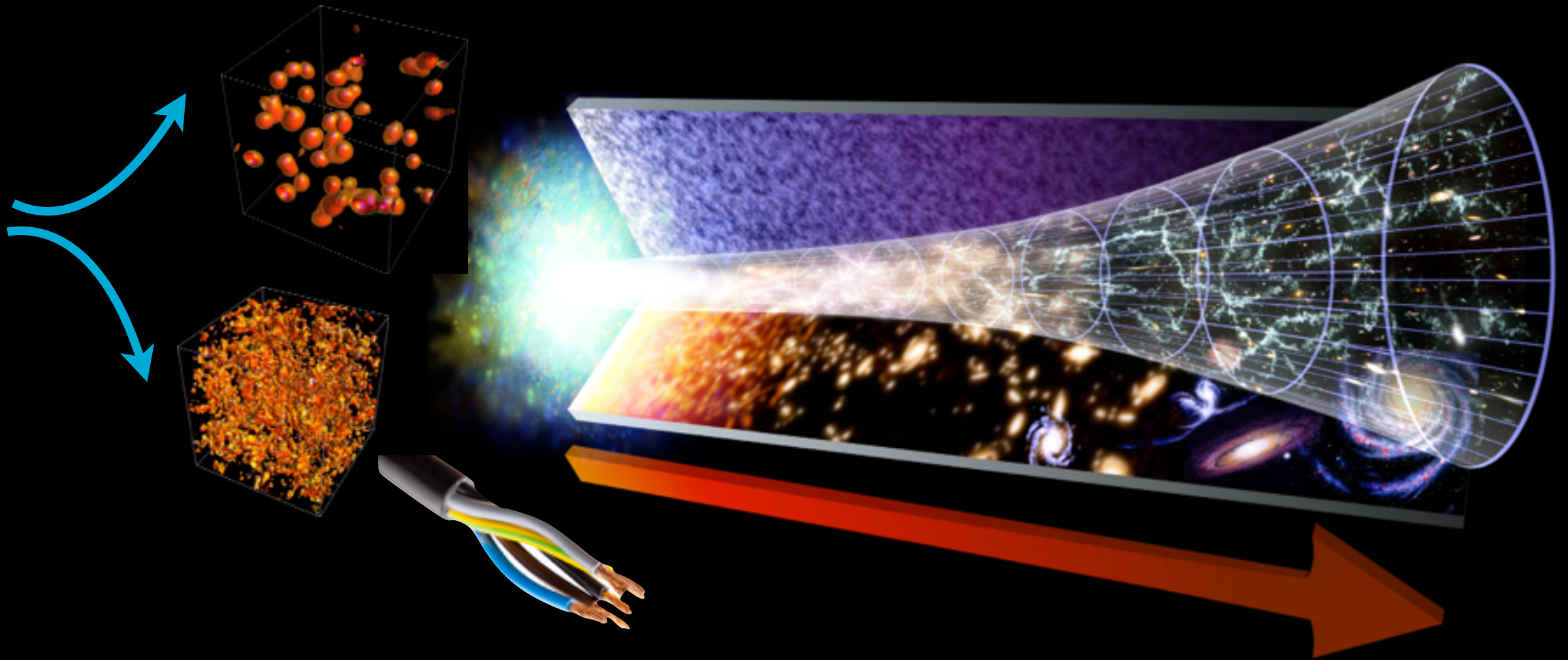


# How did the hot big bang begin?



Mustafa Amin



# Outline

- our origins ?

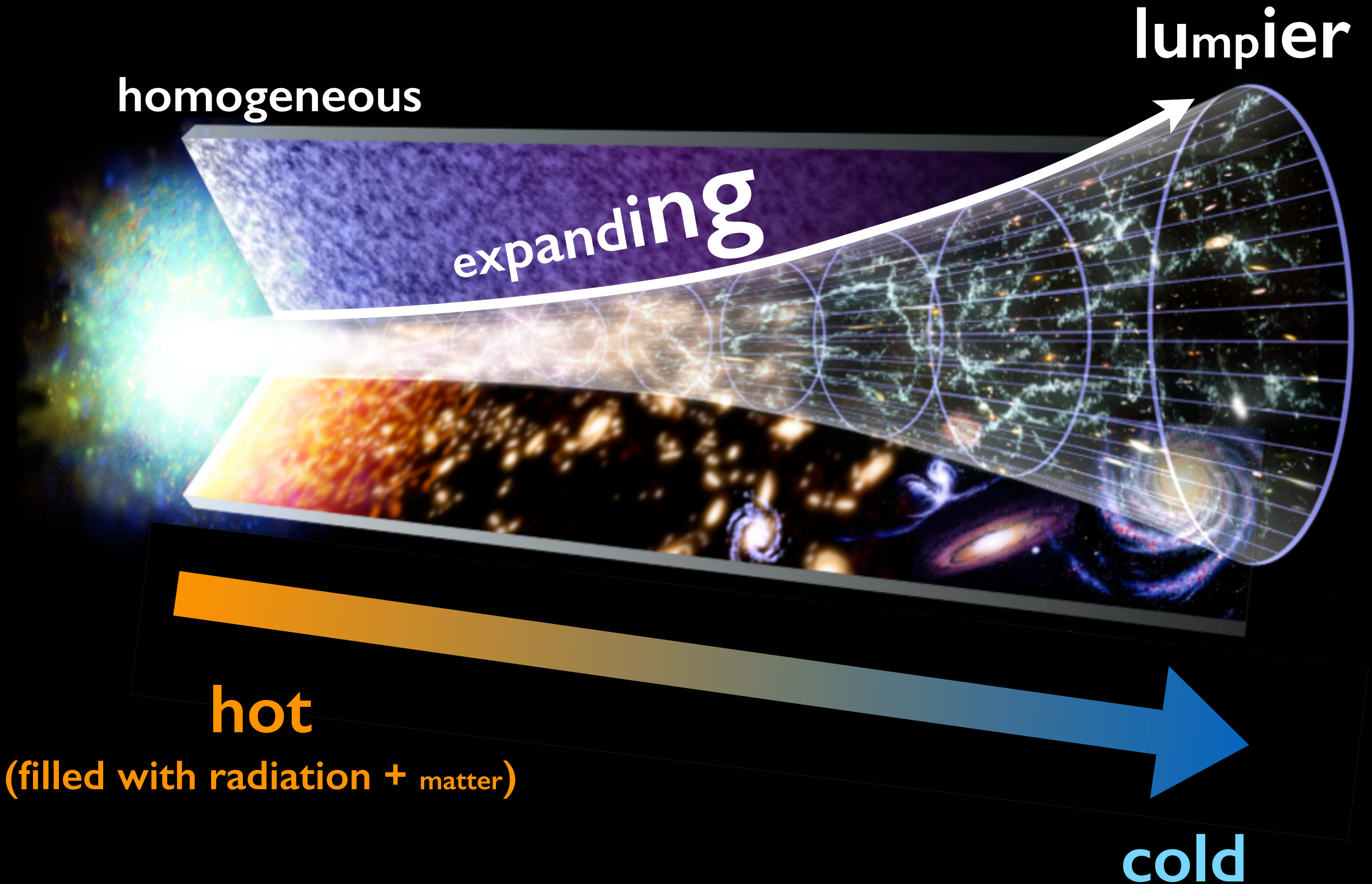
➔ inflation & reheating

◎ two approaches:

1. simple enough models — general predictions
2. complex enough models — new statistical approach



# The Standard Big Bang Cosmology



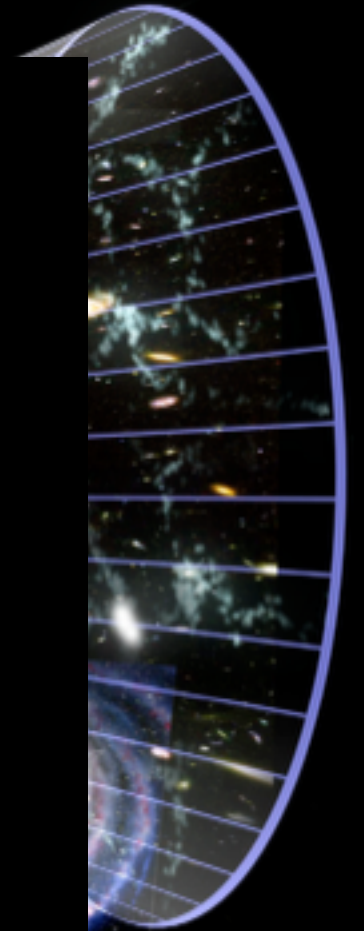
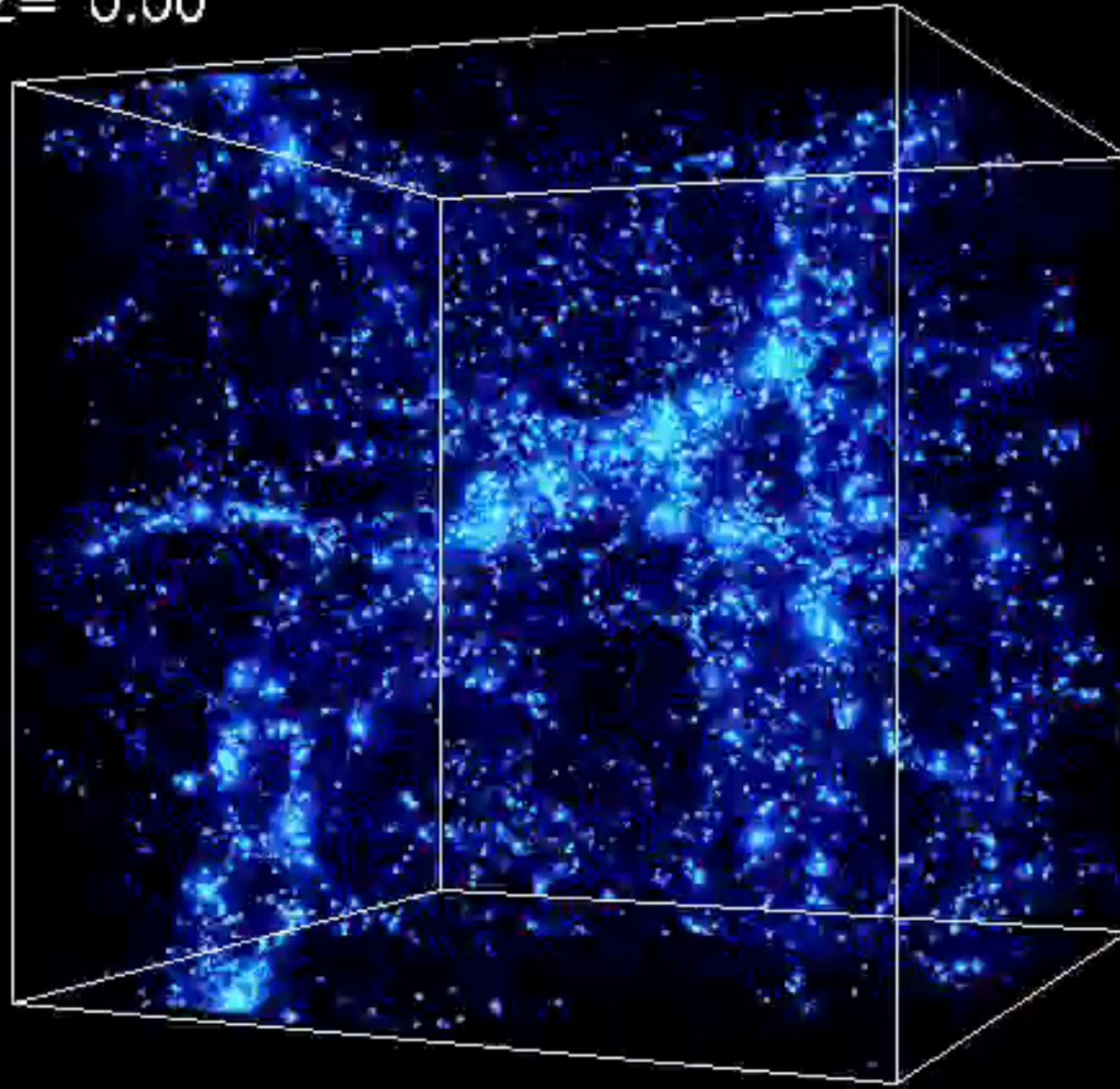


# The Standard Big Bang Cosmology

homogeneous

lumpier

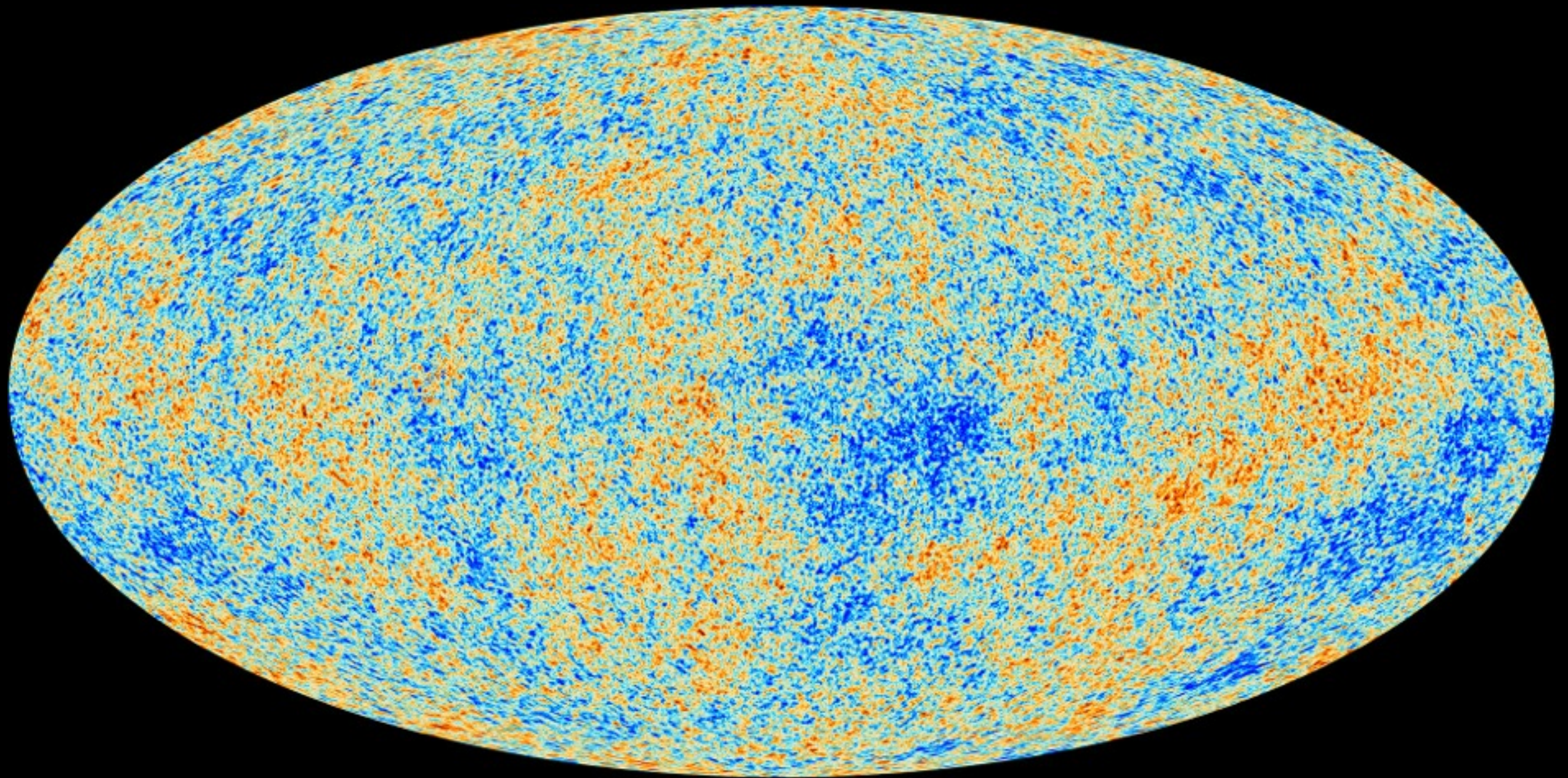
$z = 0.00$



**initial conditions  
for density perturbations ?**



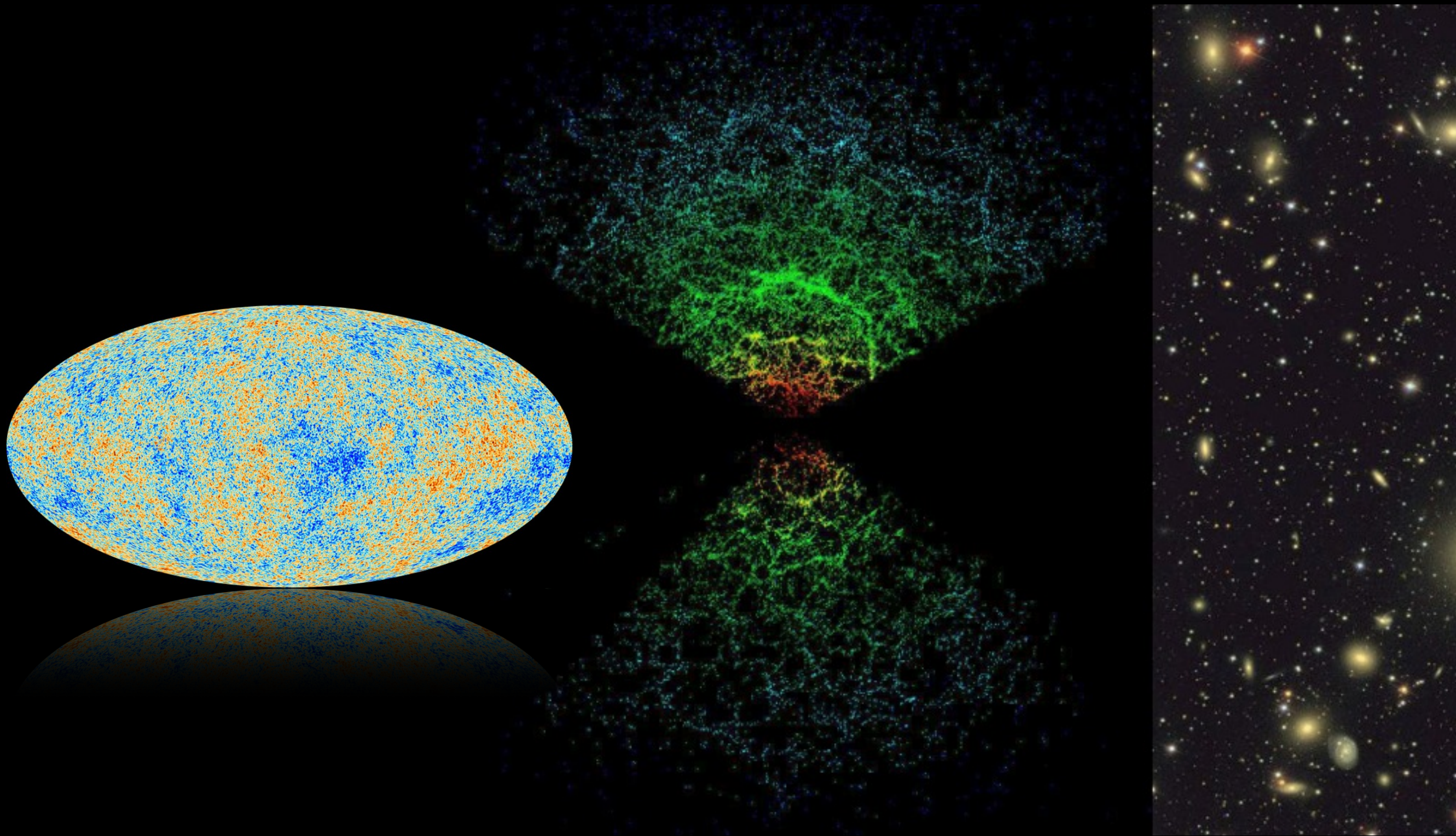
# CMB temperature anisotropies



$$\delta T/T \sim 10^{-5}$$



# initial conditions — gravity — observed structure

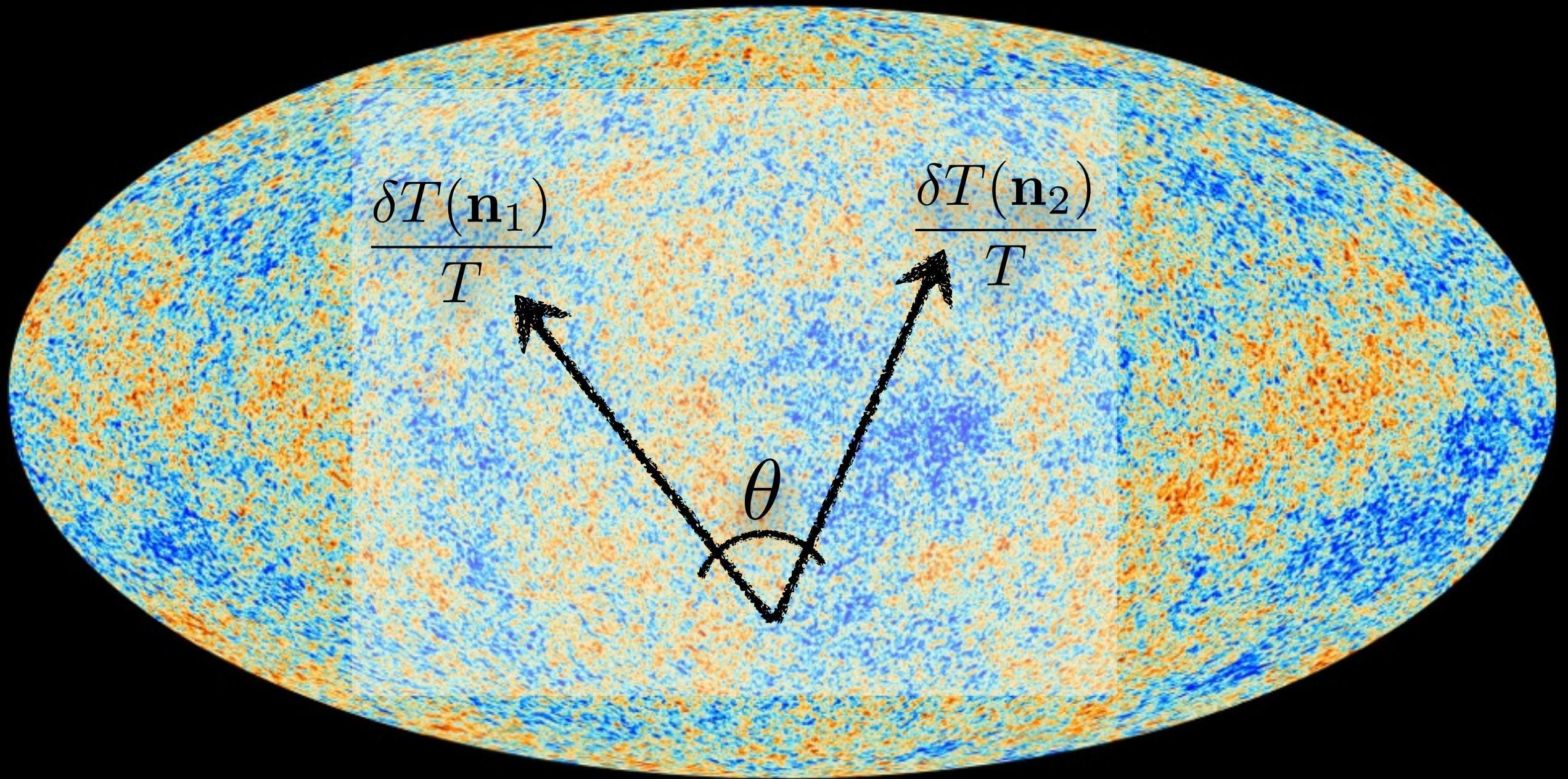


caveat: need dark matter to make this work

**a conundrum ... and a solution**



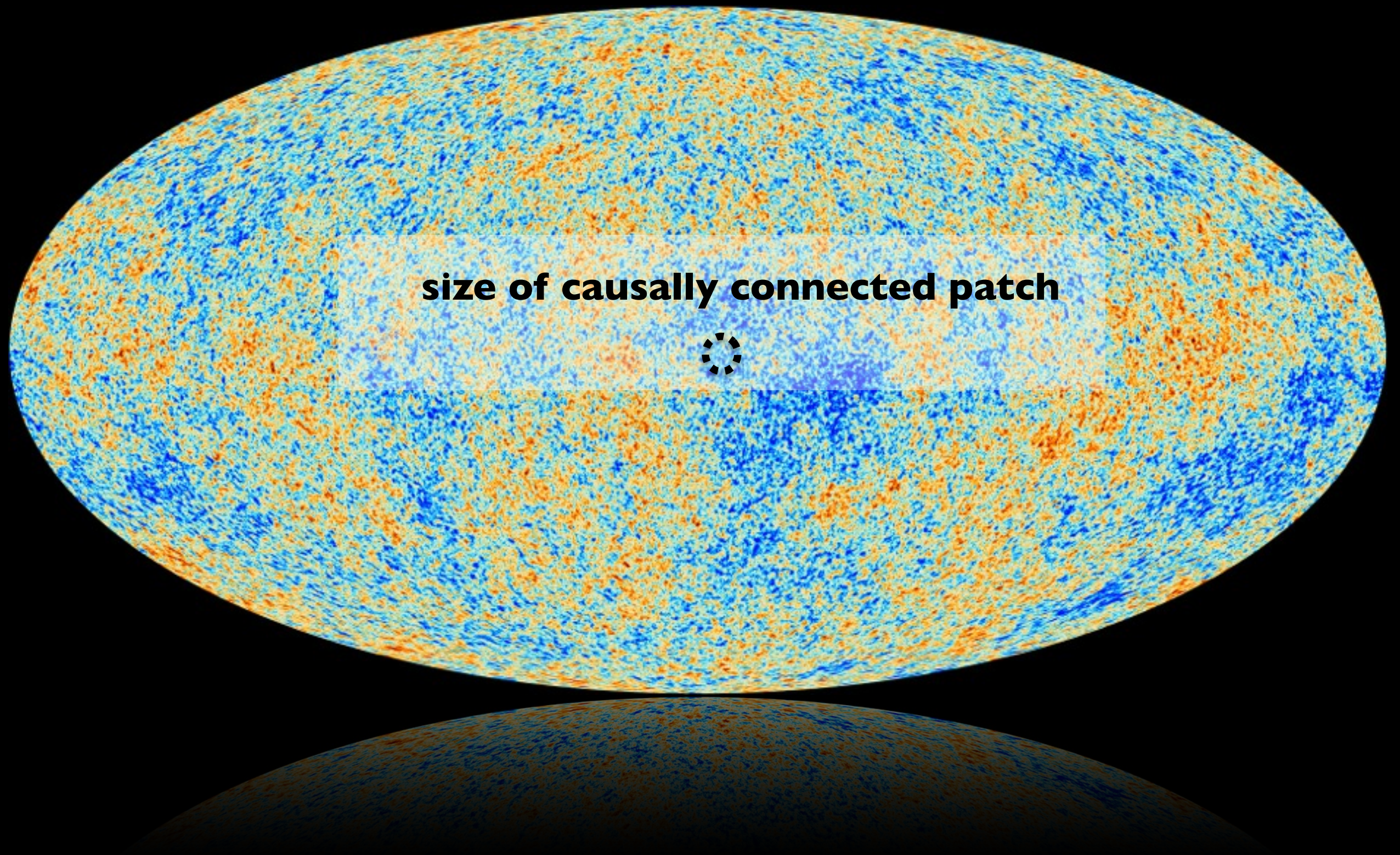
# large angle correlations?



$$C(\theta) = \left\langle \frac{\delta T(\mathbf{n}_1)}{T} \frac{\delta T(\mathbf{n}_2)}{T} \right\rangle$$

