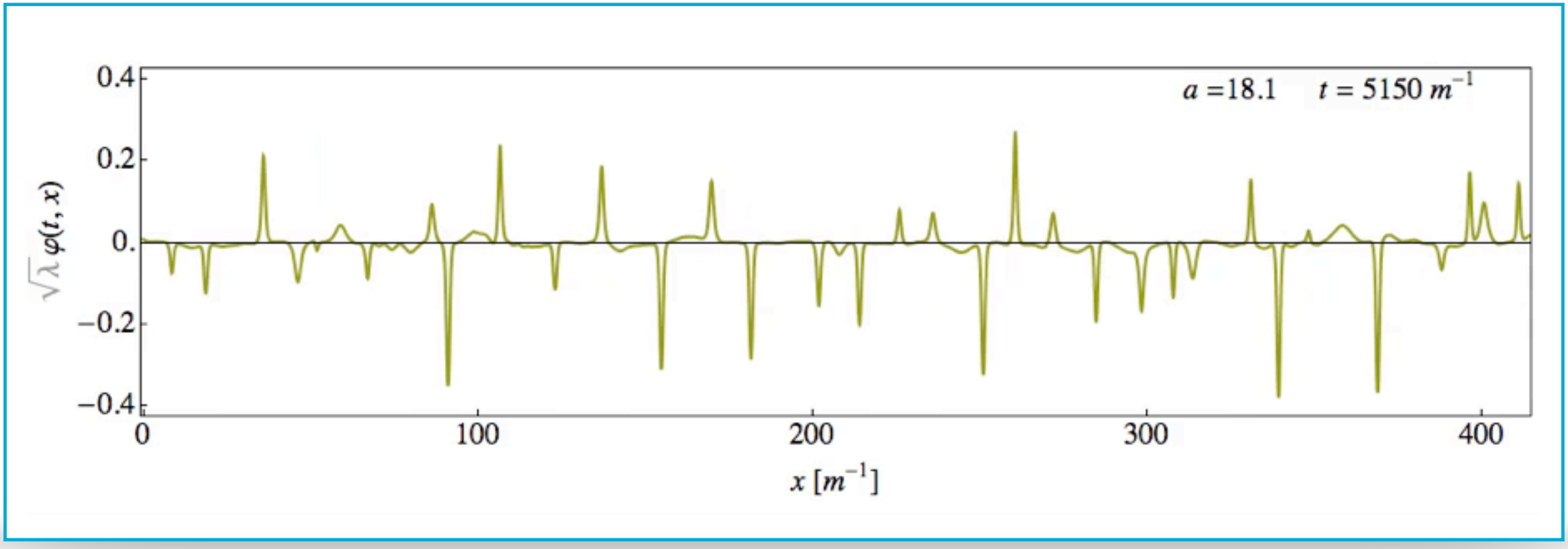

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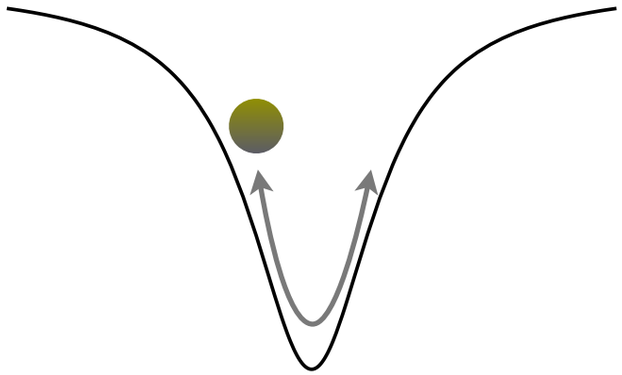
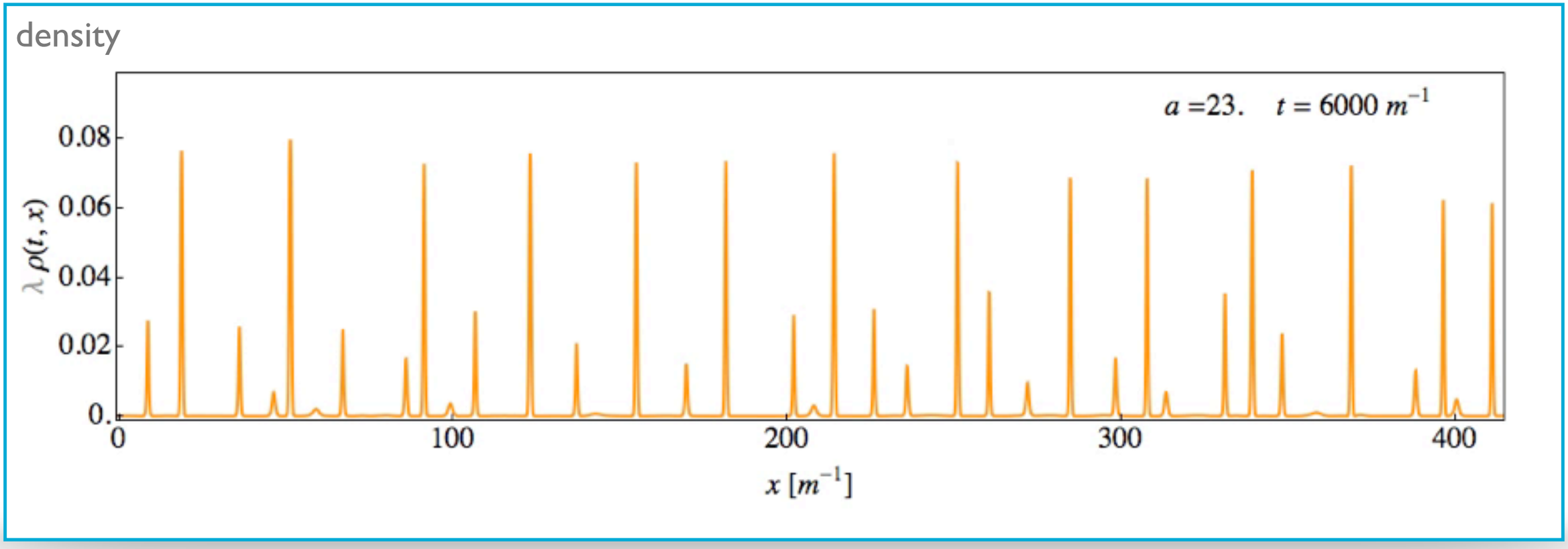
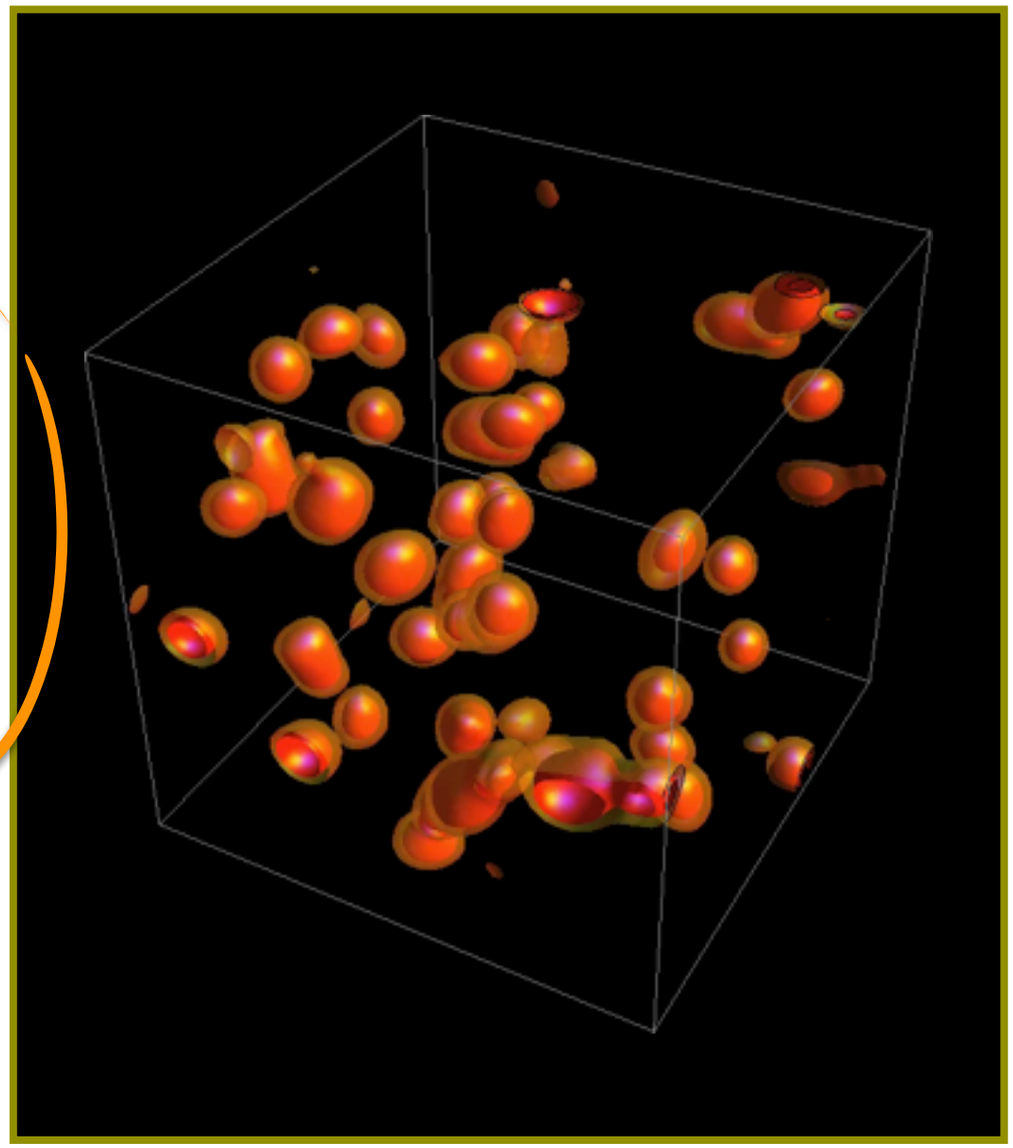




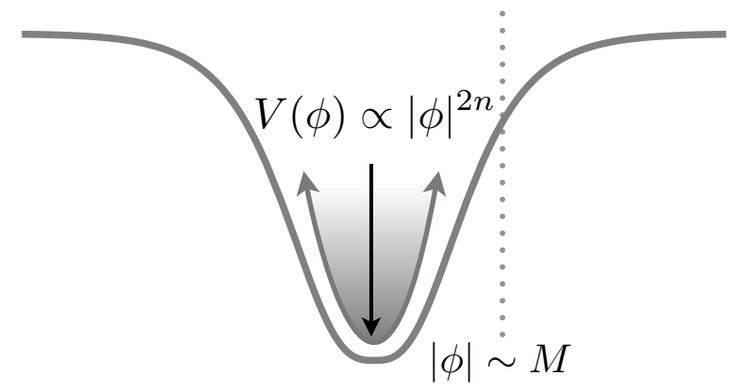
now in 3D

$n = 1$

$M \ll m_{\rm pl}$

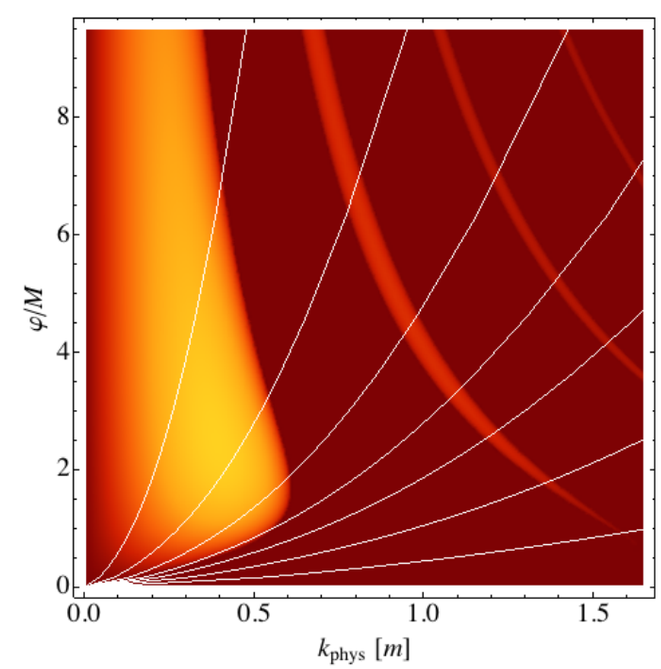


MA, Easther, Finkel, Flaugher & Hertzberg (2011)

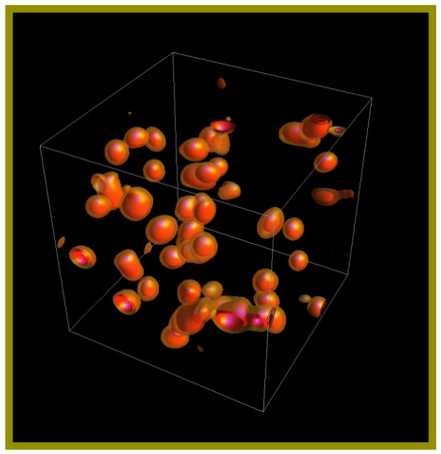


$$\frac{\mu_k}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

Floquet Chart



oscillons ?



(1) oscillatory (2) spatially localized (3) **very long lived**

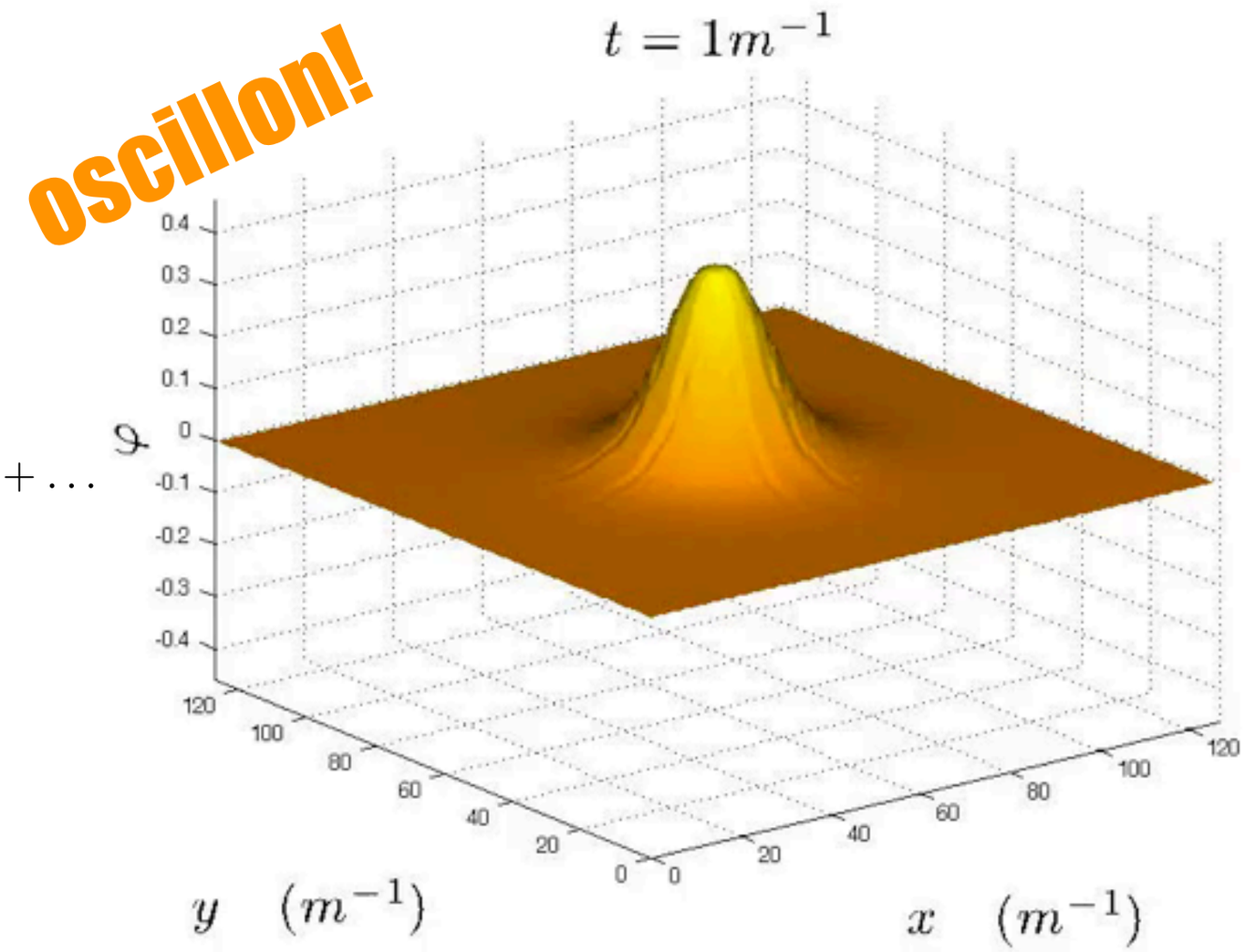
$$\mathcal{L} = T(X,\varphi) - V(\varphi)$$

$$T(X,\varphi) = X + \xi_2 X^2 + \xi_3 \varphi X^2 + \dots$$

$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{\lambda_3}{3}\varphi^3 + \frac{\lambda_4}{4}\varphi^4 + \frac{\lambda_5}{5}\varphi^5 + \dots$$

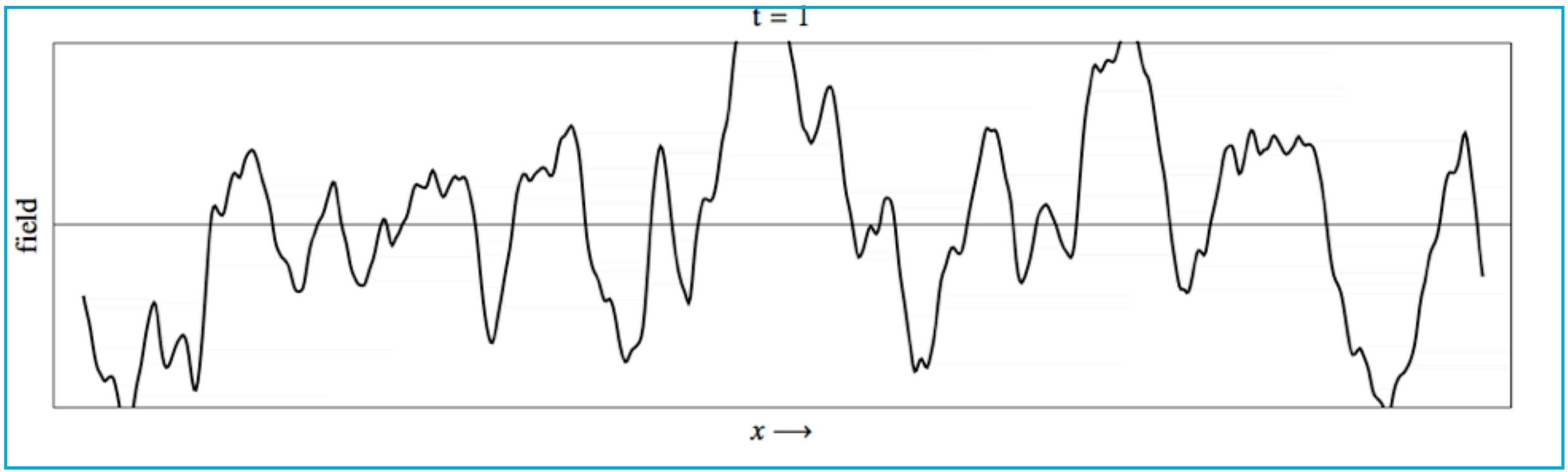
$$\boxed{\Delta = \xi_2 - \lambda_4 + \frac{10}{9}\lambda_3^2 > 0.}$$

existence and stability:
MA (2013)
MA & Shirokoff (2010)
Hertzberg (2011)



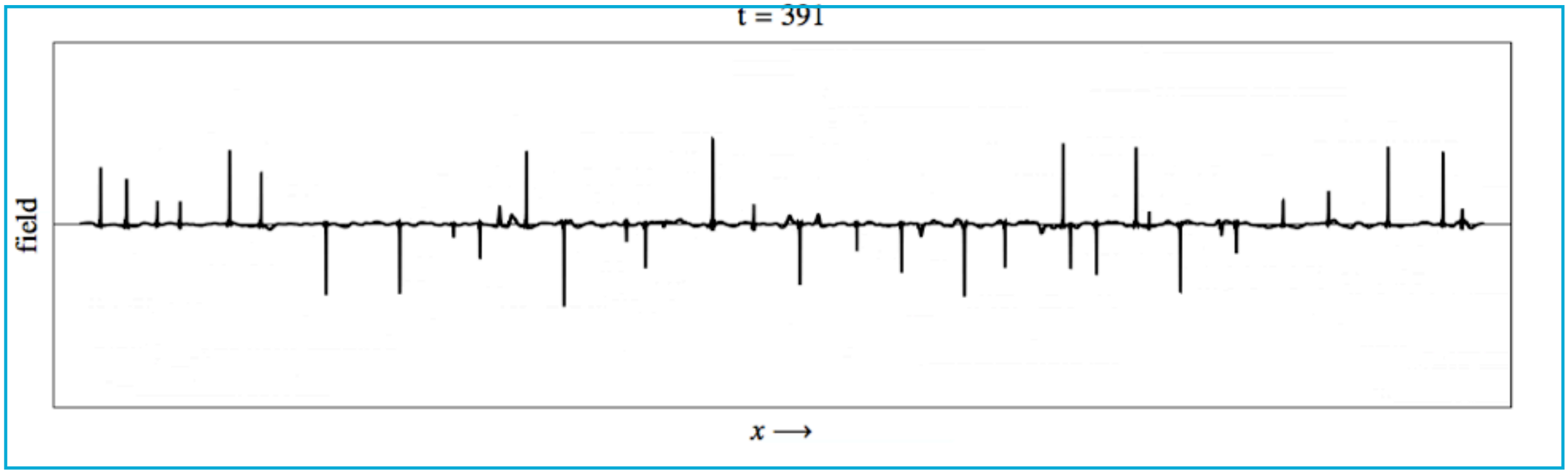
Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland, Gleiser and Mueller et al. 1995 …

insensitive to initial conditions



simulation of "quasi-thermal" example in Farhi et. al 2008

insensitive to initial conditions



simulation of "quasi-thermal" example in Farhi et. al 2008

consequences ?

