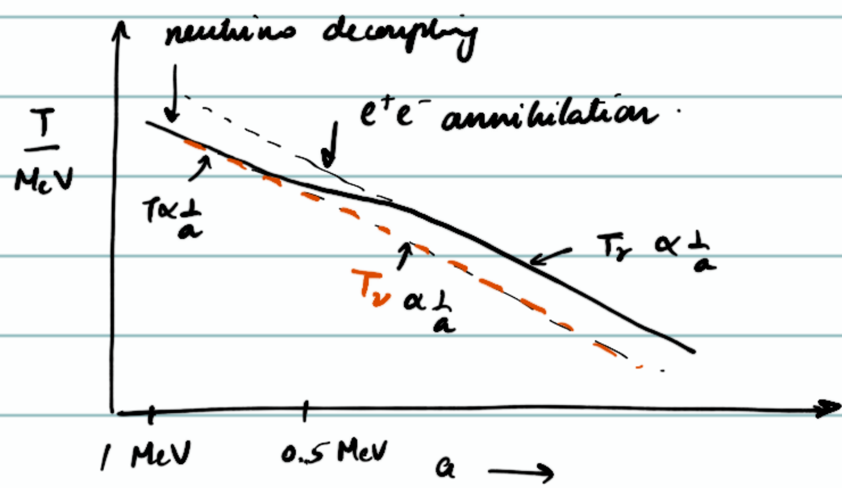
Lecture 11

2. e^+e^- annihilation

Shortly after neutrino decoupling
e^+e^- annihilate into photons efficiently.
(However photons can no longer efficiently generate
e^+e^- pairs). This is because at $T \sim 0.5$ MeV
$m_e \sim T$.
electron mass
or positron

- The extra photons from the annihilation contribute to an increase in temperature of the photons.

- However the neutrinos have decoupled, and no longer get a boost in temperature!
Hence we should expect



To actually calculate the difference in temperature, we will use the conservation of entropy:

$$[s a^3]_{before} = [s a^3]_{after}.$$

before = before e^+e^- annihilation.

$$[s a^3]_{before} = \Big[\underbrace{(\tilde{s} a^3)}_{\text{everything relativistic other than } \nu} + s_\nu a^3 \Big]_{before}.$$

because decoupled so lets count it separately.

$$= \left[\frac{2\pi^2}{15} g_* T^3 a^3 + s_\nu a^3 \right]_{before}.$$

where $g_* = \underset{\gamma}{2} + \frac{7}{8}\underbrace{\Big(\underset{e^+}{2} + \underset{e^-}{2}\Big)}_{\text{fermions}} + \dots$

Hence

$$[s a^3]_{before} = \left[\frac{2\pi^2}{15} \left(\frac{11}{2}\right) T^3 a^3 \right]_{before} + [s_\nu a^3]_{before}.$$

Similarly

$$[s a^3]_{after} = \left[\frac{2\pi^2}{15} (2 + 0 + \dots) T^3 a^3 \right]_{after} + [s_\nu a^3]_{after}.$$

e^+, e^- are non relativistic

$\therefore$

$$\therefore \quad T_{after} = \left(\frac{11}{4}\right)^{1/3} T_{before}.$$

$$\Rightarrow \boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma}$$

after

where we used the fact that after decoupling
$[S_\nu a^3]$ is conserved by it-self (free streaming).

& that before e^+e^- annihilation $T_\gamma = T_\nu = T$

* Notes: - When referring to T, we are using the photon temperature.
  - the e^+e^- transition as well as decoupling of neutrinos is not quite instantaneous.

IMP

$\Gamma \sim H$: decoupling of ν = ν stop interacting with the rest of the plasma.

$T \sim m_e$: e^+e^- annihilation = not enough energy in plasma to regenerate e^+e^- pairs

Beyond Equilibrium.

We can no longer use $f(p) = \frac{1}{e^{\frac{E}{T}-\frac{\mu}{T}} \pm 1}$

Instead we have to solve the Boltzmann equation. Still assuming homogeneity and isotropy, the Boltzmann equation is given by

The RHS is due to interactions.

Instead of dealing with f, it is convenient (and sufficient for our purposes) to deal with the number density.

In absence of interactions the number density of a species i, $n_i \propto a^{-3}$

$$\therefore \quad \frac{1}{a^3}\frac{d}{dt}(n_i a^3) = 0 .$$

In presence of interactions

$$\frac{1}{a^3}\frac{d}{dt}(n_i a^3) = C[\{n_j\}]$$

interactions

Let us consider $1 + 2 \rightleftarrows 3+4$

type interactions (those involving higher # of particles are less likely).

Focus on 1.

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_1) = C[\{n_1, n_2, n_3, n_4\}]$$

What is the form of C?