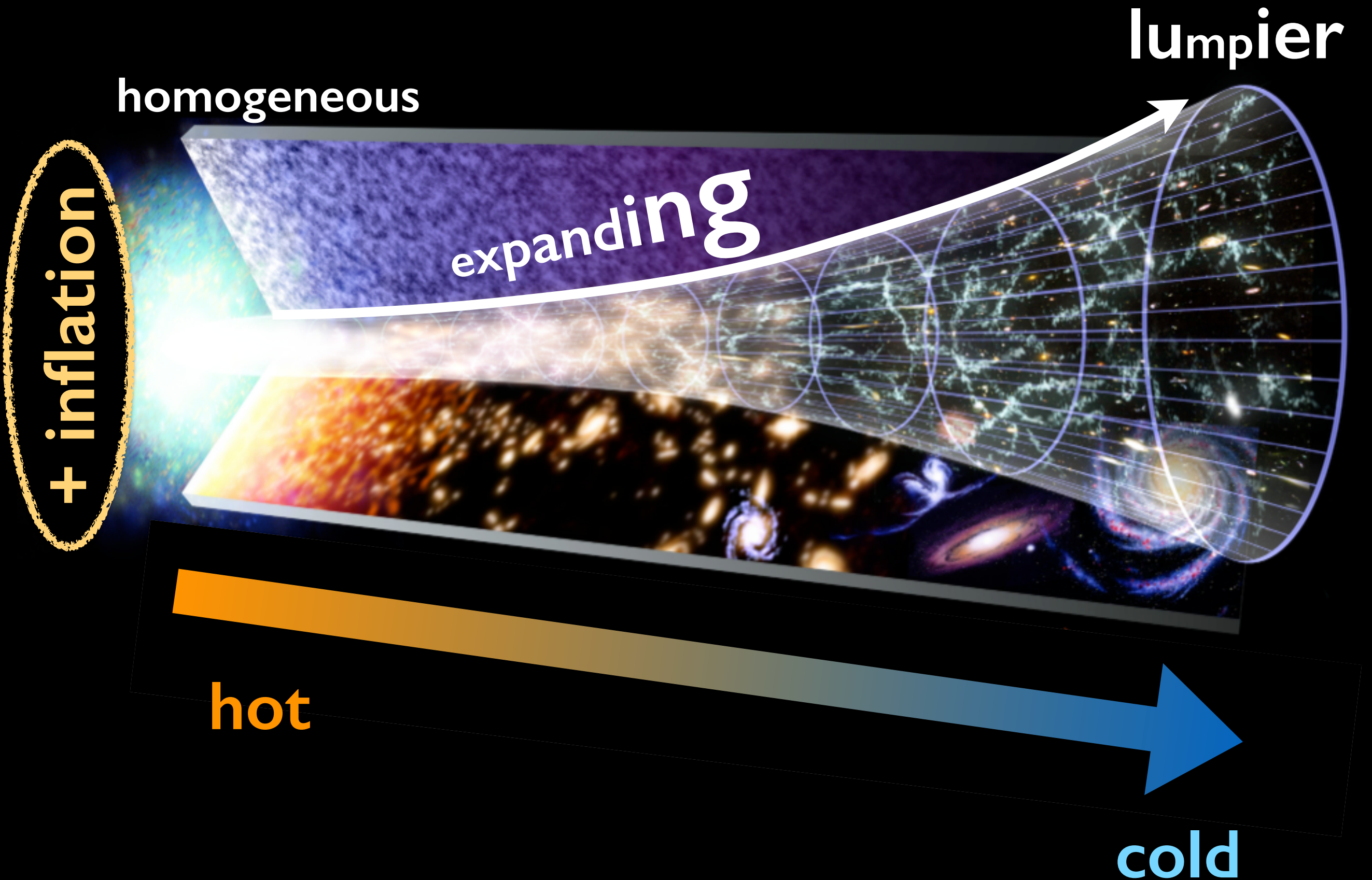


# standard big bang cosmology



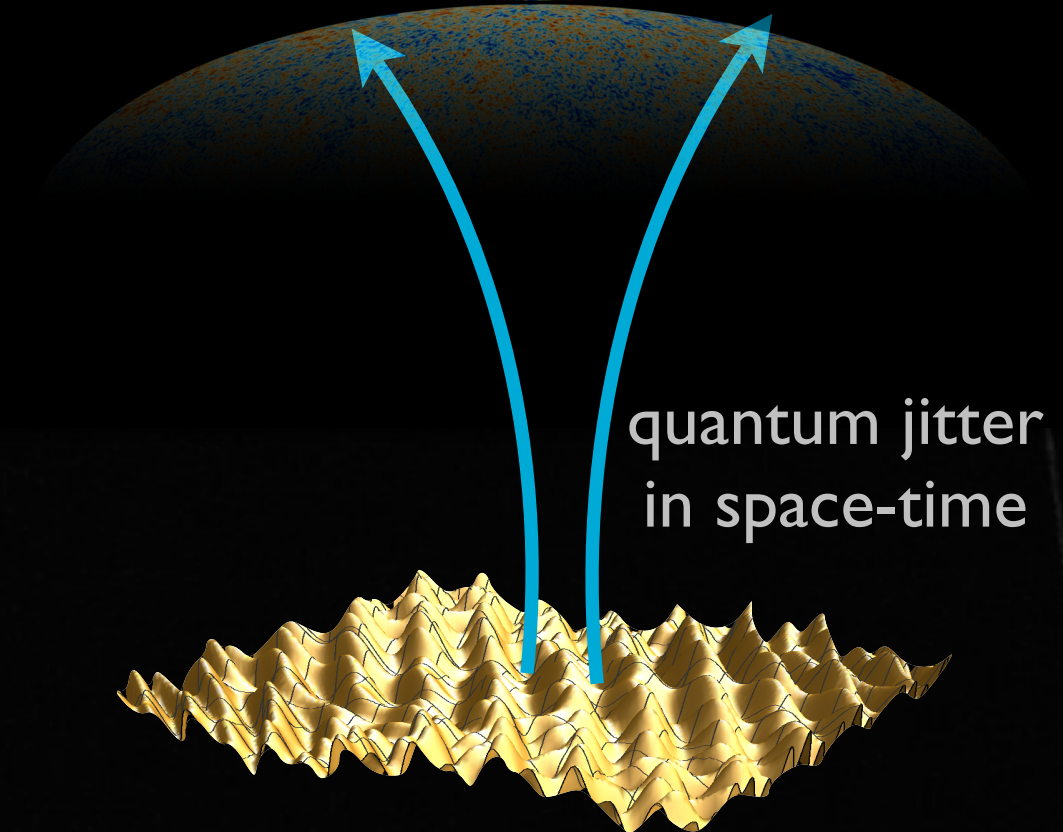
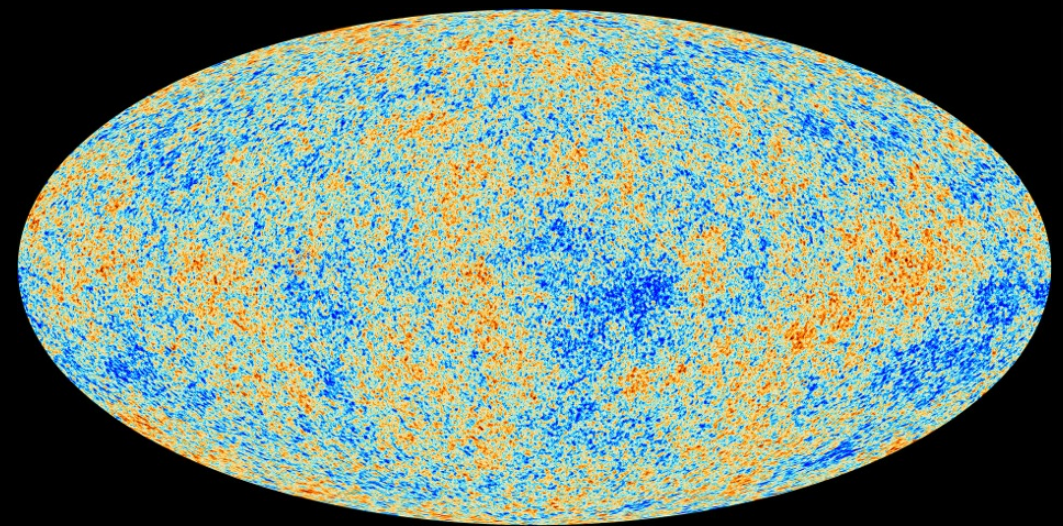
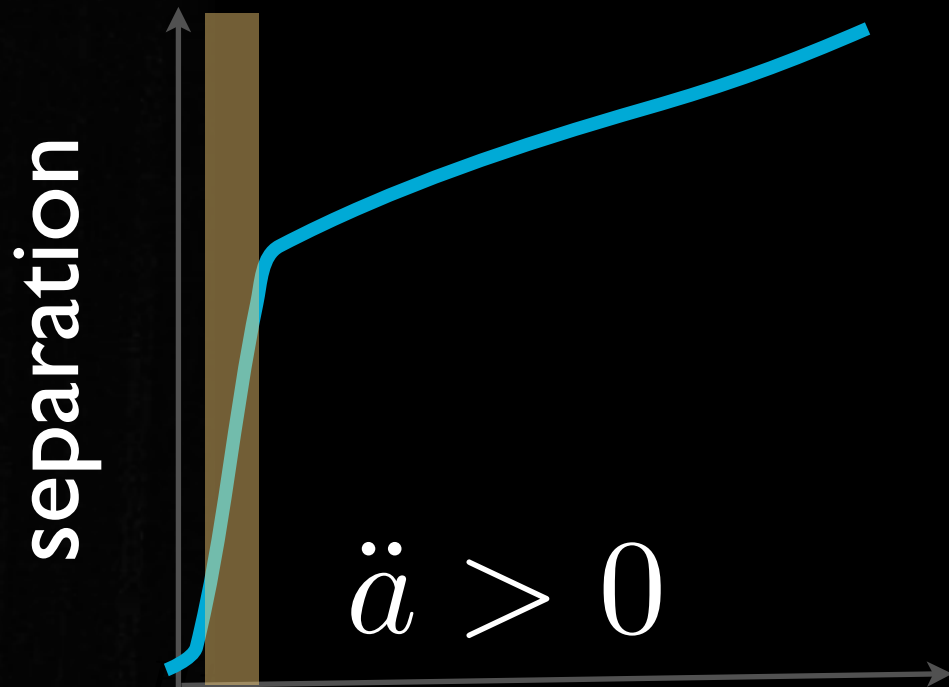


# Inflationary Cosmology



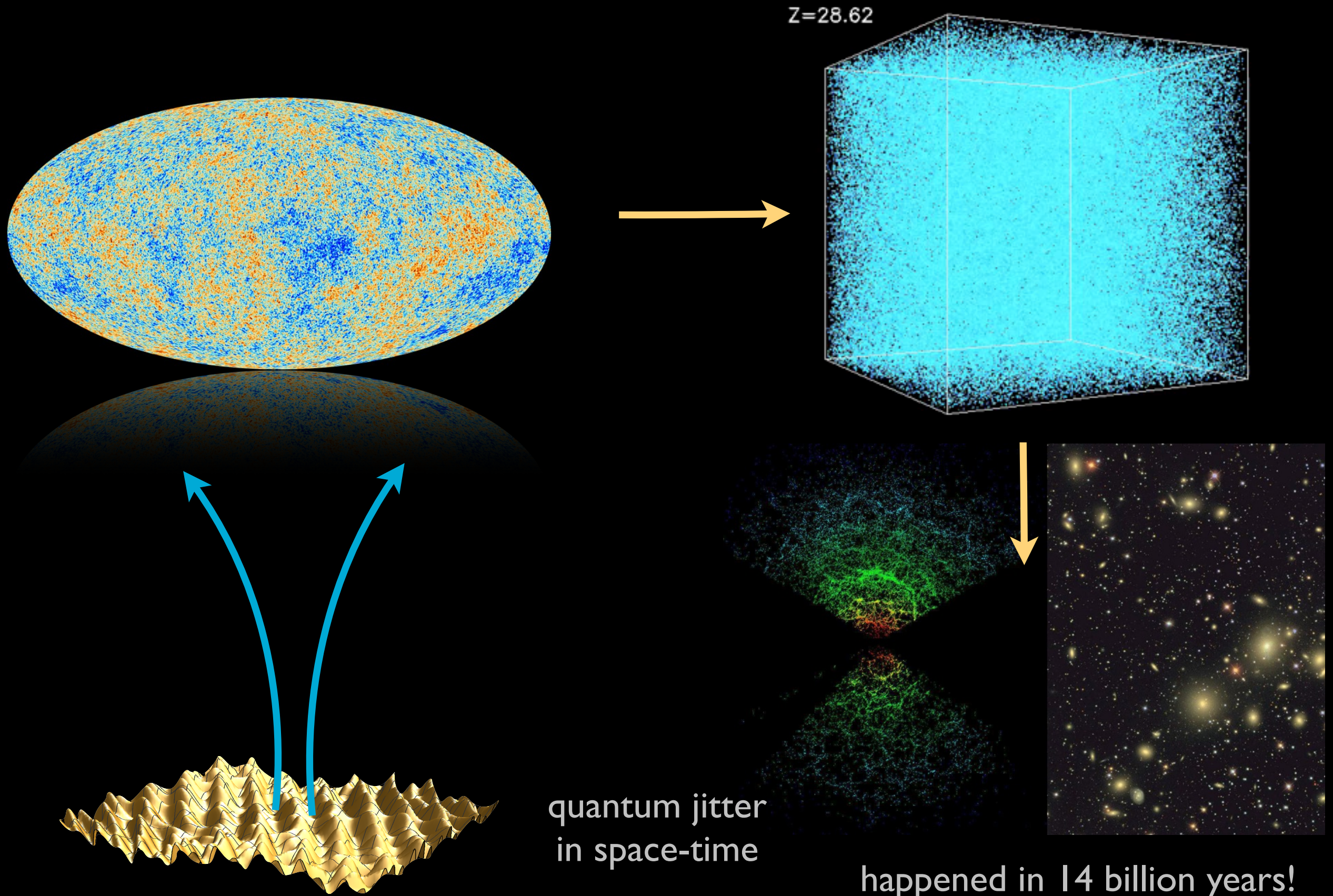


# inflation and long distance correlations

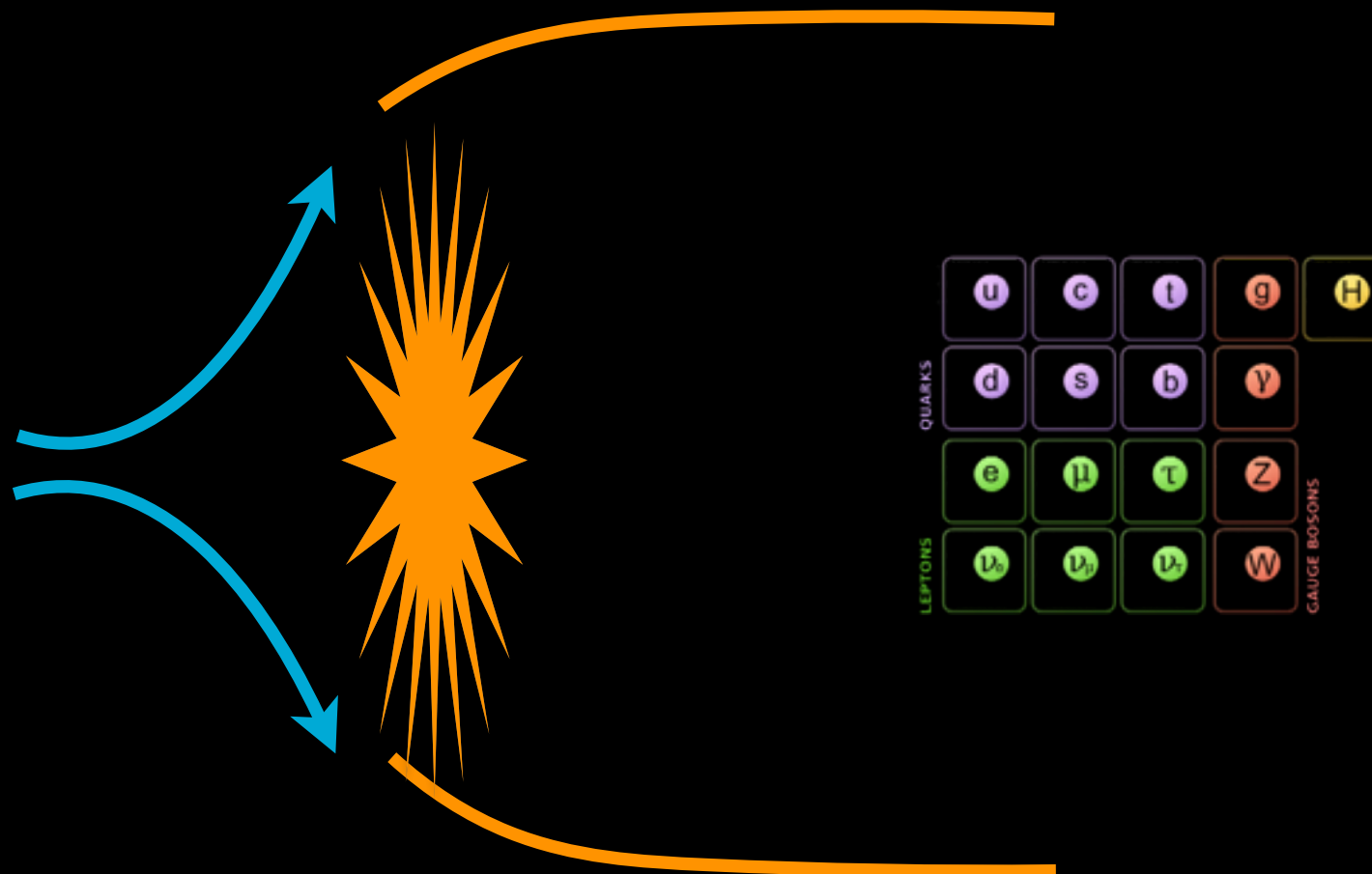




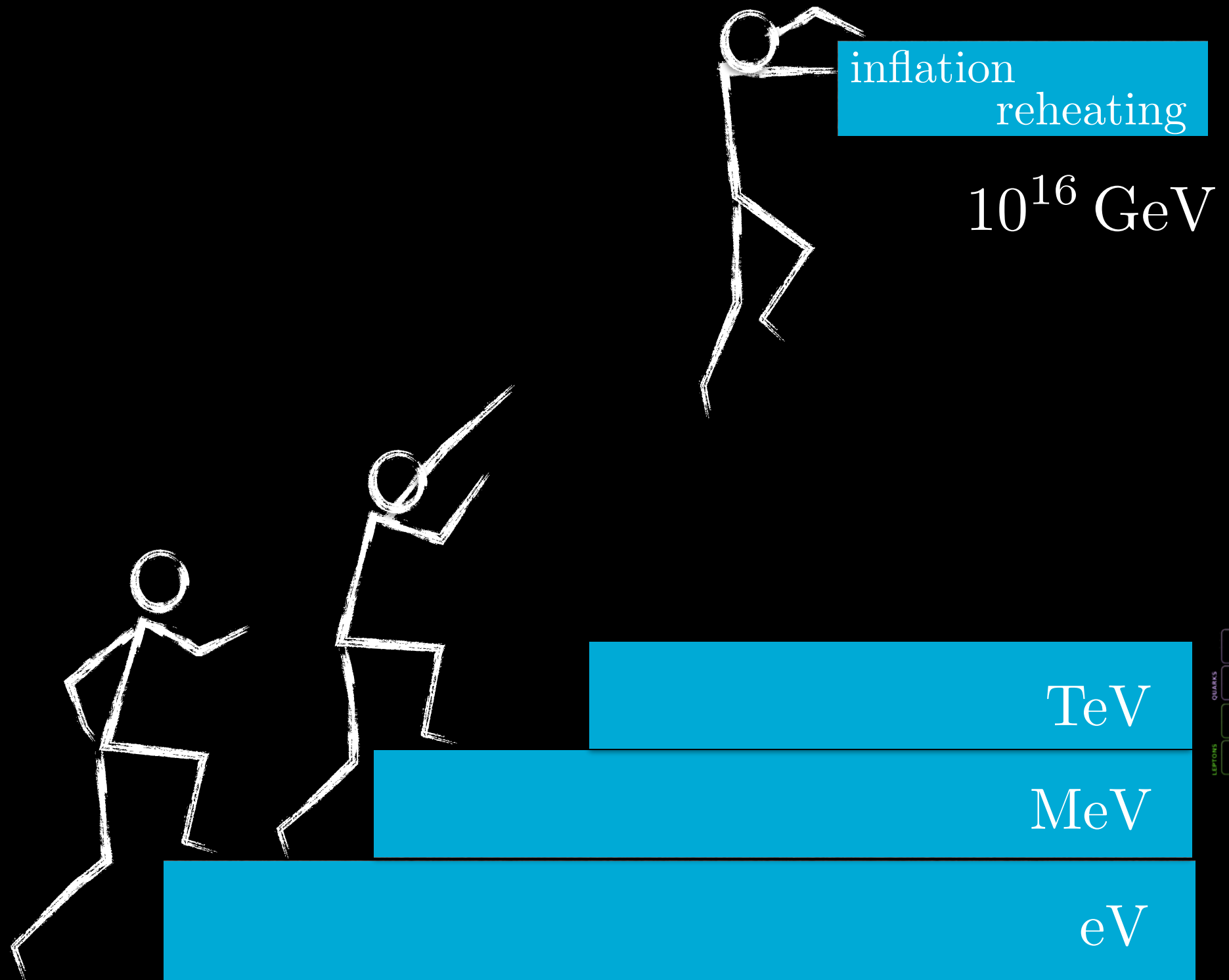
# gravity — correlations in galaxies



- what is the physics of inflation ?
- how did inflation end ? (reheating)
- Standard model? (or an UV complete theory)







QUARKS	u	c	t	g	H
	d	s	b	γ	
	e	μ	τ	Z	
LEPTONS	ν <sub>e</sub>	ν <sub>μ</sub>	ν <sub>τ</sub>	W	
GAUGE BOSONS					

# two approaches

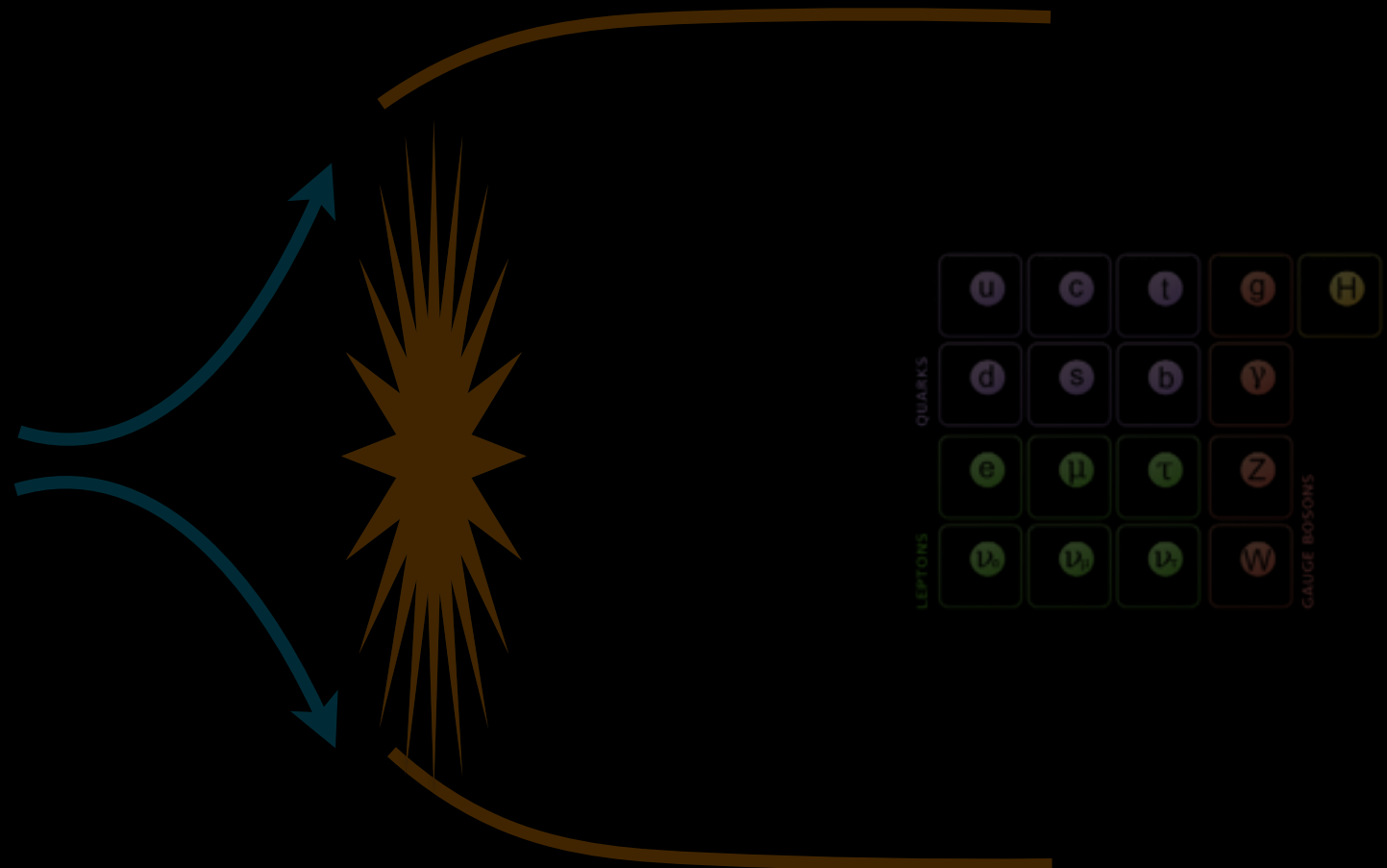
**SIMPLE enough**

**COMPLEX enough**

# I. Simple models & their general phenomenology

\* simplest things I can get away with

- what drives inflation ?
- how did inflation end ?
- Standard model?





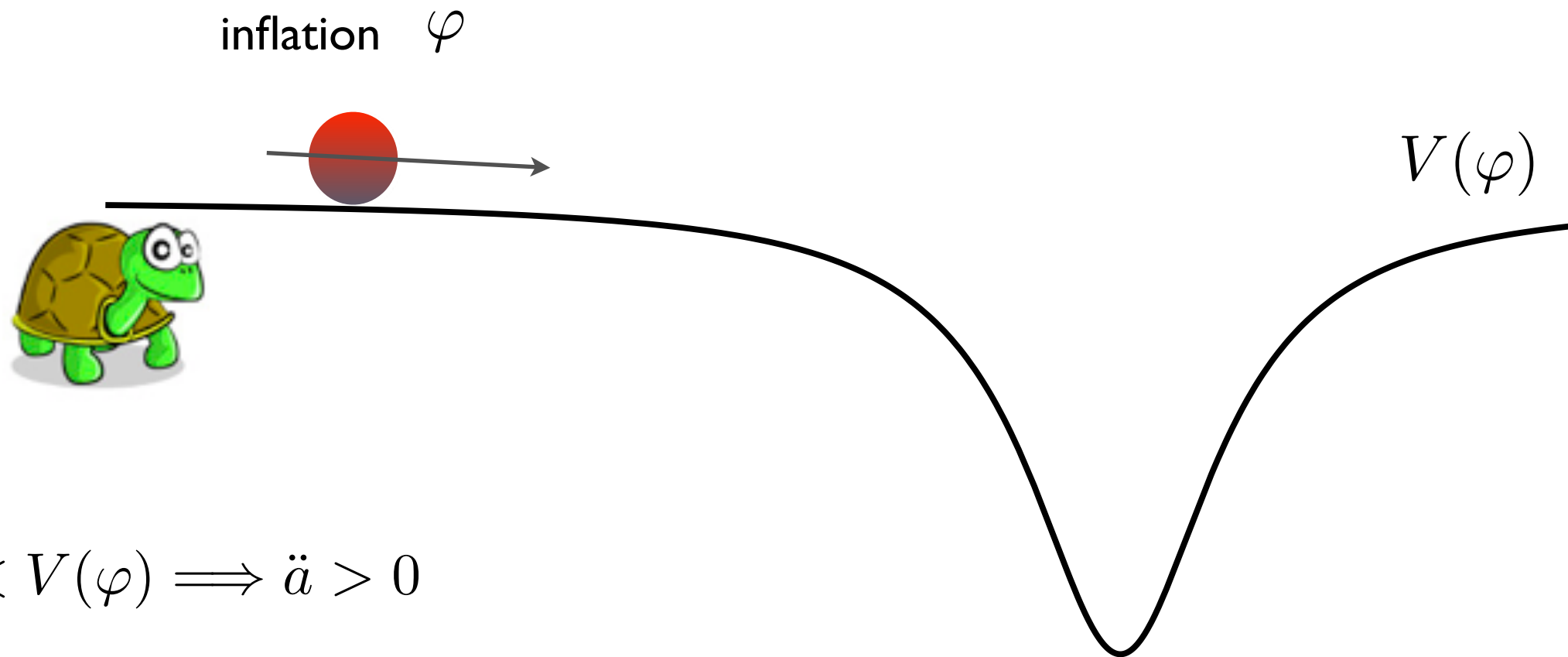
# a scalar field drives inflation

$$\varphi(t, \mathbf{x})$$

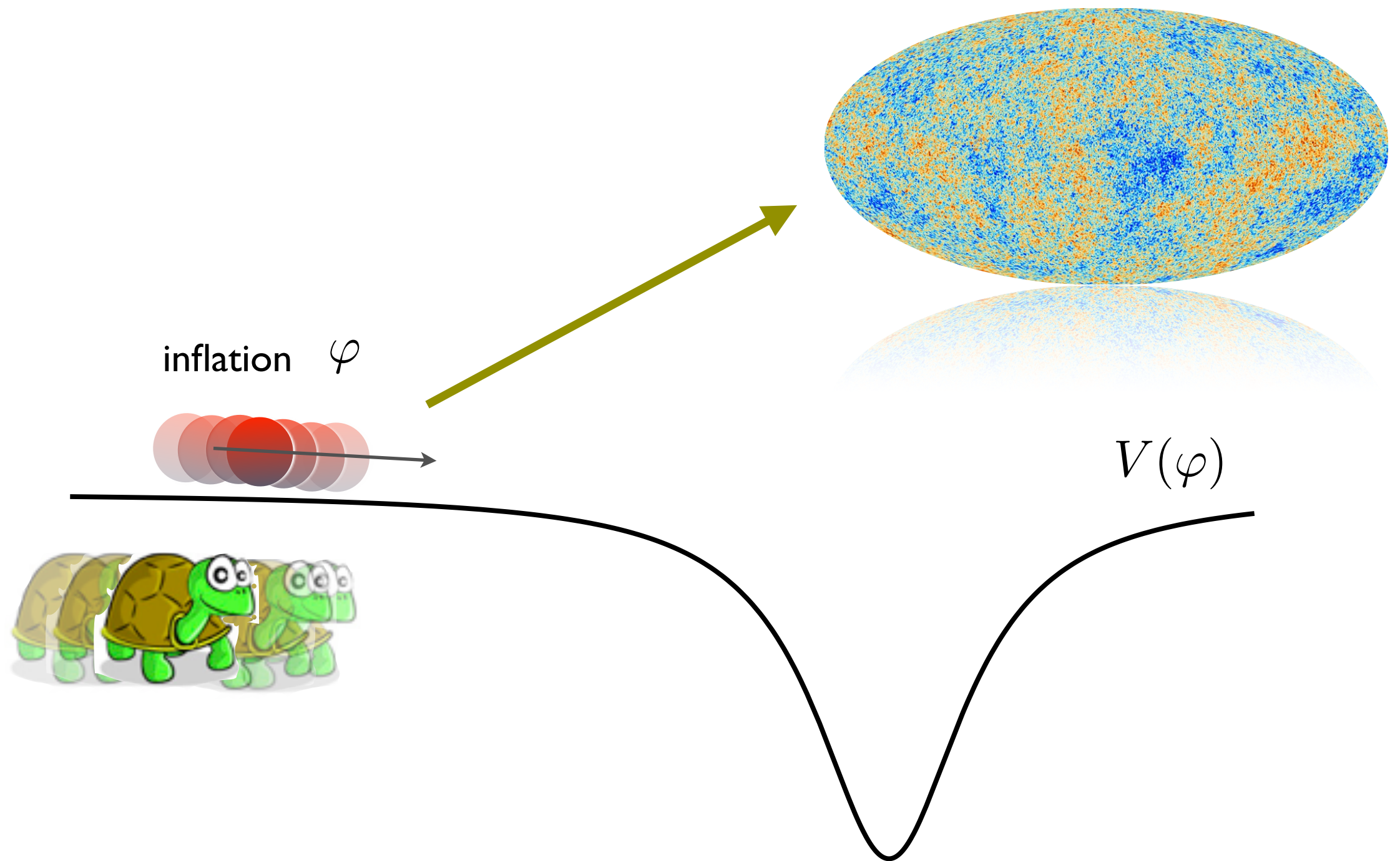
Lagrangian: 
$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