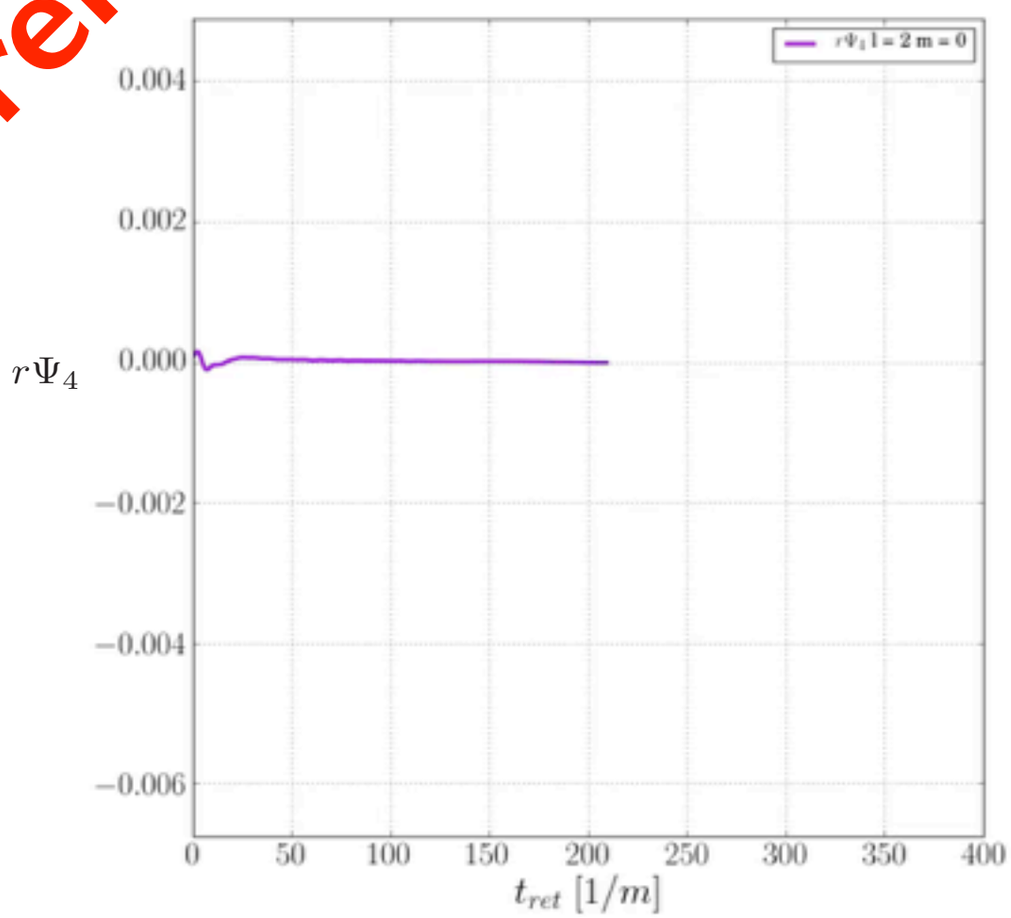
- delay in radiation domination ? (if coupled to other fields)
- black holes ? (upcoming paper with *K. Lozanov*)
- baryogengesis ? (*K. Lozanov & MA 2014*)
- gravitational waves ?

  Zhou, Copeland, Easther, Finkel, Mou & Saffin (2013)
  Antusch, Cefala, Orani (2016)
  Antusch, Cefala, Krippendorf, Muia, Orani, Quevedo (2017)
  Bond, Braden & Mersini-Houghton (2015)

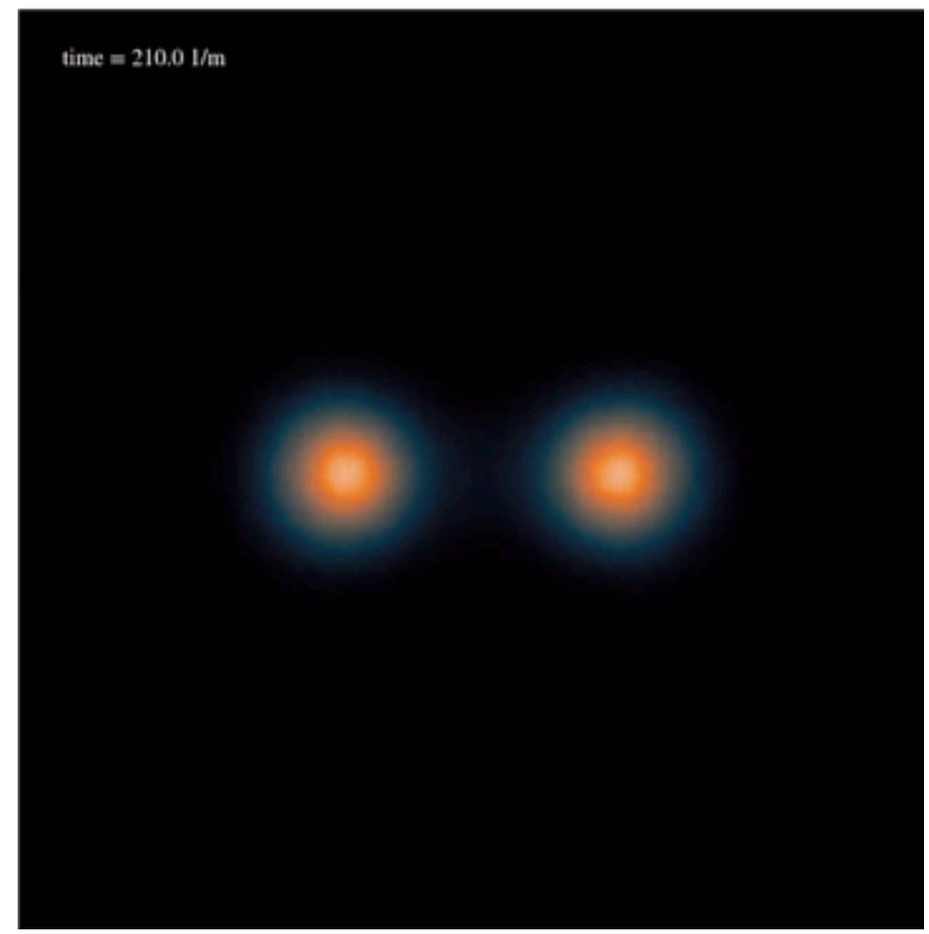
gravitational waves from scalar field lumps with full numerical GR

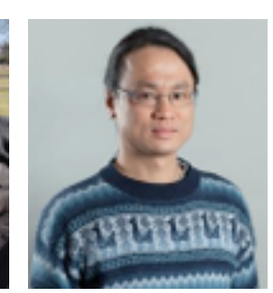
preliminary

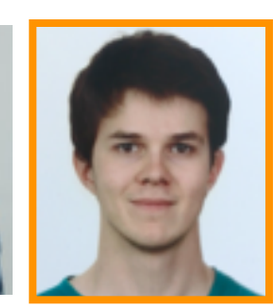
relevance for late universe processes — axion stars



MA, Garcia, **Helfer** & Lim (in prep)







gravitational waves from scalar field lumps with full numerical GR

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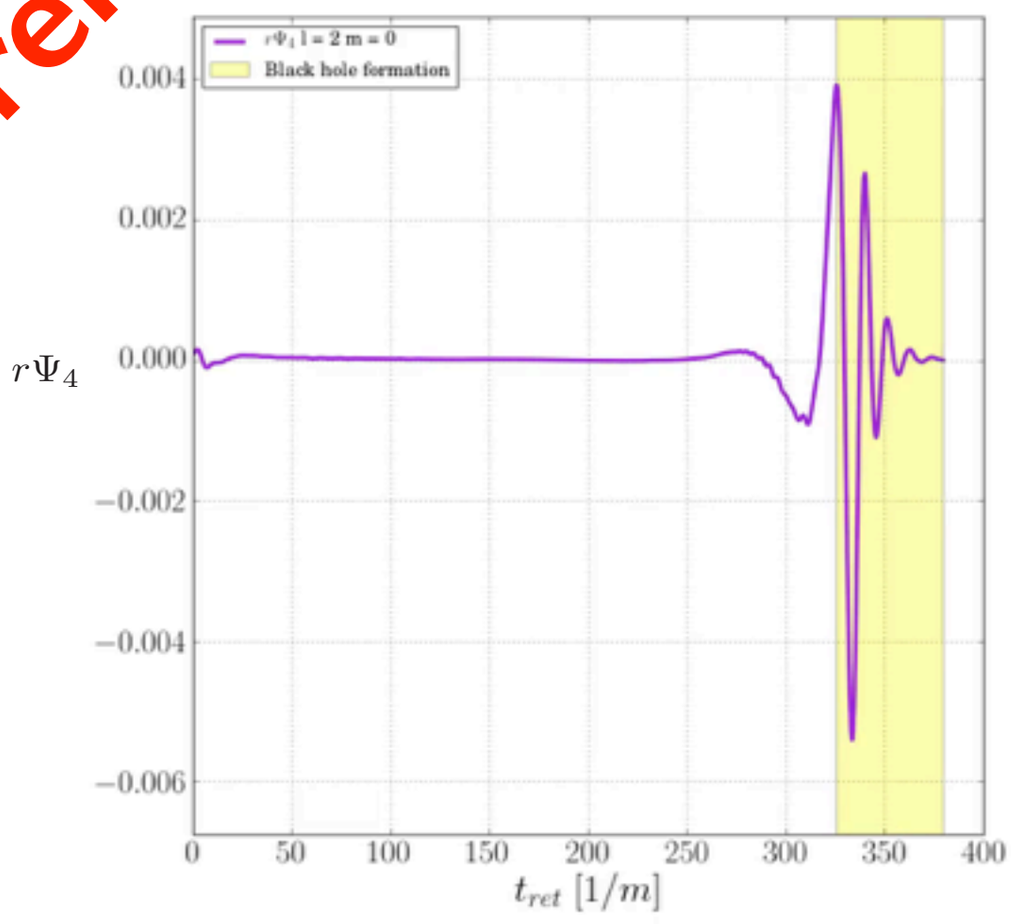

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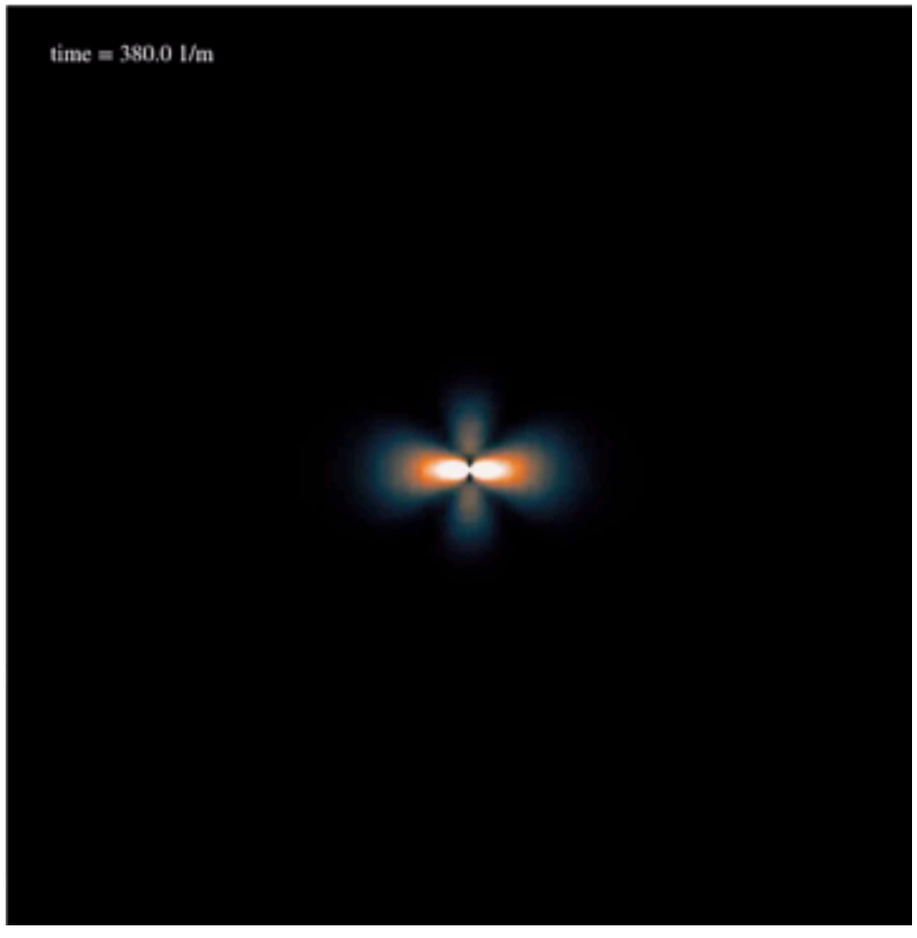
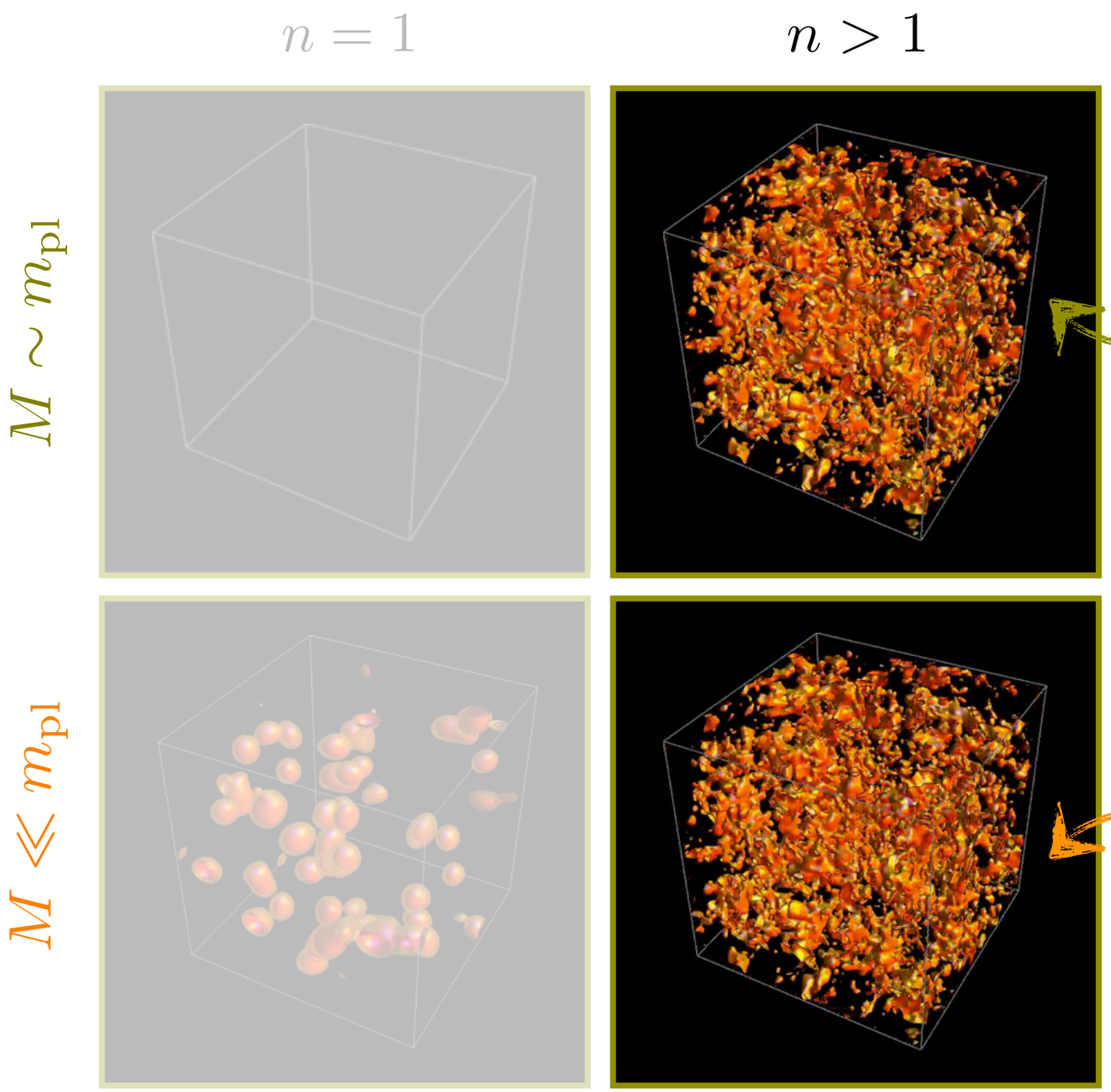
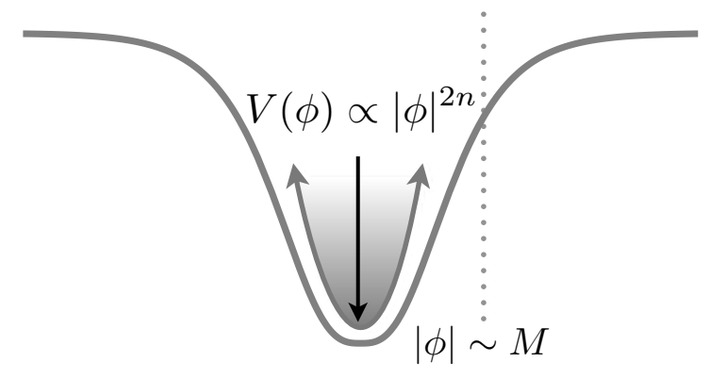


eq. of state $n > 1$

* after sufficient time



slowly!



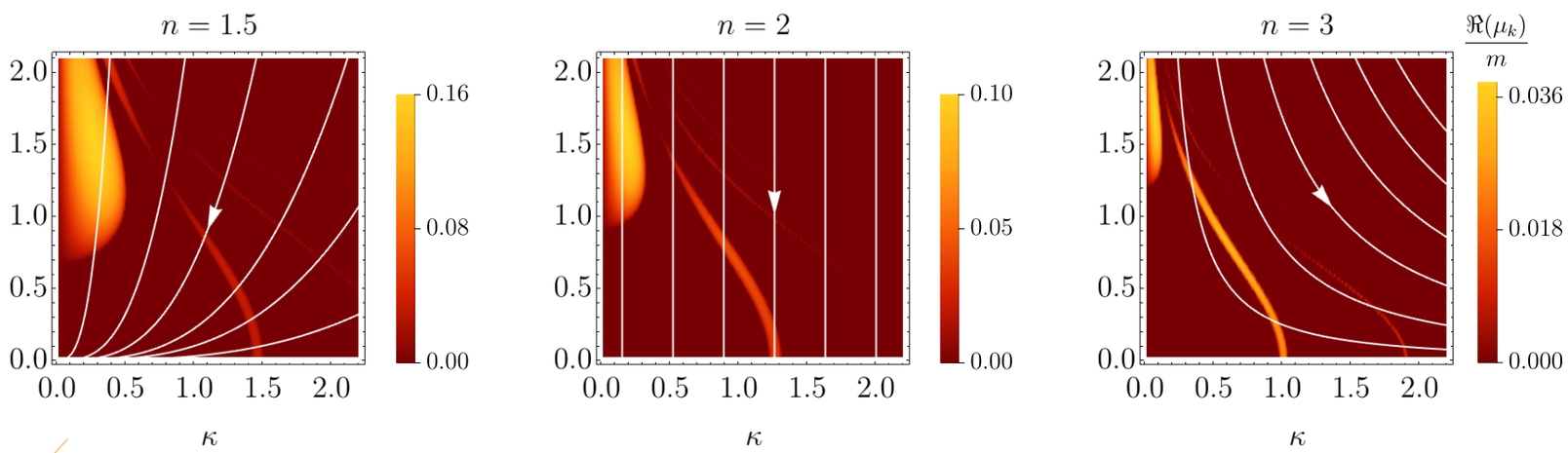
$$w \to \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

