Lecture 12

Plan : 1) Dark - Matter Freezout
2) Big - Bang Nucleosynthesis (First nuclei).
3) Recombination + Decoupling (First atoms)

Review : $1 + 2 \rightleftharpoons 3 + 4$.

Boltzmann equation

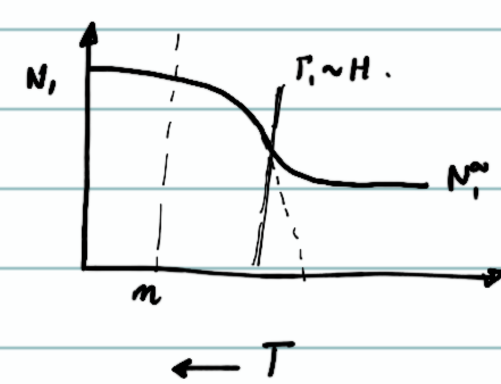
$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = -\langle \sigma v \rangle \left[n_1 n_2 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} n_3 n_4 \right].$$

$$\frac{d\ln N_1}{d\ln a} = -\frac{\Gamma_1}{H}\left[1 - \left(\frac{N_1 N_2}{N_3 N_4}\right)_{eq} \frac{N_3 N_4}{N_1 N_2}\right]$$

where $N_i = \frac{n_i}{s} \propto n_i a^3$; $\Gamma_1 = \langle \sigma v \rangle n_2$.

$\frac{\Gamma_1}{H} \gg 1 \Rightarrow N_1 \rightarrow N_1^{eq}$

$\frac{\Gamma_1}{H} \ll 1 \Rightarrow N_1 \rightarrow N_1^{\infty} = \text{const}$.



1) Dark matter freezout.

Consider a neutral dark matter particle mass M_X which can annihilate to massless leptons:

$$X + \bar{X} \rightleftarrows l + \bar{l}$$

$\bar{X}$ antiparticle, $\bar{l}$ antiparticle

Assume $n_X = n_{\bar{X}}$ initially

$l, \bar{l}$ are charged & in equilibrium with photons $n_l = (n_l)_{eq} = n_{\bar{l}}$

In this case the Boltzmann eq becomes.

$$\frac{dN_X}{d\ln a} = \frac{-\Gamma}{H}\left[1 - \frac{(N_X^{eq})^2}{N_X^2}\right]$$

Re-arranging, & defining $x = \frac{M_X}{T}$, as well as $H = \frac{H(M_X)}{x^2}$

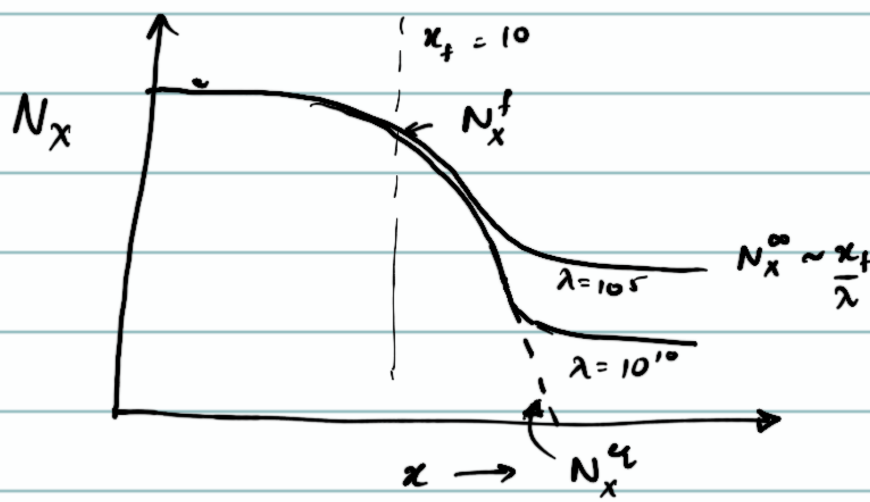
$\because$ we are in radiation domination.