# scalar field driven inflation

$$\frac{\ddot{a}}{a} = -\frac{1}{3m_{\text{pl}}^2}(\dot{\varphi}^2 - V(\varphi))$$



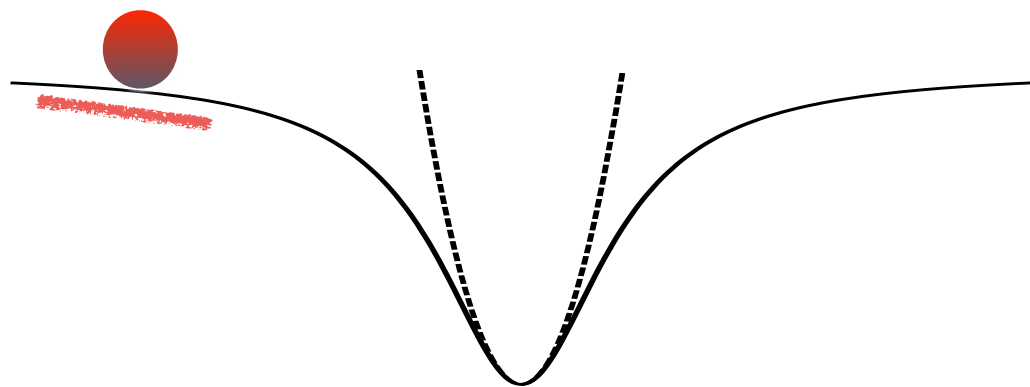
# inflationary quantum perturbations





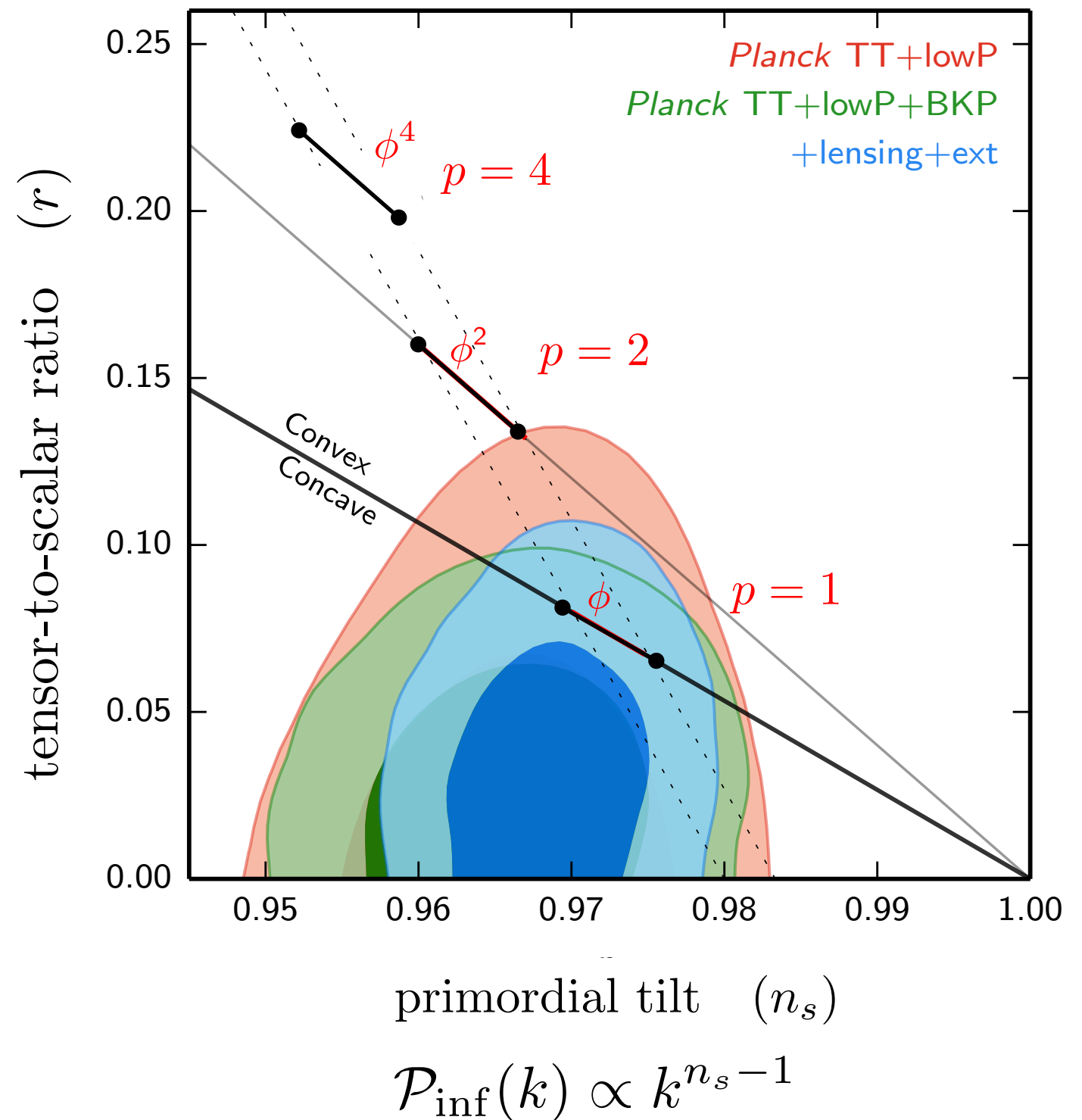
# constraints from observations

$$V(\phi) \propto \phi^p$$

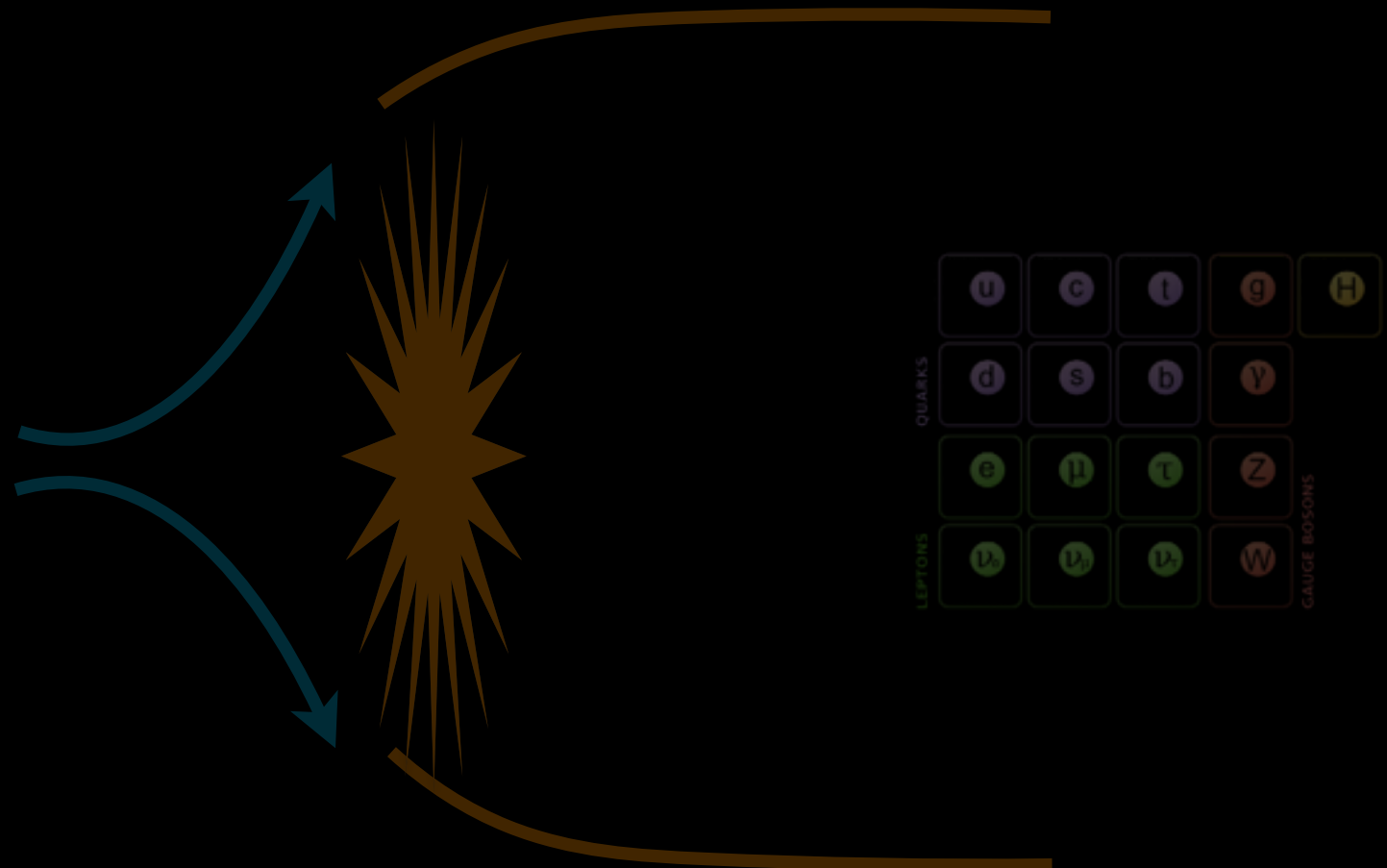


for example:

Silverstein & Westphal (2008)  
 McAllister et. al (2014)  
 Kallosh & Linde (2014)

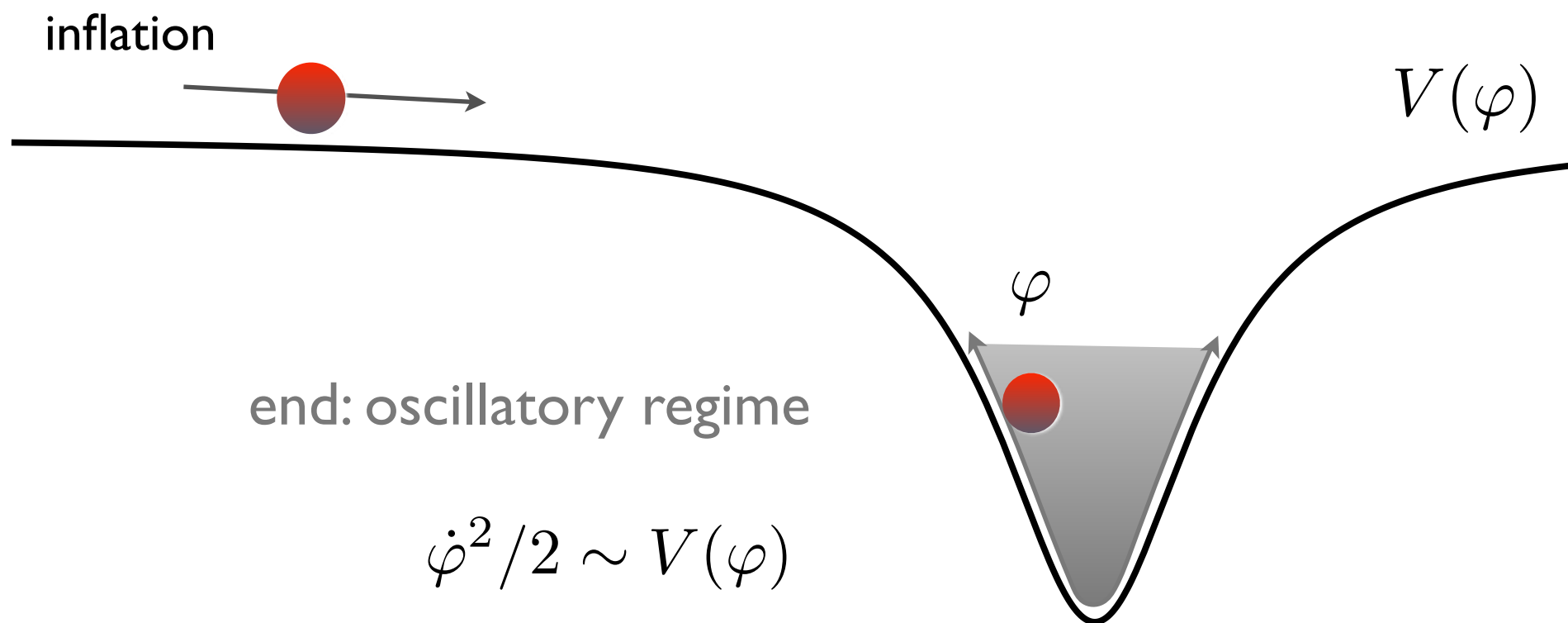


- what drives inflation ?
- how did inflation end ?
- Standard model?



# ending inflation

$$\frac{\ddot{a}}{a} = -\frac{1}{3m_{\text{pl}}^2}(\dot{\varphi}^2 - V(\varphi))$$

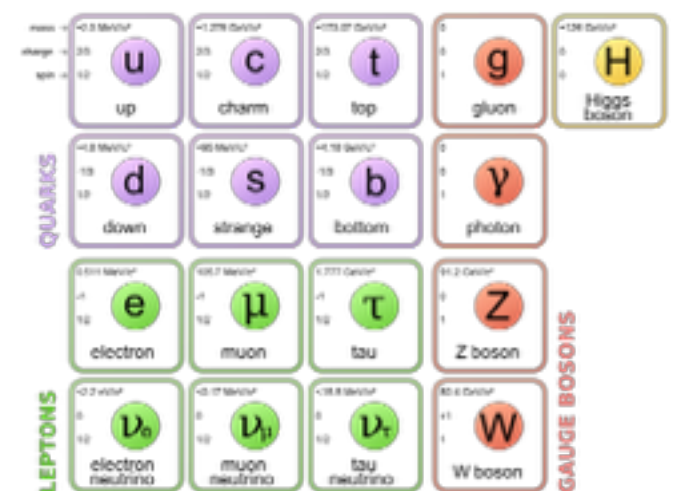




# energy transfer: “reheating”

- shape of the potential (self couplings)
- couplings to other fields

$\chi, \psi$



for example:

Kofman, Linde & Starobinsky (1994)

review: MA, Kaiser, Karouby & Hertzberg (2014)

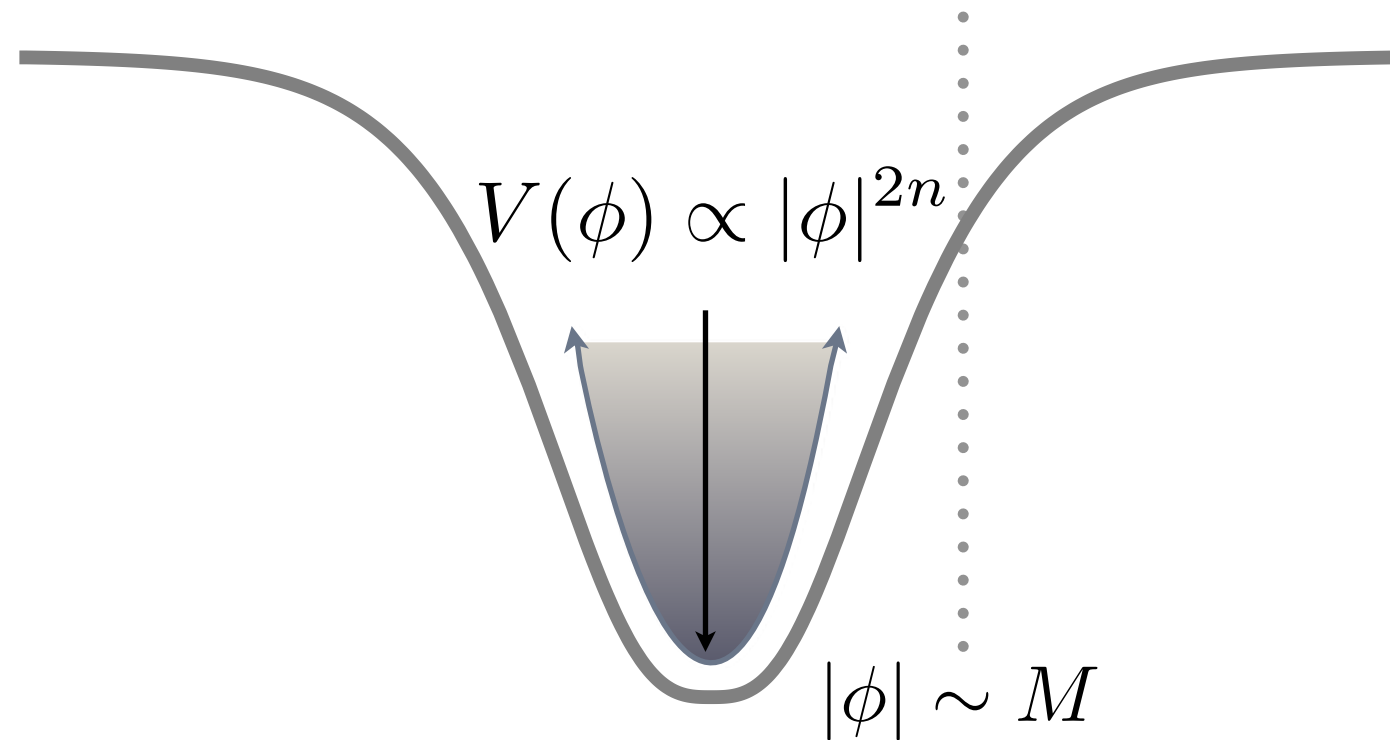
# end of inflation in “simple” models

for example:

Silverstein & Westphal (2008)

McAllister et. al (2014)

Kallosch & Linde (2014)



- shape of the potential (self couplings)

- ~~couplings to other fields~~



~~$\chi, \psi$~~



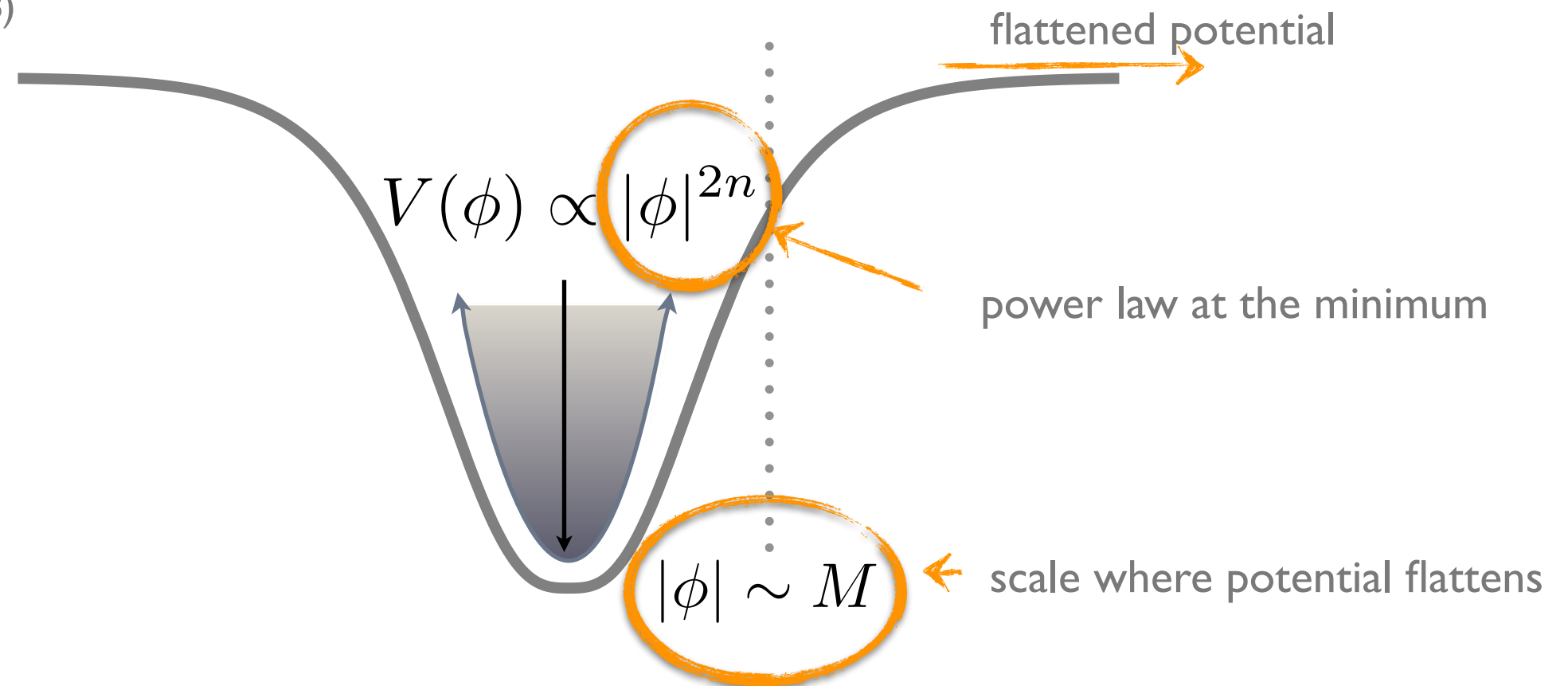
# end of inflation in “simple” models

for example:

Silverstein & Westphal (2008)

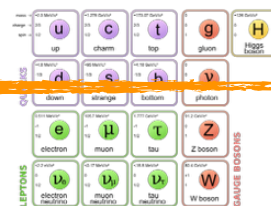
McAllister et. al (2014)

Kallosch & Linde (2014)



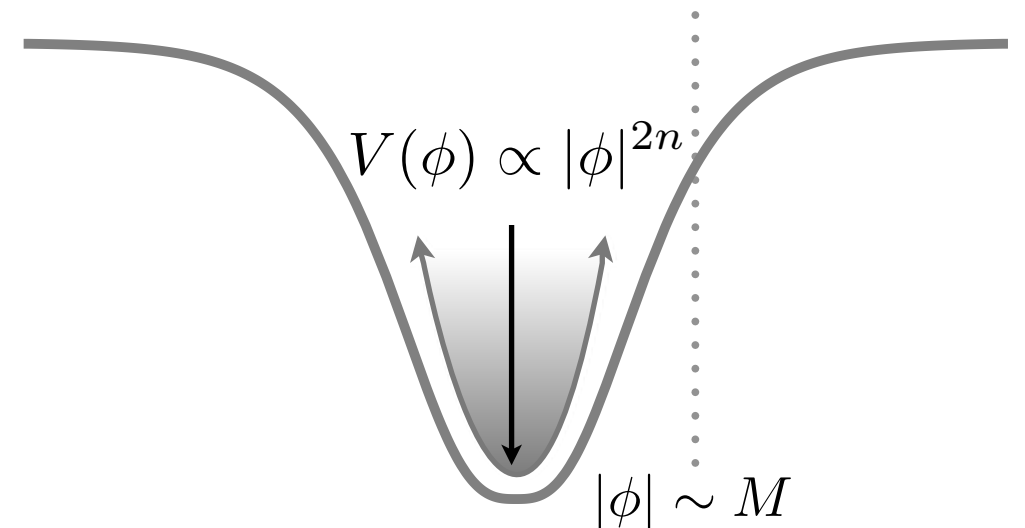
- shape of the potential (self couplings)

- ~~couplings to other fields~~



$\chi, \psi$

# end of inflation in “simple” models

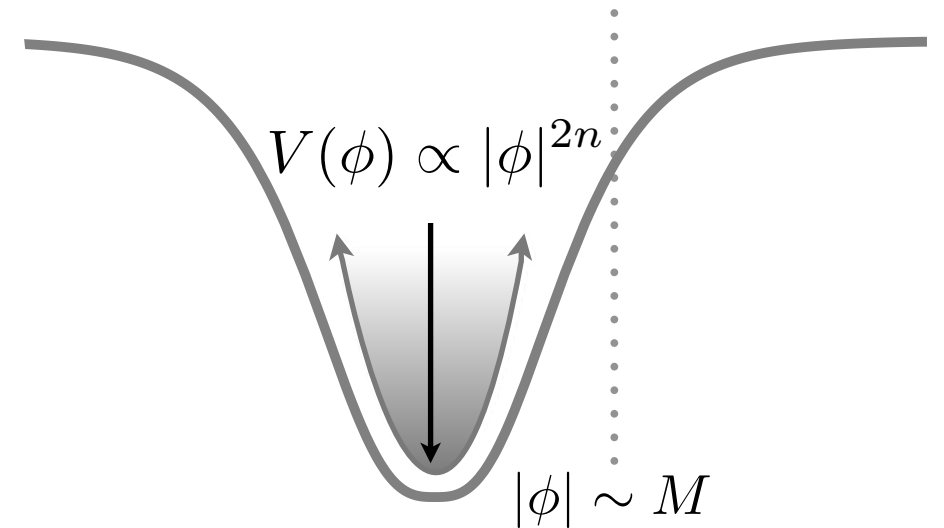
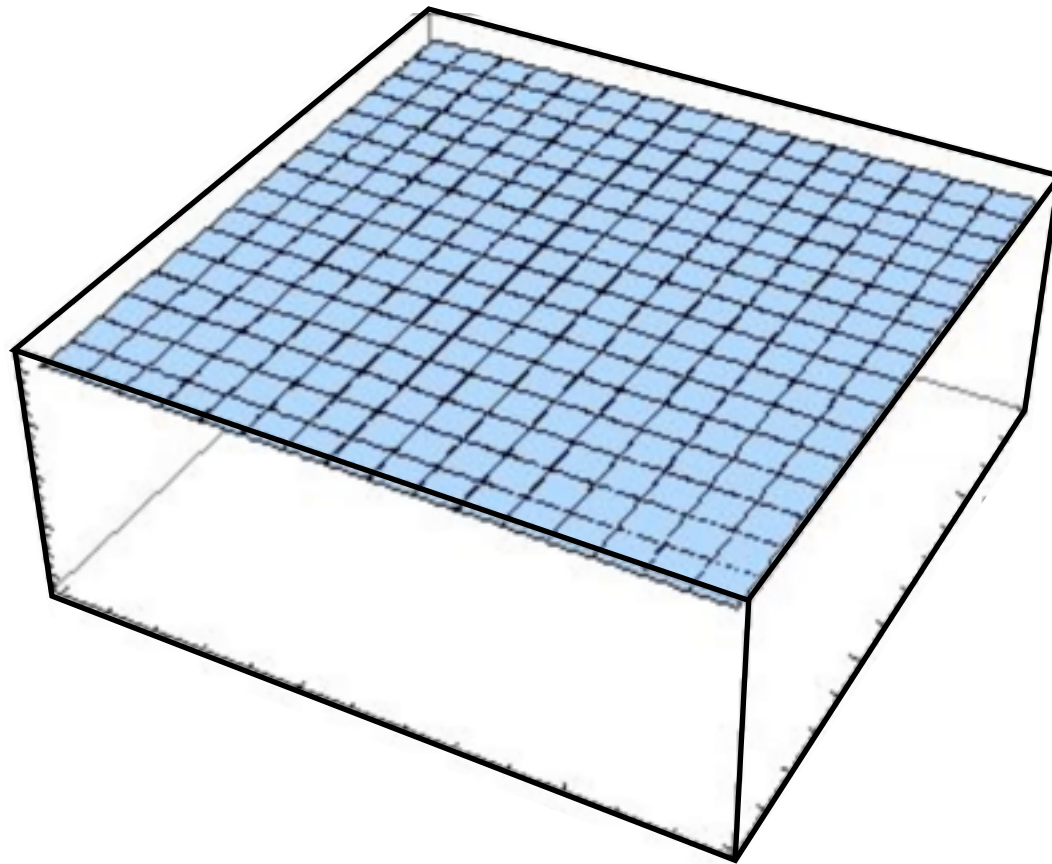


- (i) what are the dynamics ?
- (ii) eq. of state & how long to radiation domination ?
- (iii) obs. consequences ?