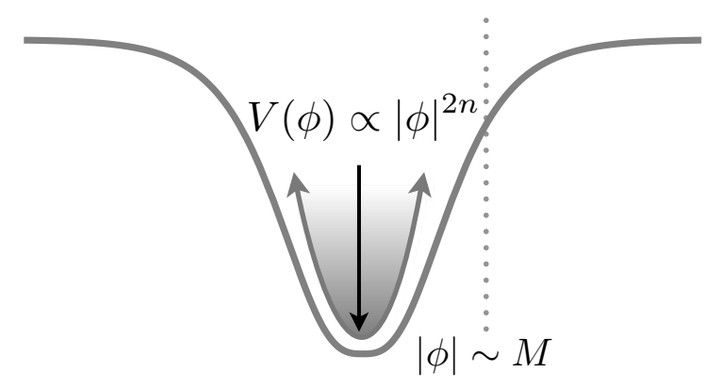
quickly

$$w \neq \frac{n-1}{n+1}$$

see Micha & Tkachev 2002, for quartic case

duration to radiation domination
* non-quadratic minima



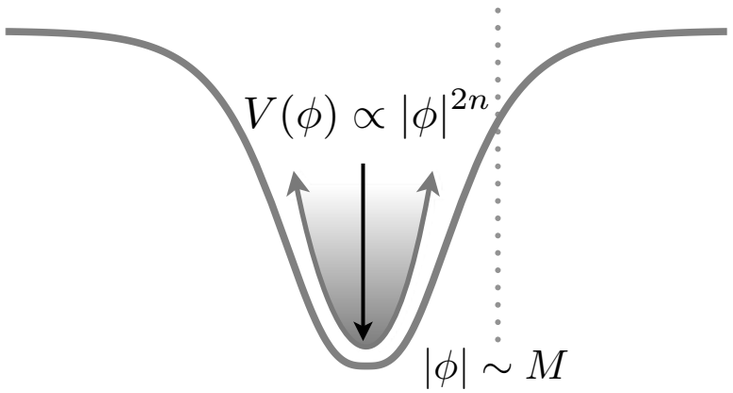


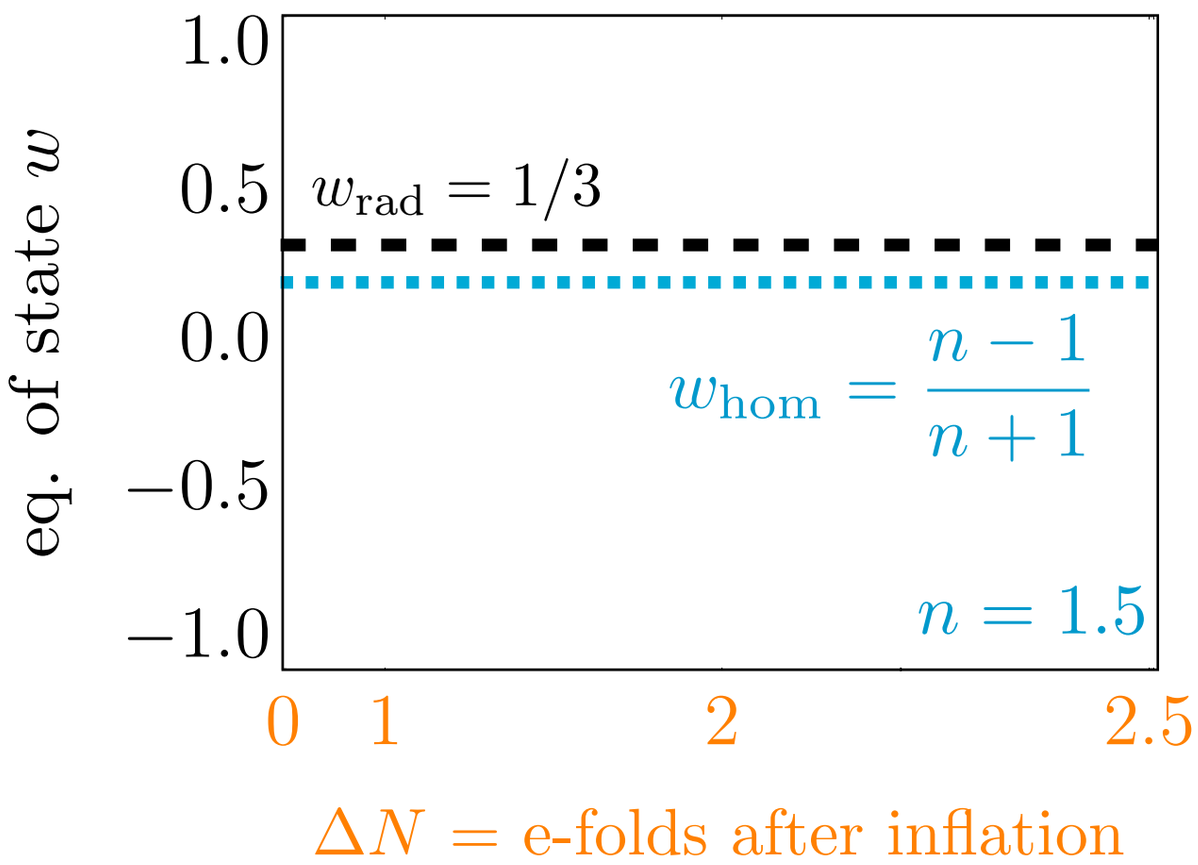
e-folds to radiation domination

$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

$$\Delta N_{\rm rad} \equiv \int_{a_{\rm end}}^{a_{\rm rad}} d\ln a$$

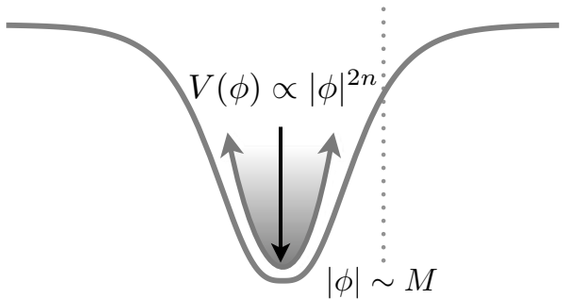
duration to radiation domination
* non-quadratic minima

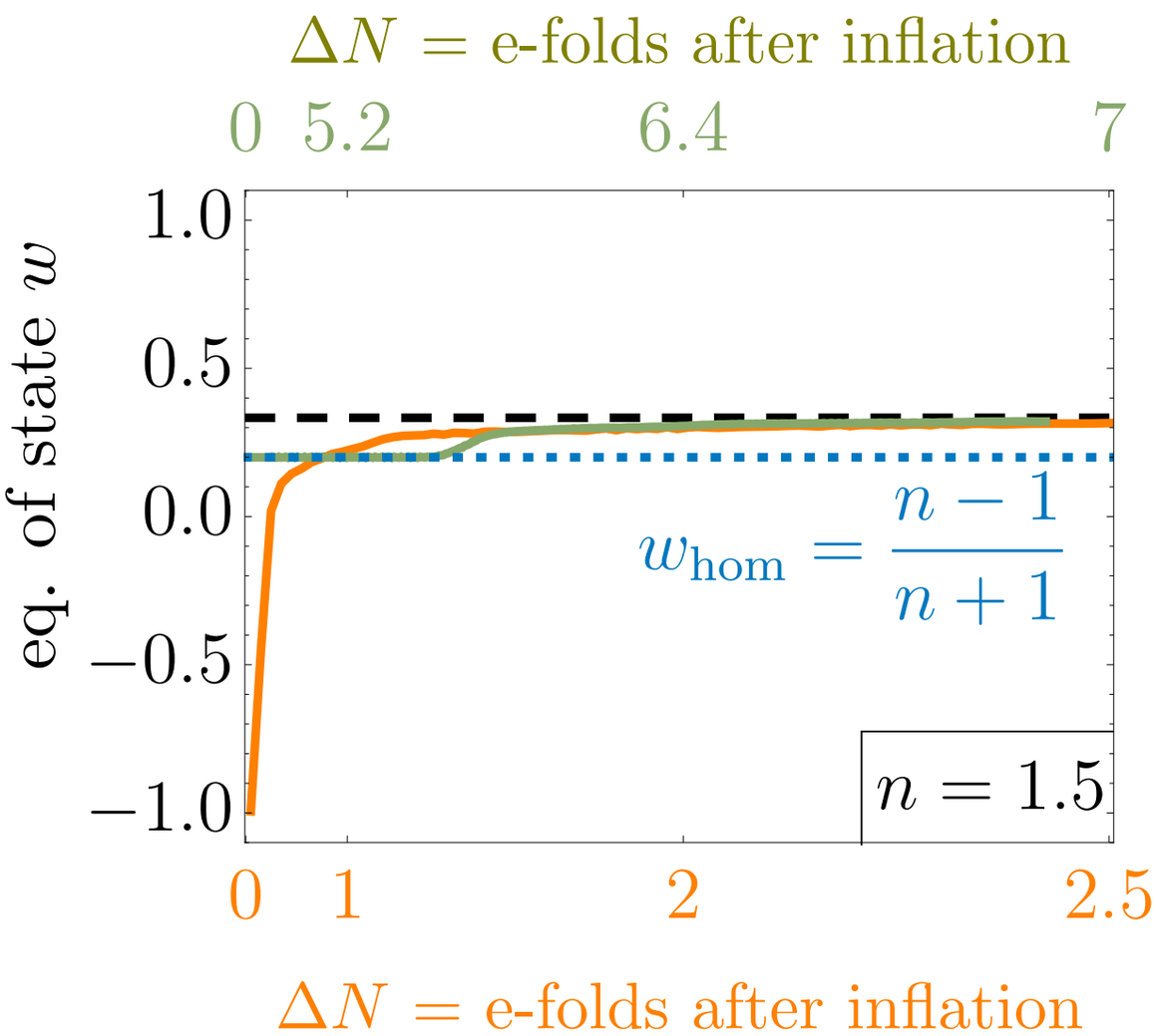




duration to radiation domination
* non-quadratic minima

from detailed 3+1 dimensional lattice simulations





$M \sim m_{\rm pl}$

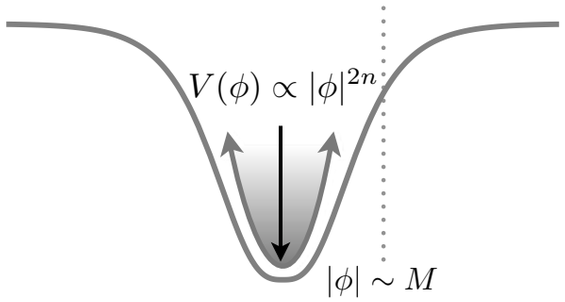
inefficient initial resonance

$M \ll m_{\rm pl}$

efficient initial resonance

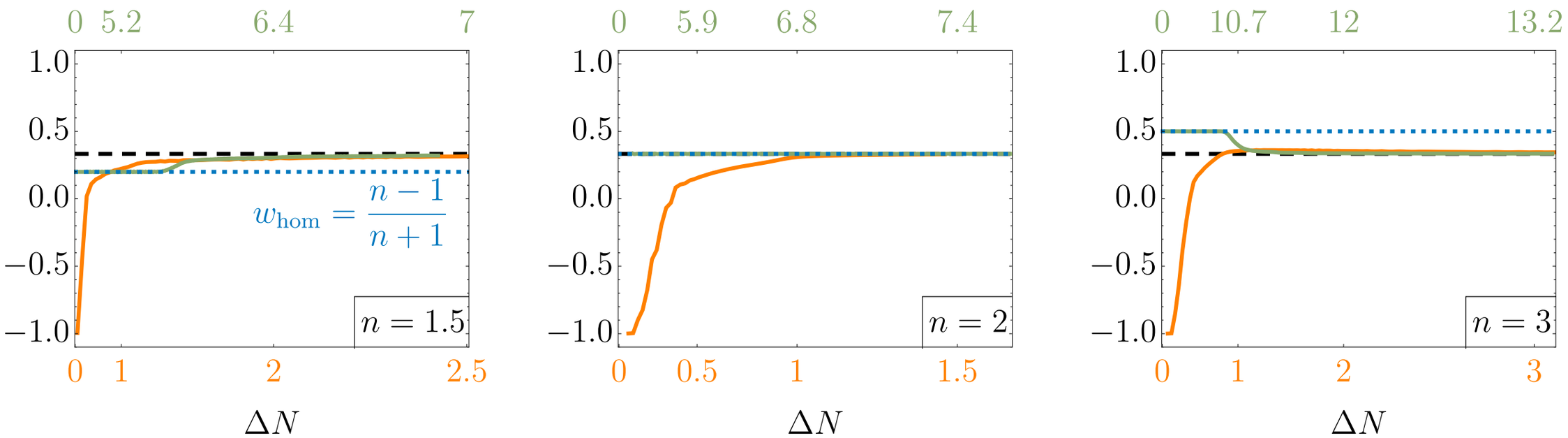
duration to radiation domination
* non-quadratic minima

from detailed 3+1 dimensional lattice simulations



green = inefficient initial resonance
orange = efficient initial resonance

---- $w_{\rm rad} = 1/3$

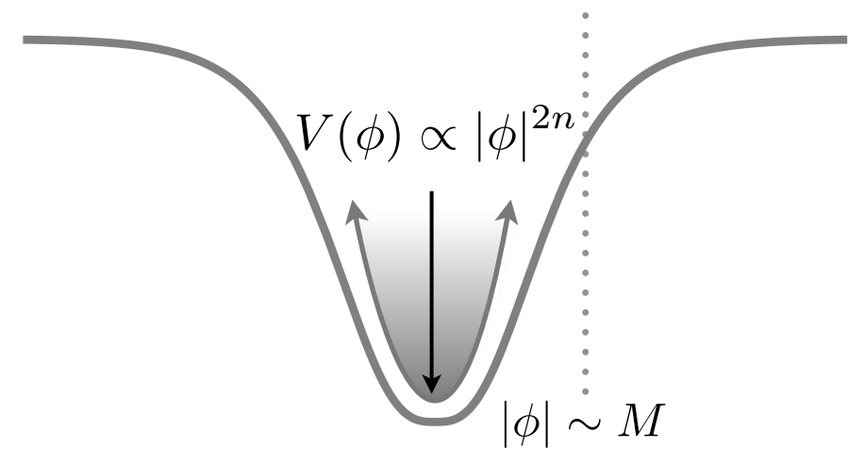


from analytic considerations

$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \frac{n+1}{3} \ln\left(\frac{\kappa}{\Delta\kappa}\frac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

an upper bound on duration to radiation domination

$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$



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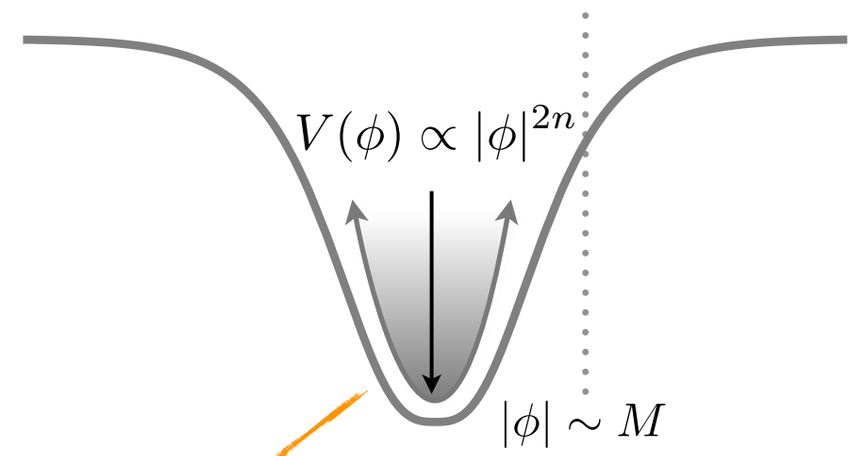
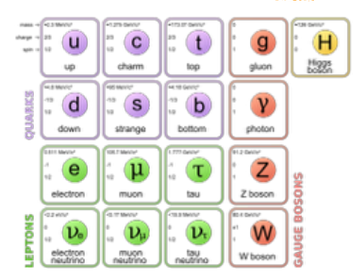




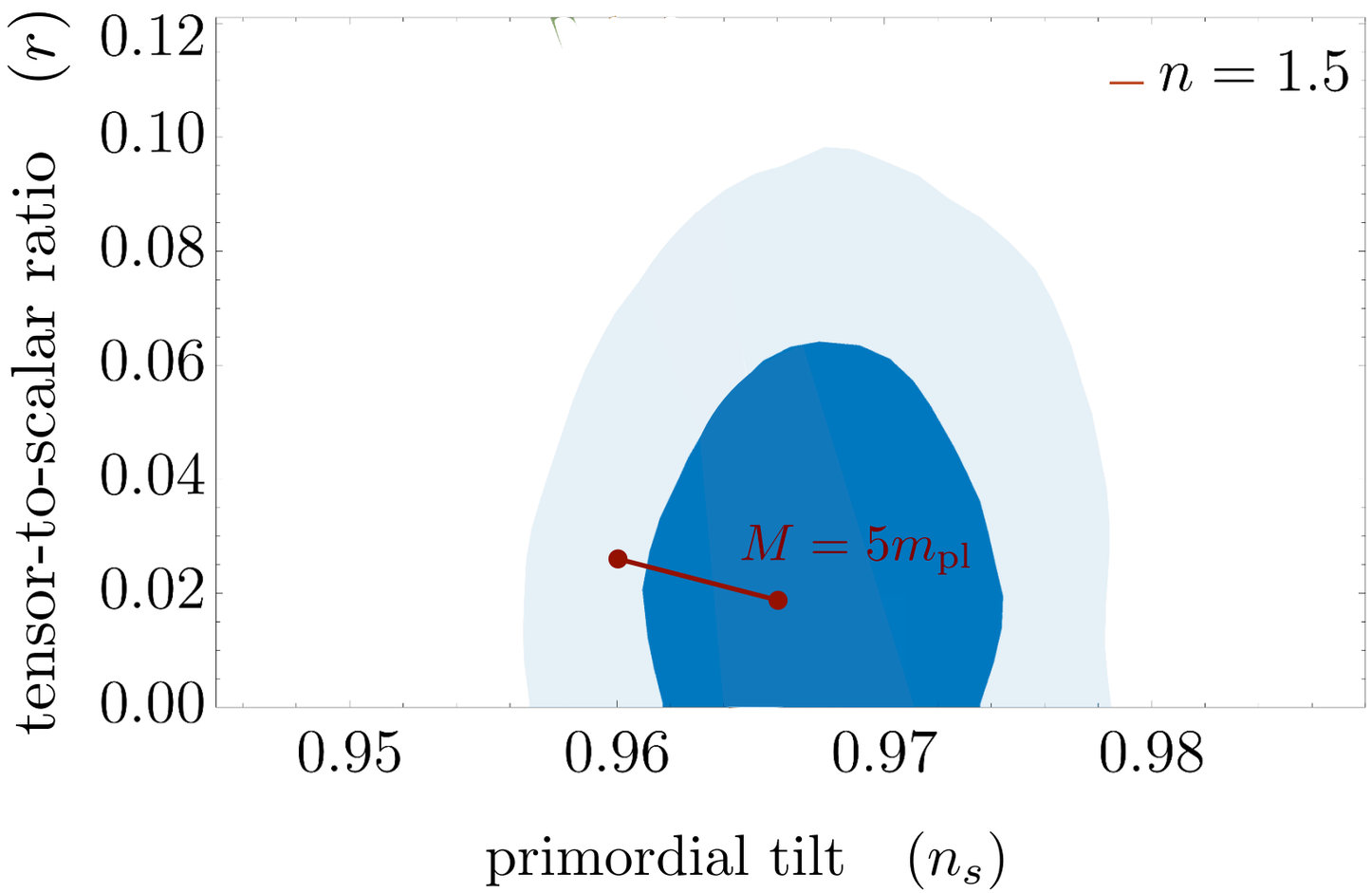
additional *light (massless) fields* can only decrease the duration!

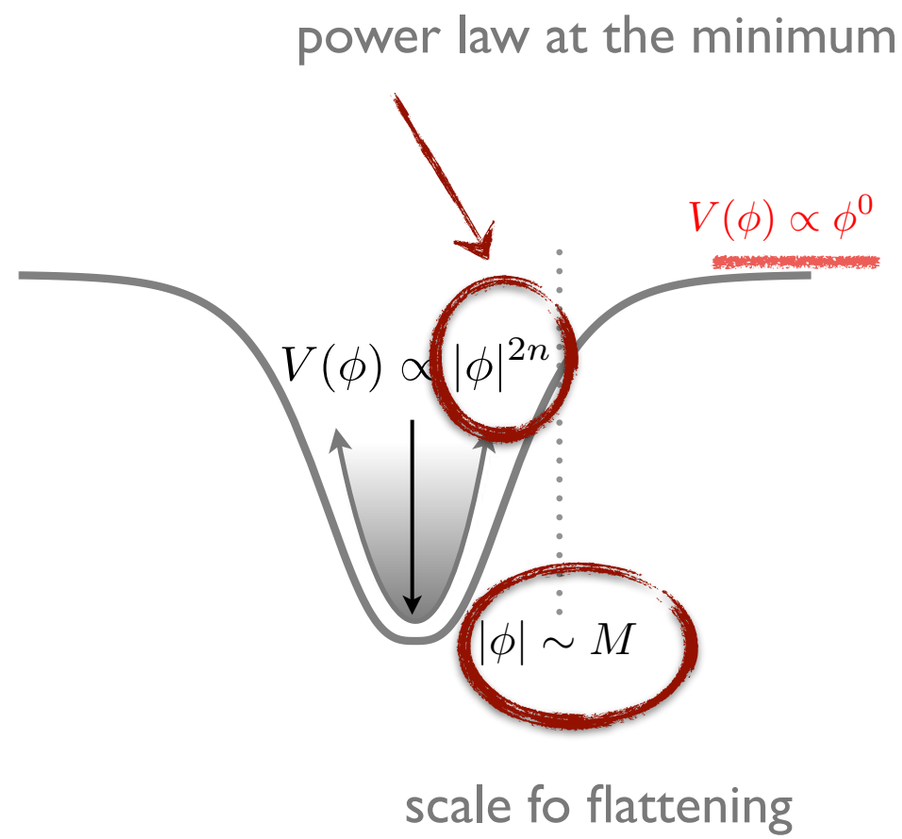
* decay to significantly massive fields can change this conclusion

caveats

- effectively massless daughter fields
- non-perturbative dynamics daughter fields ?
- long term, gravitational clustering ?