we get

$$\frac{dN_\chi}{dx} = -\frac{\lambda}{x^2}\left[N_\chi^2 - (N_\chi^{eq})^2\right]$$

$$\text{where} \quad \lambda = \frac{2\pi^2}{45} g_* \frac{\overbrace{M_\chi^3 \langle \sigma v \rangle}^{\text{particle physics}}}{\underbrace{H(M_\chi)}_{\text{cosmology!}}}$$

We can solve the equation numerically, to get (assuming $\lambda = $ const).



$\Gamma(x_f) \sim H(x_f)$.

For $x \gtrsim x_f$, $N_\chi^{eq} \ll N_\chi \Rightarrow \quad \frac{dN_\chi}{dx} \approx -\frac{\lambda}{x^2} N_\chi^2$

integrate

$$\Rightarrow \quad \frac{1}{N_\chi^\infty} - \frac{1}{N_\chi^f} \approx \frac{\lambda}{x_f}$$

Since $N_\chi^\infty \ll N_\chi^f \quad \Rightarrow \quad N_\chi^\infty \approx \frac{x_f}{\lambda}$

What remains, is estimating x_f. You will do this in your homework.

It turns out $x_f \sim 10$ (reasonably independent of λ!).

Thus $N_\chi^\infty \sim \frac{10}{\lambda}$

We can convert N_χ^∞ into Ω_χ = fraction of energy density of the universe in χ. today.

$$\Omega_\chi \sim 0.2 \left(\frac{x_f}{10}\right)\left(\frac{10}{g_*(M_\chi)}\right)^{1/2} \frac{10^{-8}\,\mathrm{GeV}^{-2}}{\langle\sigma v\rangle}.$$

Amazingly, if $\langle\sigma v\rangle \sim 10^{-4}\,\mathrm{GeV}^{-2}$ (typical weak interaction cross section)
$\sim 0.\sqrt{G_F}$

the abundance of dark matter is roughly what we observe. This is called the WIMP miracle.

(weakly interacting massive particle)

Big Bang Nucleosynthesis $(T \sim 0.1\,\text{MeV})$

Synthesis of light element : Hydrogen H
Helium . He
Lithium Li
⋮

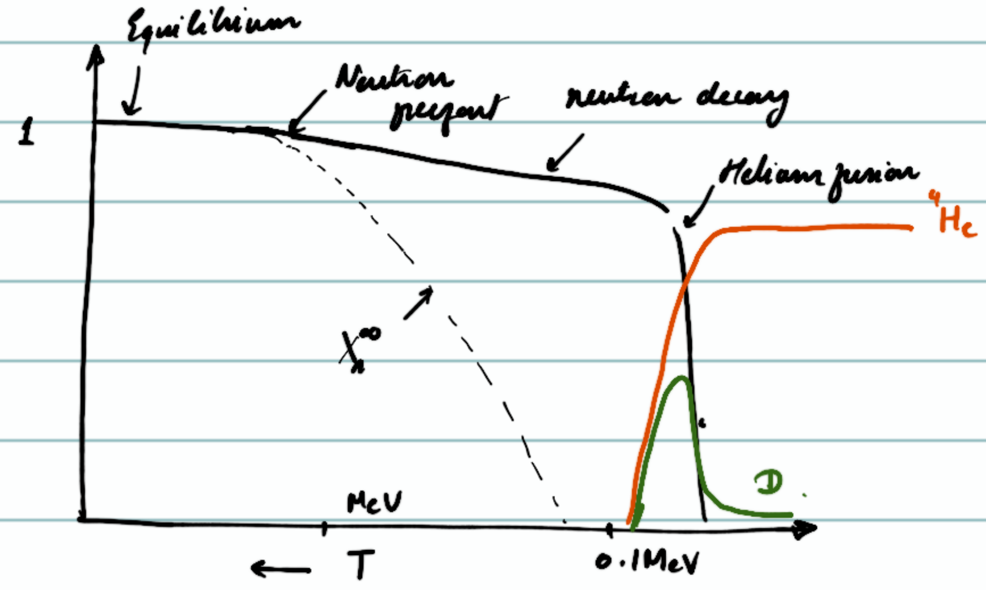
Important steps : (see figure).