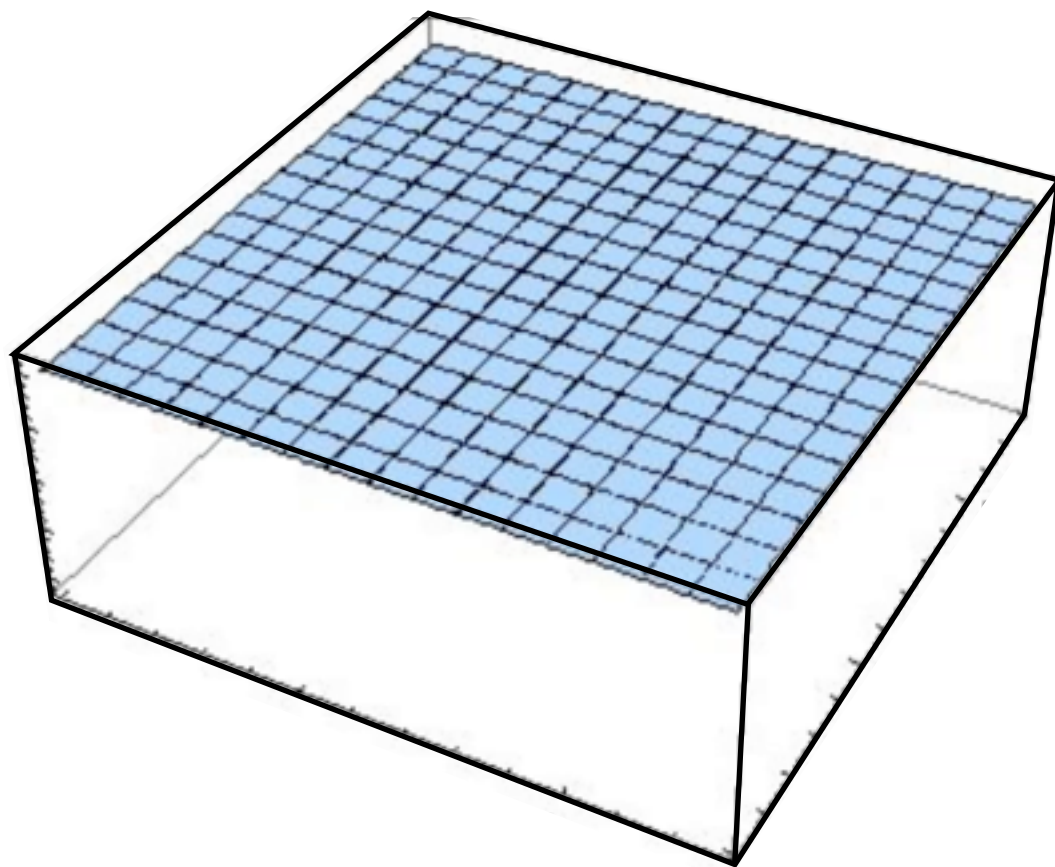




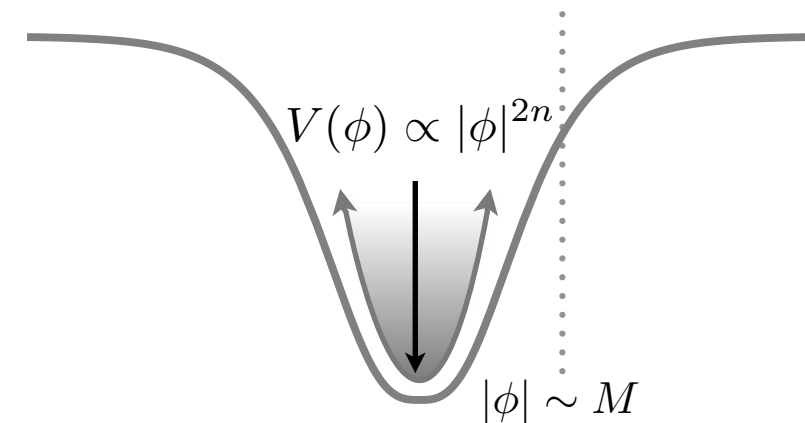
# homogeneous dynamics



# homogeneous eq. of state



eq. of state  $w = \frac{\text{pressure}}{\text{density}}$



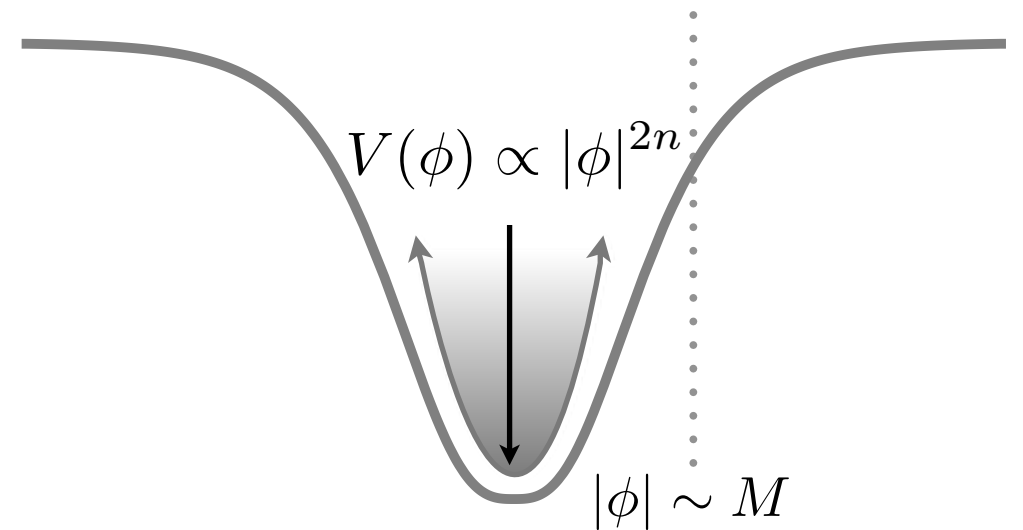
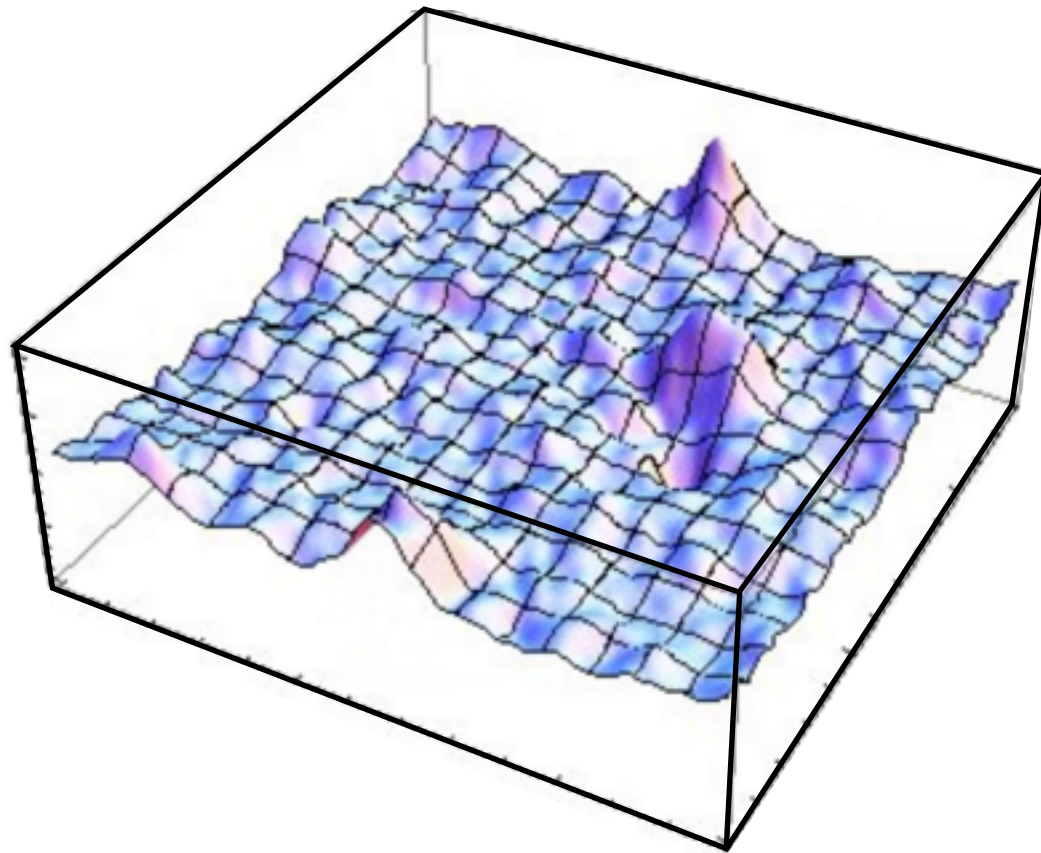
$$w \equiv \frac{\langle p \rangle_s}{\langle \rho \rangle_s} = \frac{\langle \dot{\phi}^2/2 - (\nabla \phi)^2/6a^2 - V \rangle_s}{\langle \dot{\phi}^2/2 + (\nabla \phi)^2/2a^2 + V \rangle_s} \approx \frac{n-1}{n+1}$$

Turner (1983)

\* can be obtained from a viral theorem



# fragmentation is (almost) inevitable



(i) existence of wings (self-couplings)  $M \lesssim m_{\text{pl}}$

and/ or

(ii) non-quadratic minimum  $n > 1$

\* but might take a while depending on parameters

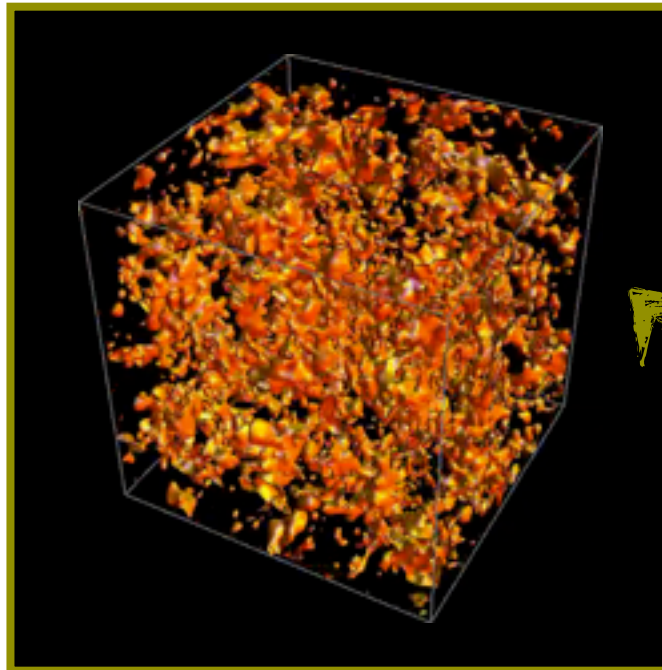
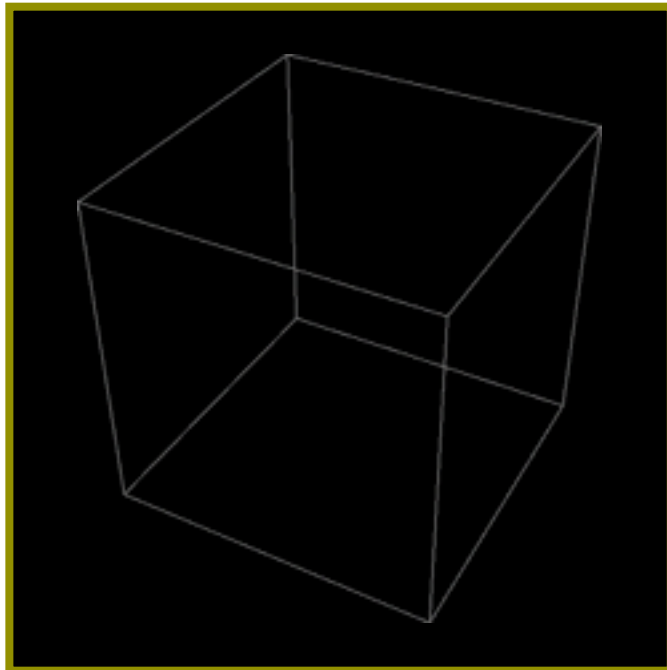
# result of fragmented dynamics

\* after sufficient time

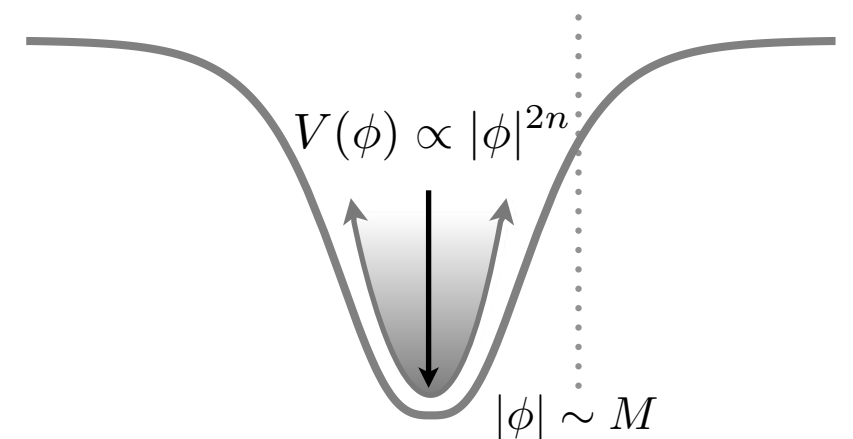
$n = 1$

$n > 1$

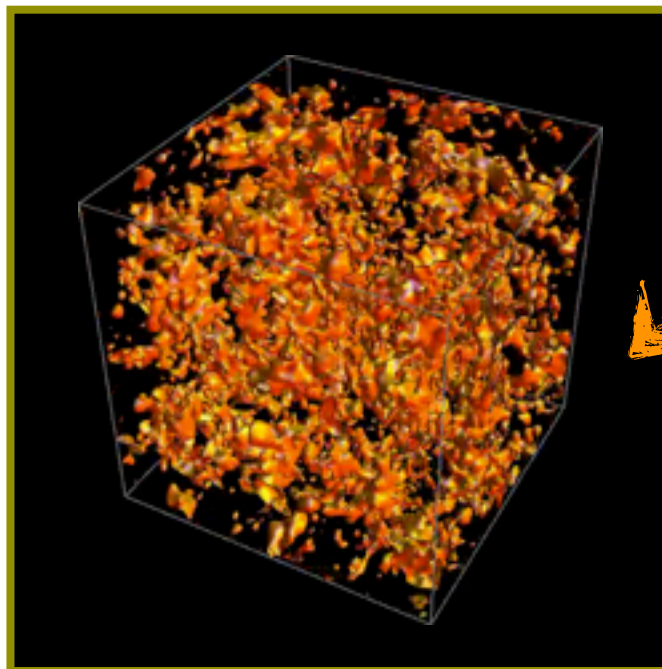
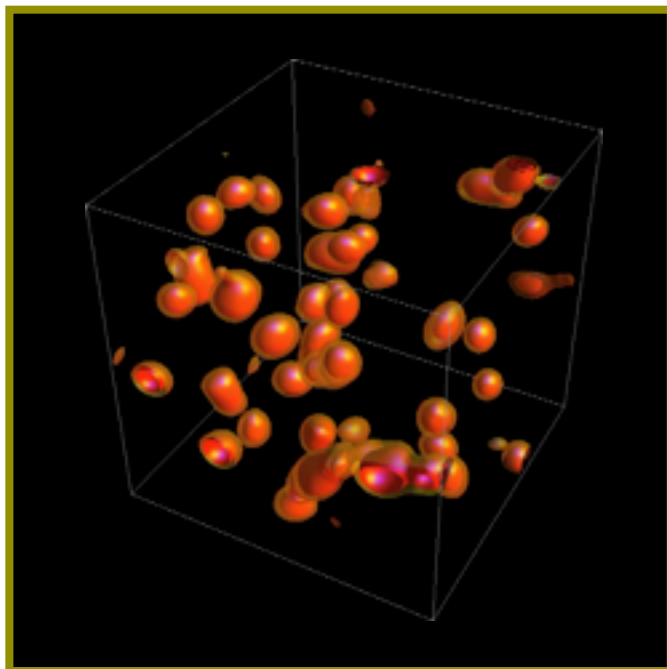
$M \sim m_{\text{pl}}$



slowly!



$M \ll m_{\text{pl}}$



quickly



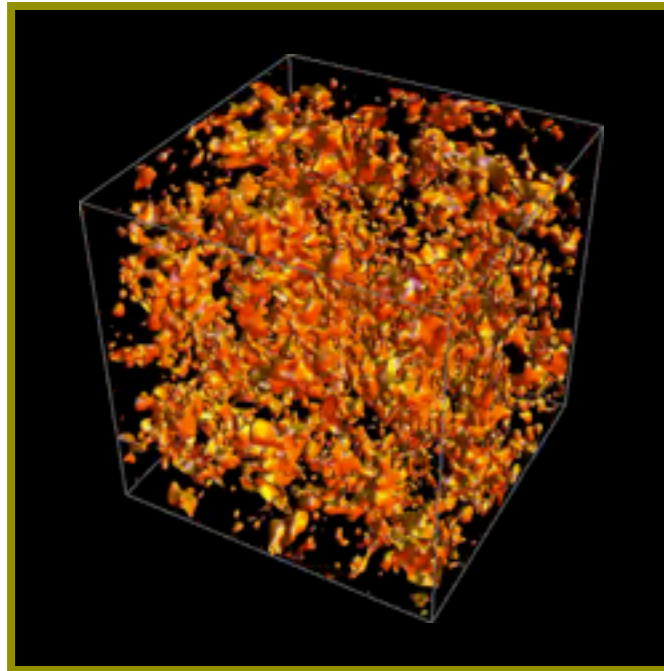
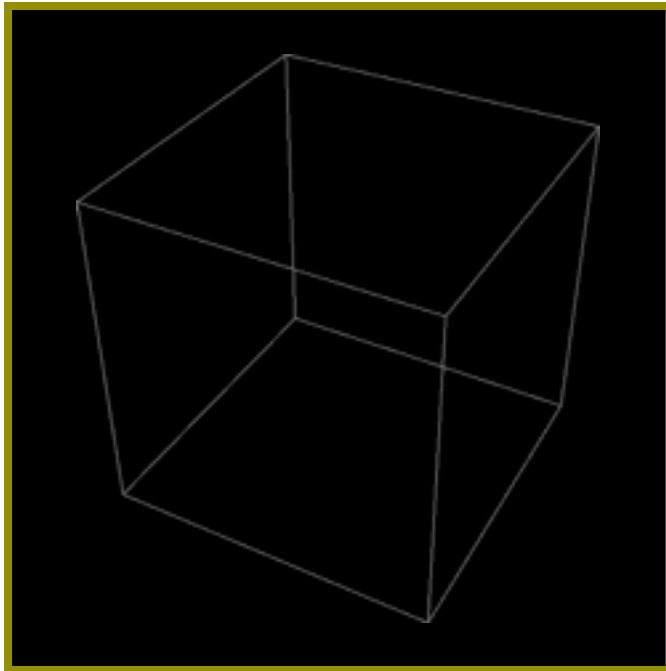
# eq. of state

\* after sufficient time

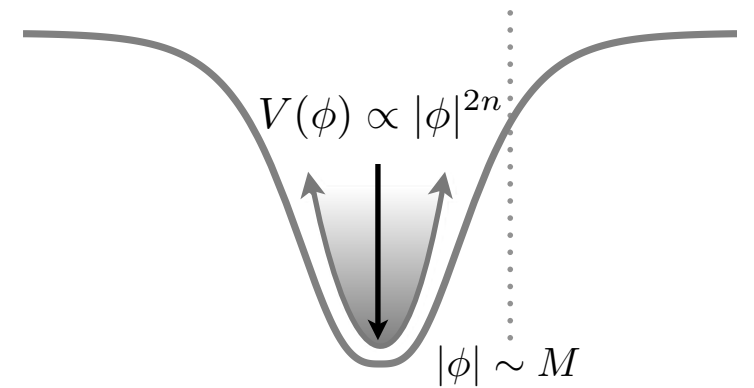
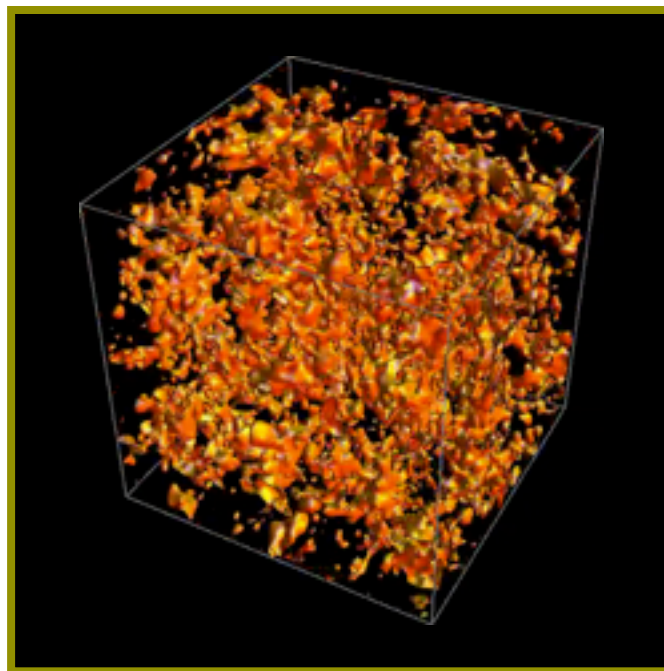
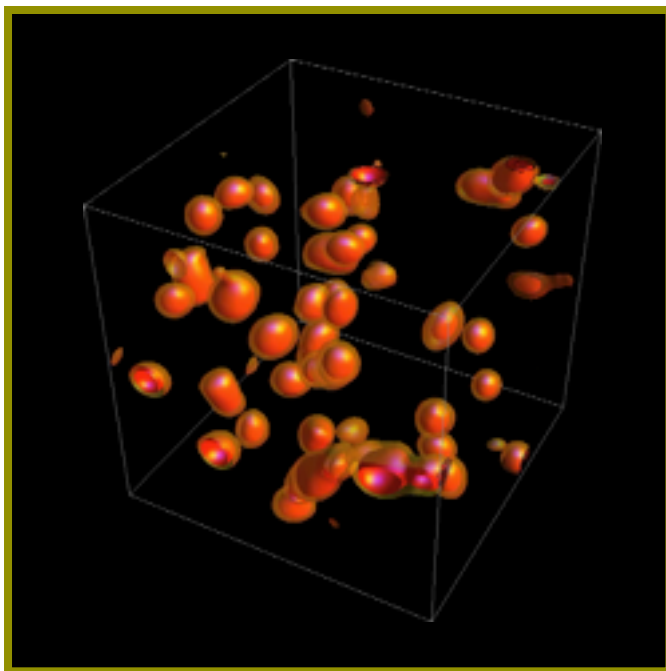
$n = 1$

$n > 1$

$M \sim m_{\text{pl}}$



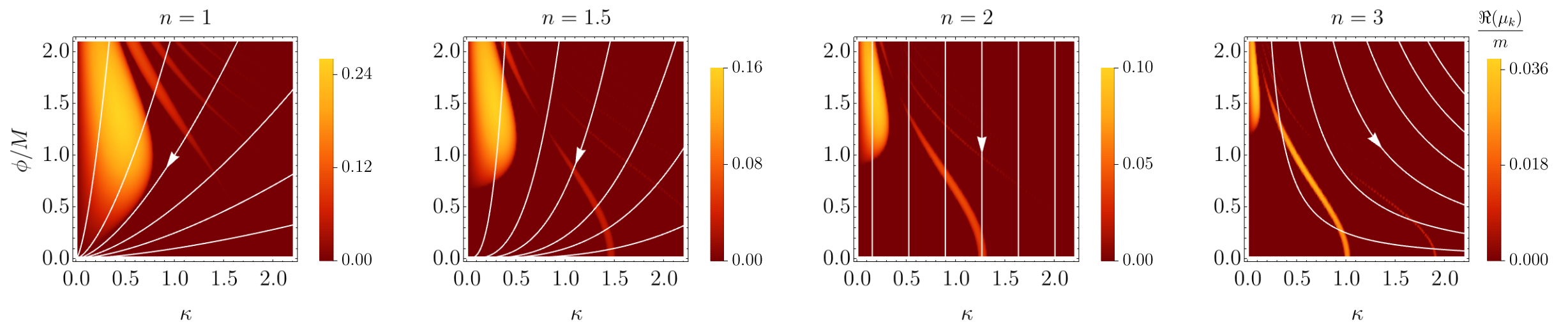
$M \ll m_{\text{pl}}$



$$w \rightarrow \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

$$w \neq \frac{n-1}{n+1}$$

# eq. of state — Floquet & expansion



rapid fragmentation due to broad band

importance of the narrow band

getting stuck in the instability band

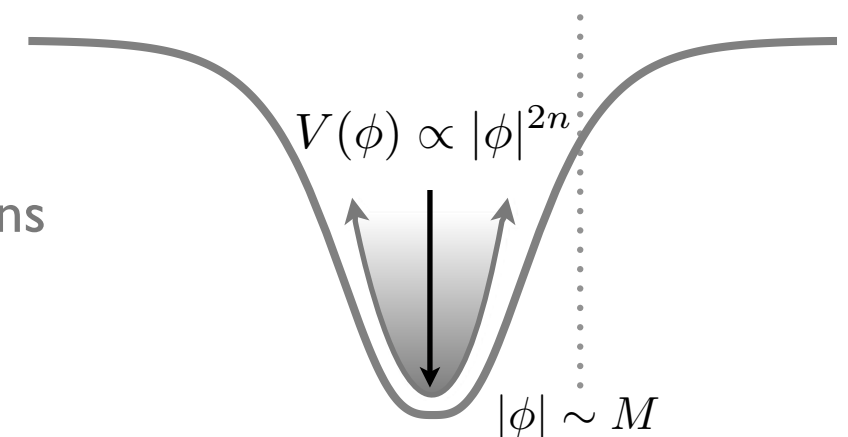
how gradients redshift compared to the potential energy

$$[|\Re(\mu_k)|/H]_{\max}^0 = f(n)(m_{\text{Pl}}/M) \quad M \ll m_{\text{Pl}}$$

$$[\Re(\mu_k)/H]^1 \propto m_{\text{Pl}}/|\bar{\phi}| \quad |\bar{\phi}| \ll M$$

$$|\dot{\kappa}| \sim H\kappa$$

$$w \rightarrow \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases} \quad * \text{ formation of solitons}$$





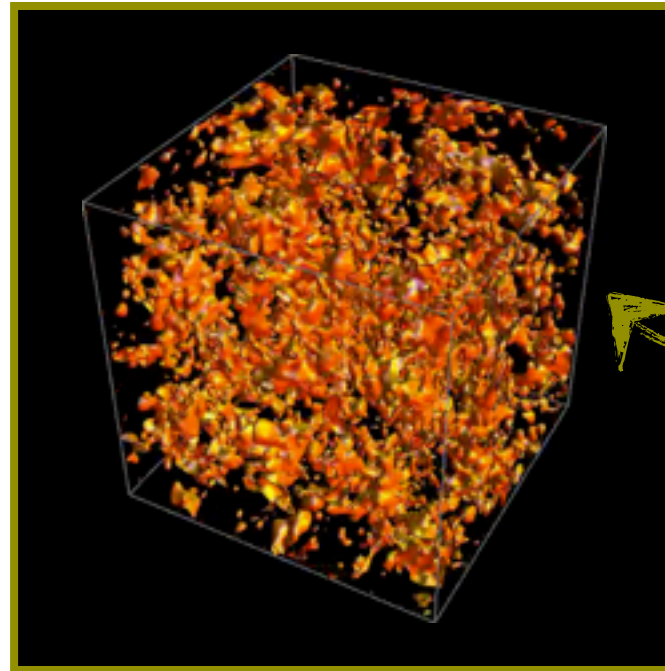
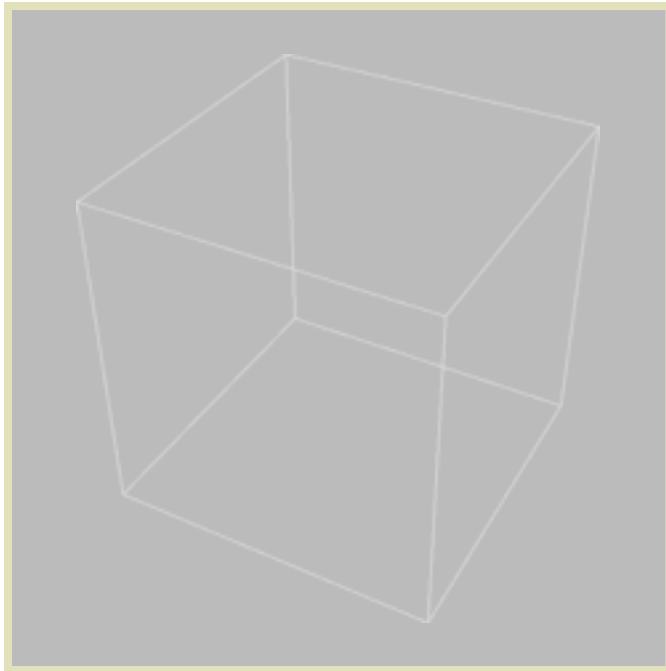
# eq. of state

\* after sufficient time

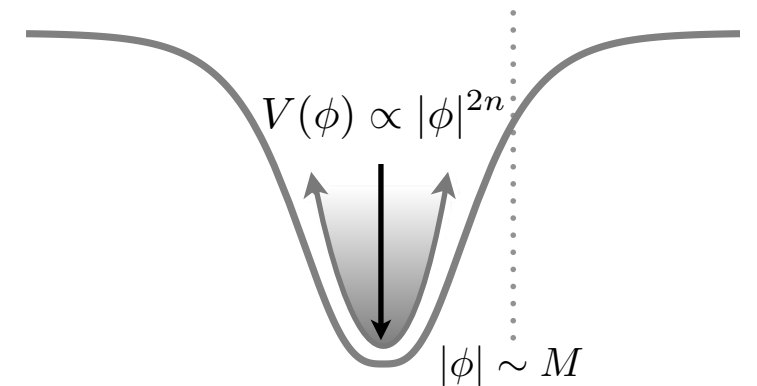
$n = 1$

$n > 1$

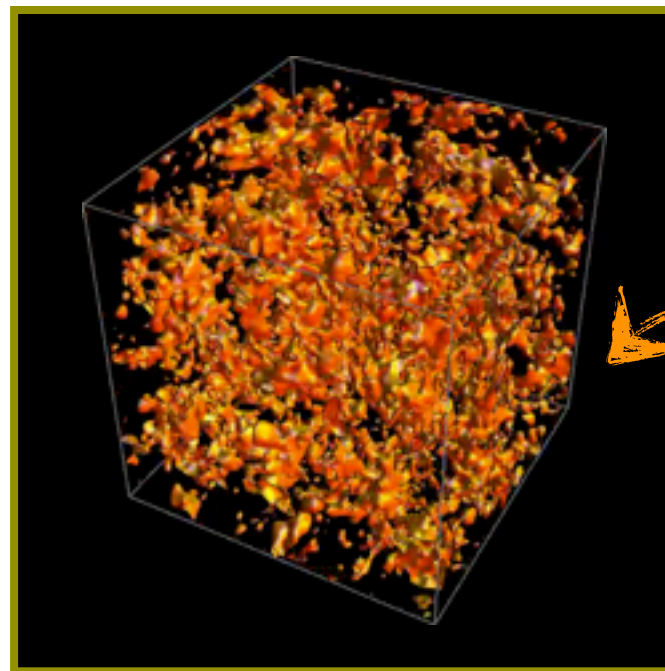
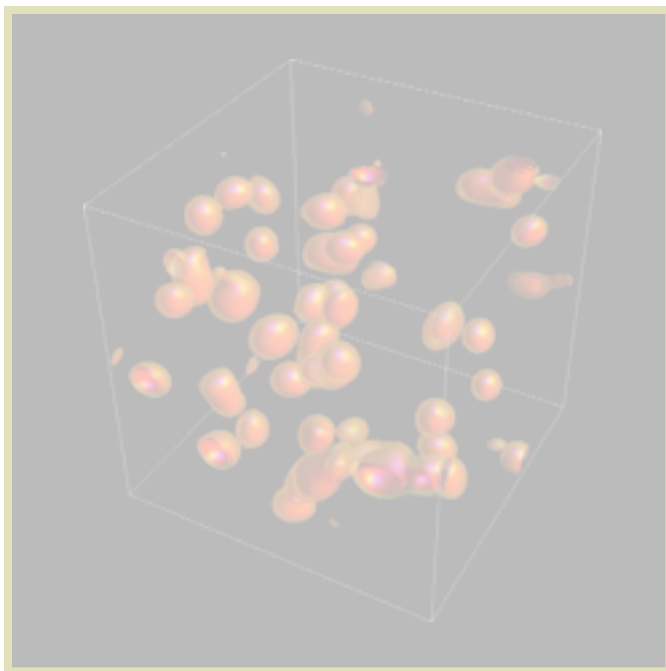
$M \sim m_{\text{pl}}$



slowly!