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_1) = \underbrace{-\alpha n_1 n_2}_{\text{destroys } 1} + \underbrace{\beta n_3 n_4}_{\text{Generates } 1.}$$

One can arrive at the above general form, as well as determine α & β by considering a more detailed derivation (see for example, Dodelson's Textbook).

I will state the answer here, and then justify that it is reasonable.

$\alpha = \langle \sigma v \rangle =$ thermally averaged cross-section $1+2 \to 3+4$.

$\beta = \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq}$ where "eq" stands for the equilibrium expressions for the respective # densities.

Note that the chemical potentials $\mu_1 + \mu_2 = \mu_3 + \mu_4$.

ie.

$$\frac{1}{a^3}\frac{d}{dt}(n_1 a^3) = -\langle\sigma v\rangle n_2 n_1 \left[1 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} \frac{n_3 n_4}{n_1 n_2}\right]$$

Now $\Gamma_1 \equiv n_2 \langle\sigma v\rangle$

$$\therefore \frac{1}{a^3}\frac{d}{dt}(n_1 a^3) = -\Gamma_1 n_1 \left[1 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} \frac{n_3 n_4}{n_1 n_2}\right]$$

Let us make sure that this equation makes sense.
We expect that if $\Gamma_1 \gg H$, then $n_i \approx n_i^{eq}$ should be an "approximate" solution of the above equation.
This is indeed the case.
You should check that regardless of $m_1 \ll T$ or $m_1 \gg T$, as long as $\frac{\Gamma_1}{H}$ is large enough, $n_i \sim n_i^{eq}$ will be an approximate solution to the above equation.

A convenient re-writing of the above equation is

using $\frac{d\ln a}{dt} = H$

Hence

$$\frac{d\ln(a^3 n_1)}{d\ln a} = \frac{-\Gamma_1}{H}\left[1 - \left(\frac{n_1 n_2}{n_3 n_4}\right)_{eq} \frac{n_3 n_4}{n_1 n_2}\right]$$

It is convenient to define

$$N_i = \frac{n_i}{S} \propto n_i a^3 \quad (\text{Recall } s \propto a^{-3})$$
$$\propto n_i / T^3$$

Then we have:

$$\frac{d\ln N_1}{d\ln a} = -\frac{\Gamma_1}{H}\left[1 - \left(\frac{N_1 N_2}{N_3 N_4}\right)_{eq} \frac{N_3 N_4}{N_1 N_2}\right]$$

Another sanity check for this equation.

<u>Case 1</u>: $\Gamma_1 >> H$

If $N_1 >> N_{1eq}$, $N_i \sim N_{i\,eq}$ $i = 2,3,4$.

then $\frac{d\ln N_1}{d\ln a} < 0 \Rightarrow N_1$ decreases (towards the eq value)

If $N_1 << N_{eq}$

$\frac{d\ln N_1}{d\ln a} > 0 \Rightarrow N_1$ increase (towards the eq. value)

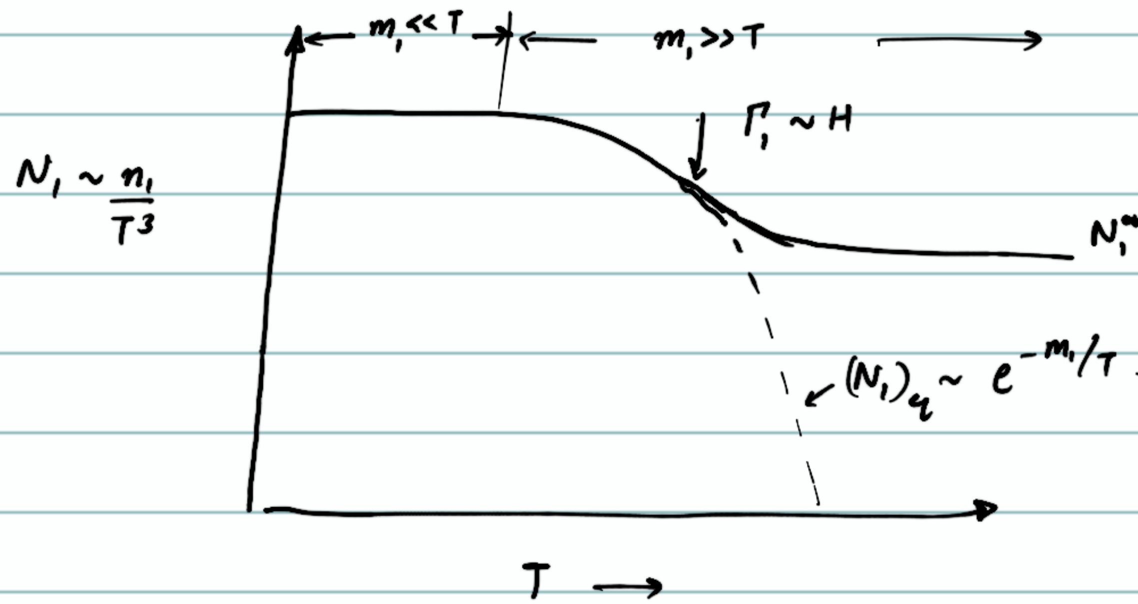
Case 2 $\quad \Gamma_1 \ll H$.

$$\frac{d\ln N_1}{d\ln a} \approx 0 \Rightarrow N_1 \approx \text{constant} \leftarrow \text{Freezout}.$$

Thus if $\Gamma_1 \gg H$, $N_1 \to (N_1)_{eq}$.