1) Neutron Freezout : $n + \nu_e \rightleftharpoons p + e \Rightarrow n_n^\infty \sim \frac{1}{6} n_p^\infty$
2) Neutron decay : $n_n(t) \simeq n_n^\infty e^{-t/\tau_n}$ $\tau_n \approx 900\,\text{sec}$.
3) Helium fusion.

Helium can only form after deuterium
$\Rightarrow$ deuterium "Bottleneck" (Deuterium takes a while because of small binding energy and large photon/baryon ratio).

$$n + p \rightarrow D + \gamma$$
$$\downarrow$$
$$D + p \longrightarrow {}^3He + \gamma$$
$$D + {}^3He \longrightarrow {}^4He + p.$$

Higher elements built from lighter elements. Plasma is too dilute for heavier elements to form by > 2 body interactions.



$$X_n = \frac{n_n}{n_n + n_p} = \text{neutron fraction.}$$

Some useful simplifications that make the calculation tractable:

1) only track elements lighter than Helium.
2) for $T > 0.1$ MeV, we only need to track protons & neutrons, all others negligible.

* The binding energy for Deuterium is ~1 MeV. Why do we have to wait till 0.1 MeV for it to form.

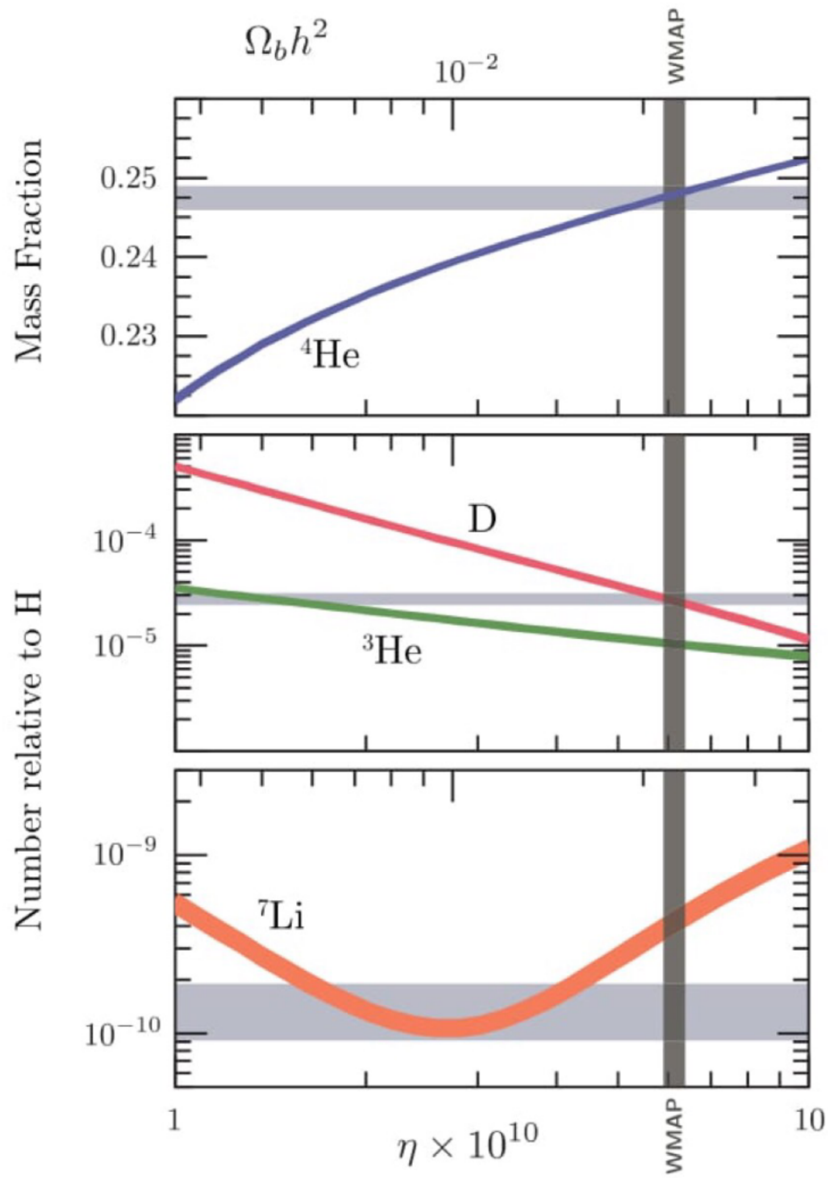
Ans. Too many photons.