$M \ll m_{\text{pl}}$



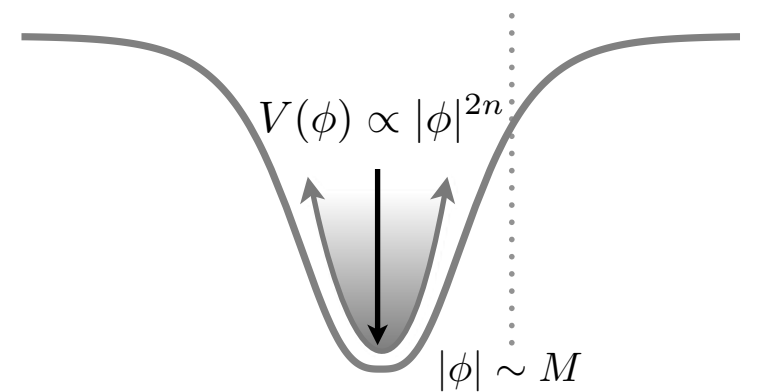
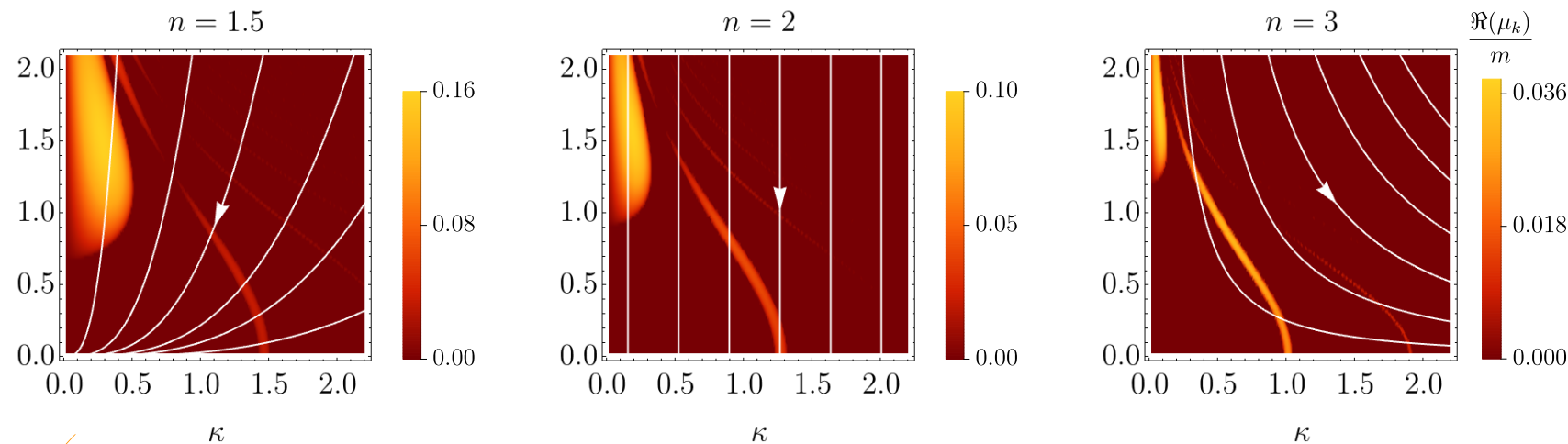
$$w \rightarrow \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}$$

quickly

$$w \neq \frac{n-1}{n+1}$$

# duration to radiation domination

## \* non-quadratic minima



e-folds to radiation domination

$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

$$\Delta N_{\text{rad}} \equiv \int_{a_{\text{end}}}^{a_{\text{rad}}} d \ln a$$

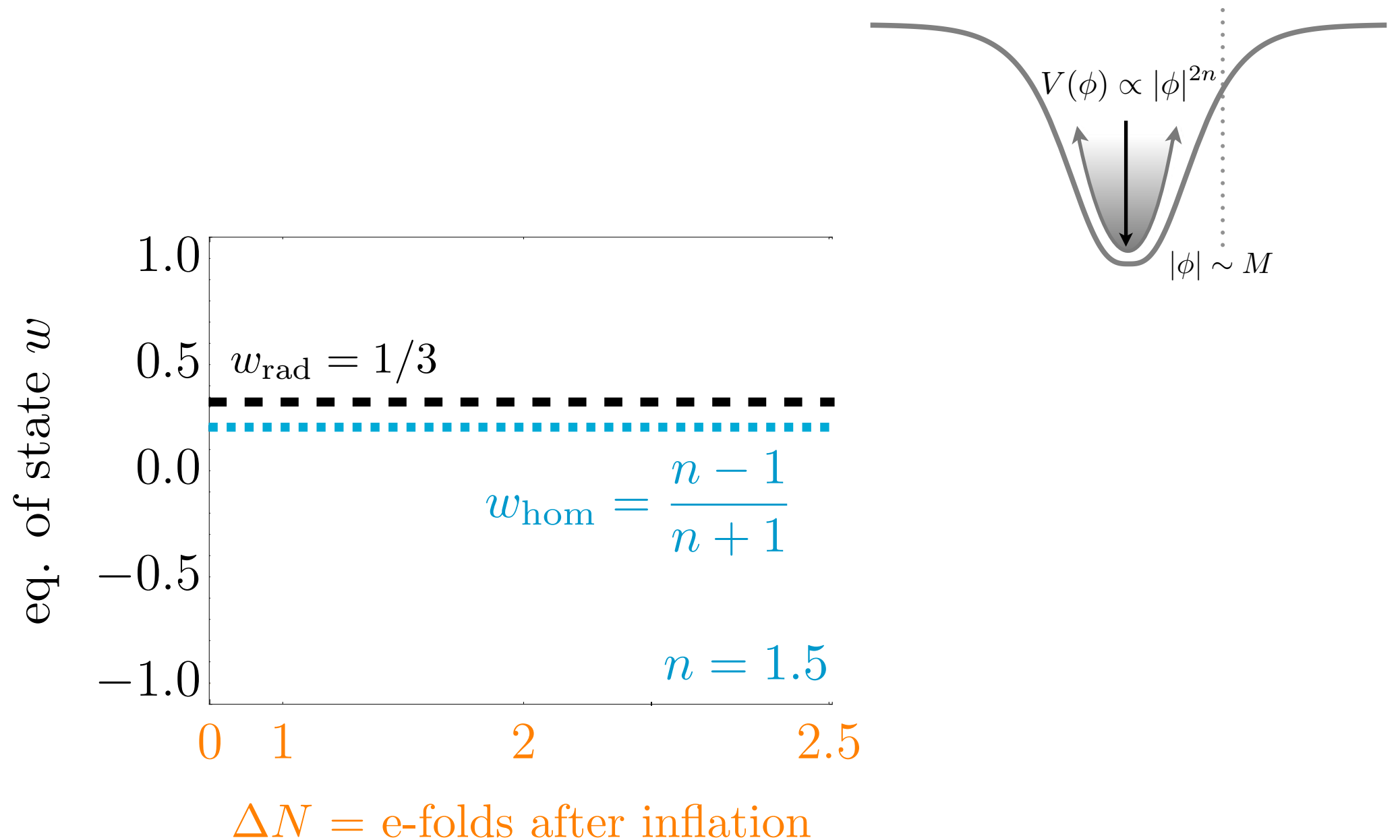
# summary of simulations

№	$n$	$\alpha$	$w$	$N$	$L_{\text{phys}}H_{\text{init}}$	$\Delta N_{\text{efolds}}^{\text{reh}}$	$\Delta N_{\text{efolds}}^{\text{br}}$	$\Delta N_{\text{efolds}}^{\text{final}}$	$\delta_{\text{energy}}^{\text{conserv}}$	$\frac{\Delta_{\text{Grad,kmax}}^2}{\Delta_{\text{Grad,peak}}^2}$	CPUhrs	Clockhrs
1	1	$10^{-5}$	0.03	512	0.18	—	$< 1$	2.2	$10^{-4}$	$10^{-4}$	320	2.5
2	1	$10^{-3}$	0	128	2.5	—	$< 1$	4.4	$10^{-4}$	$< 10^{-20}$	16	0.5
3	1.5	$10^{-5}$	0.316	512	0.12	1.2	$< 1$	2.5	$10^{-3}$	$10^{-3}$	1536	12
4	1.5	$10^{-5}$	0.316	512	0.06	1.2	$< 1$	2.5	$10^{-3}$	$10^{-5}$	1536	12
5	1.5	$10^{-5}$	0.327	512	0.06	1.2	$< 1$	3.3	$10^{-3}$	$10^{-3}$	+3072	+24
6	1.5	$10^{-3}$	0.324	256	0.077	4.3	3.9	5.3	$10^{-5}$	$10^{-5}$	3072	48
7	1.5	$10^{-2}$	0.321	256	0.17	4.8	4.5	5.9	$10^{-5}$	$10^{-5}$	3072	48
8	1.5	1	0.320	256	1.1	5.9	5.8	6.9	$10^{-5}$	$10^{-4}$	2304	36
9	1.5	1	0.320	256	0.55	5.9	5.8	6.9	$10^{-5}$	$10^{-5}$	2304	36
10	1.5	1	0.329	256	0.55	5.9	5.8	7.8	$10^{-5}$	$10^{-4}$	+6144	+96
11	1.5	1	—	512	1.1	—	—	—	—	—	—	—
12	2	$10^{-5}$	0.330	512	0.36	1.0	$< 1$	1.5	$10^{-3}$	$10^{-4}$	192	12
13	2	$10^{-5}$	0.330	512	0.36	1.0	$< 1$	1.5	$10^{-3}$	$< 10^{-3}$	192	12
14	2	$10^{-5}$	0.330	1024	0.73	1.0	$< 1$	1.5	$10^{-3}$	$< 10^{-3}$	1536	12
15	2	$10^{-5}$	0.330	256	0.18	1.0	$< 1$	1.5	$10^{-3}$	$< 10^{-3}$	48	3
16	2	1	0.333	128	10.7	$< 1$	5.0	8.2	$10^{-4}$	$10^{-5}$	132	132
17	3	$10^{-5}$	0.341	256	0.20	$< 1$	$< 1$	3.6	$10^{-4}$	$10^{-4}$	320	10
18	3	$10^{-5}$	0.361	1024	0.79	$< 1$	$< 1$	1.4	$10^{-3}$	$< 10^{-4}$	3072	12
19	3	$10^{-5}$	0.361	512	0.39	$< 1$	$< 1$	1.4	$10^{-3}$	$< 10^{-4}$	384	12
20	3	$10^{-5}$	0.361	256	0.20	$< 1$	$< 1$	1.4	$10^{-3}$	$< 10^{-4}$	48	1.5
21	3	$10^{-3}$	0.338	256	6.6	6.7	6.1	9.0	$10^{-3}$	$< 10^{-5}$	3072	48
22	3	1	0.336	256	1740	10.7	10.5	13.2	$10^{-3}$	$< 10^{-5}$	5120	120
23	4	$10^{-5}$	0.345	256	0.21	$< 1$	$< 1$	3.7	$10^{-3}$	$< 10^{-4}$	768	12
24	6	$10^{-5}$	0.348	256	0.17	$< 1$	$< 1$	3.7	$10^{-3}$	$< 10^{-4}$	768	12



# duration to radiation domination

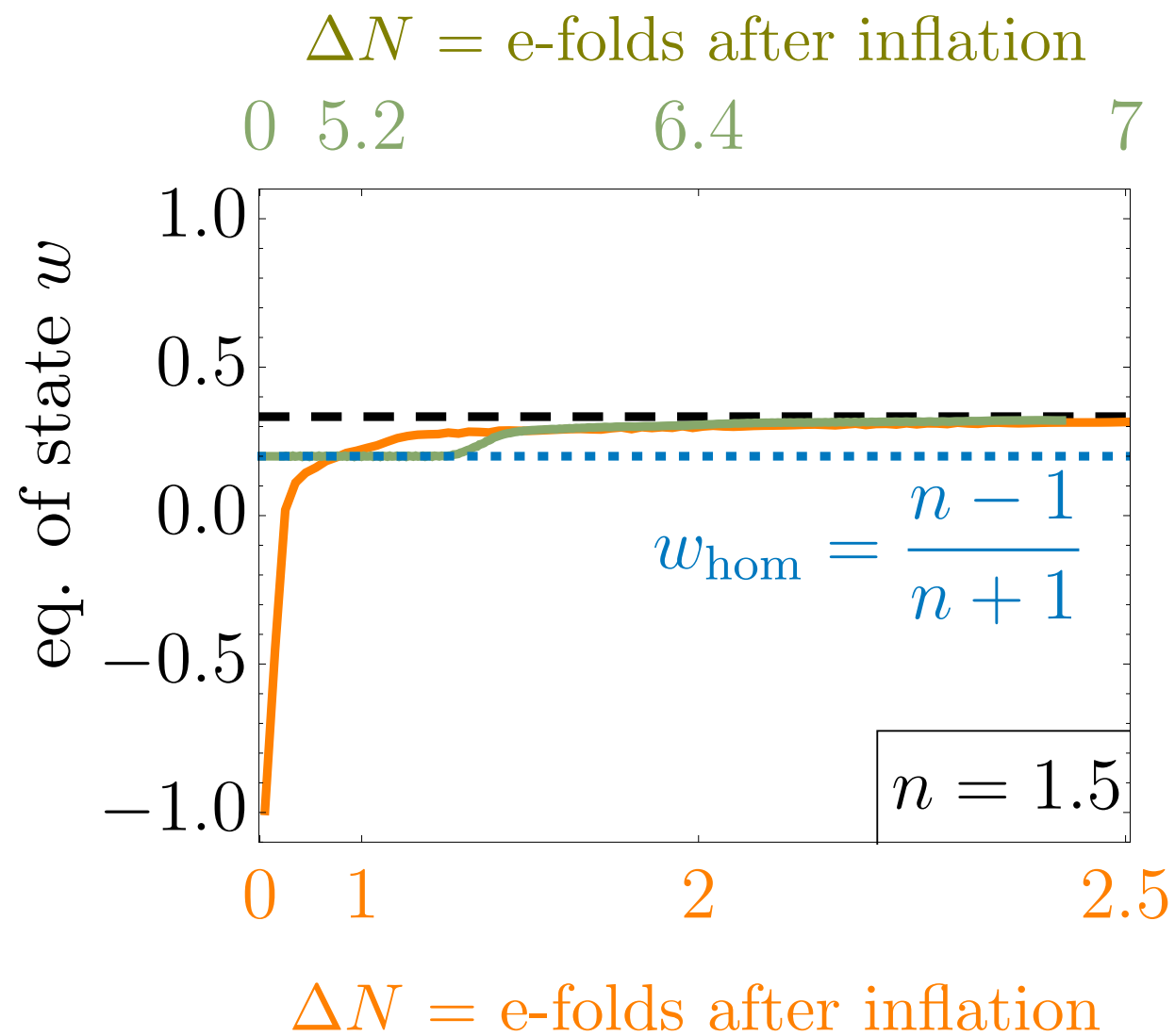
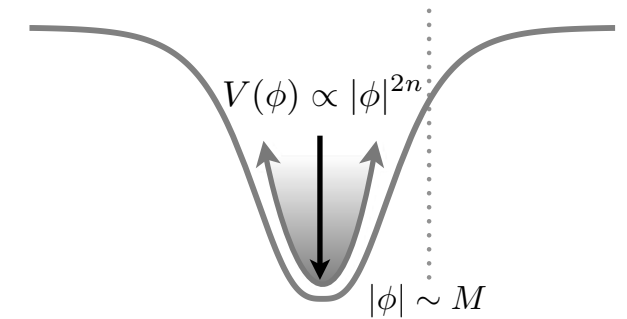
\* non-quadratic minima



# duration to radiation domination

## \* non-quadratic minima

from detailed 3+1 dimensional lattice simulations



$M \sim m_{\text{pl}}$   
inefficient initial resonance

$M \ll m_{\text{pl}}$   
efficient initial resonance

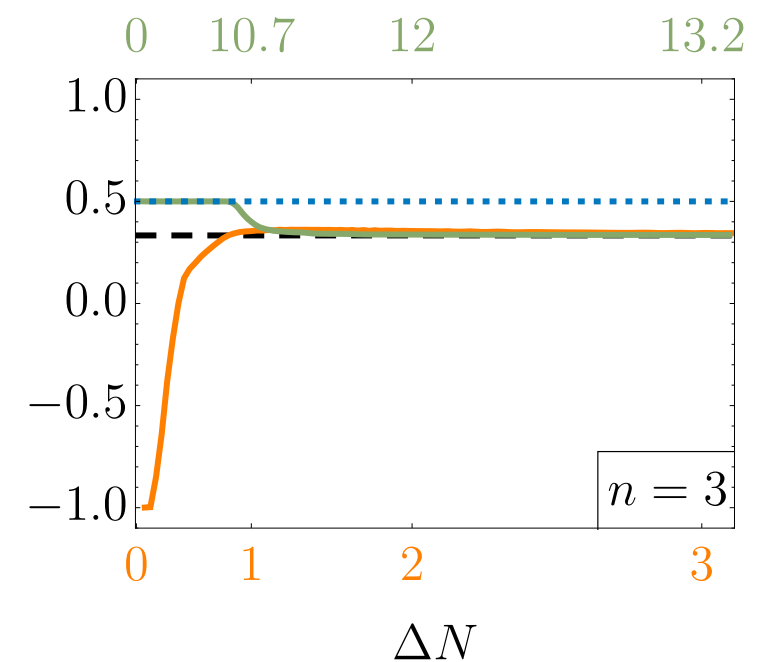
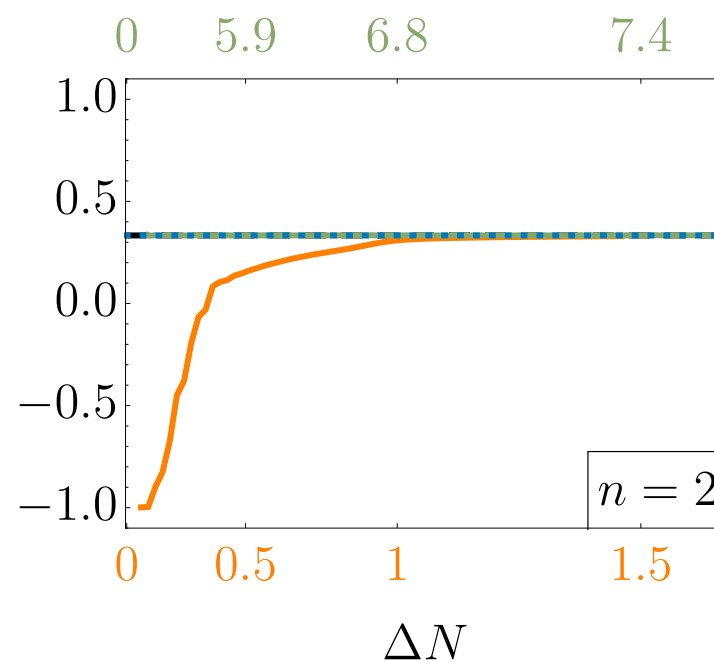
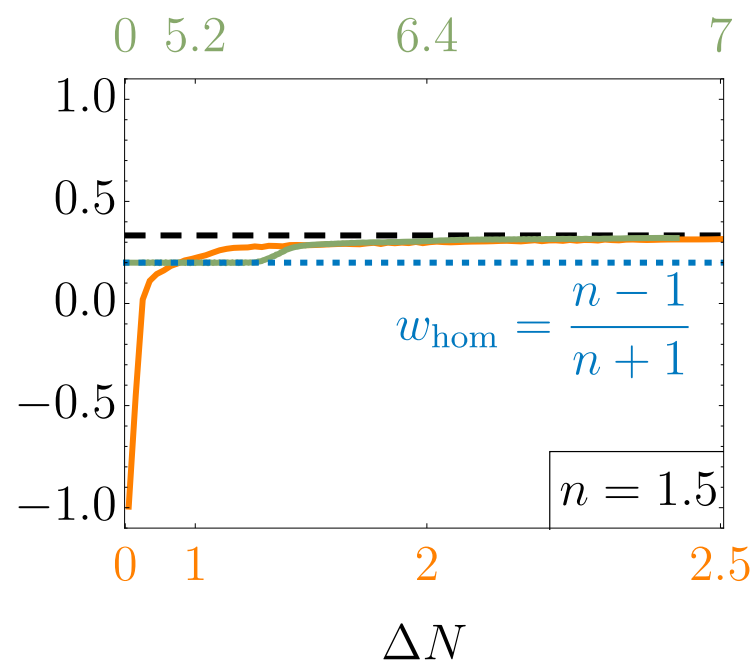
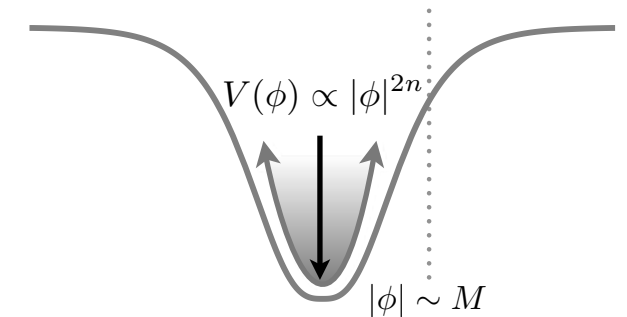
# duration to radiation domination

## \* non-quadratic minima

from detailed 3+1 dimensional lattice simulations

green = inefficient initial resonance  
orange = efficient initial resonance

---  $w_{\text{rad}} = 1/3$



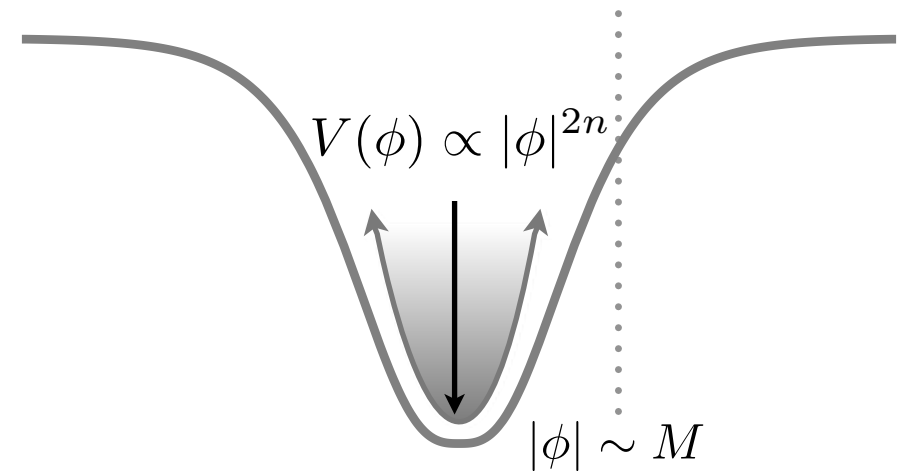
from analytic considerations

$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} \end{cases}$$



# an upper bound on duration to radiation domination

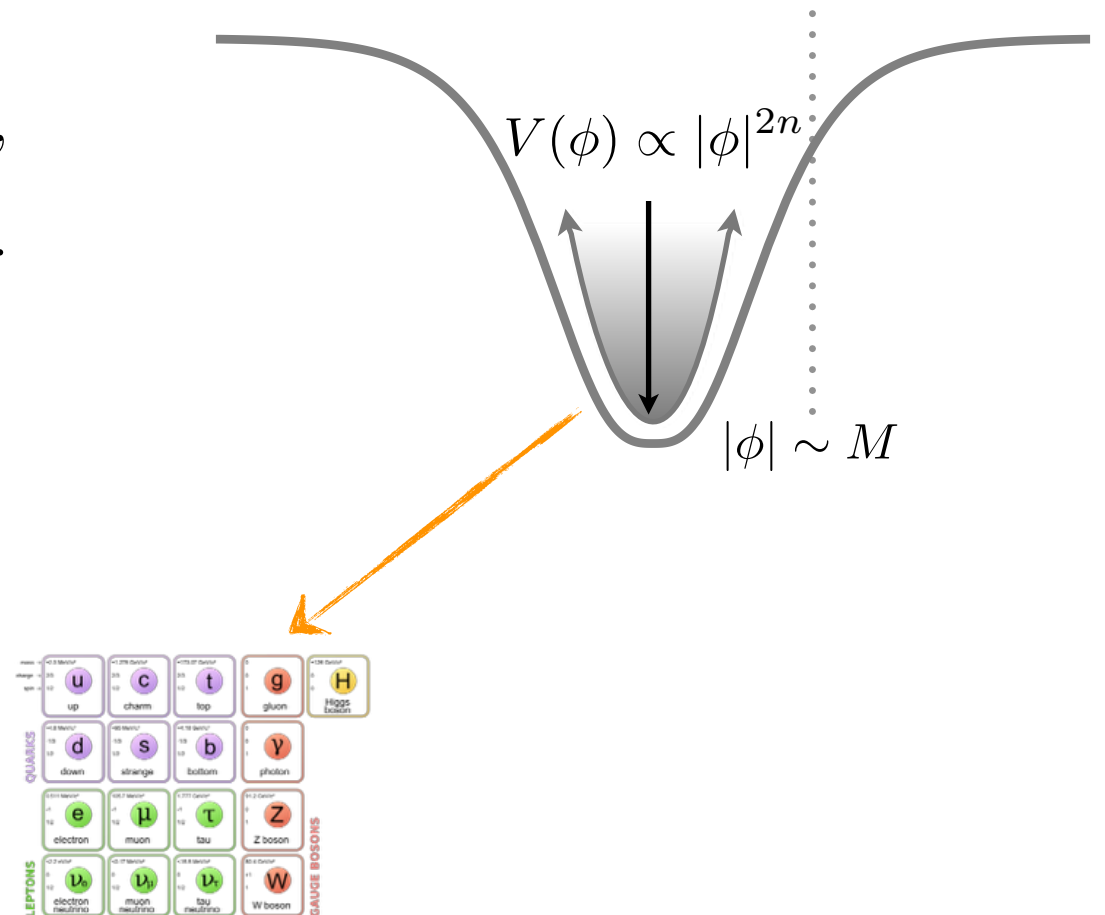
$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$



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$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta\kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

additional *light (massless) fields* can  
only decrease the duration!