implications for CMB observables

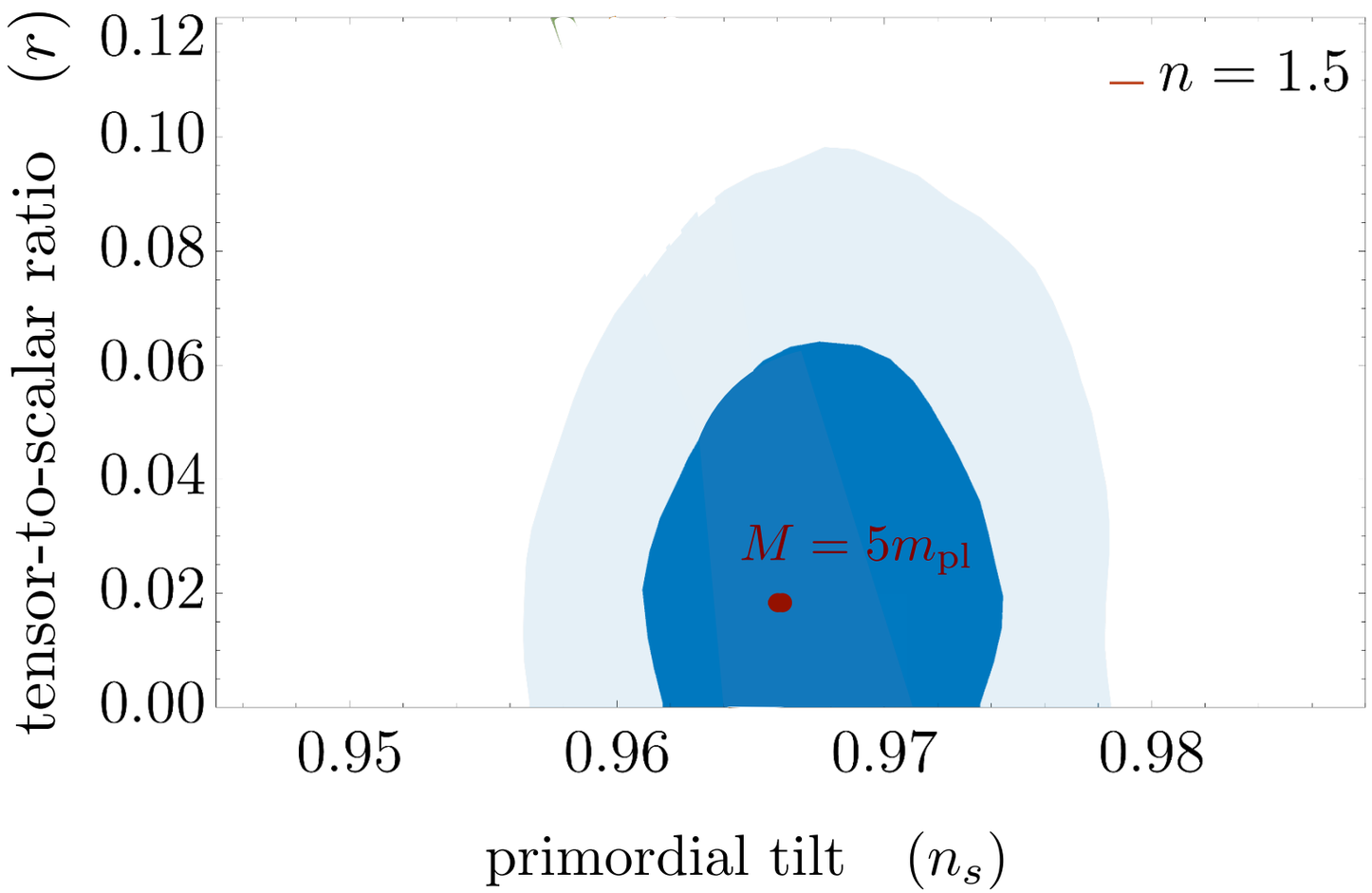


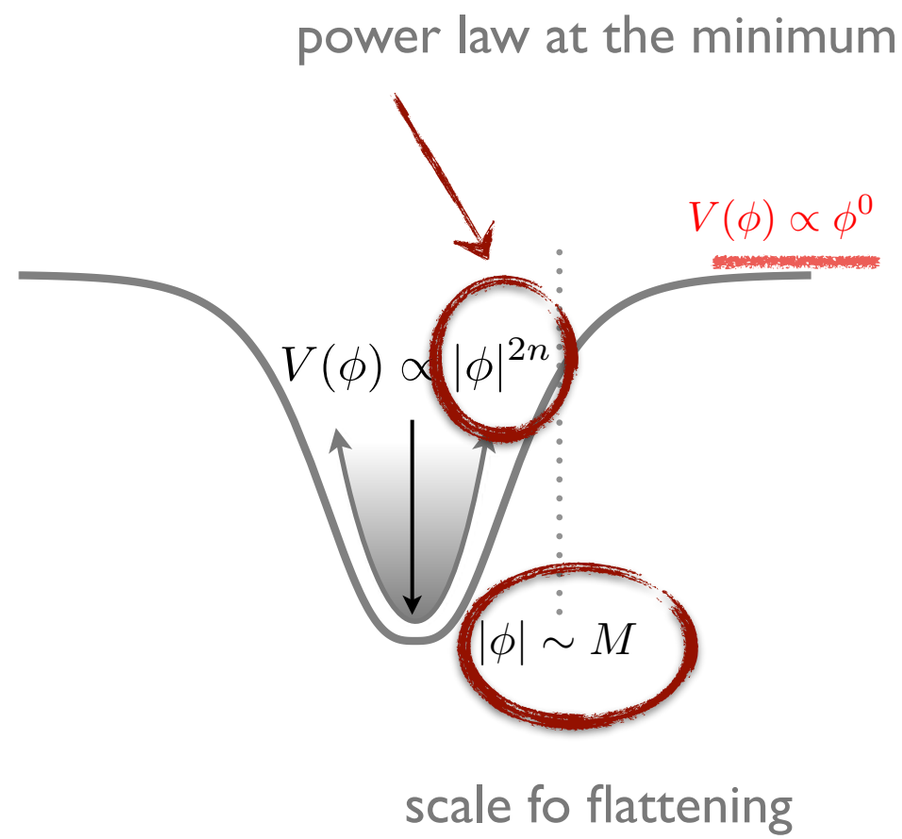


* width of the lines account for couplings to other light fields

* non-quadratic minimum

reduction in uncertainty!





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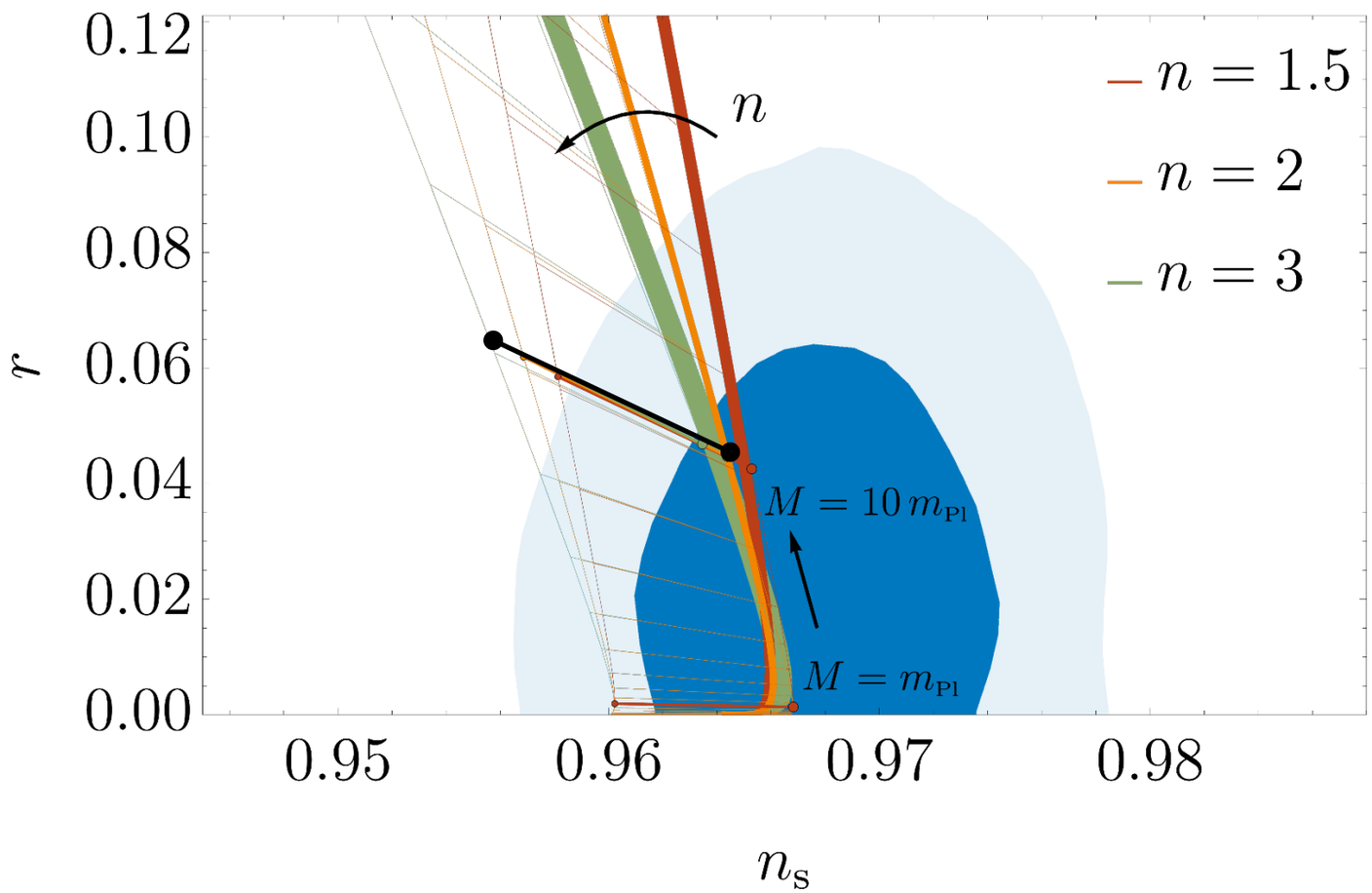
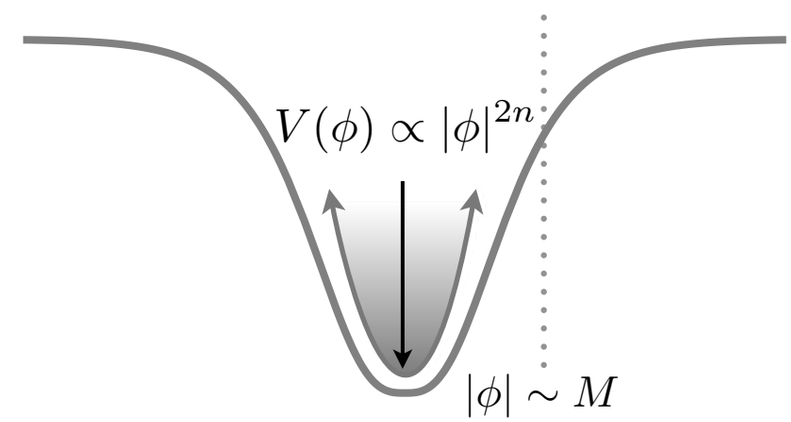
* width of the lines account for couplings to other light fields

* non-quadratic minimum

including upper bound
— significant reduction in uncertainty !





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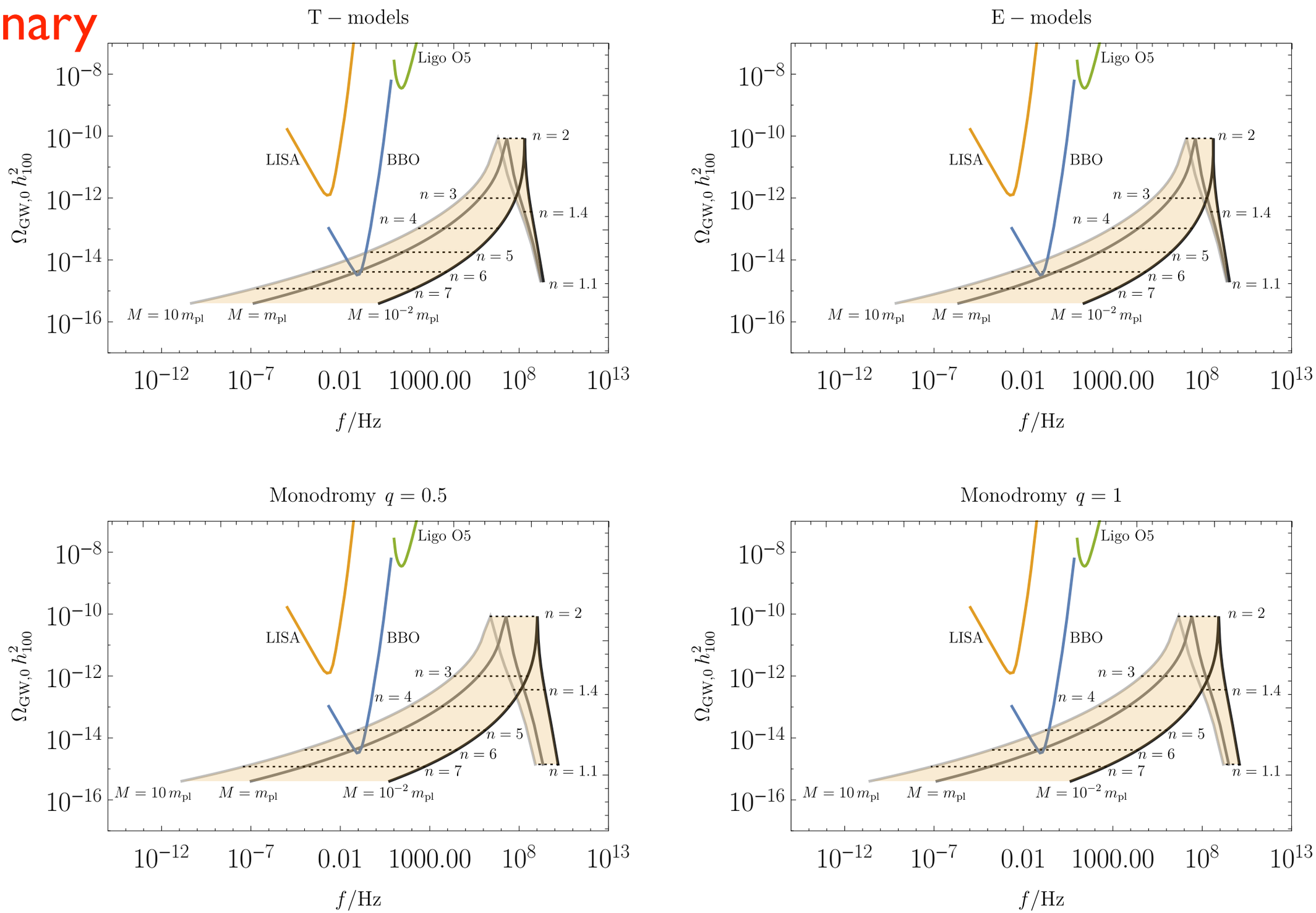
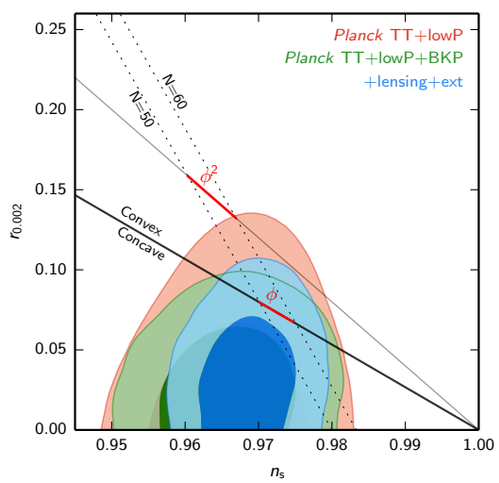
* non-quadratic minimum

gravitational waves — $\Omega_{\rm gw} \sim \Omega_{r0}\delta_\pi^2 (H/k_\star)^2$

preliminary



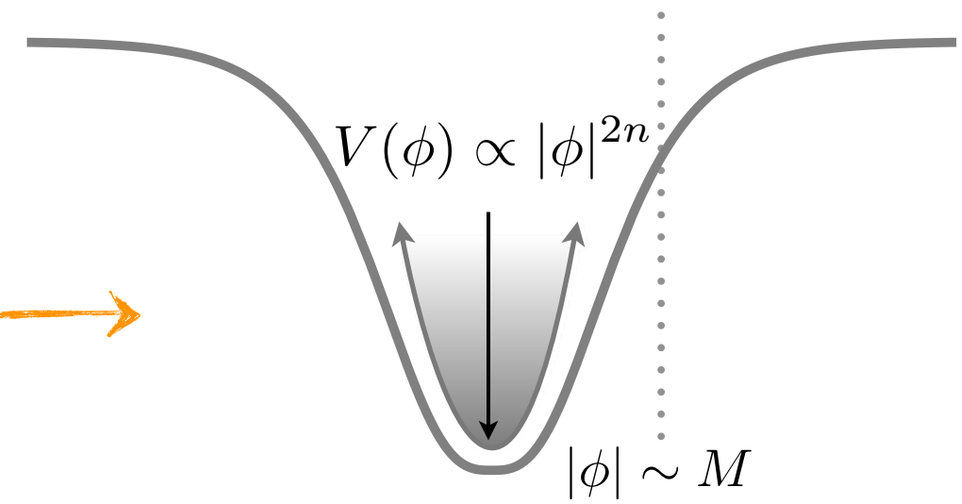
inflation and its end
"simple" models



+

for example:

Silverstein & Westhpal (2008)
McAllister et. al (2014)
Kallosh & Linde (2014)
Scalisi (2016)

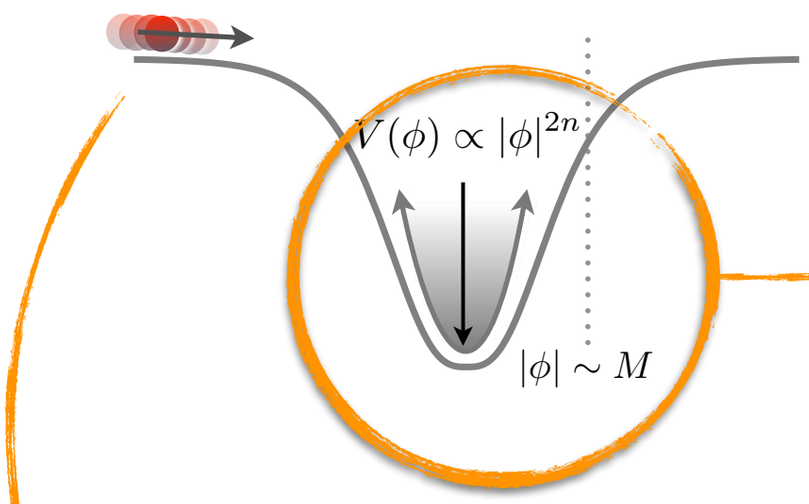


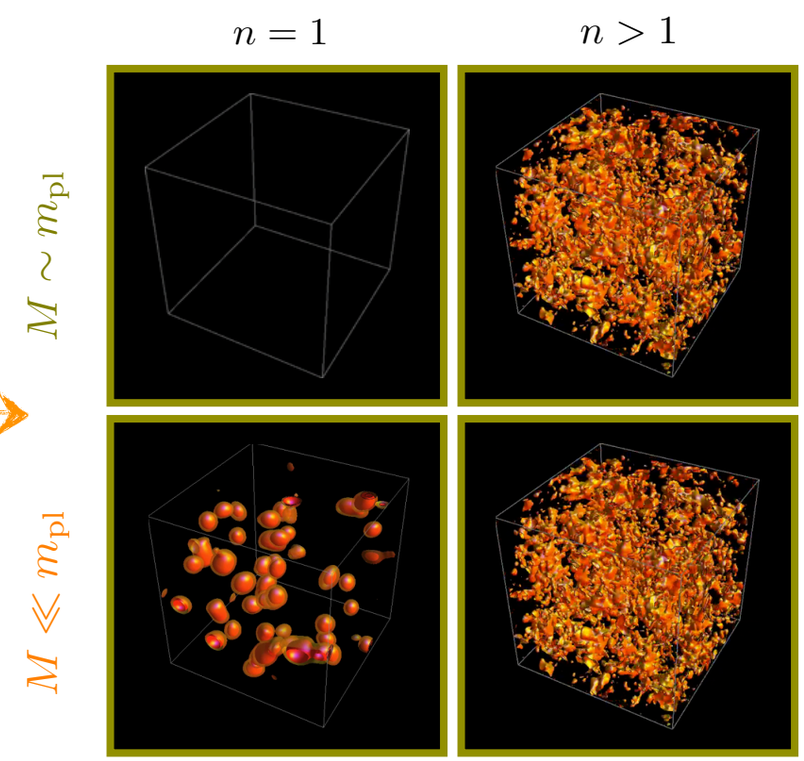
(i) what are the dynamics ?

(ii) eq. of state & how long to radiation domination ?