$\frac{\text{photons}}{\text{baryon}} \simeq \frac{1}{\eta_b} \simeq 10^{10}$ ⇒ there are enough high energy photons in the tail of the distribution that can disrupt the nucleus!

At the end of the day one finds

$$\boxed{\frac{4\,n_{He}}{n_H} \simeq \frac{1}{4}}$$, agrees beautifully with observations.

(Other light element abundances can also be calculated)



Theoretical predictions (colored bands) and observational constraints (grey bands).

From DB Notes.

Recombination: $(T \sim 0.3\,\text{eV})$. [ignore Helium]

For $T \gtrsim \text{eV}$ $e + p \rightleftharpoons H + \gamma$ (equilibrium).

Since $T < m_i$ $i = \{e, p, H\}$.

$$n_i^{eq} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{m_i}{T}} e^{\mu_i/T}$$ (re-introducing the chemical potential).

In equilibrium. $\mu_e + \mu_p = \mu_H$. $(\because \mu_\gamma = 0)$.

$$\therefore \left(\frac{n_H}{n_e n_p}\right)_{eq} = \frac{g_H}{g_e g_p}\left(\frac{m_H}{m_e m_p}\frac{2\pi}{T}\right)^{3/2} e^{(m_p + m_e - m_H)/T}.$$

$$\Rightarrow \left(\frac{n_H}{n_e^2}\right)_{eq} \approx \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}.$$

where $B_H = m_p + m_e - m_H \approx 13.6\,\text{eV}$ (Binding energy)
$g_H = 4$, $g_e = g_p = 2$. & we used $n_e = n_p$ (neutrality).
Note that it is safe to set $m_p \approx m_H$ in the co-efficient, but not in B_H.

Let us follow the free electron-fraction

$$X_e = \frac{n_e}{n_b} = \frac{n_e}{n_p + n_H}$$ n_b = # density of "baryons".

Assuming $n_b \approx n_p + n_H = n_e + n_H$ (no Helium).

yields.

$$\therefore \quad \boxed{\left(\frac{1-X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2}\,\eta_b\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}.}$$ ← SAHA Eq. (Equilibrium).

Note $n_b = \eta_b n_\gamma = \eta_b \dfrac{2\zeta(3)}{\pi^2} T^3$ where $\eta_b \approx 10^{-10}$

When does $X_e = 10^{-1}$. Solve the Saha eq to get T_{rec} = Hydrogen recombination temperature

$T_{rec} \approx 0.3\ eV$

Note again $T_{rec} = 0.3\,eV \ll B_H = 13.6\,eV$ because of smallness of η_b.