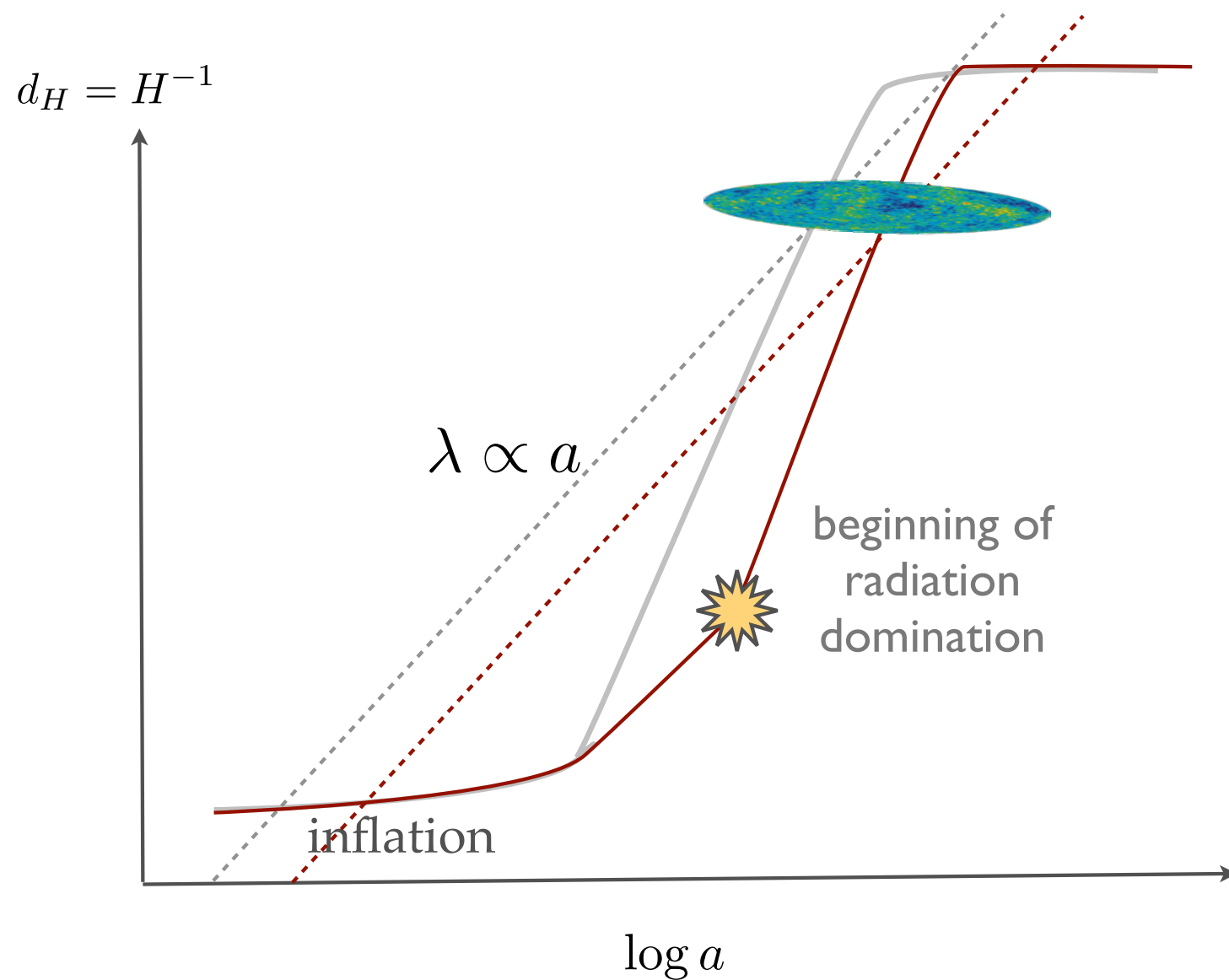


\* decay to significantly massive fields can change this conclusion

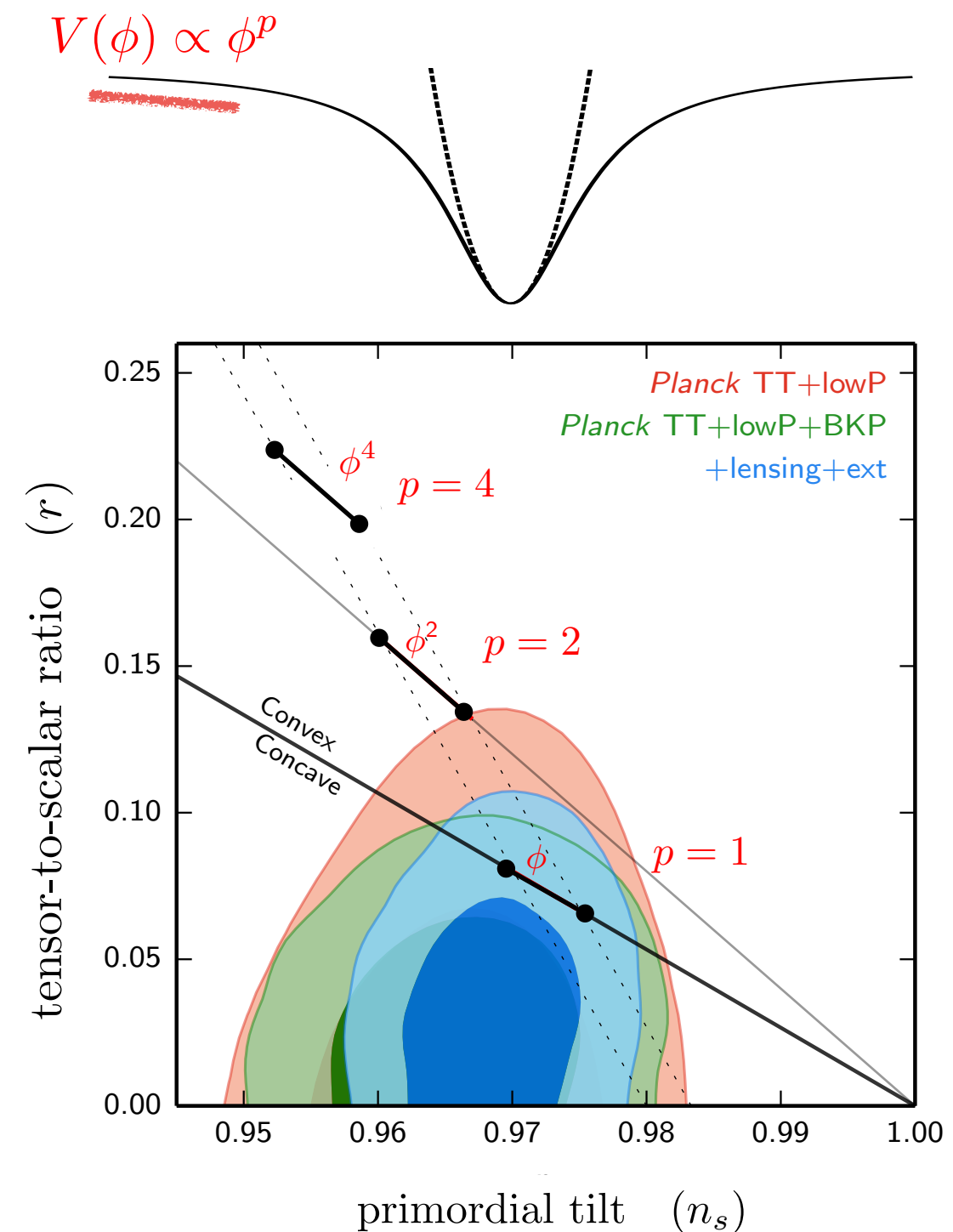
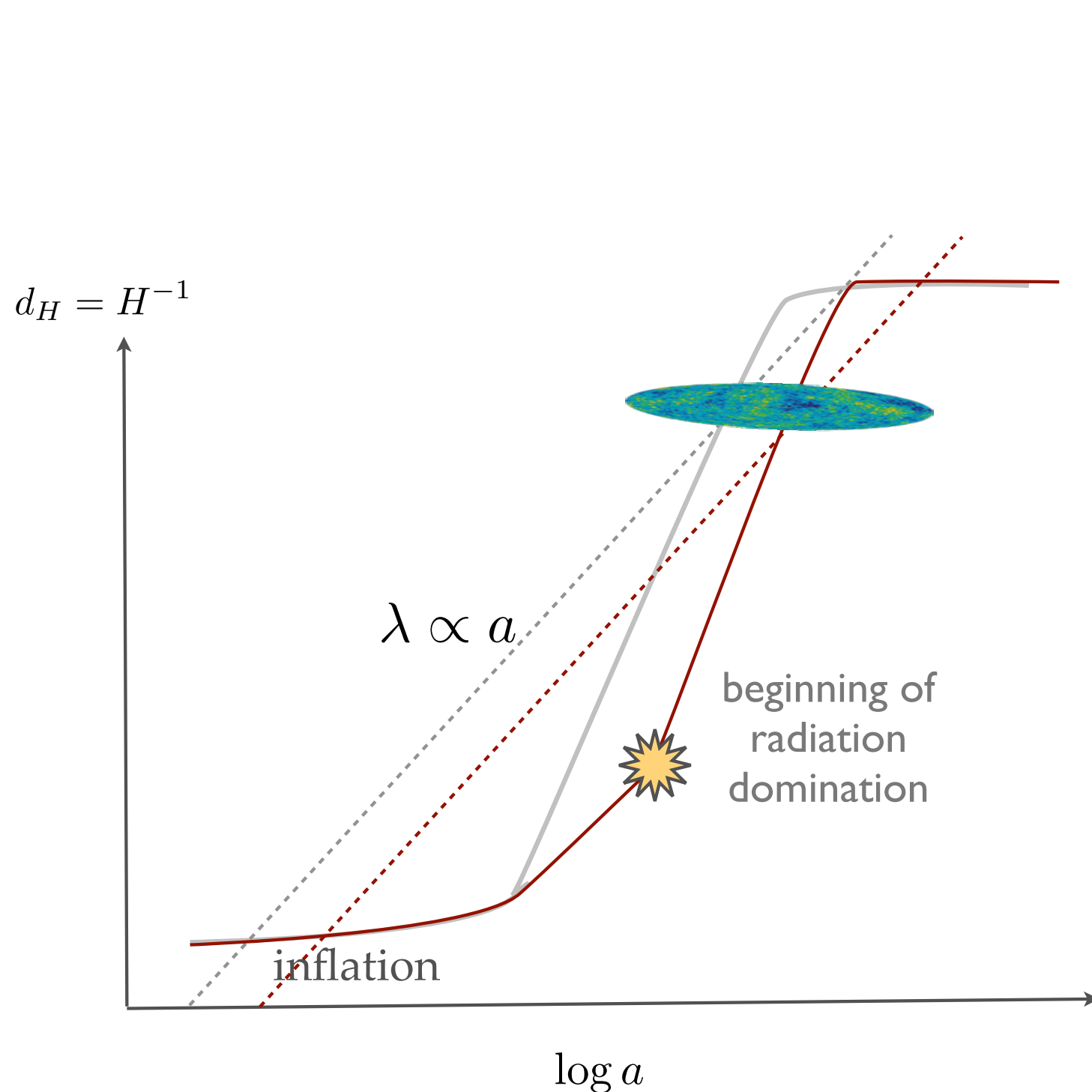
- **what drives inflation ?**
- **how did inflation end ?**
- **observable implications ?**



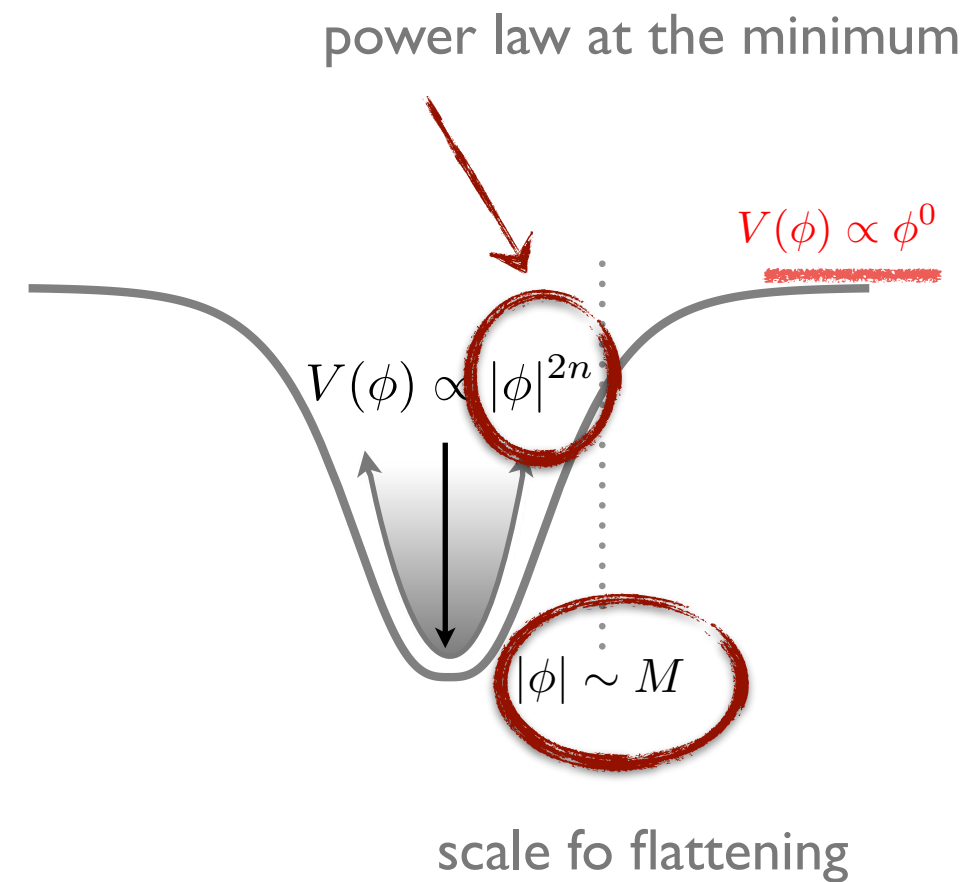
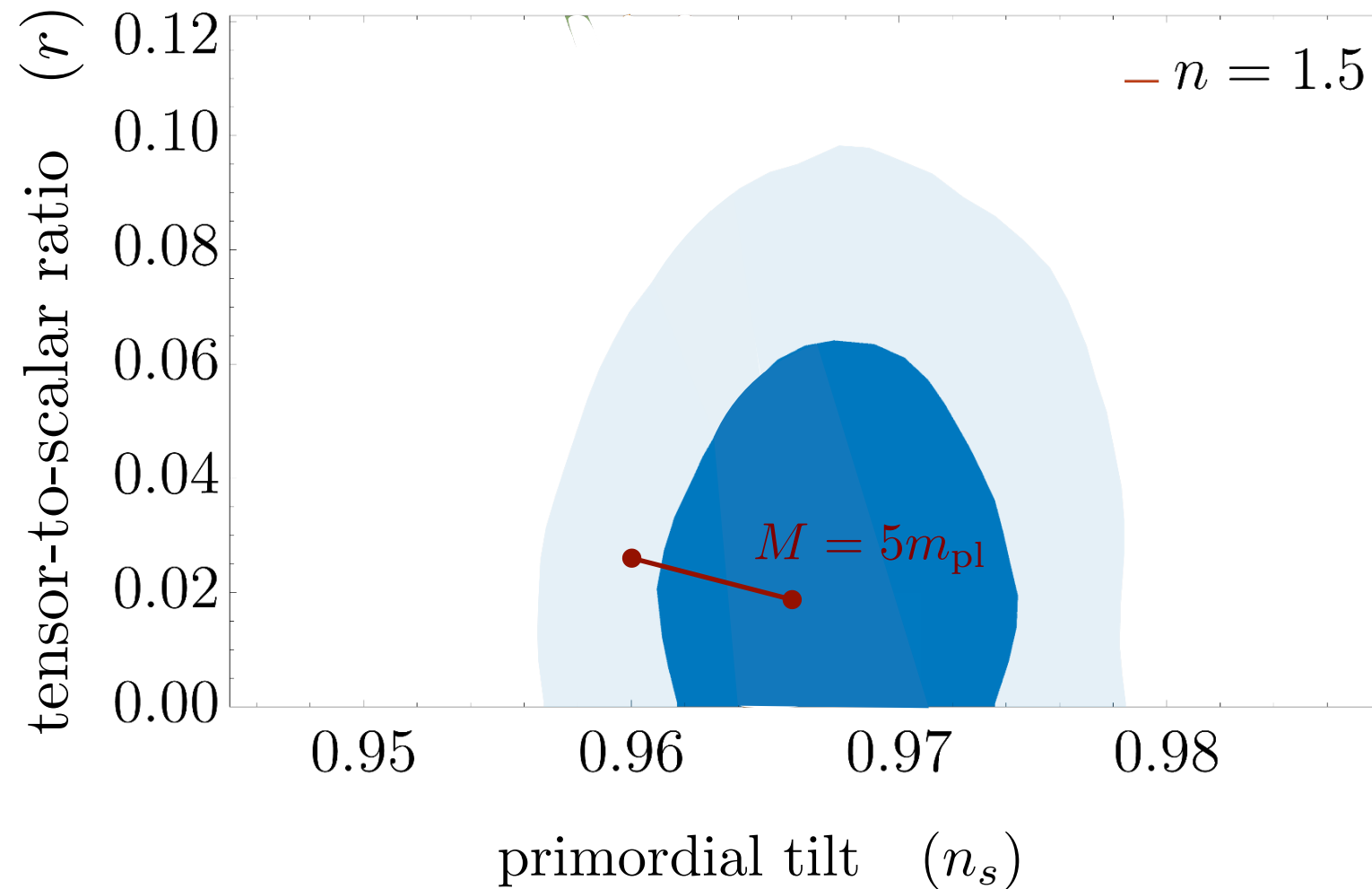
# eq. state — expansion history — CMB observables



# eq. state — expansion history — CMB observables



# implications for CMB observables

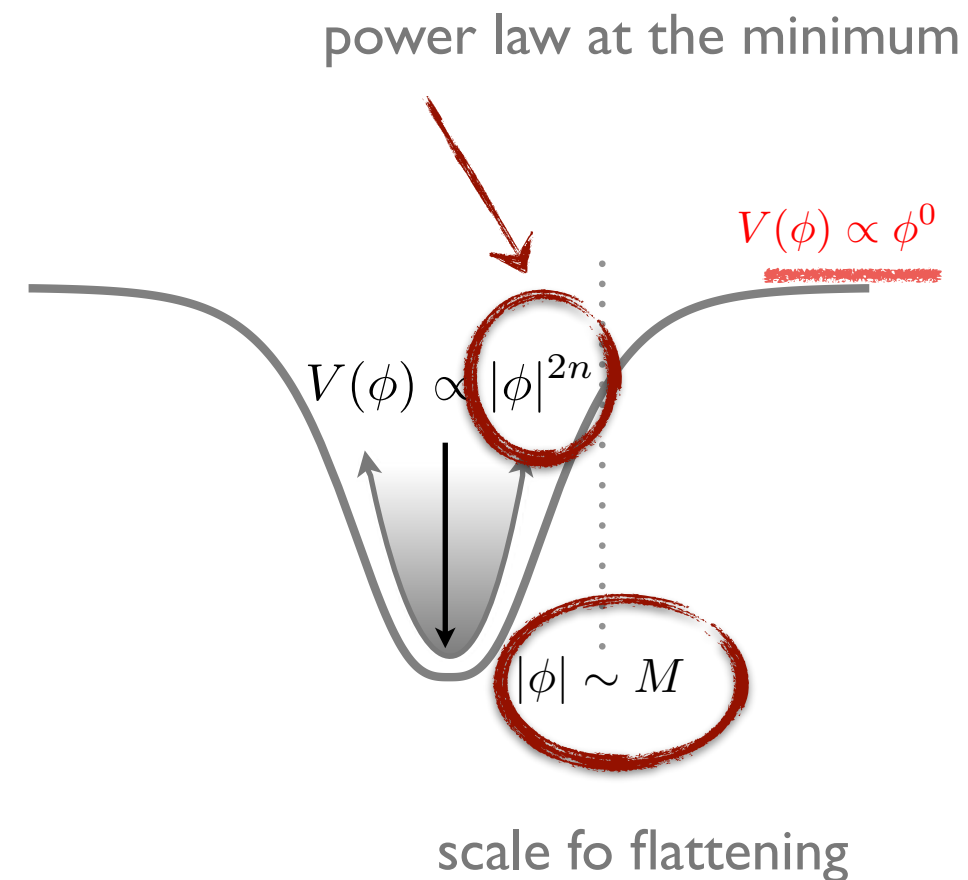
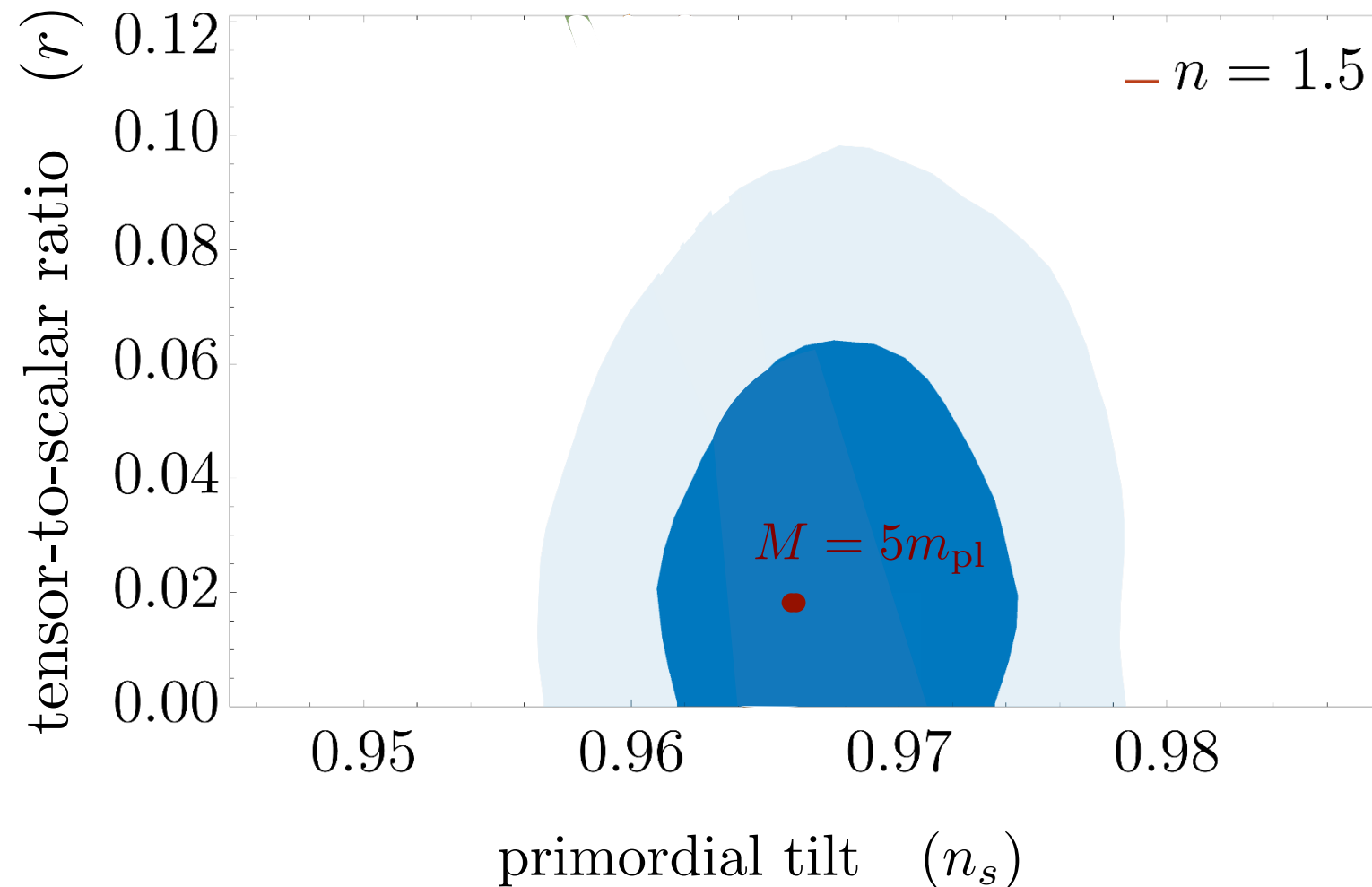


\* width of the lines account for couplings to other light fields

\* non-quadratic minimum



# reduction in uncertainty!

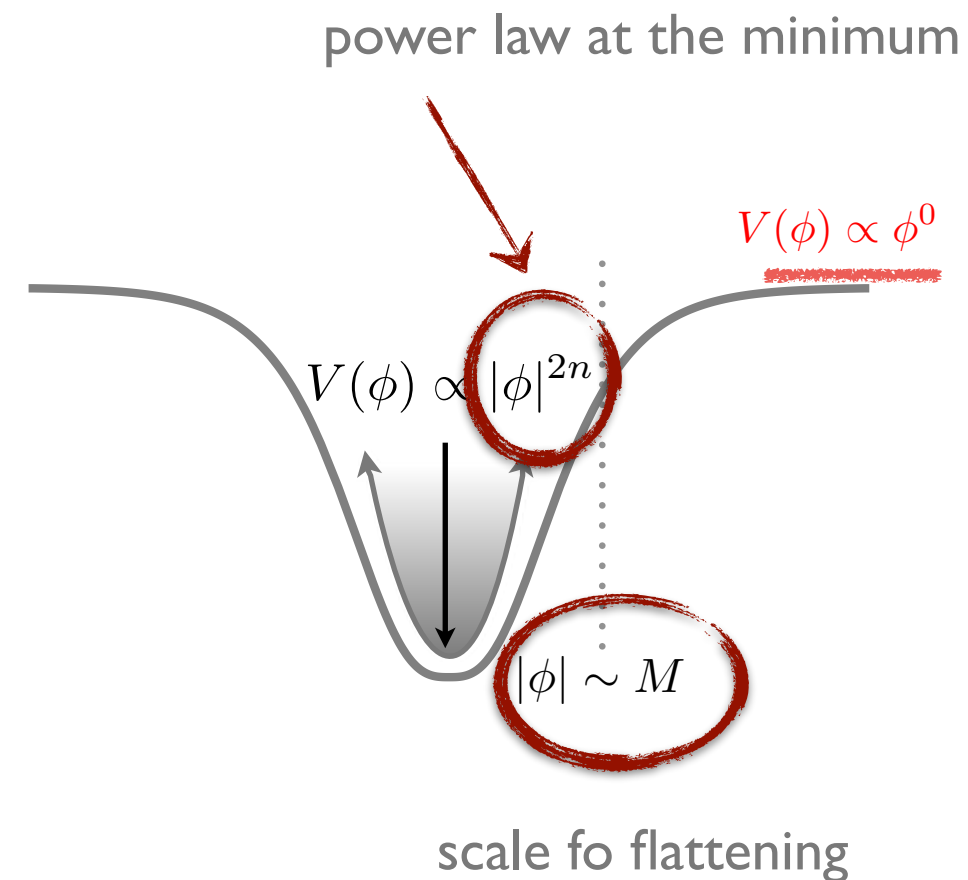
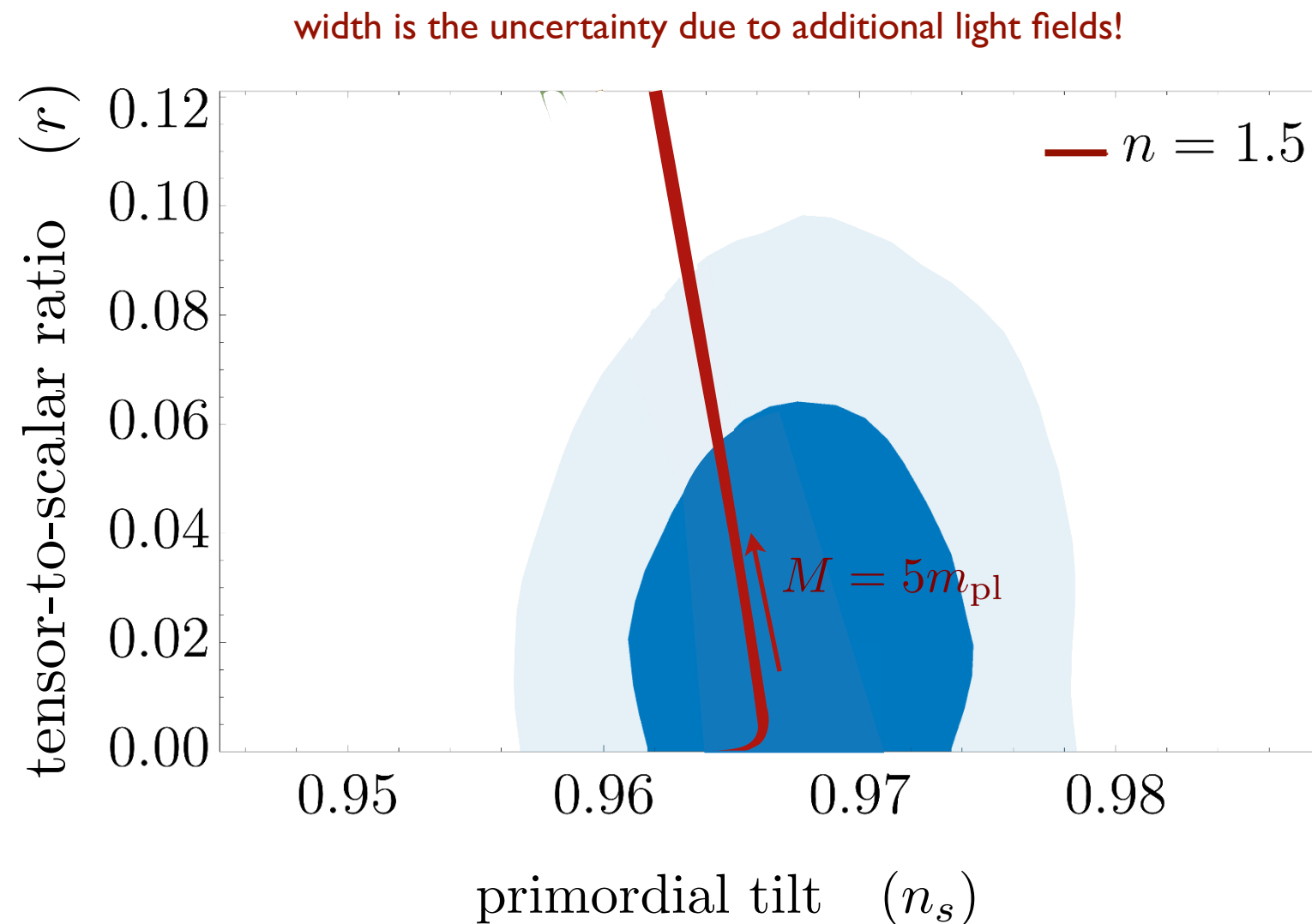


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\* width of the lines account for couplings to other light fields

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# implications for CMB observables