(iii) obs. consequences ?

summary: “simple” models of cosmological scalar field dynamics





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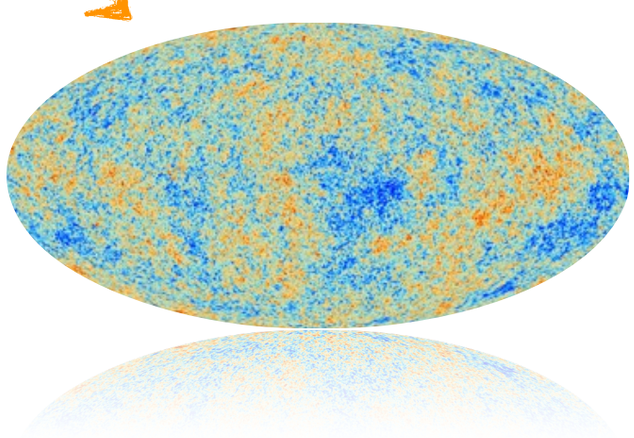

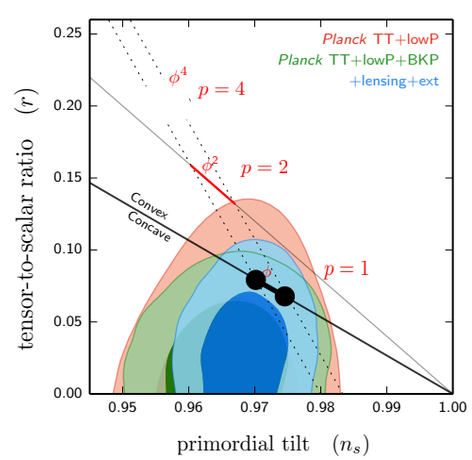

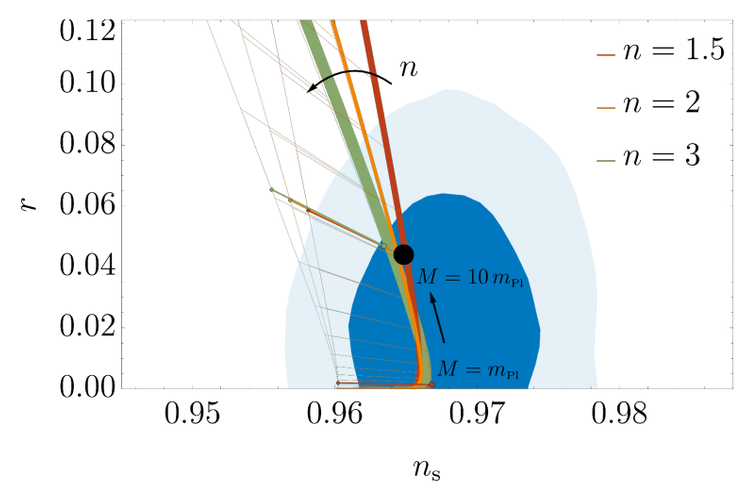

two approaches

SIMPLE enough

Lozanov & MA (2016) + earlier works

COMPLEX enough

MA & Baumann (2015),
MA, Garcia, Xie & Wen (2017)

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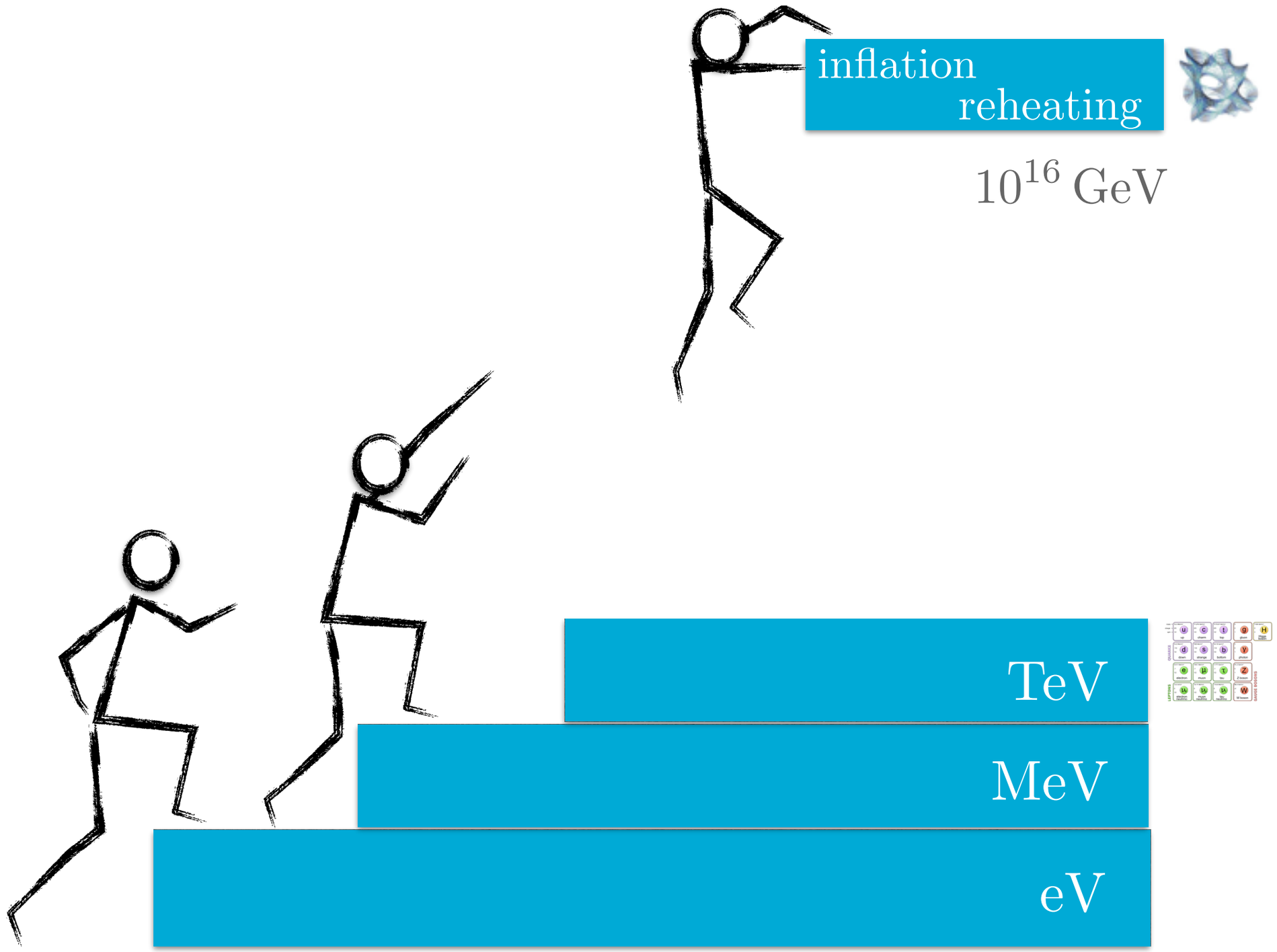
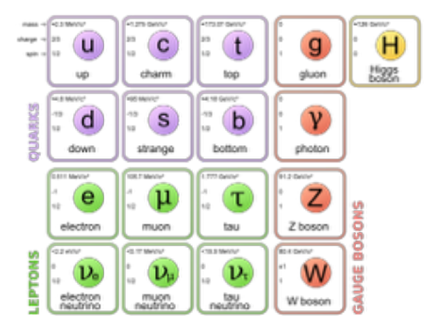

inflation
reheating
10^{16} GeV
TeV
MeV
eV

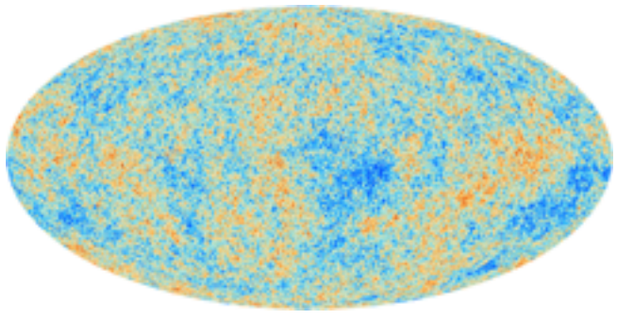
theory : its complicated (probably)

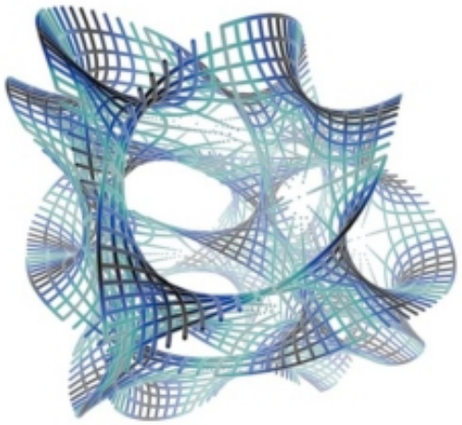
- inflation
- reheating after inflation

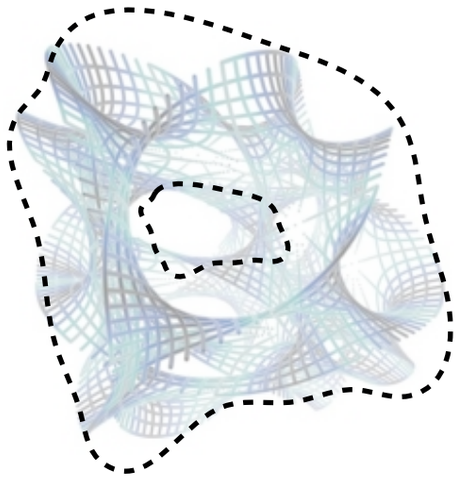


a statistical approach?

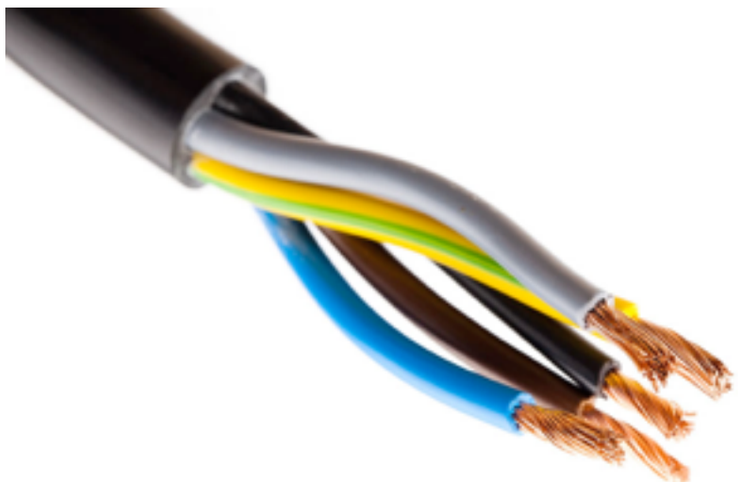
- **observations**: early universe is simple
- **theory**: not so much …
- **coarse grained view ?**
- **calculational tools ?**

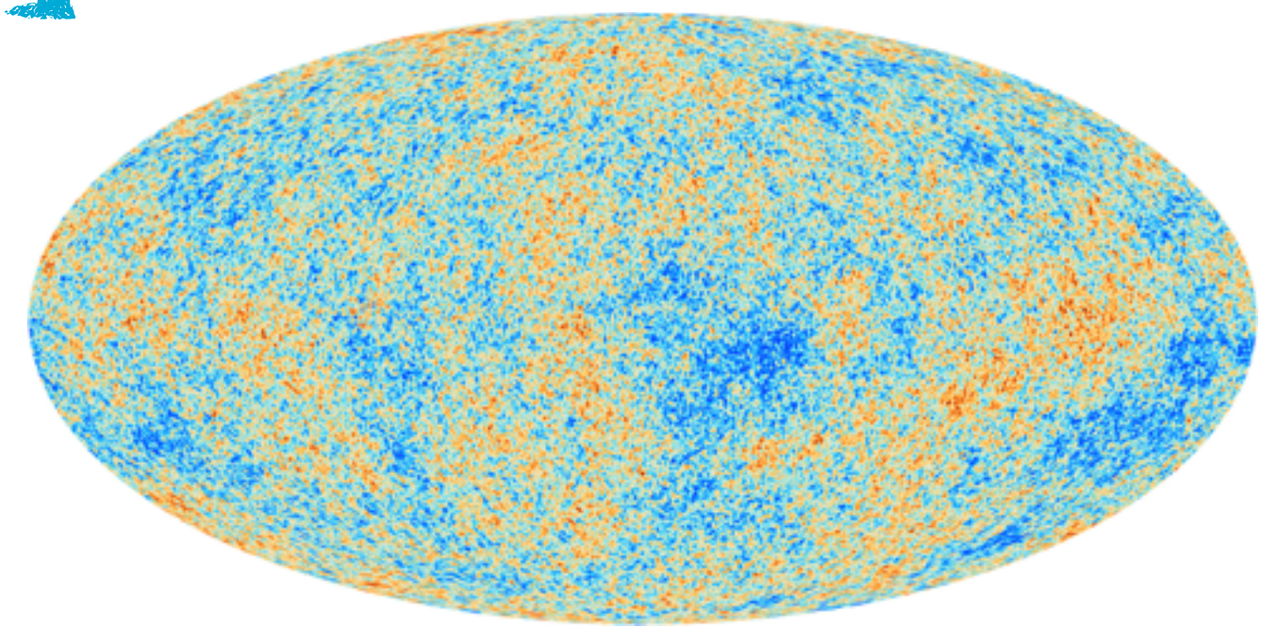






inspiration from disordered wires

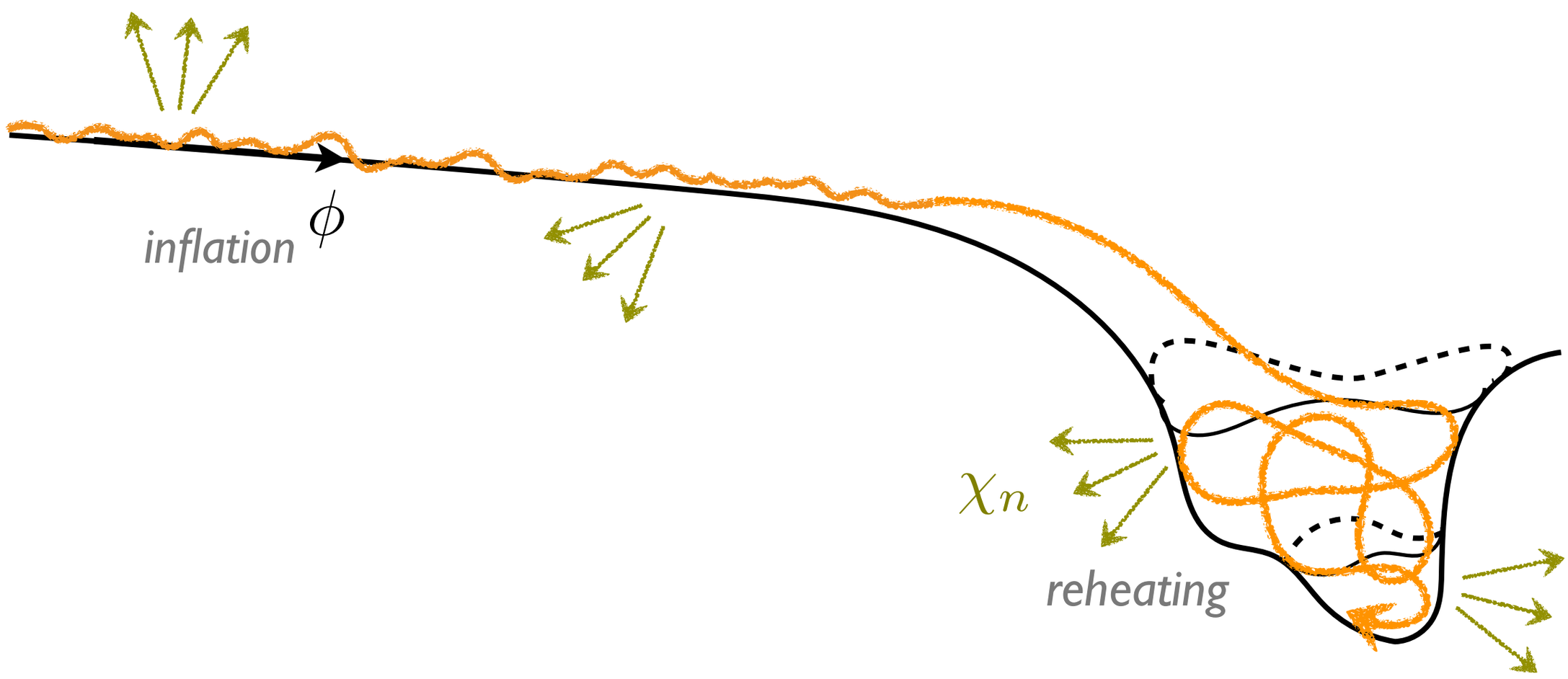




the framework

multifield inflation/reheating

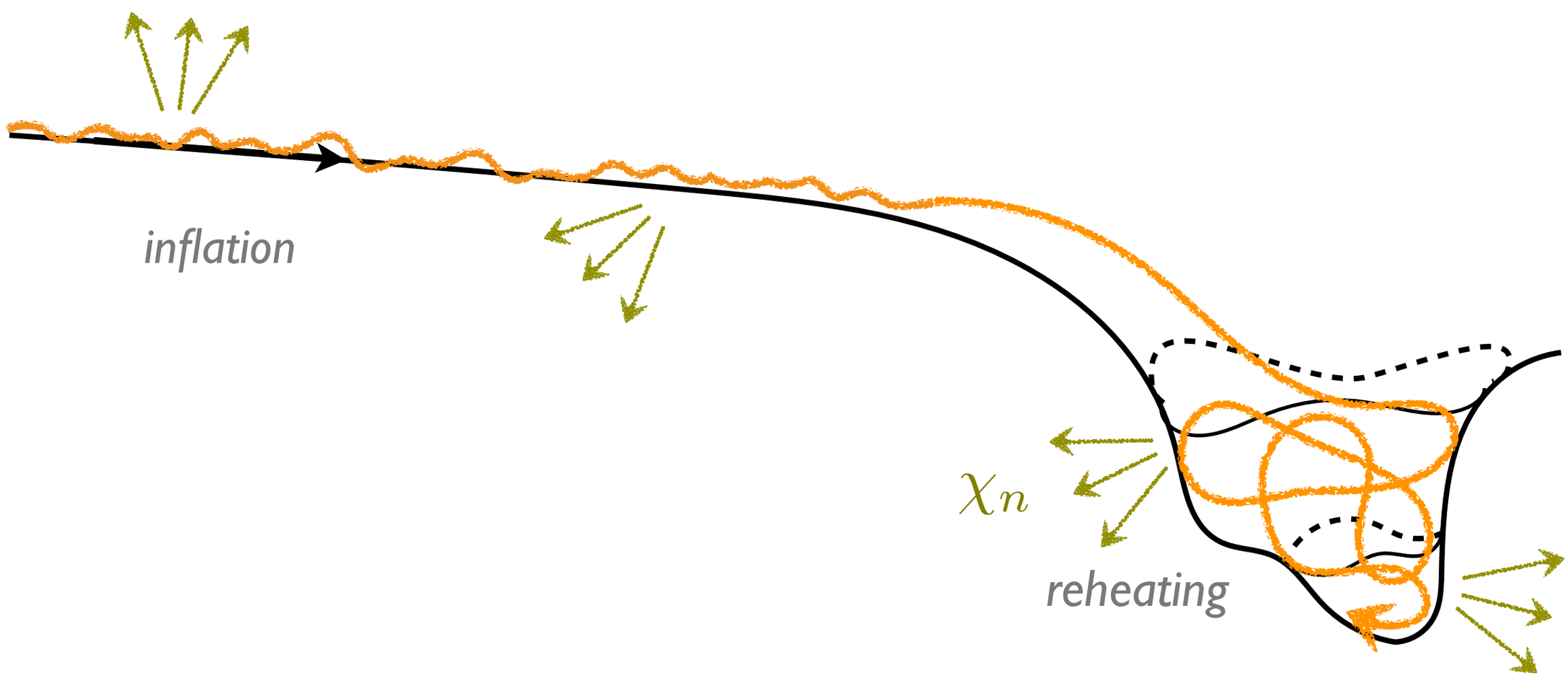
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}G_{ab}(\phi^c)\partial^\mu\phi^a\partial_\mu\phi^b - V(\phi^c) + \cdots\right]$$



focus on perturbations

$$S^{(2)} = \int d^4x \mathcal{L} = \int d^4x \sum_{I,J=1}^{N_\mathrm{f}} \left(\frac{1}{2}\delta_{IJ}\partial_\mu \chi^I \partial \chi^J - \frac{1}{2}\mathcal{M}_{IJ}(\tau)\chi^I\chi^J \right)$$

$$\mathcal{M}_{IJ}(\tau) = m_I^2 \delta_{IJ} + m_{IJ}^\mathrm{s}(\tau)\,.$$

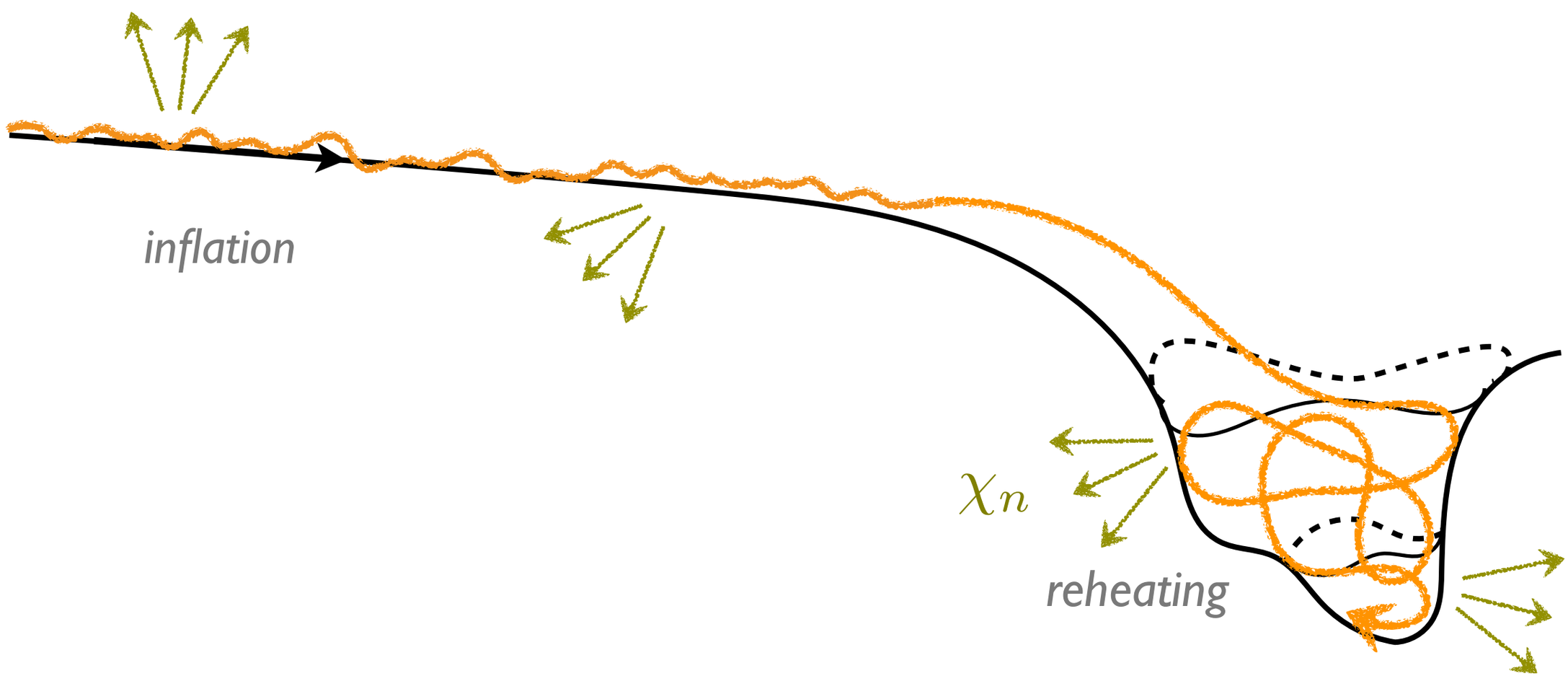


focus on perturbations

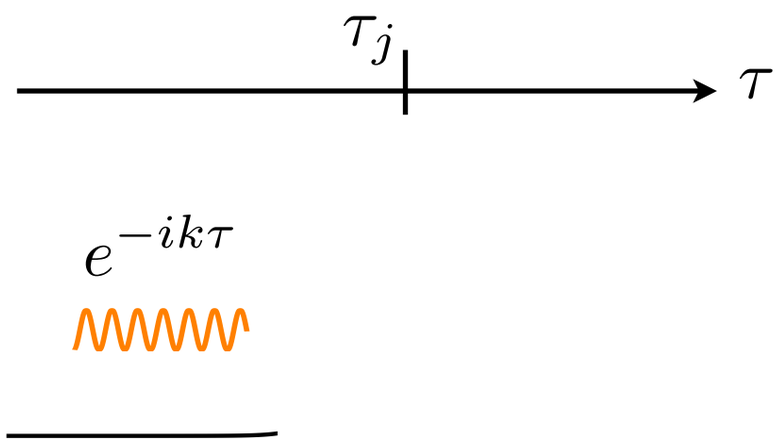
mode functions in Fourier space

$$\left(\frac{d^2}{d\tau^2}+\omega_I^2\right)\chi_k^I(\tau)+\sum_{J=1}^{N_{\rm f}} m_{IJ}^{\rm s}(\tau)\chi_k^J(\tau)=0\,,$$

$$\omega_I^2(k)=k^2+m_I^2\,,$$



particle production as "scattering"



$$|\chi_k(\text{after})\rangle = \underbrace{\begin{pmatrix} 1/t_j^* & -r_j^*/t_j^* \\ -r_j/t_j & 1/t_j \end{pmatrix}}_{\mathsf{M}_j} |\chi_k(\text{before})\rangle$$