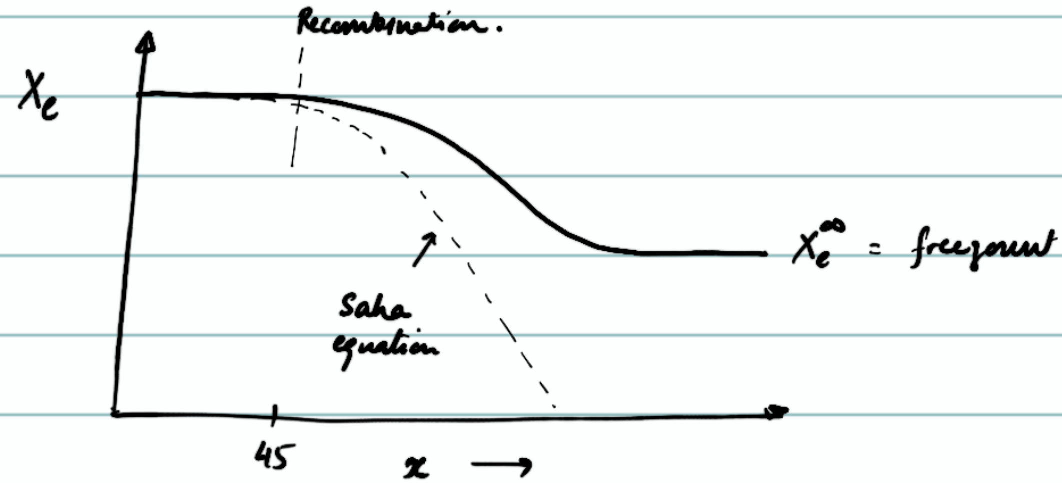
* Redshift of recombination : $z_{rec} \approx 1320$.

This was all done using equilibrium expressions. By now we know that this is an incomplete story. Eventually $p + e \leftarrow H + \gamma$ becomes too inefficient and the free electron density freezes out.

The equation describing the process. (see Baumann's notes for details

$$\frac{dX_e}{dx} = -\frac{\lambda}{x^2}\left[X_e^2 - \left(X_e^{eq}\right)^2 \right]$$

when $x = \frac{B_H}{T}$ $\lambda = \left[\frac{n_b \langle \sigma v \rangle}{xH}\right]_{x=1}$



There is another important event, squeezed in between Hydrogen recombination and free electron freezout.

Photon Decoupling:

Consider the reaction $e + \gamma \rightleftarrows e + \gamma$

$$\Gamma_\gamma = \langle \sigma v \rangle n_e = \sigma_T n_e .$$

$\sigma_T \equiv$ thompson cross section.

Since the free electron fraction is dropping

$\frac{\Gamma_\gamma}{H} \lesssim 1$ will happen! Using the equilibrium abundances, we find $\Gamma_\gamma(T_{dec}) \simeq H(T_{dec})$

$$\Rightarrow \boxed{T_{dec} \simeq 0.27 \text{ eV}}.$$

After this point the photons barely interact with the electrons and are free to stream to us. This is the cosmic microwave Background. (CMB) !!!

The redshift $\boxed{z_{dec} \simeq 1100}$.

Note the ordering

① Recombination : $T_{rec} \simeq 0.3\,eV$, $z_{rec} \simeq 1320$, $t_{rec} \simeq 290{,}000$ yrs

② Photon decoupling : $T_{dec} \simeq 0.27\,eV$, $z_{dec} \simeq 1100$, $t_{dec} \simeq 380{,}000$ yrs.