$\Gamma_1 \ll H$, $N_1 \to$ constant.

Here is a typical behavior.



IMPORTANT.

Applications 1) Dark matter freezout
2) BBN
3) Recombination & Decoupling.

1) Dark matter freezout.

Consider a neutral dark matter particle mass M_X which can annihilate to massless leptons.

$$X + \bar{X} \rightleftarrows l + \bar{l}$$

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Assume $n_X = n_{\bar{X}}$ initially
$l, \bar{l}$ are charged & in equilibrium with photons
$n_l = (n_l)_{eq} = n_{\bar{l}}$

In this case the Boltzmann eq becomes.

$$\frac{dN_X}{d\ln a} = \frac{-\Gamma}{H}\left[1 - \frac{(N_X^{eq})^2}{N_X^2}\right]$$

Re-arranging, & defining $x = \frac{M_X}{T}$, as well as $H = \frac{H(M_X)}{x^2}$

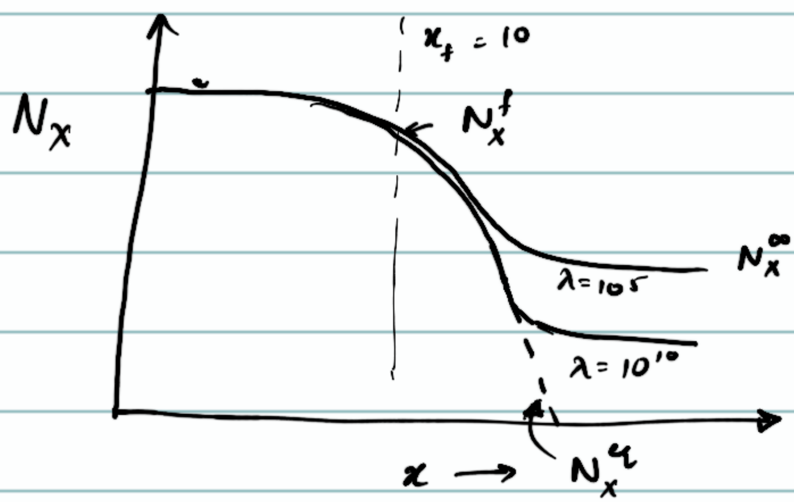
$\because$ we are in radiation domination.

we get

$$\frac{dN_X}{dx} = -\frac{\lambda}{x^2}\left[N_X^2 - (N_X^{eq})^2 \right]$$

$$\text{where} \quad \lambda = \frac{2\pi^2}{45} g_* \frac{\overbrace{M_X^3 \langle \sigma v \rangle}^{\text{particle physics}}}{\underbrace{H(M_X)}_{\text{cosmology!}}}$$

We can solve the equation numerically, to get .
(assuming $\lambda = \text{const}$).



For $x \gtrsim x_f$, $N_X^{eq} \ll N_X \Rightarrow \quad \dfrac{dN_X}{dx} \approx -\dfrac{\lambda}{x^2} N_X^2$

integrate

$$\Rightarrow \quad \frac{1}{N_X^\infty} - \frac{1}{N_X^f} \approx \frac{\lambda}{x_f}$$

Since $N_X^\infty \ll N_X^f \Rightarrow \quad N_X^\infty \approx \dfrac{x_f}{\lambda}$

What remains, is estimating x_f. You will do this in your homework.

It turns out $x_f \sim 10$ (reasonably independent of λ!).

Thus $N_\chi^\infty \sim \frac{10}{\lambda}$

We can convert N_χ^∞ into Ω_χ = fraction of energy density of the universe in χ today.

$$\Omega_\chi \sim 0.2 \left(\frac{x_f}{10}\right)\left(\frac{10}{g_*(M_\chi)}\right)^{1/2} \frac{10^{-8}\,\mathrm{GeV}^{-2}}{\langle \sigma v \rangle}.$$

Amazingly, if $\langle \sigma v \rangle \sim 10^{-4}\,\mathrm{GeV}^{-2}$ (typical weak interaction cross section
$\sim 0.\sqrt{G_F}$

the abundance of dark matter is roughly what we observe. This is called the WIMP miracle.
(weakly interacting massive particle)