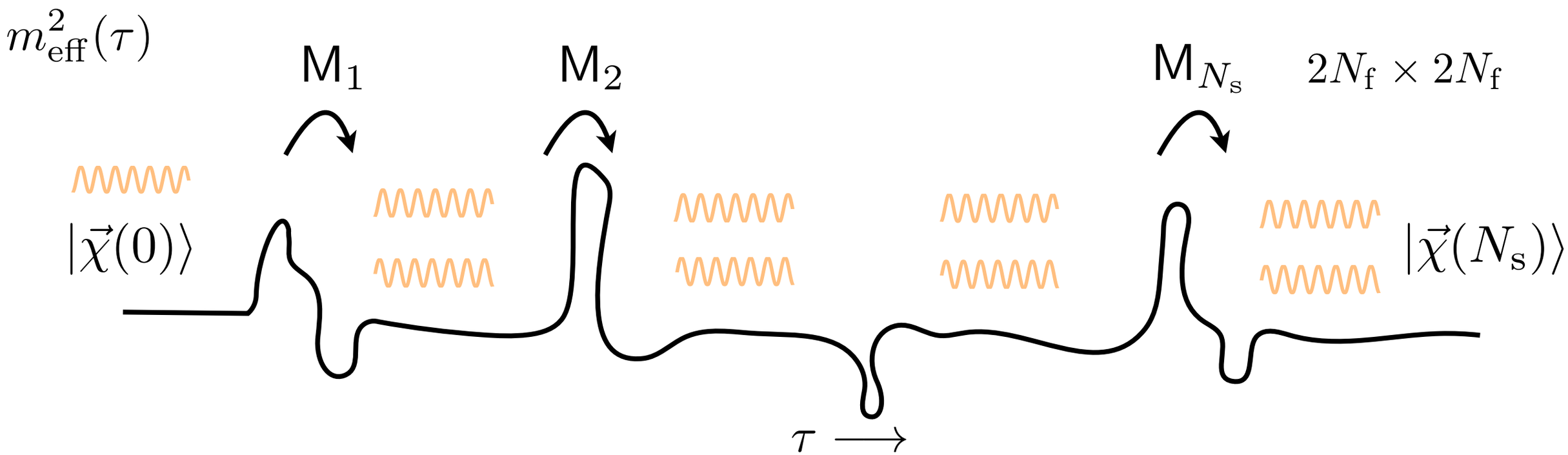
occupation number per mode

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$

$$T_j = |t_j|^2 \qquad n_j \equiv \frac{|r_j|^2}{|t_j|^2} = T_j^{-1} - 1$$

multifield particle production as scattering

$$|\vec{\chi}(N_\mathrm{s})\rangle \;=\; \mathsf{M}\,|\vec{\chi}(0)\rangle \quad \text{where} \quad \mathsf{M} \equiv \mathsf{M}_{N_\mathrm{s}} \cdots \mathsf{M}_2 \mathsf{M}_1$$



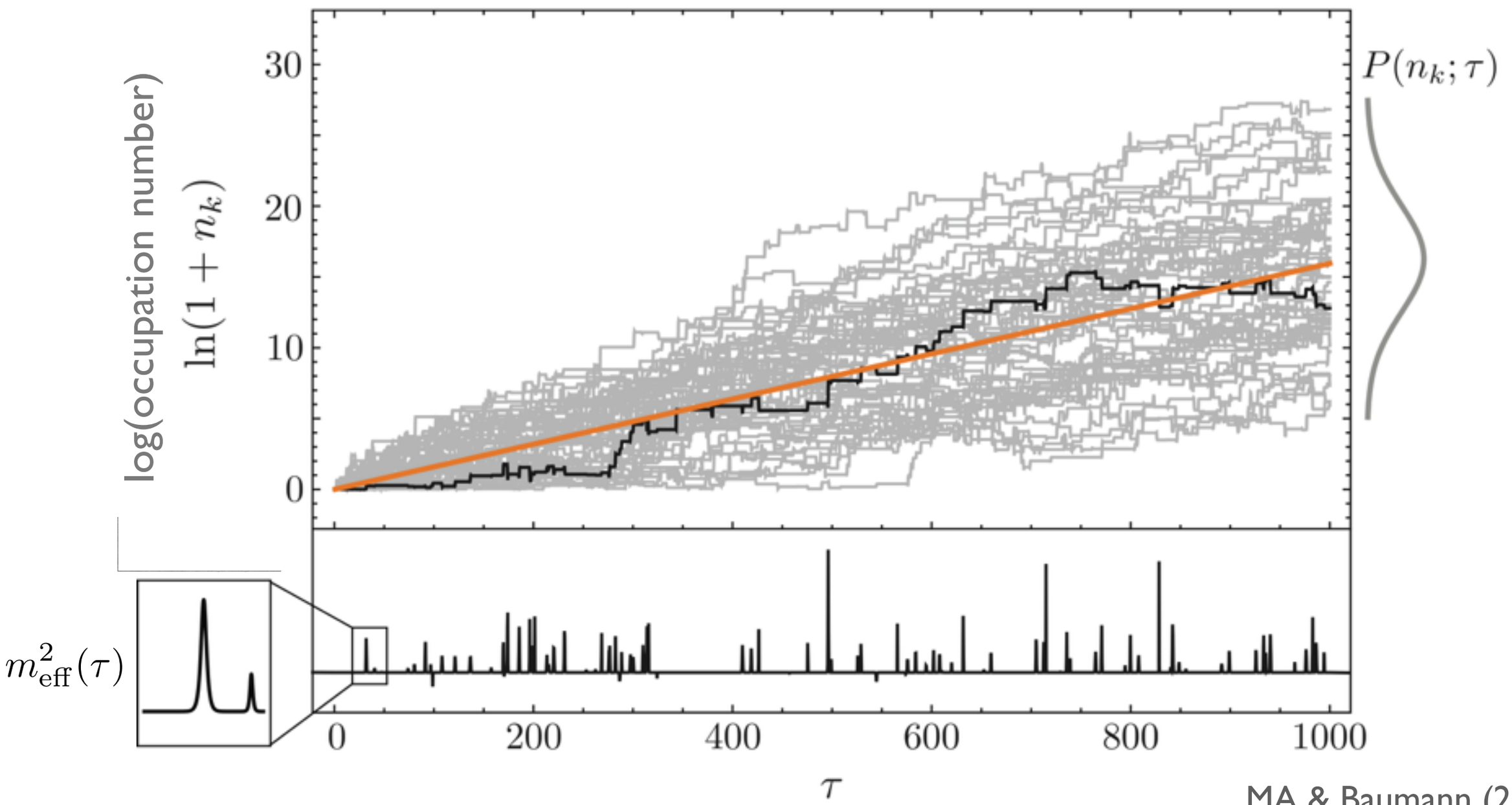
total occupation number

$$n = \mathrm{Tr}(\mathsf{n}) = \sum_{a=1}^{N_\mathrm{f}} n_a \qquad \text{where} \quad \mathsf{n} \sim \mathsf{M}\mathsf{M}^\dagger \,.$$

particles in each "field" (eignevalues)

occupation number performs a drifted random walk

$$n(k,\tau) = \frac{1}{2\omega_k}\left(|\dot{\chi}_k|^2 + \omega_k^2|\chi_k|^2\right)$$



MA & Baumann (2015)

multifield Fokker Planck equation

joint probability for occupation numbers satisfies the a Fokker Planck-like equation:

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n_a;\tau) = \sum_{a=1}^{N_\mathrm{f}}\left[(1+2n_a)+\frac{1}{N_\mathrm{f}+1}\sum_{b\neq a}\frac{n_a+n_b+2n_an_b}{n_a-n_b}\right]\frac{\partial P}{\partial n_a}$$

$$+\frac{2}{N_\mathrm{f}+1}\sum_{a=1}^{N_\mathrm{f}}n_a(1+n_a)\frac{\partial^2 P}{\partial n_a^2}$$

Dokhorov, Mello, Pereyra & Kumar = DMPK eq.

MA & Baumann 2015

local mean particle production rate

$$\mu_k \equiv \frac{1}{N_\mathrm{f}}\lim_{\delta\tau\to 0}\frac{\langle n\rangle}{\delta\tau} \qquad \text{where} \qquad n=\sum_{a=1}^{N_\mathrm{f}}n_a$$

* more general results in
* MA, Garcia, Xie and Wen 2017

moments: Fokker Planck equation

$$\langle n \rangle = \frac{N_\mathrm{f}}{2}\left(e^{2\mu_k \tau} - 1\right)$$

$$\frac{\mathrm{Var}[n]}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \gg 1} \left(\frac{1+N_\mathrm{f}}{3N_\mathrm{f}}\right) e^{\frac{4}{1+N_\mathrm{f}}\mu_k \tau}$$

$$\langle \ln(1+n) \rangle = \mu_k \tau$$

$$\frac{\mathrm{Var}[\ln(1+n)]}{\langle \ln(1+n) \rangle^2} \xrightarrow{\mu_k \tau \gg 1} \frac{N_\mathrm{f}+1}{N_\mathrm{f}^2}\frac{1}{\mu_k \tau}$$

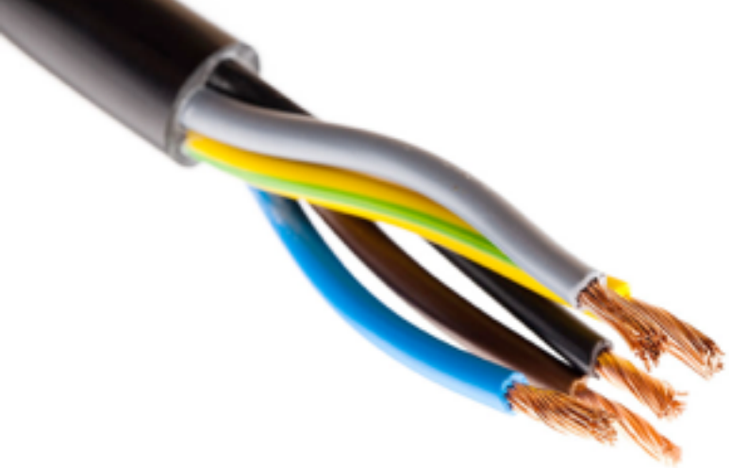
most probable total occupation number

$$n_\mathrm{typ} \equiv e^{\langle \ln(1+n) \rangle} \longrightarrow e^{\frac{2N_\mathrm{f}}{1+N_\mathrm{f}}\mu_k \tau}$$

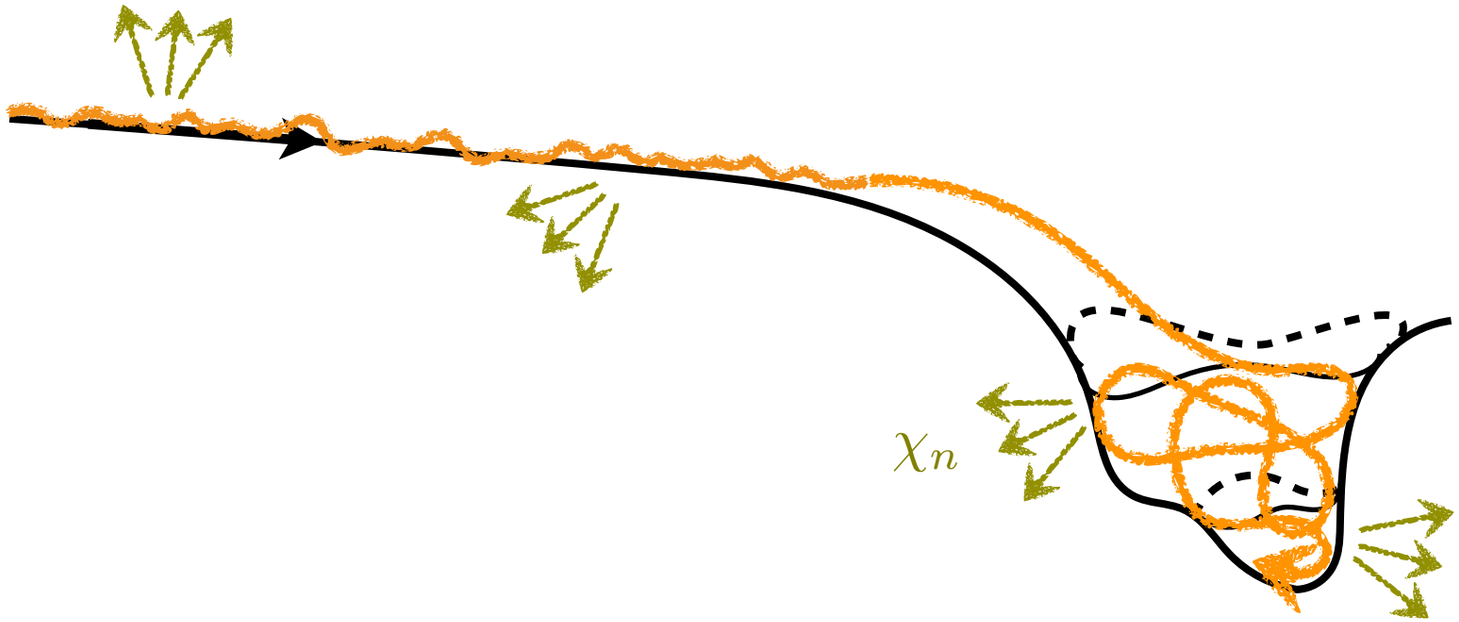
Log-Normal Distribution!

MA & Baumann (2015)

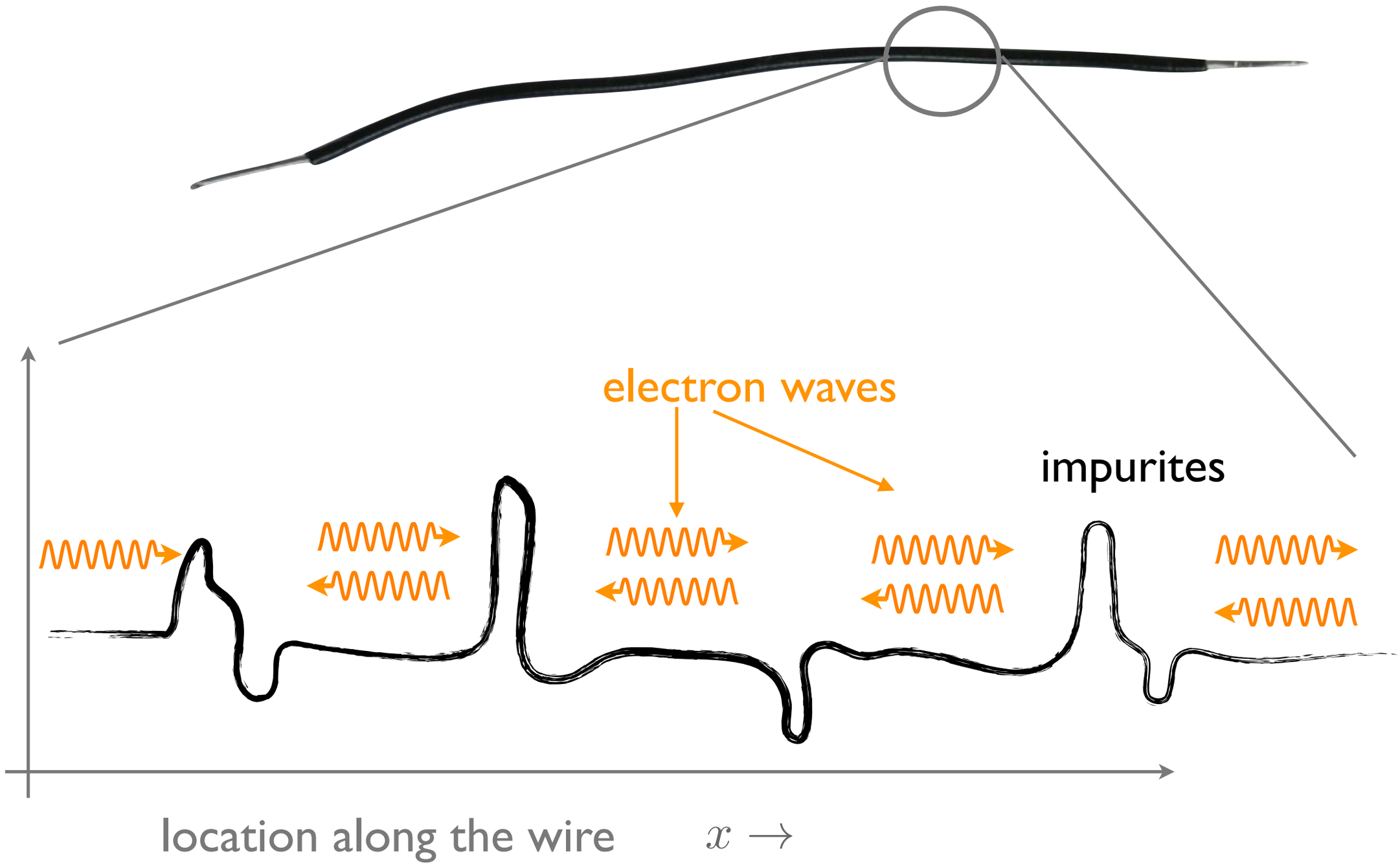
* result for statistical similar fields. More general result in MA, Garcia, Xie and Wen (2017)



what is the connection to wires?

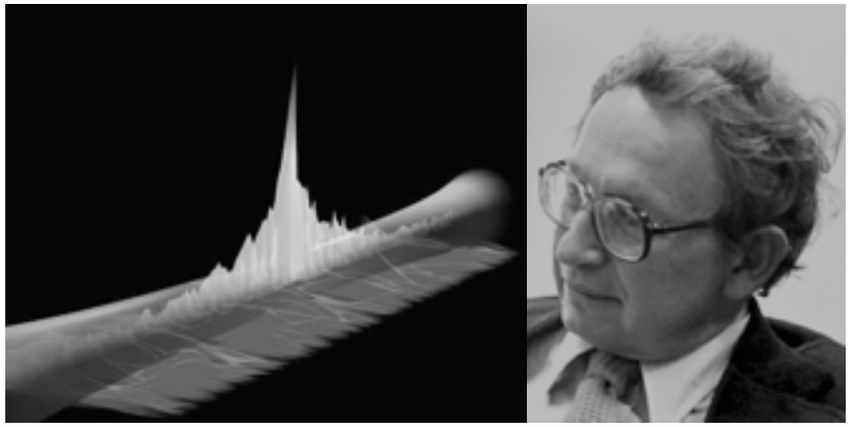


electron wave function: disordered wires

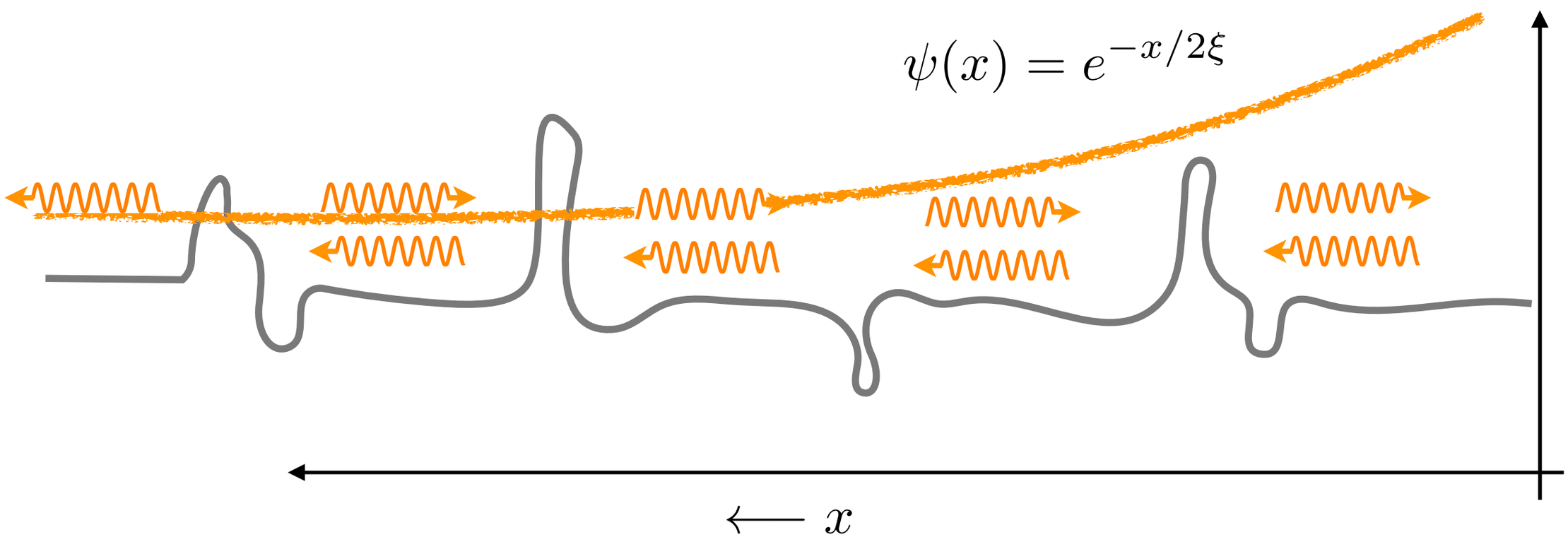


Anderson localization !