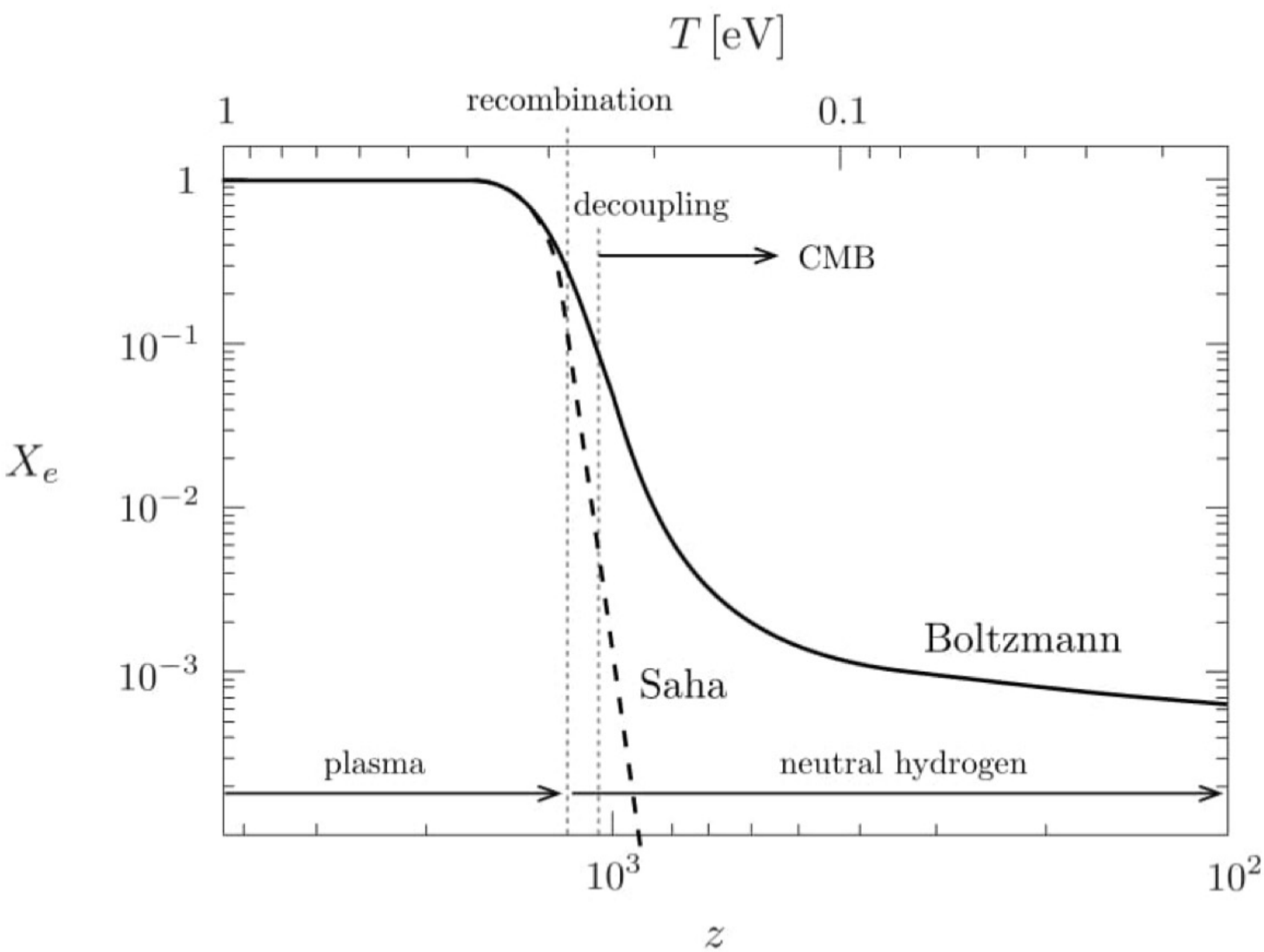


Figure 3.8: *Free electron fraction as a function of redshift.*

From DB Notes.

Summary of the Chapter

thermal history

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
✓ Dark matter freeze-out	?	?	?
✓ Neutrino decoupling	1 s	6×10^9	1 MeV
✓ Electron-positron annihilation	6 s	2×10^9	500 keV
✓ Big Bang nucleosynthesis	3 min	4×10^8	100 keV
✓ Matter-radiation equality	60 kyr	3400	0.75 eV
✓ Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
✓ Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV