

Lecture 9

Plan :

- 1) Thermal unwin timeline
- 2) Important concepts
- 3) Review Eq. Thermodynamics.

thermal history

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	—	—
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μ s	10^{12}	150 MeV
✓ Dark matter freeze-out	?	?	?
✓ Neutrino decoupling	1 s	6×10^9	1 MeV
✓ Electron-positron annihilation	6 s	2×10^9	500 keV
✓ Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
✓ Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
✓ Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

[Imp]

$\Gamma \gg H$ (eq)

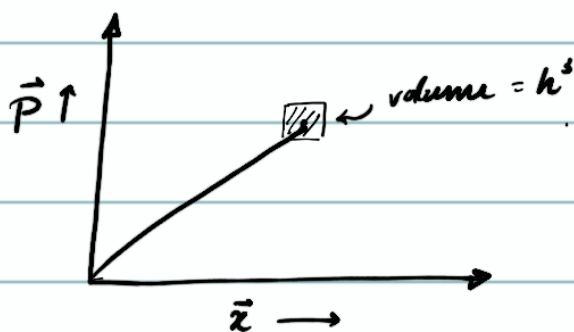
$\Gamma \sim H$ (dec.)

$\Gamma \ll H$ (freezeout)

Review of Statistical Mechanics

Consider a gas of N identical particles, each with " g " internal degrees of freedom (d.o.f.).

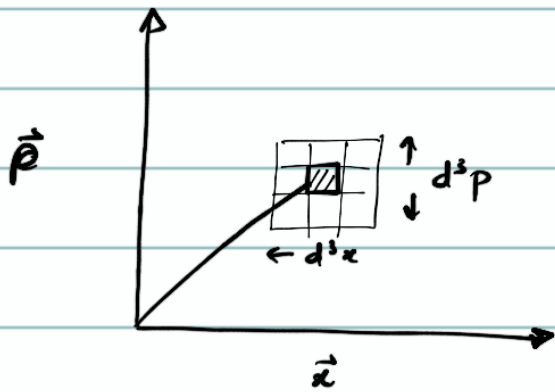
Consider the single particle phase space.



The state of a particle is given by (\vec{p}, \vec{x}, s) where $s = 1, \dots, g$ (for photons $g = 2$ and corresponds to two polarizations)
 $\pm \Delta p \quad \pm \Delta x$

Note that Heisenberg's uncertainty principle tells us that phase space is coarsely grained. That is the state occupies a volume $= h^3$. We cannot specify the position and momentum to better than $\Delta p \Delta x \sim h$.

[For more discussion on why the volume is h^3 , see the end of this lecture]



Now consider a volume $d^3p d^3x$ in phase space.

Q: How many single particle states are contained in this volume.

Ans $g \frac{d^3p d^3x}{h^3}$

Q: How many particles, ^(typically) occupy this volume of phase space at time t ?

Ans $f(\vec{p}, \vec{x}; t) \times g \frac{d^3p d^3x}{h^3}$

prob. of # of particles/state # of states

$f(\vec{p}, \vec{x}; t) \equiv$ distribution function
 $=$ occupation number

Note that we are assuming that f does not depend on $s = 1, \dots, g$.

Homogeneity + isotropy $\Rightarrow f(\vec{p}, \vec{x}; t) \rightarrow f(p; t)$

We will drop the time dependence "t" to reduce clutter.

Note the total # of particles in a volume V

$$N = \frac{g}{h^3} \int d^3x d^3p f(\vec{p}, \vec{x}; t) \stackrel{h=1}{=} \frac{g}{(2\pi)^3} \int d^3x d^3p f(\vec{p}, \vec{x}, t)$$
$$\stackrel{\text{homogeneity} \& \text{isotropy}}{=} \frac{g}{(2\pi)^3} V \int d^3p f(p)$$

\therefore Number density

$$n = \frac{g}{(2\pi)^3} \int d^3p f(p)$$

If the energy of a single particle momentum state $E(\vec{p}) = E(p)$, then the energy density

$$\rho = \frac{g}{(2\pi)^3} \int d^3p E(p) f(p) \quad \text{Energy density}$$

Similarly pressure.

$$P = \frac{g}{(2\pi)^3} \int d^3p \frac{p^2}{3E(p)} f(p).$$

For small interaction energies

$$E(p) = \sqrt{p^2 + m^2} + \overset{\text{small} \approx 0}{\cancel{\epsilon_{\text{int}}}}$$

What is $f(p)$? Depends on the situation.
Important case:

Thermal Equilibrium: ($\hbar = k_B = c = 1$).

$$f(p) = \frac{1}{\exp\left[\frac{E(p) - \mu(T)}{T}\right] \pm 1} \quad \begin{array}{l} + \text{ for fermions} \\ - \text{ for bosons.} \end{array}$$

$T(t)$ = temperature

$\mu(T)$ = chemical potential

[Recall: $dU = TdS - PdV + \mu dN$]

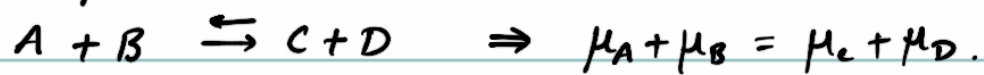
If there are different species, each will have its own m_i (hence $E_i(p)$), μ_i etc. Hence each has a different $f_i(p)$.

However in thermal equilibrium.

T = same for all species.

μ_i 's are related to each other.

In particular



Useful to note that:

$$\mu_\gamma = 0$$

photons. (see why?)

$$\mu_A = -\mu_{\bar{A}}$$

A, \bar{A} are particle/antiparticle pair

Also at early enough times $\frac{\mu}{T} \rightarrow 0$, so
for now we will ignore the chemical potential.

(a) From Question by Osmond:

* Why is phase space divided into (volume = h^3) elements? Why not h^3 or $2h^3$?

Ans: Start with the 3rd law of Thermodynamics (Equilibrium)

$$\underset{\substack{\uparrow \\ \text{entropy}}}{S} = k_B \ln \Omega \quad \rightarrow 0 \quad \text{as } T \rightarrow 0$$

of states in phase space = $\frac{\text{Volume of phase space}}{(?)}$

- So the (?) above must be such that $S \rightarrow 0$ as $T \rightarrow 0$
- We want this (?) to be universal (i.e. not depend on property of system)

You can carry off the calculation, for say a collection of oscillators and convince yourself that $(?) = h^3$.

Also, you can calculate the area between n and $n+1$ eigenstate of a harmonic oscillator

exp/theoretical.

[Note: Need to think more about the \wedge underlying for 3rd law]

(b) Related to question by Laura, Jake.

Relativistic species in equilibrium ($k_B = c = \hbar = 1$)

$$n = \frac{\xi(3)}{\pi^2} g T^3 \quad \text{bosons.}$$

→ Only dimensionful parameter is T , must be T^3 to get right dims.

Re-introducing factors of \hbar, c, k_B .

$$[k_B T] = \text{energy}$$

$$[\hbar c] = \text{energy} \times \text{length}$$

$$\therefore n = \frac{\xi(3)}{\pi^2} g \frac{(k_B T)^3}{(\hbar c)^3} \quad \text{which has units of } (\text{length}^{-1})^3.$$