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

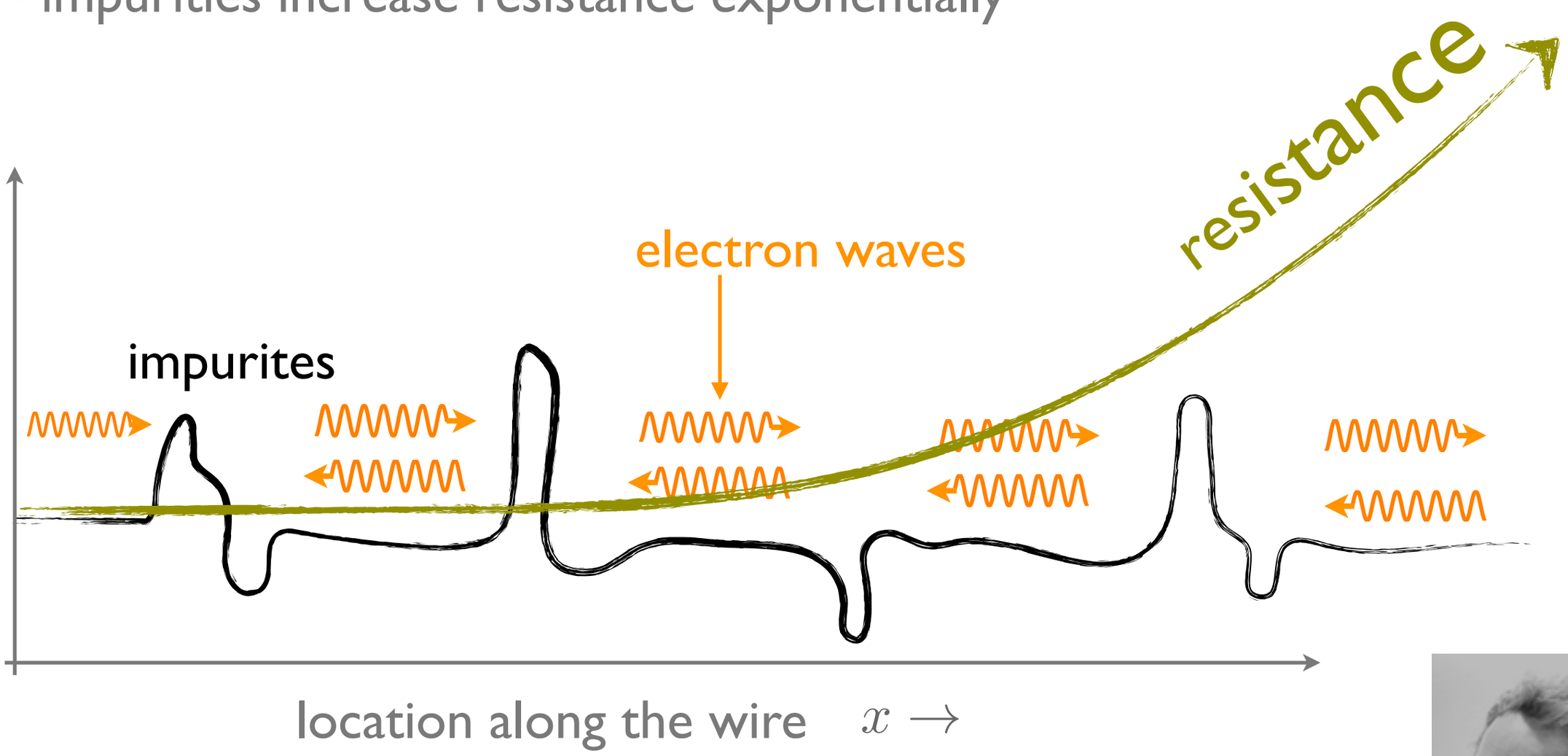


Anderson 1957



universal behavior

- impurities increase resistance exponentially



at low temperatures, one dimensional wires are insulators

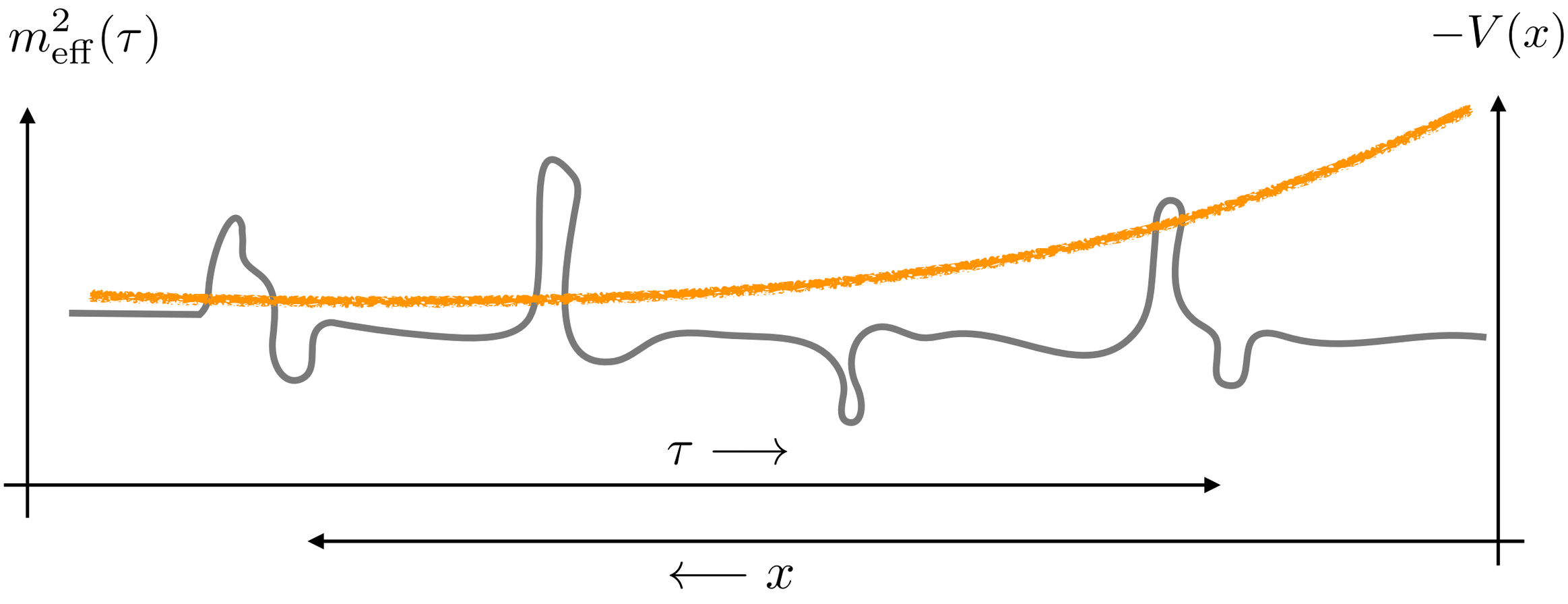
complexity in time cosmology $\longleftrightarrow$ complexity in space wires

exponential growth in occupation number

Anderson localization

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0 \quad \longleftrightarrow \quad \psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

simplified version!

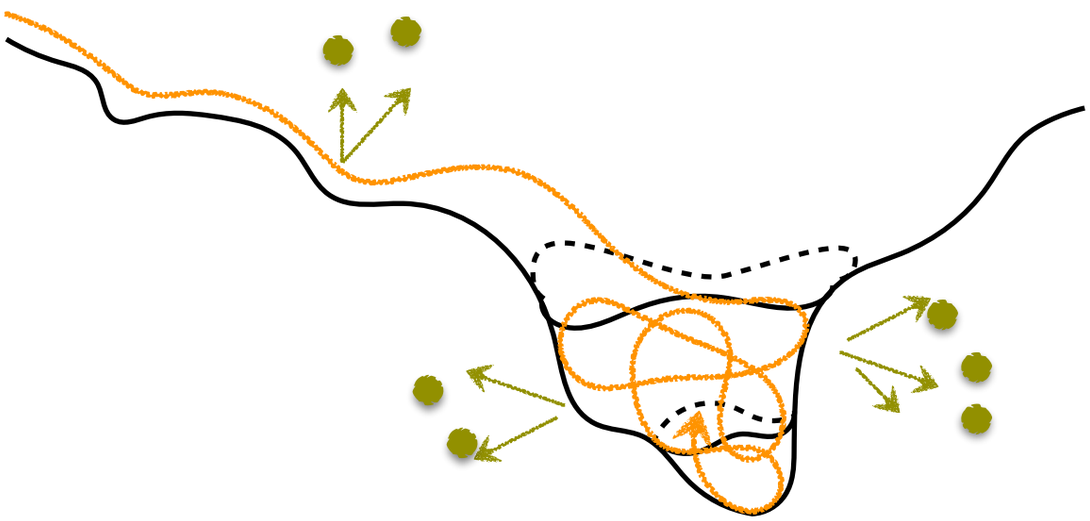


for periodic case with noise see Zanchin et. al 1998, Brandenberger & Craig 2008

simplicity/universality

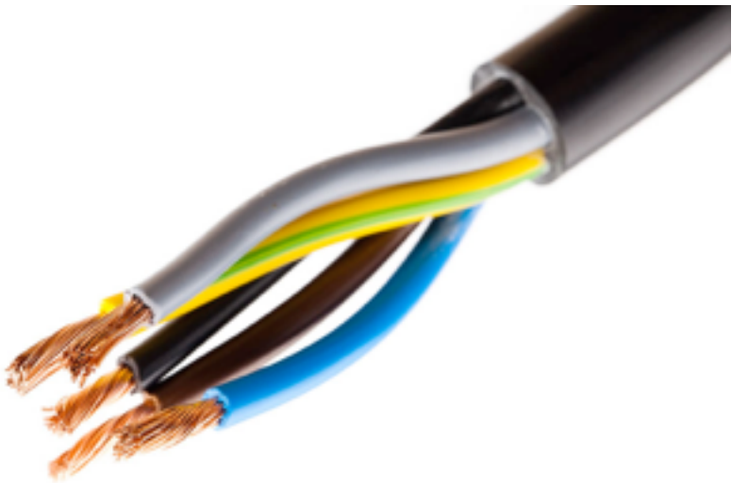
μ_k local mean particle production rate

N_f number of fields



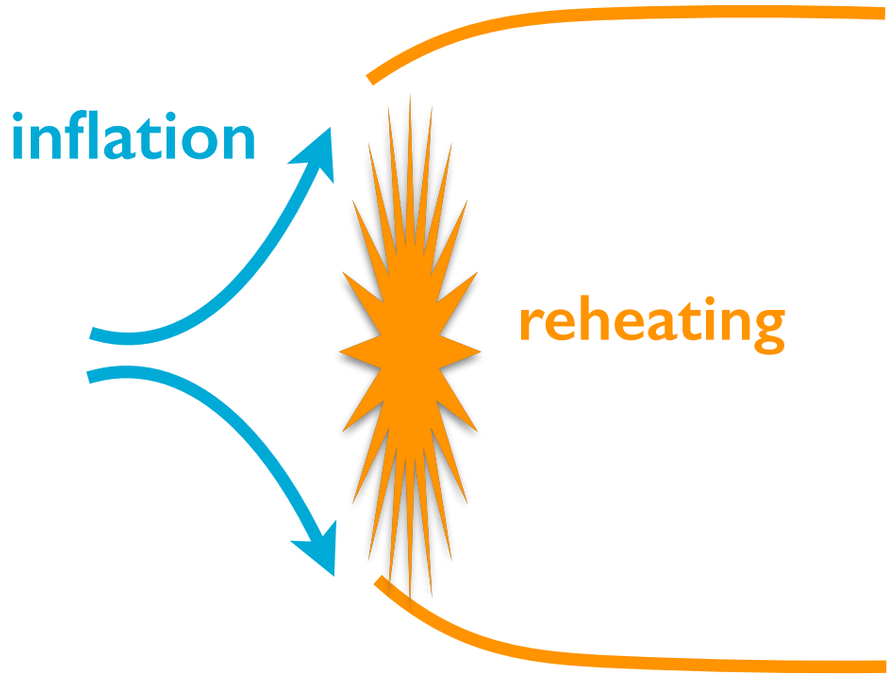
l_mf mean ballistic mean free path

N_c number of channels

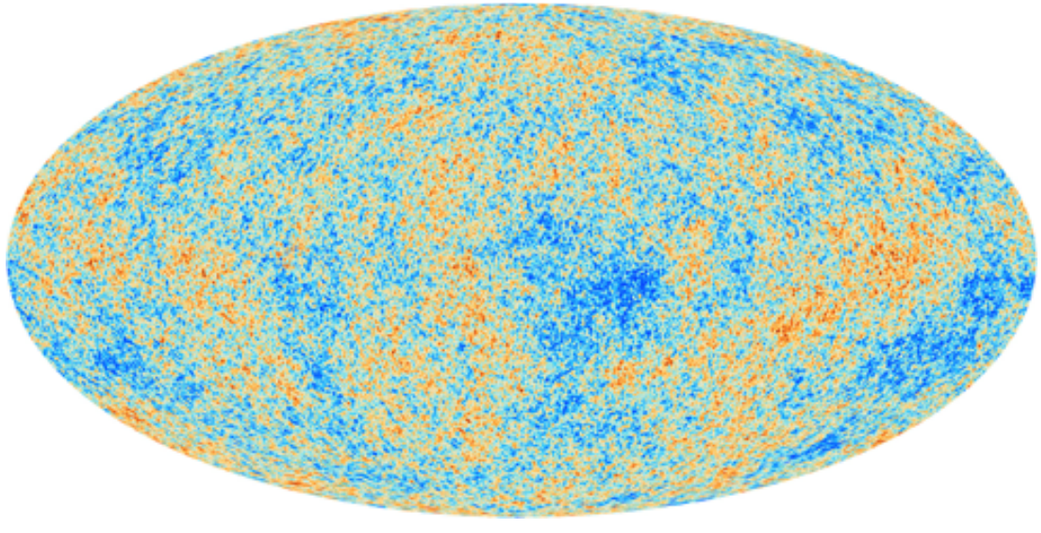


μ_k - calculate from 'local" microphysics or parametrize

N_f



applications



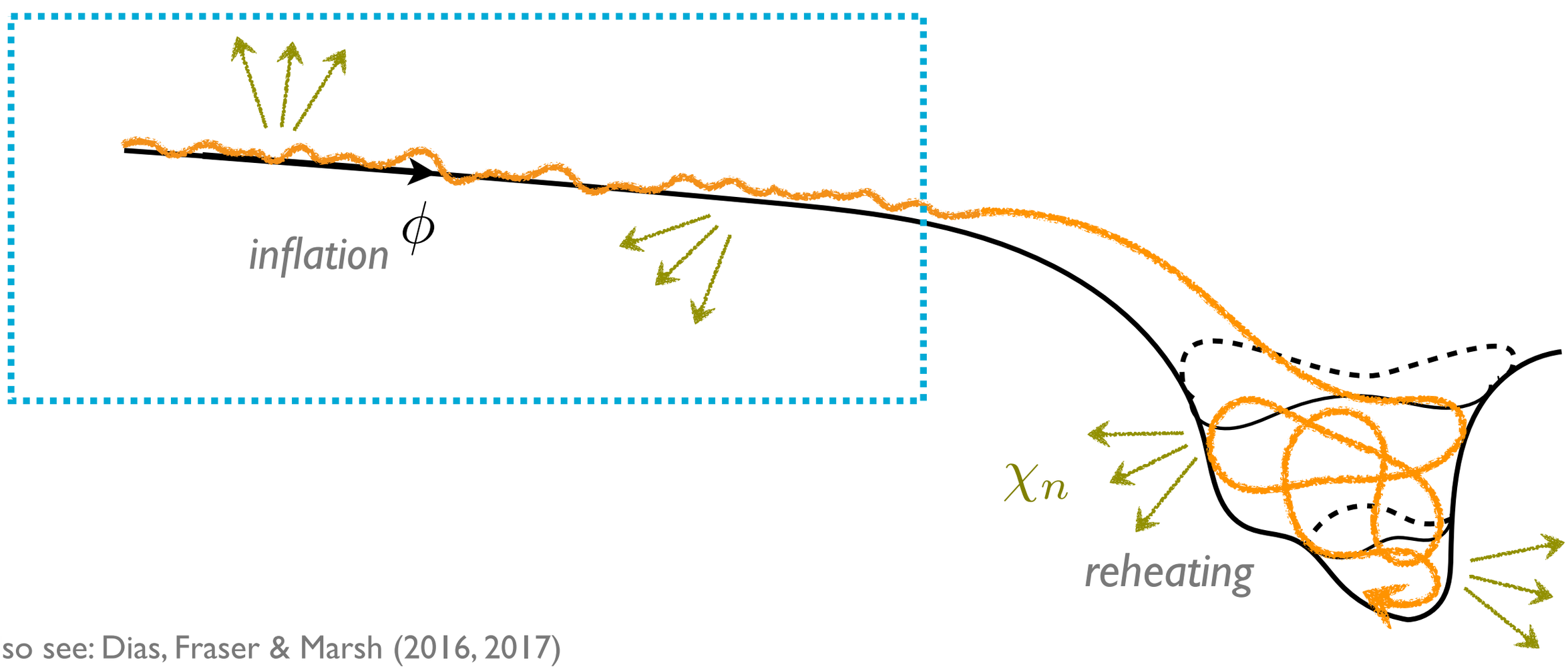
WORK IN PROGRESS

applications: inflation

MA, Garcia, Baumann, Carlsten, Chia & Green

background dynamics → particle production ↔ curvature fluctuations

$$\langle n_{k_1} n_{k_2} \ldots \rangle \qquad\qquad \langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$$



also see: Dias, Fraser & Marsh (2016, 2017)

WORK IN PROGRESS

combine particle production with driving and dissipation

background dynamics → particle production ↔ curvature fluctuations

$$\langle n_{k_1} n_{k_2} \ldots \rangle \qquad\qquad \langle \zeta_{k_1} \zeta_{k_2} \ldots \rangle$$

$$\zeta_k = -H \pi_k$$

$$\ddot{\pi}_k + [3H + \mathcal{O}_{\rm d}]\, \pi_k + \frac{k^2}{a^2} \pi_k = \mathcal{O}_{\rm s}(\langle \chi\chi \ldots \rangle_k)$$

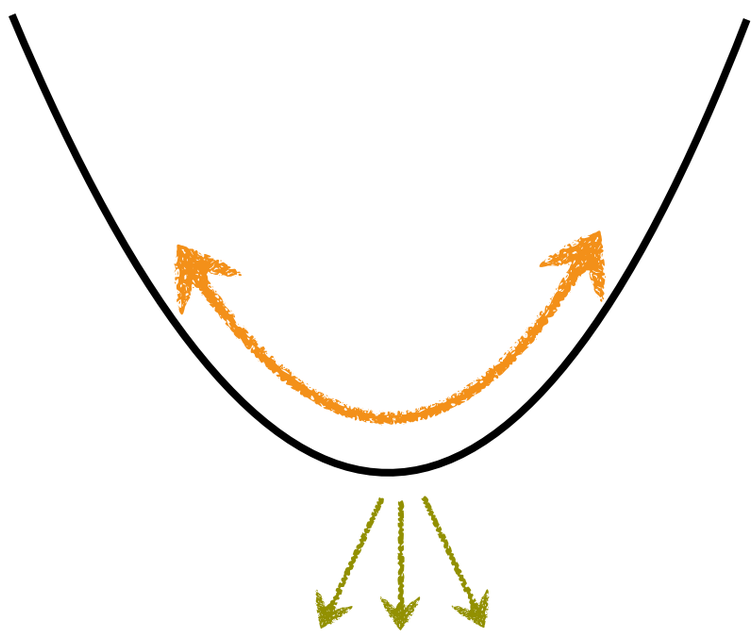
dissipation driving

MA, Garcia, Baumann, Carlsten, Chia & Green

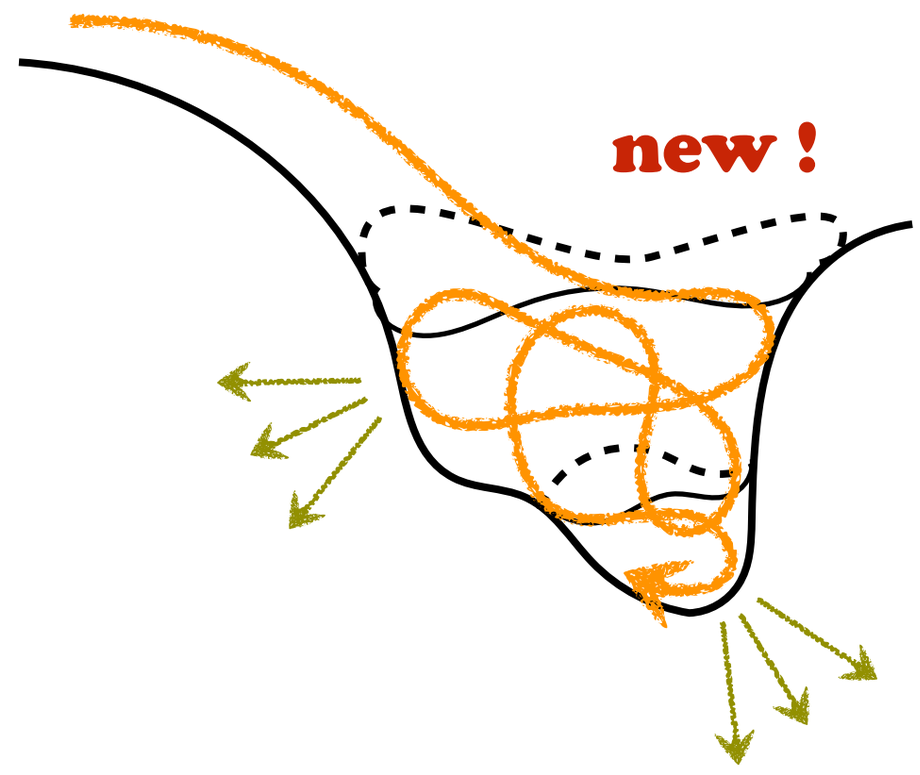
Green, Horn, Senatore, and Silverstein (2009)
Green 2014
Nacir, Porto, Senatore, and Zaldarriaga (2012)
Flauger, Mirbabayi, Senatore, Silverstein (2016)

applications : reheating

WORK IN PROGRESS



multichannel — multifield — statistical



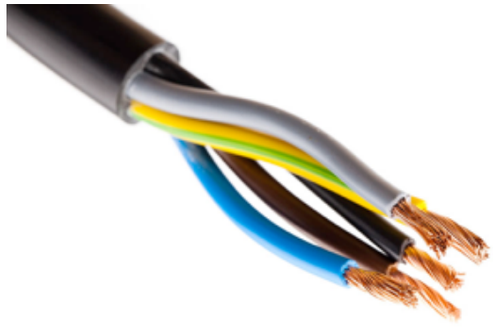
model-insensitive description of a complicated reheating process.

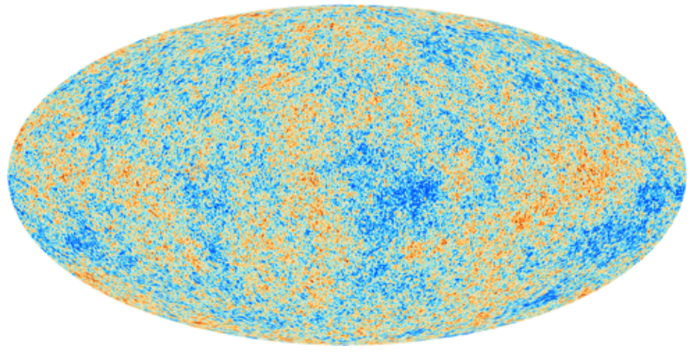
for example:
Shtanov, Traschen & Brandenberger (1995)
Kofman, Linde & Starobinsky (1997)
Zanchin et. al (1998) & Bassett (1998) [with noise]
Barnaby, Kofman & Braden et. al 2010 [quasiperiodic]
Giblin, Nesbit, Ozsoy, Sengor & Watson (2016-17)

MA, Garcia & Shen

summary

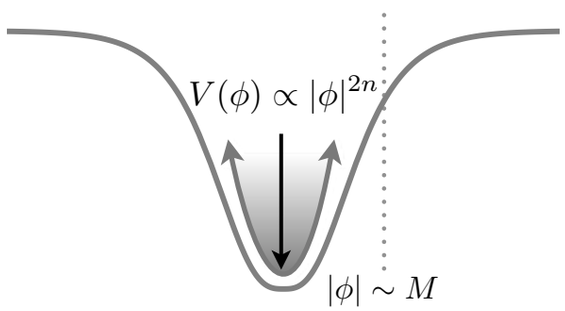
- statistical tool for theoretical complexity
- simplicity & hints of universality
- *observed simplicity* in spite of underlying complexity ?

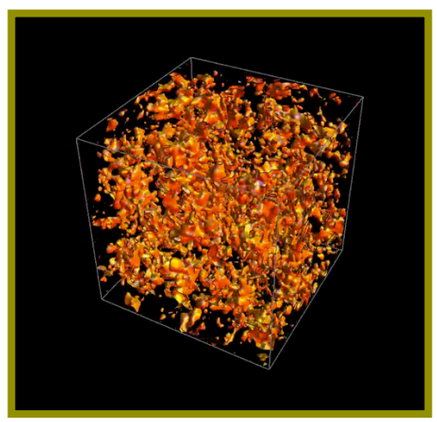


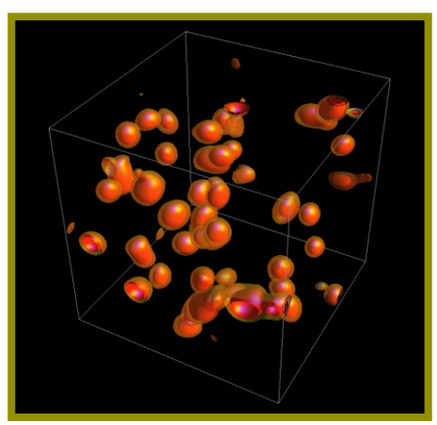


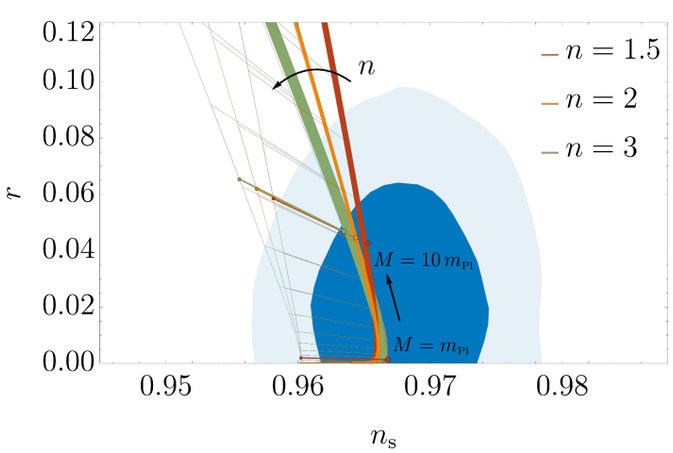
summary of 2 summaries

SIMPLE enough



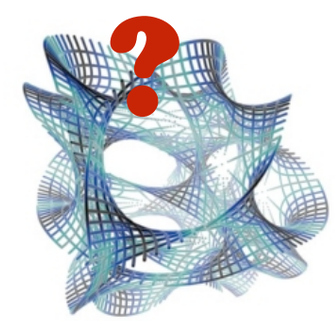




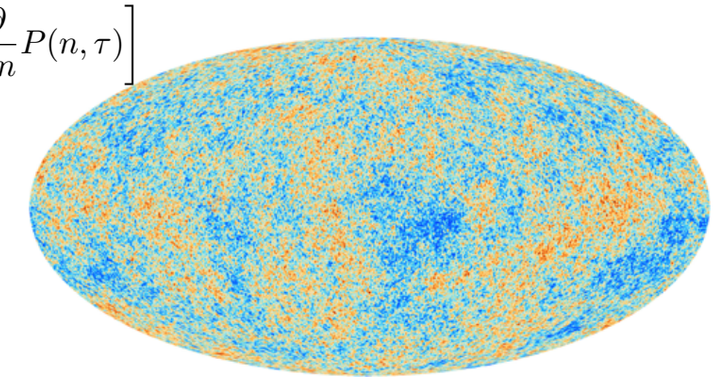


$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

COMPLEX enough



$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$



extra slides

Assistant Professor in Theoretical Astro-Particle Physics/Cosmology Position

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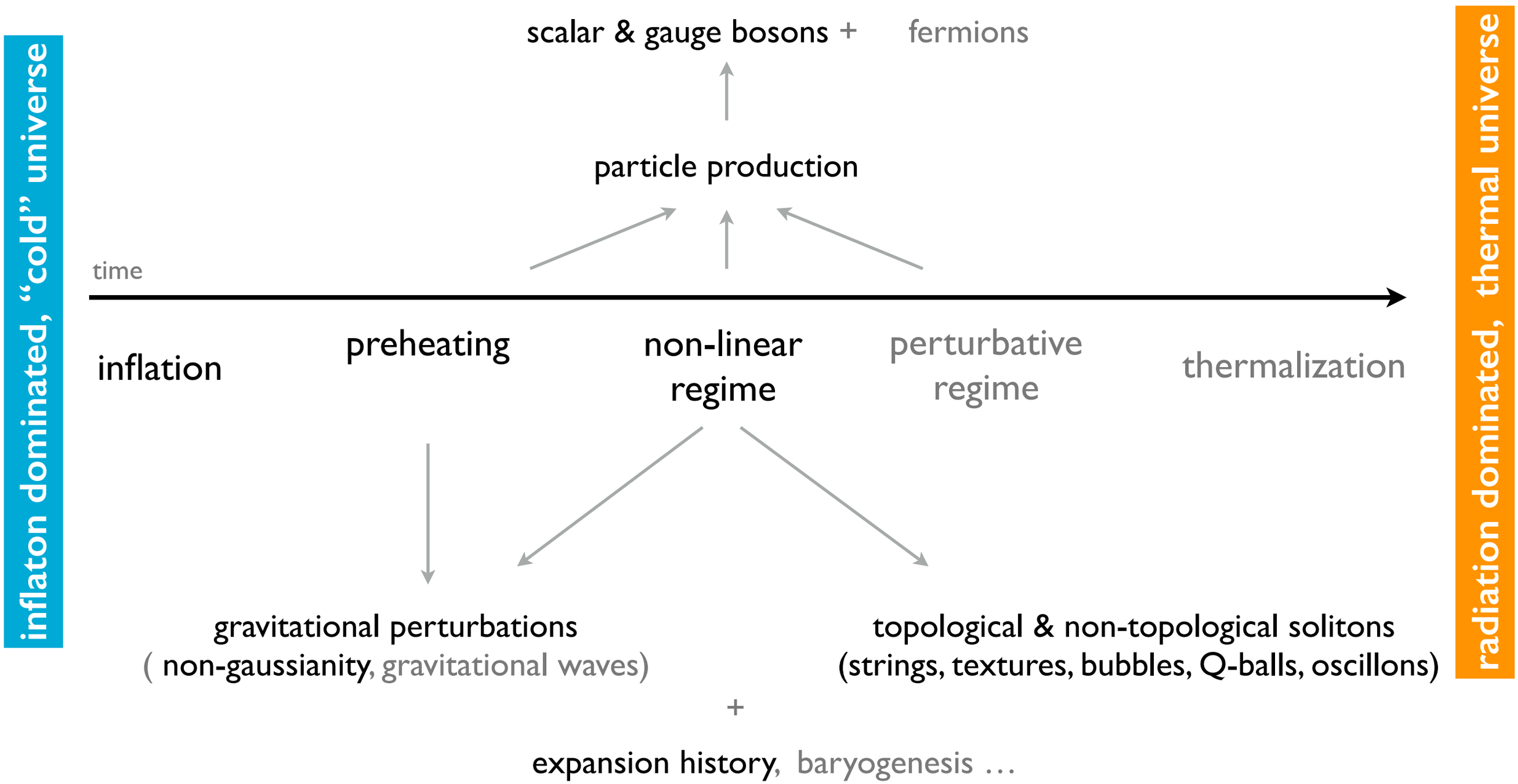
inflation — thermalization



from review: MA, Kaiser, Karouby & Hertzberg (2013)

details of spectrum

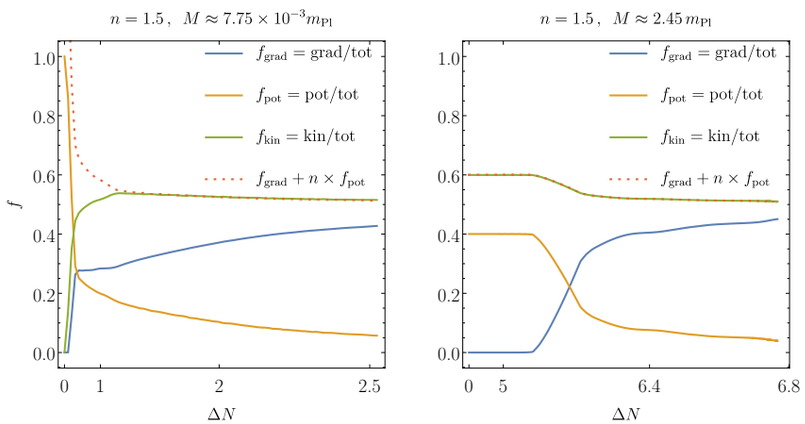


FIG. 8. The evolution of the fraction of energy, f, stored in gradient (blue), potential (orange) and kinetic (green) terms. The red (dotted) line is the right-hand side of the virial expression in eq. (17) divided by the total energy. All curves represent time averages over many oscillations and spatial averages over the simulation volume. In the case on the left, the condensate fragments rapidly into transient objects, which survive for about an e-fold of expansion as evident from the plateau near $\Delta N = 1$ in $f_{\rm grad}$. After that the transients decay away and the inflaton field becomes virialised. In the right panel, the first narrow instability band leads to slow but steady particle production. The condensate oscillates for over 5 e-folds, as indicated by the initial plateaus in the three fs, before the excited modes backreact and the condensate fragments. Interestingly, the field remains completely virialised throughout its evolution. In both cases the self-interaction energy becomes increasingly subdominant with time.

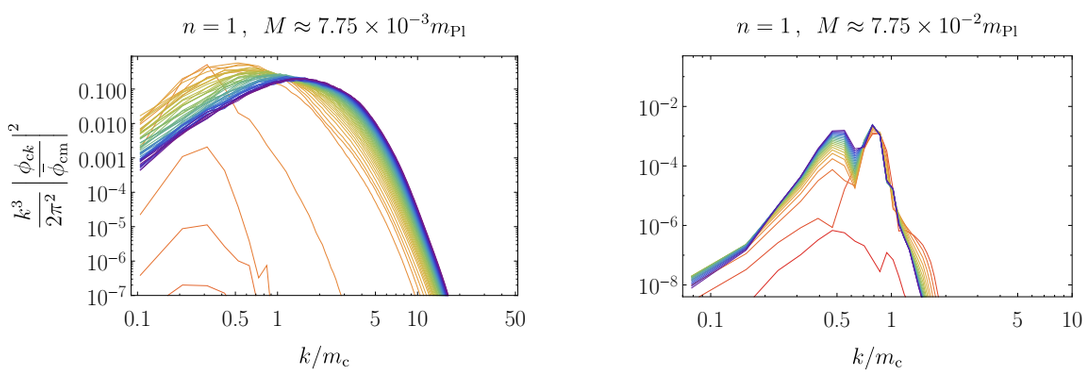


FIG. 5. The time evolution of the power spectra of the inflaton field perturbations, with time running from red to purple. In both panels, we see initial particle production due to the broad low-momentum instability band. In the left panel, where M is sufficiently small, the growth is eventually shut off by backreaction and fragmentation. The broad peak in the power spectrum is slowly shifted towards higher co-moving wavenumbers as the universe expands at late times, indicating the formation of stable objects of fixed physical size – oscillons. In the right panel, where M is not small enough, the particle production is quenched by the rapid expansion of the universe and does not lead to backreaction or fragmentation. The subscript 'c' stands for conformal – the Fourier modes, ϕ_{ck}, are rescaled by $a^{3/(n+1)}$ whereas $\bar{\phi}_{\rm cm} \approx \mathcal{O}[1]\bar{\phi}_{\rm in}$, and $m_{\rm c} \equiv m(\bar{\phi}_{\rm cm}) = m$ for $n = 1$. With these scalings, when the peak of the rescaled (by an inflaton oscillation amplitude) power spectrum reaches unity, the variance becomes comparable to the mean (as in the left panel) and indicates the start of backreaction. The data above is for the T-model.

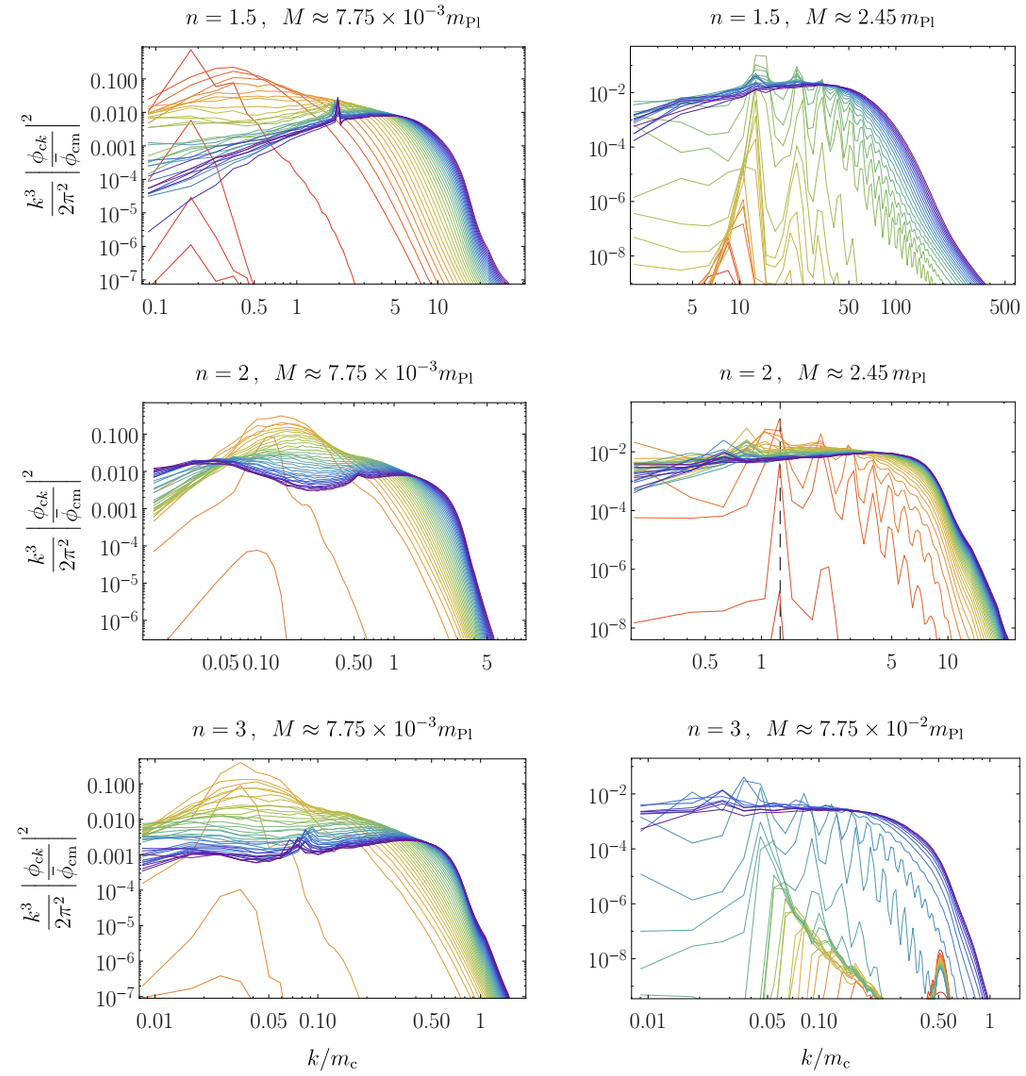


FIG. 9. Representative power spectra of inflaton fluctuations for $n > 1$. The left column is for sufficiently small M, allowing for the broad instability band to fragment the condensate and form transients. As the transient objects decay, the broad peaks in the power spectra disappear, shifting power to the UV modes. The right column is for larger M, for which the first narrow instability band leads to slow, but steady particle production in a narrow co-moving band. The peak of this band shifts with time towards higher ($n < 2$), lower ($n > 2$) co-moving modes or stays fixed ($n = 2$) at $k \approx 1.27 m_{\rm c}$. The generation of multiple re-scattering peaks is also evident in the second column. The growth is eventually shut off by backreaction and fragmentation without the formation of any transient nonlinear objects. In all six panels, power cascades slowly towards the UV at late times. Since there is a subdominant remnant oscillating condensate, some particle production from the first narrow instability band occurs at late times (clearly visible in the first column). The notation is the same as in Fig. 5.

Lozanov & MA (2017)

models considered

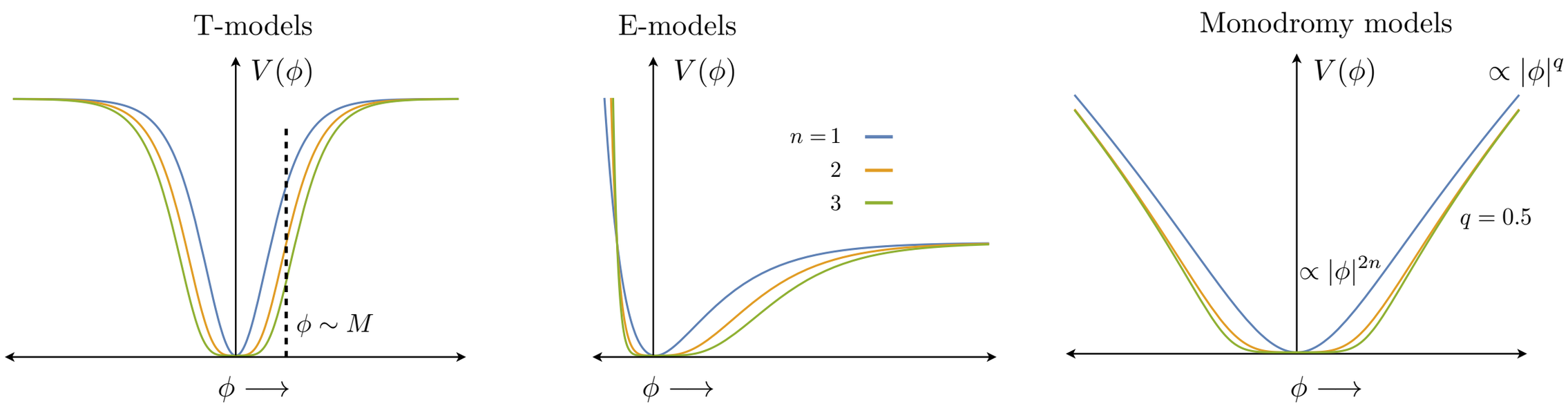


FIG. 1. The qualitatively different models used in our analysis. In all cases, the potential behaves as $|\phi|^{2n}$ close to the origin, and changes behavior (flattens at least on one side) for $\phi \gtrsim M$. The T-model and Monodromy models are symmetric about the origin, whereas the E-model is not. In the T and E-models, the potential asymptotes to a constant for large field values (at least on one side). For the Monodromy models, the potential asymtotes to a general (shallower than quadratic: $q < 2$) power law.

Lozanov & MA (2017)