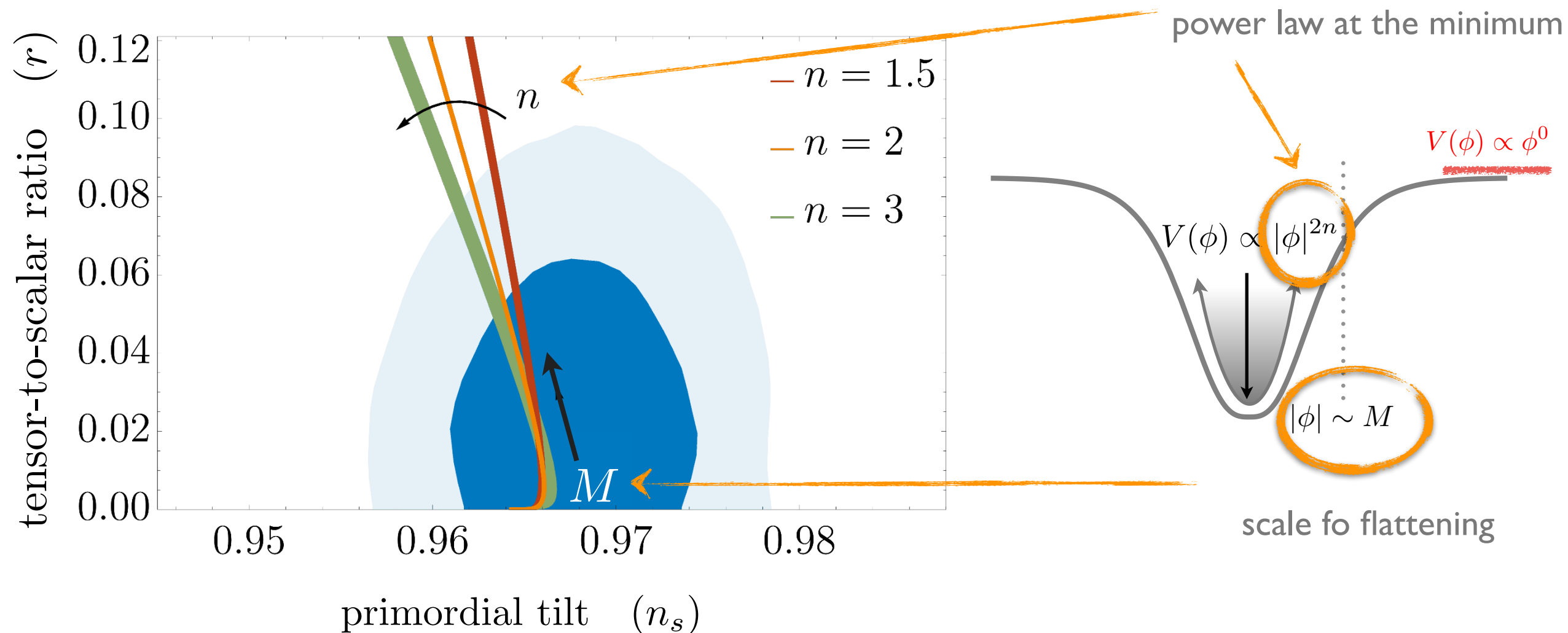


$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

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# implications for CMB observables



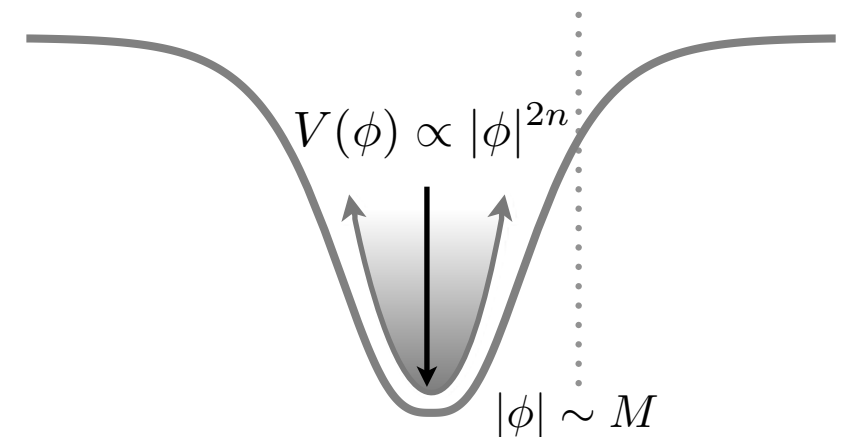
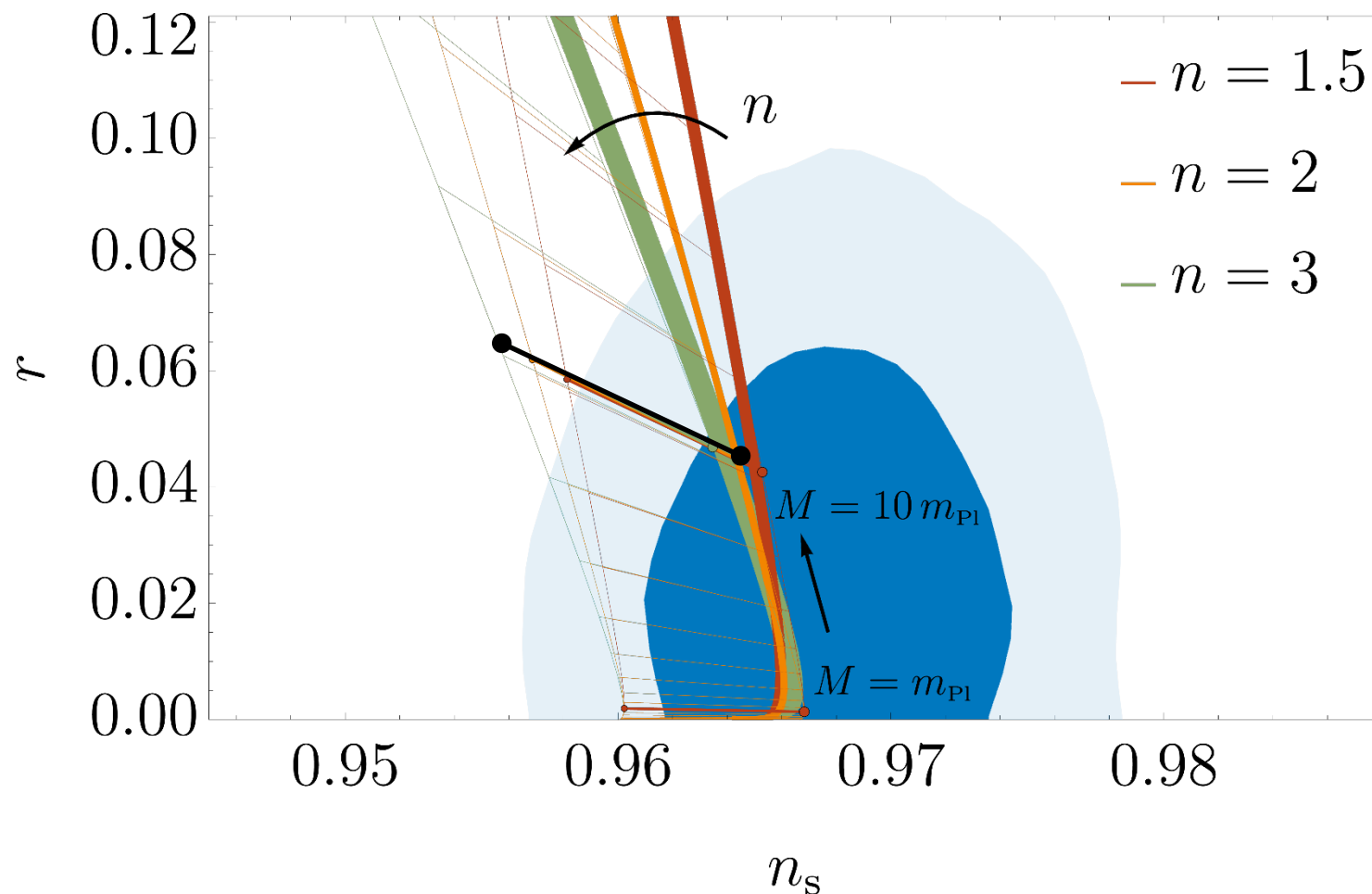
$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

\* width of the lines account for couplings to other light fields

\* non-quadratic minimum



# including upper bound — significant reduction in uncertainty !



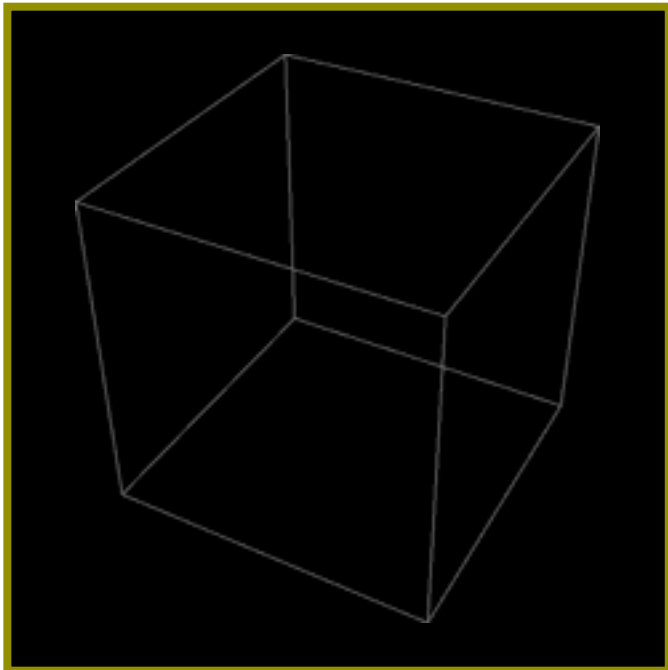
$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}} , \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}} . \end{cases}$$

\* non-quadratic minimum

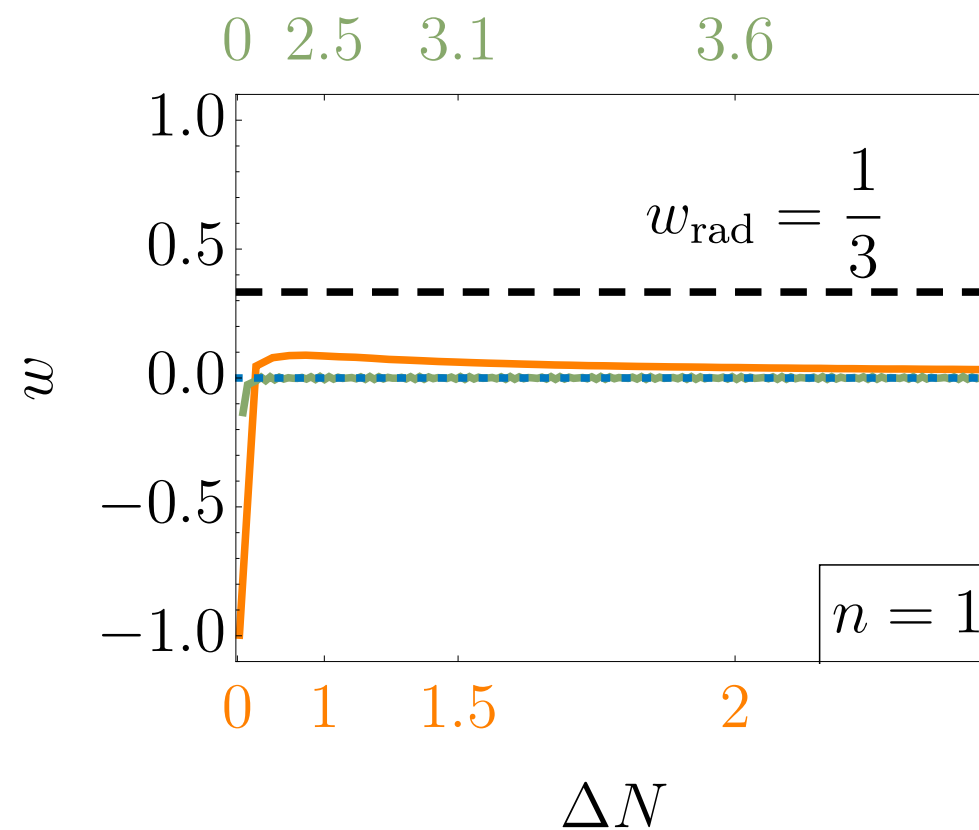
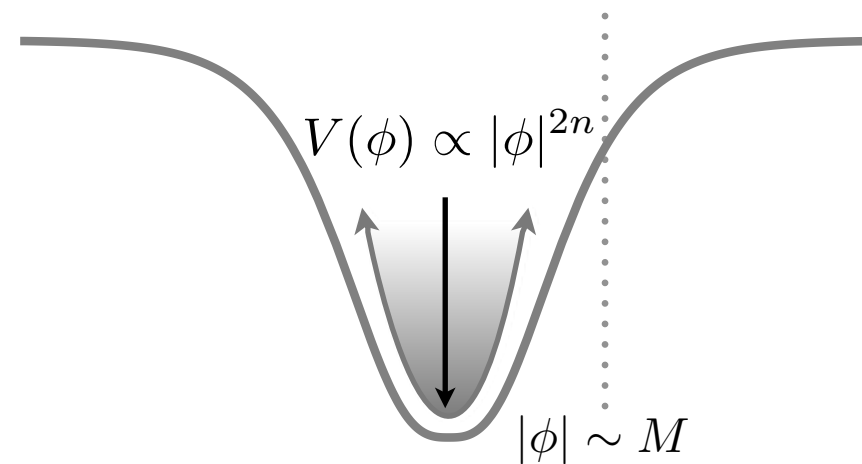
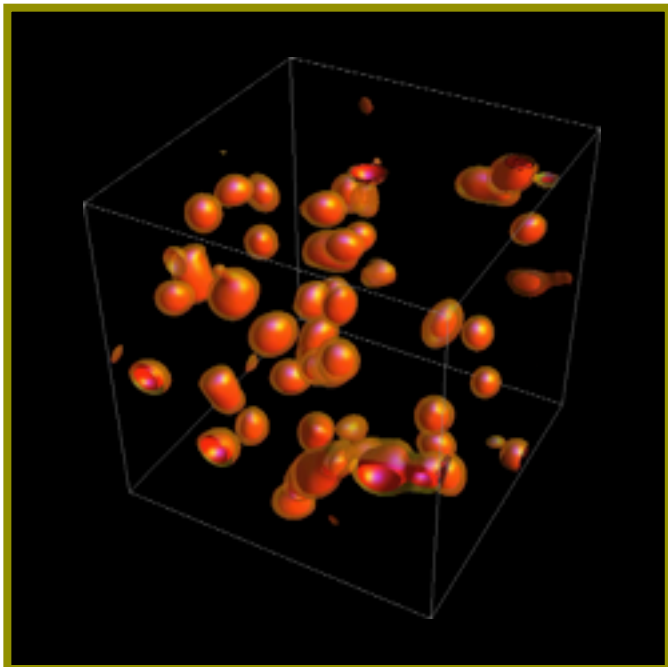
# \* quadratic minimum

$$n = 1$$

$$M \sim m_{\text{pl}}$$



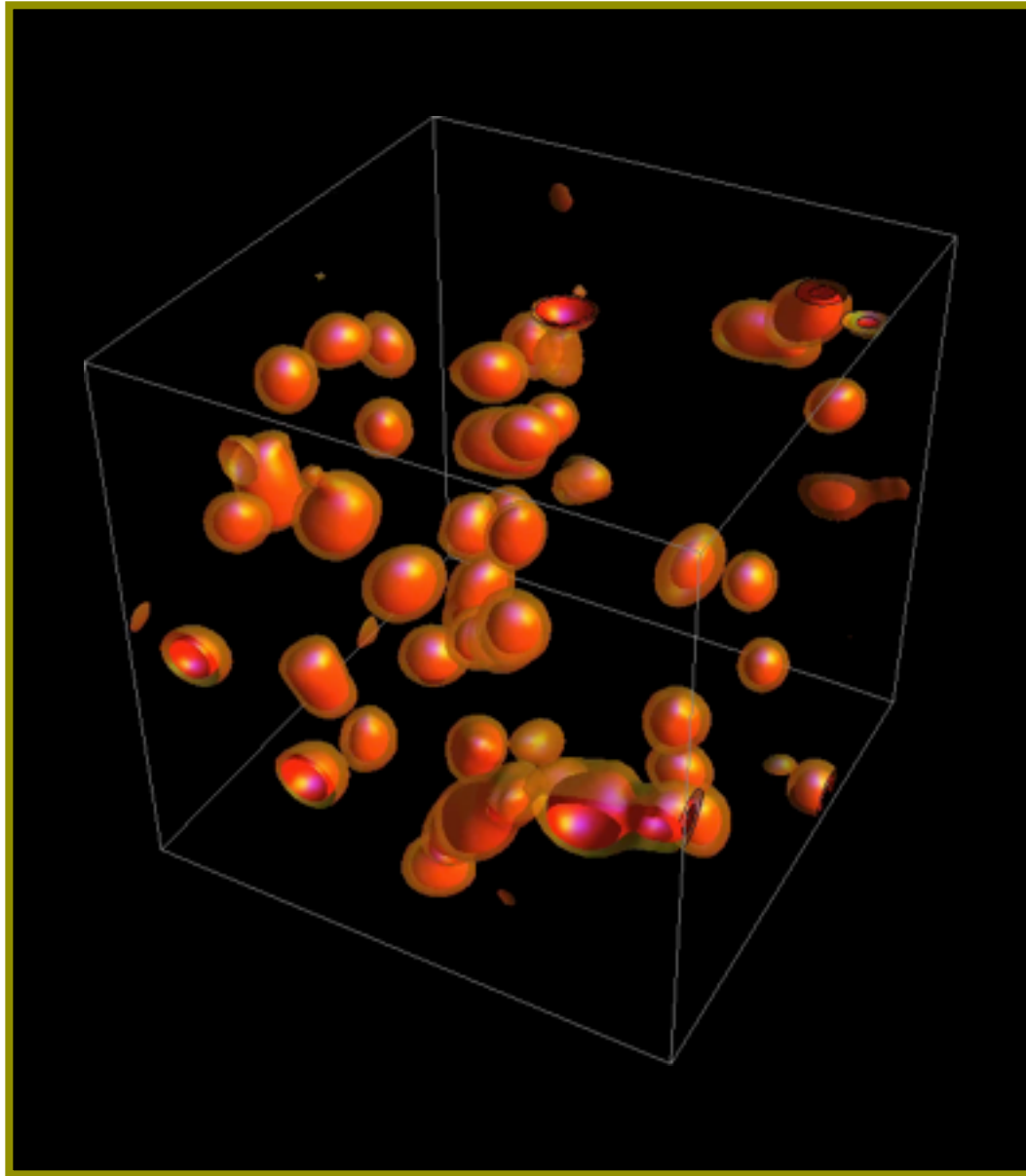
$$M \ll m_{\text{pl}}$$



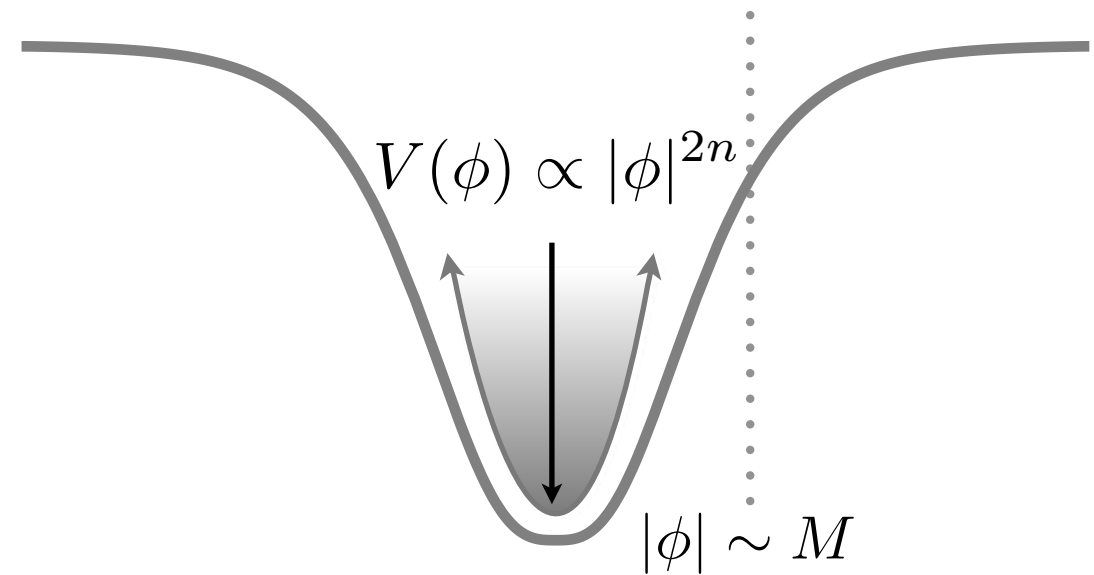
# oscillons after inflation

$$n = 1$$

$$M \ll m_{\text{pl}}$$



Oscillons — non-topological solitons!



- approach to radiation domination with additional fields can be complex when additional fields are included

see for example:

Gleiser (1994)

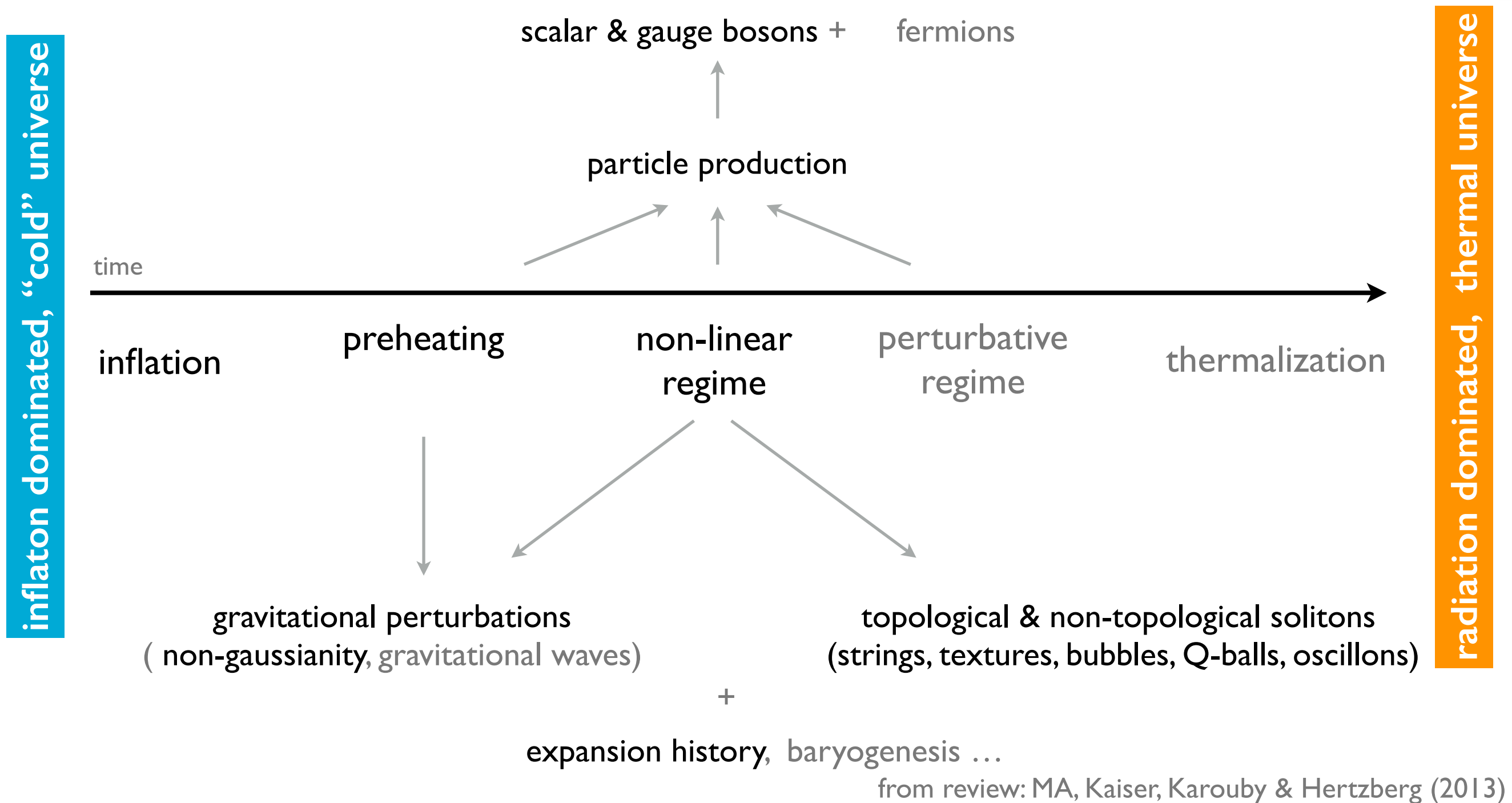
MA, Easter, Finkel, Flaugher & Hertzberg (2011)

MA (2013)

and @ UIUC Adshead, Giblin, Scully, Sfakianakis (2014)



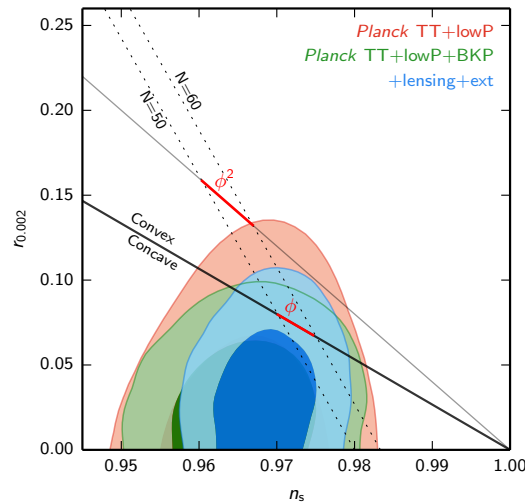
# left out ...



related work at UIUC, for example:

Adshead, Cui & Shelton (2016), Adshead, Skully, Giblin & Sfakianakis (2015, 2016), Adshead & Sfakianakis (2015)

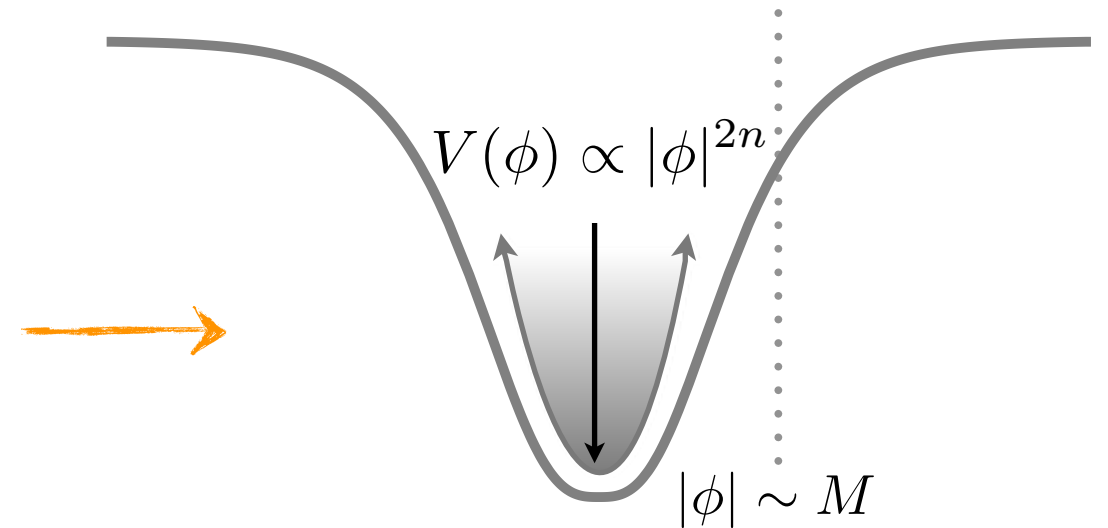
# inflation and its end “simple” models



for example:

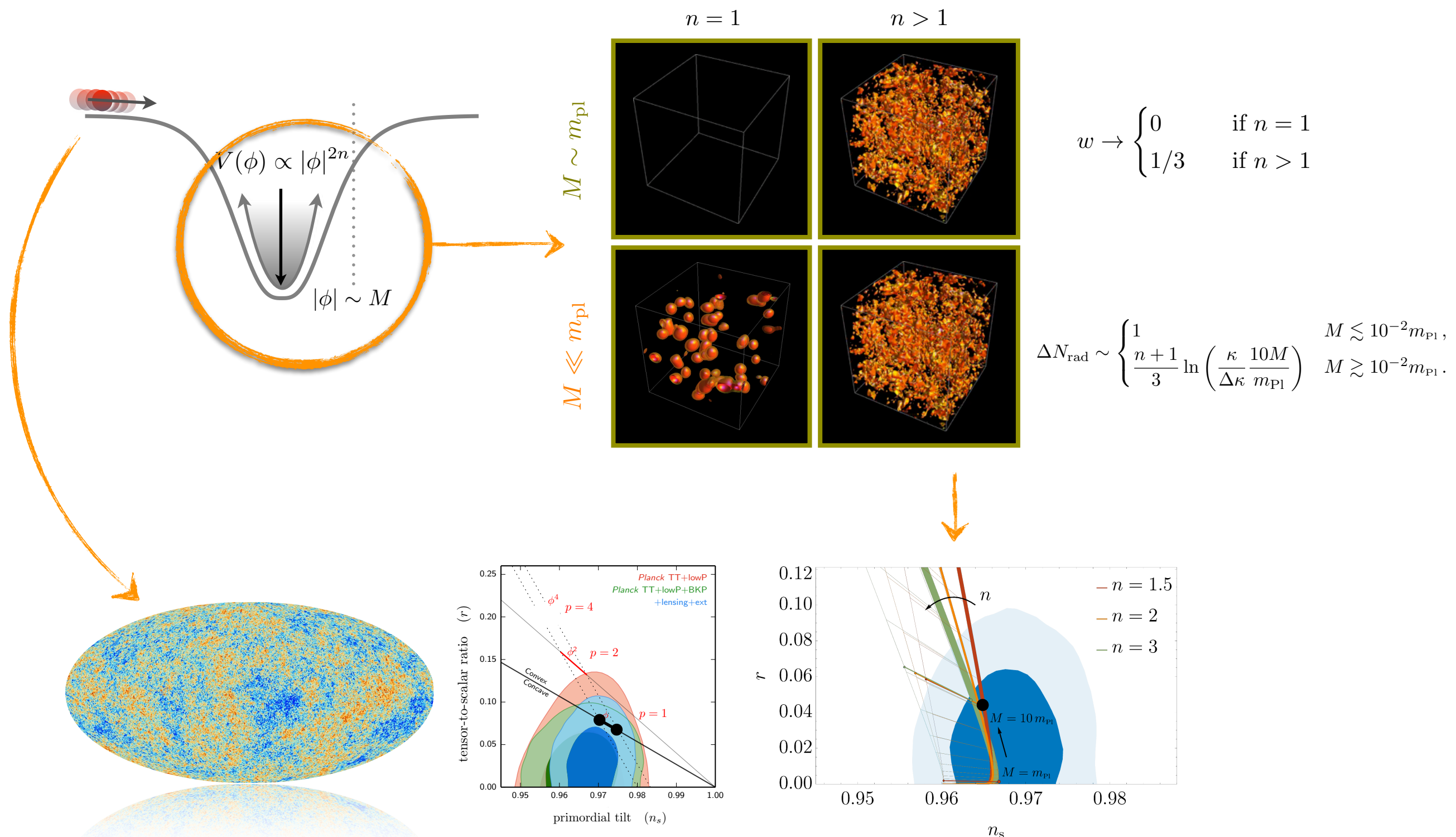
+

Silverstein & Westphal (2008)  
McAllister et. al (2014)  
Kallosh & Linde (2014)



- (i) what are the dynamics ?
- (ii) eq. of state & how long to radiation domination ?
- (iii) obs. consequences ?

# summary: “simple” models of our origins





# two approaches

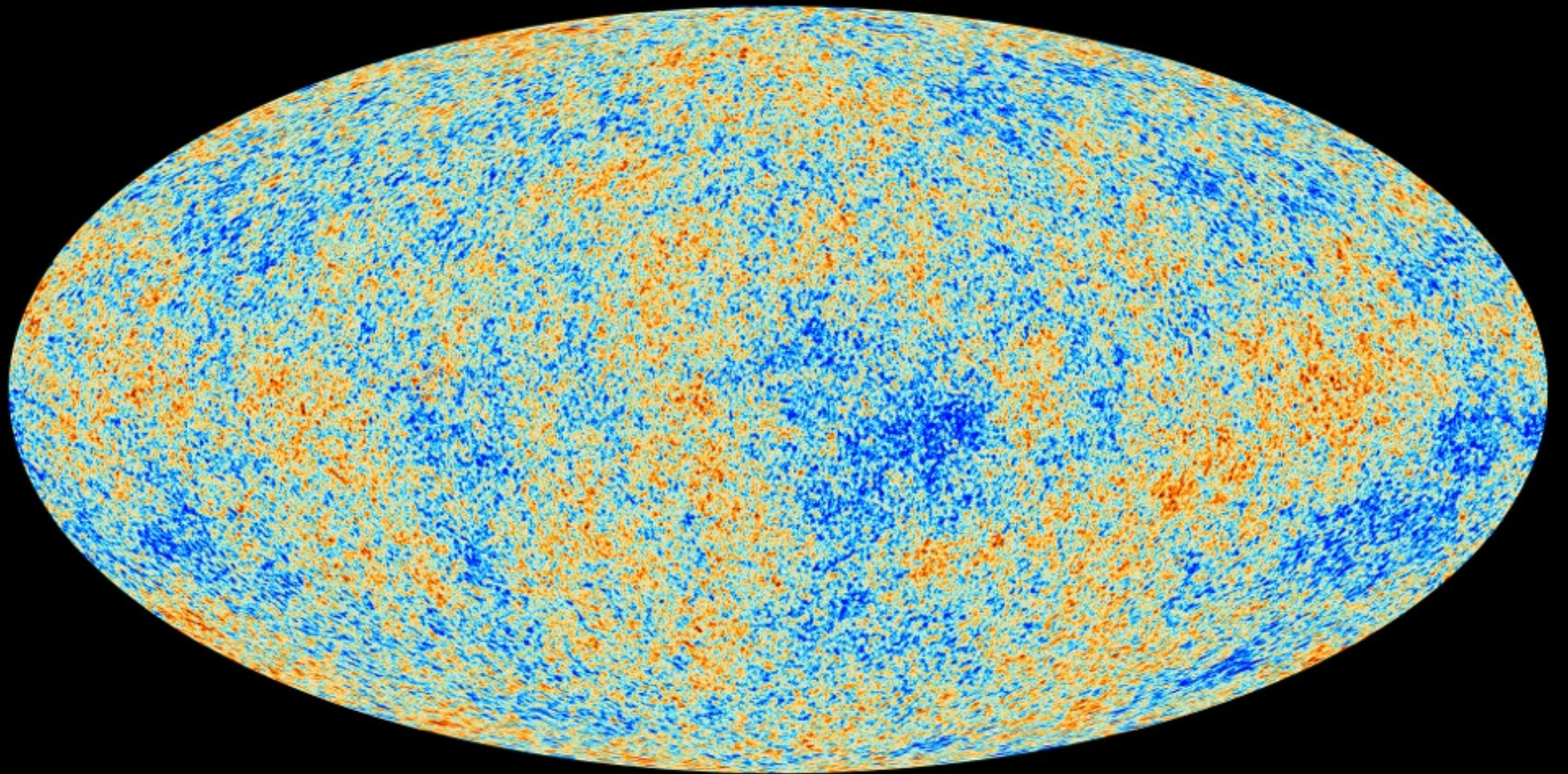


Lozanov & MA (2016) + earlier works

MA & Baumann (2015)



# simple early universe



$$\delta T/T \sim 10^{-5}$$



# theory : its complicated (probably)

- inflation
- reheating after inflation

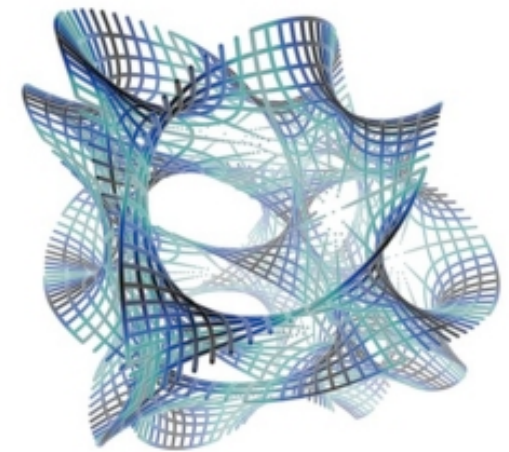
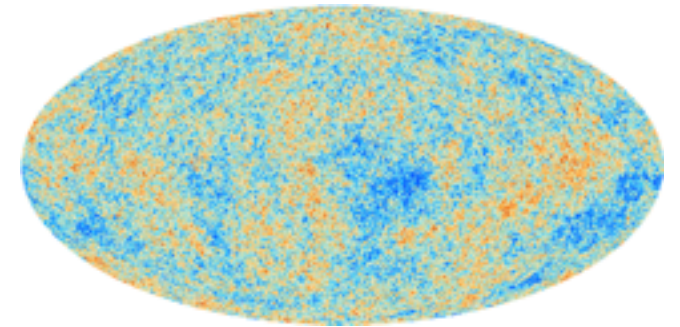


	<b>u</b> up mass = 2.3 MeV/c² charge = 2/3 spin = 1/2	<b>c</b> charm mass = 1.28 GeV/c² charge = 2/3 spin = 1/2	<b>t</b> top mass = 173.1 GeV/c² charge = 2/3 spin = 1/2	<b>g</b> gluon mass = 0 charge = 0 spin = 1	<b>H</b> Higgs boson mass = 125.1 GeV/c² charge = 0 spin = 0
<b>QUARKS</b>	<b>d</b> down mass = 4.18 MeV/c² charge = -1/3 spin = 1/2	<b>s</b> strange mass = 96 MeV/c² charge = -1/3 spin = 1/2	<b>b</b> bottom mass = 4.18 GeV/c² charge = -1/3 spin = 1/2	<b>γ</b> photon mass = 0 charge = 0 spin = 1	
	<b>e</b> electron mass = 0.511 MeV/c² charge = -1 spin = 1/2	<b>μ</b> muon mass = 105.7 MeV/c² charge = -1 spin = 1/2	<b>τ</b> tau mass = 1.777 GeV/c² charge = -1 spin = 1/2	<b>Z</b> Z boson mass = 91.187 GeV/c² charge = 0 spin = 1	
<b>LEPTONS</b>	<b>ν<sub>e</sub></b> electron neutrino mass < 2 eV/c² charge = 0 spin = 1/2	<b>ν<sub>μ</sub></b> muon neutrino mass < 17 MeV/c² charge = 0 spin = 1/2	<b>ν<sub>τ</sub></b> tau neutrino mass < 18.8 MeV/c² charge = 0 spin = 1/2	<b>W</b> W boson mass = 80.379 GeV/c² charge = ±1 spin = 1	<b>GAUGE BOSONS</b>



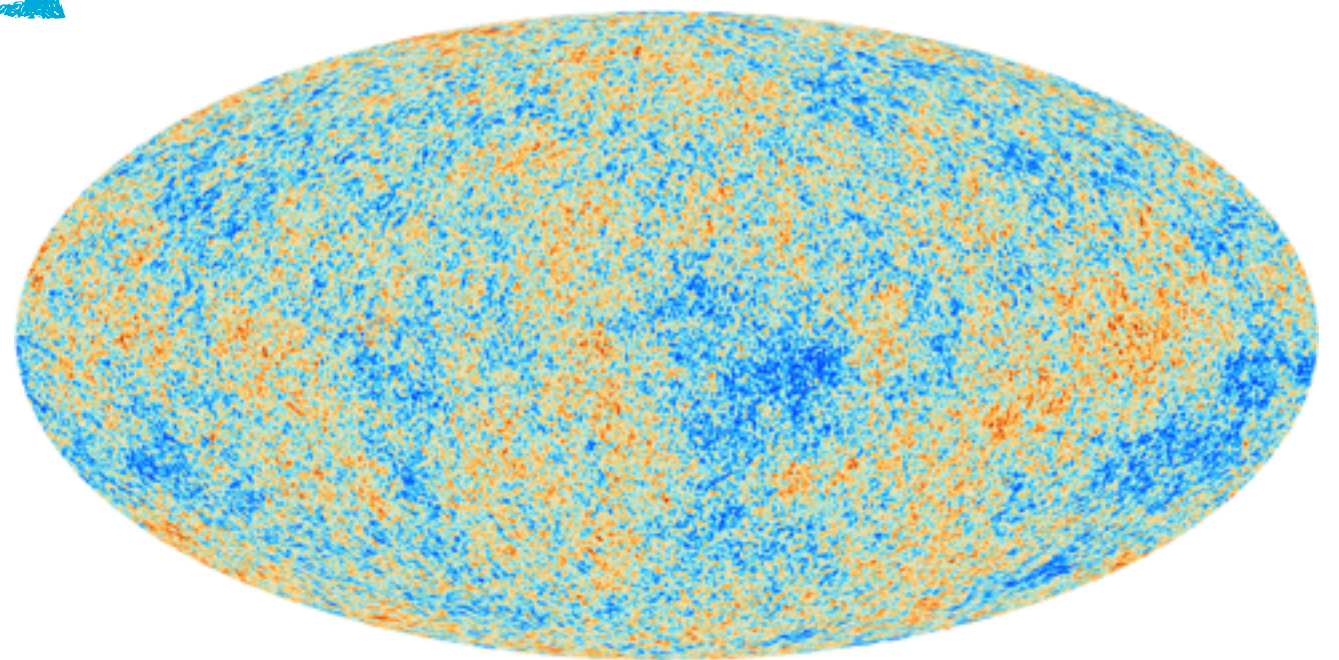
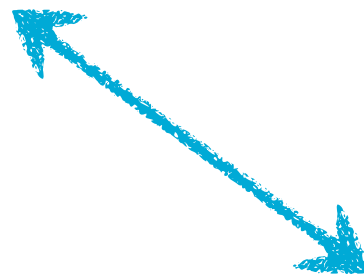
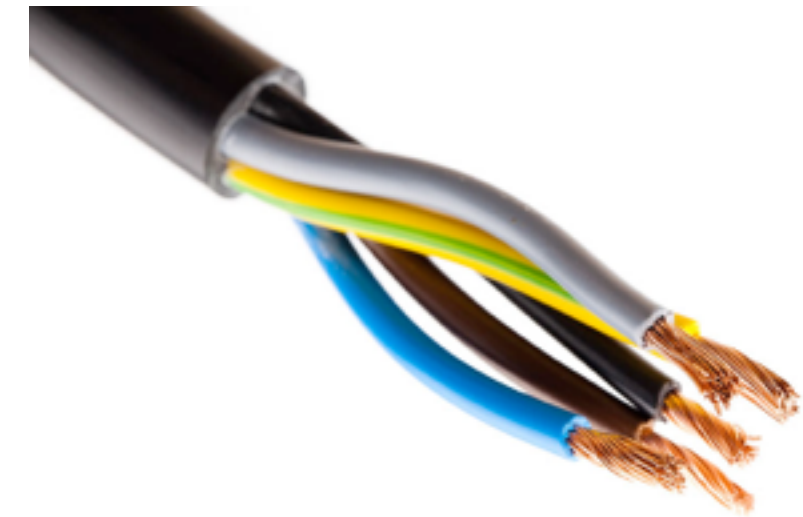
# a statistical approach?

- observations: early universe is simple
- fundamental theory: not so much ...
- **coarse grained view ?**
- **calculational tools ?**



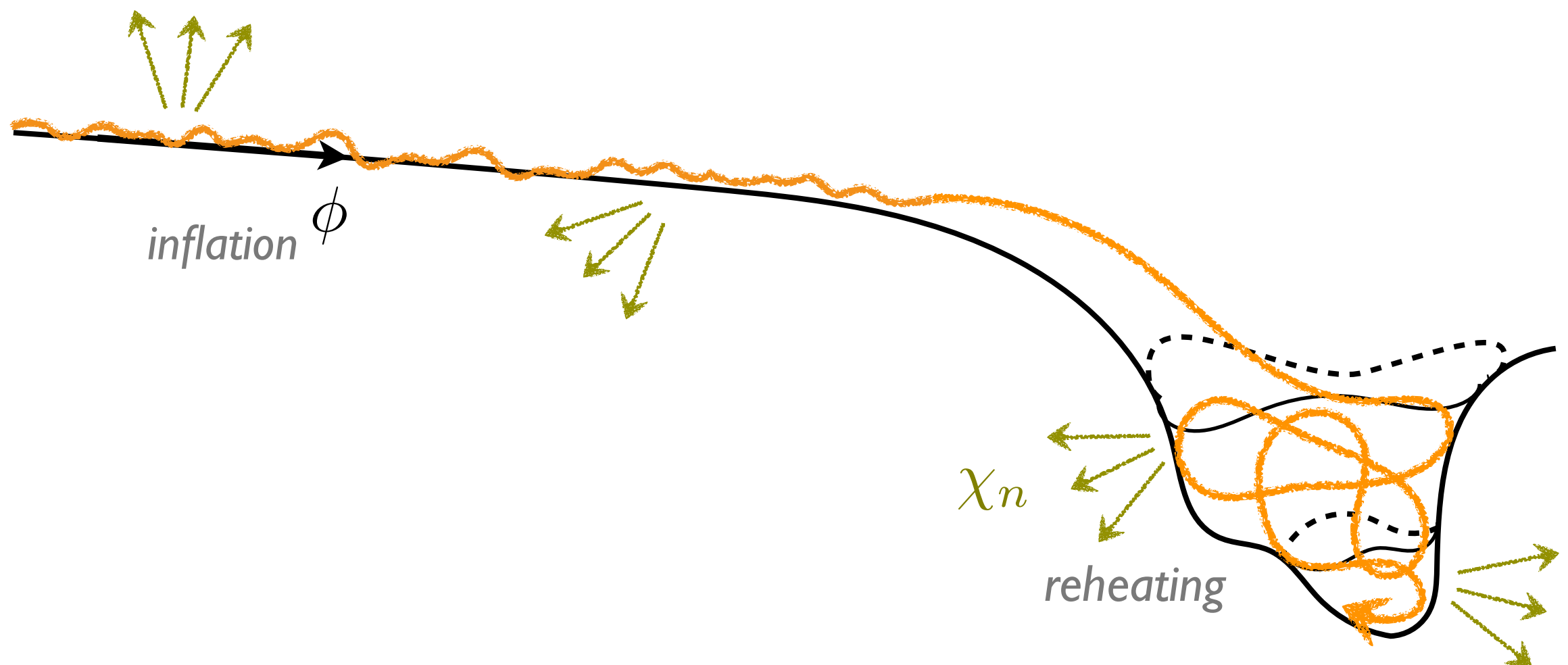
# from wires to cosmology

MA & Baumann (2015)



# multifield inflation/reheating

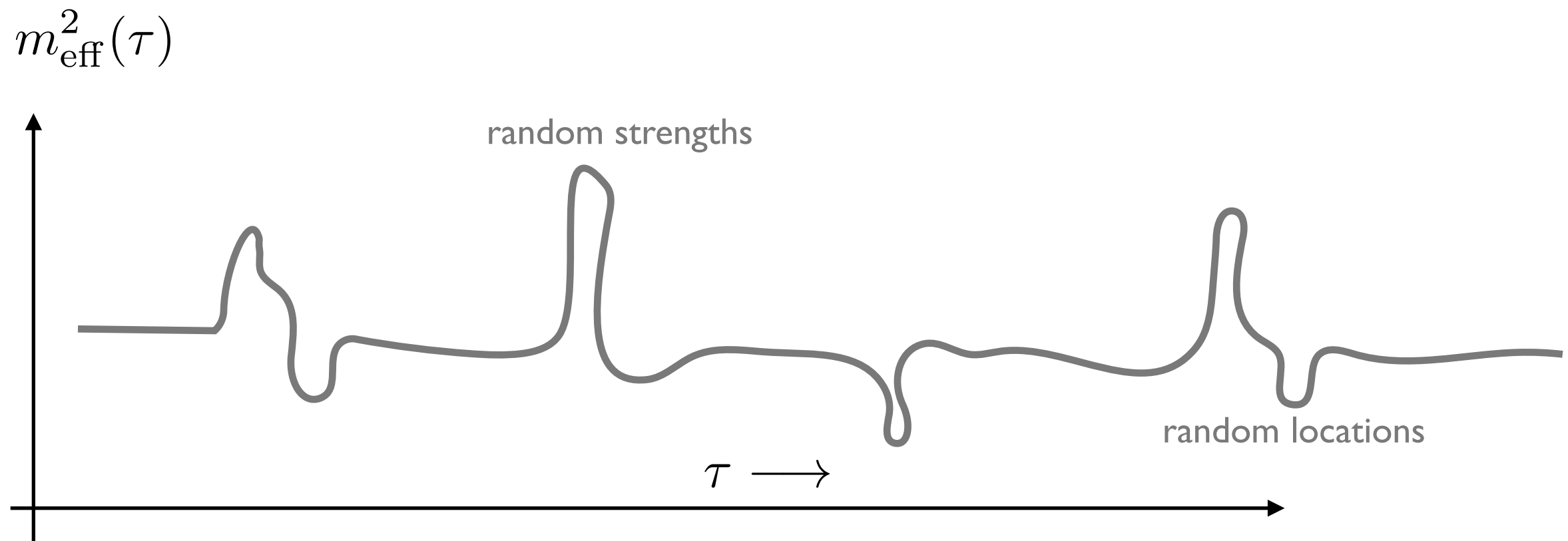
- inflation/reheating: many interacting fields
- fluctuations: coupled, non-perturbative



# complexity in time: cosmology

$$\ddot{\chi}_k(\tau) + [k^2 + m_{\text{eff}}^2(\tau)] \chi_k(\tau) = 0$$

$$m_{\text{eff}}^2(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)m_\varphi^2 + a^2(\tau)g^2(\phi(\tau) - \phi_*)^2 + \dots$$



simplified version!



complexity in time  
cosmology



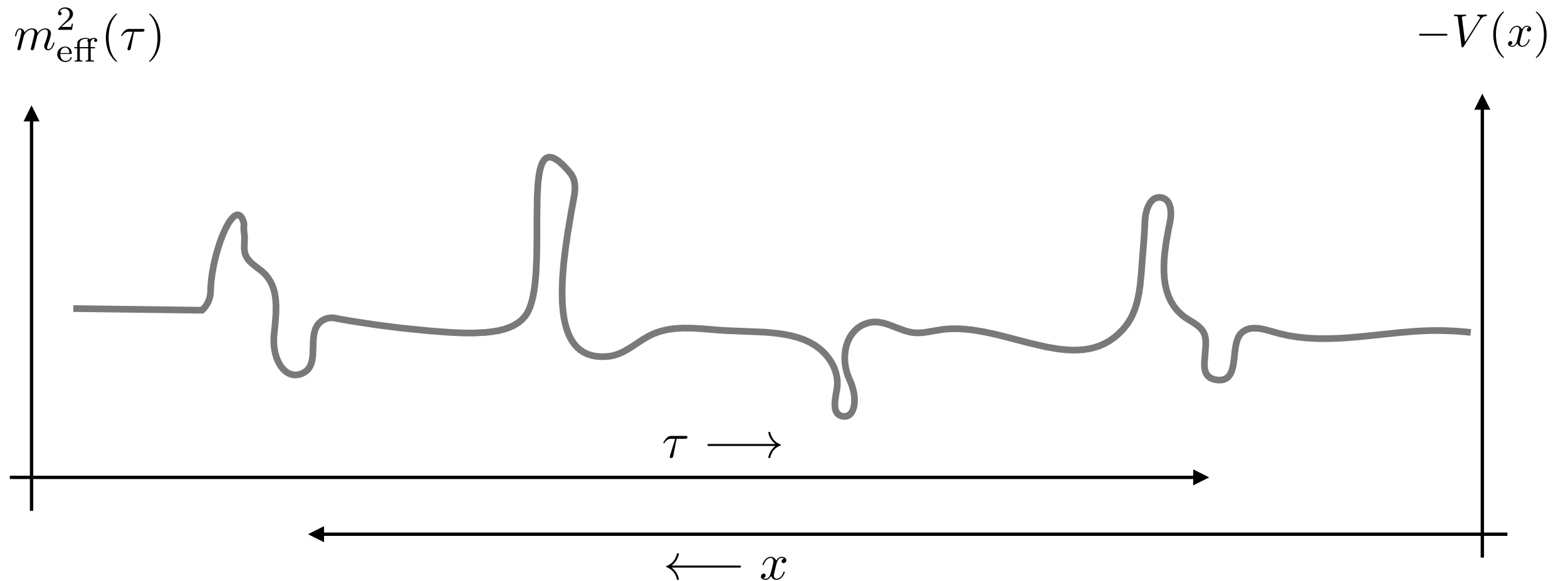
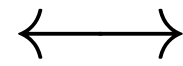
complexity in space  
wires

particle production

$$\ddot{\chi}_k(\tau) + [k^2 + m_{\text{eff}}^2(\tau)] \chi_k(\tau) = 0$$

Schrodinger

$$\psi''(x) + [k^2 - V(x)] \psi(x) = 0$$



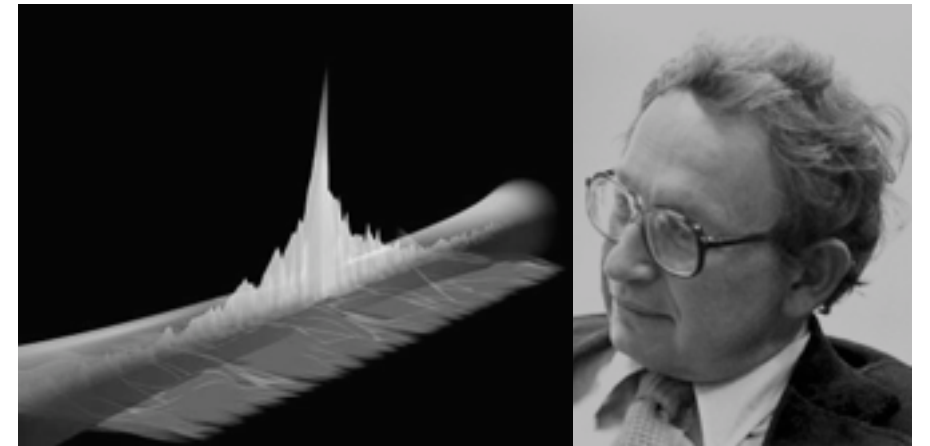
simplified version!

# Anderson localization!

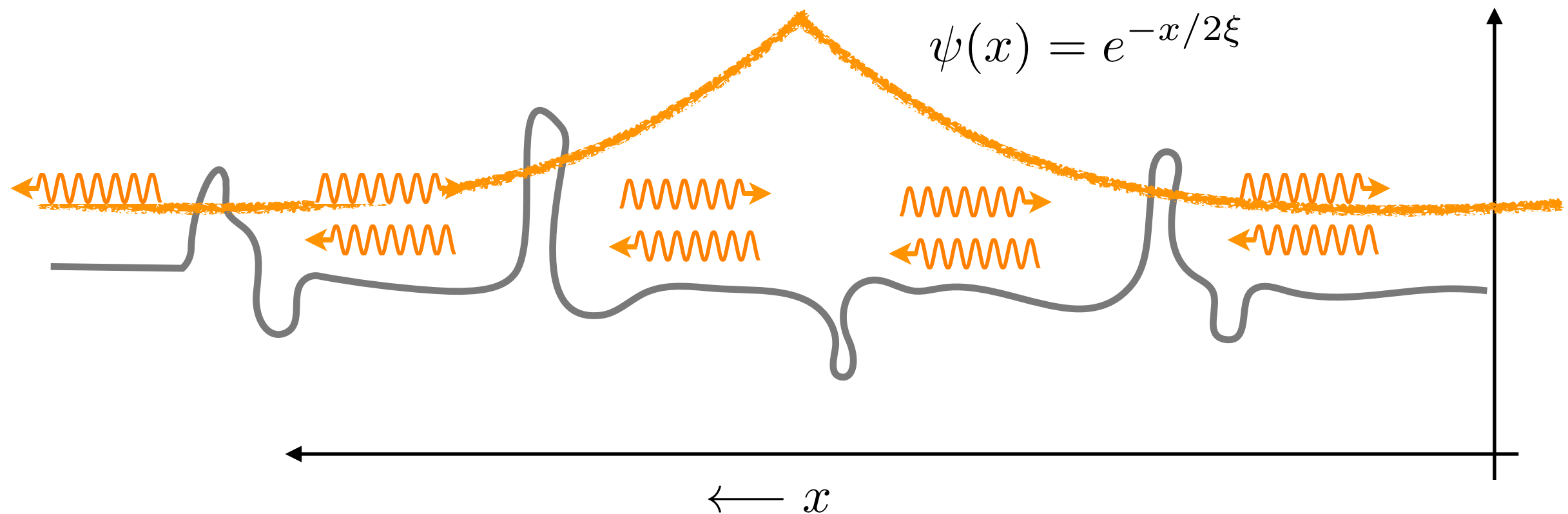
## complexity in space — emergent simplicity

$$\psi''(x) + [k^2 - V(x)] \psi(x) = 0$$

at low temperatures, one dimensional wires are insulators



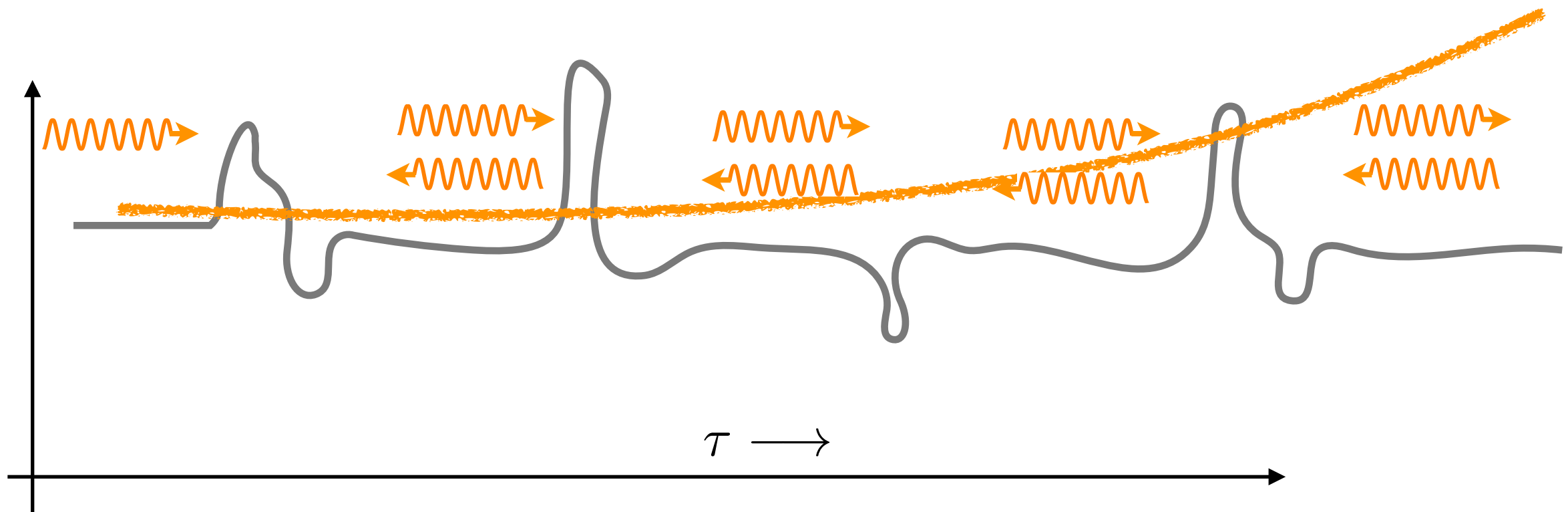
Anderson 1957



# complexity in time — exponential particle production

$$\ddot{\chi}_k(\tau) + [k^2 + m_{\text{eff}}^2(\tau)] \chi_k(\tau) = 0$$

$$\chi_k(\tau) \sim e^{\mu_k \tau / 2}$$



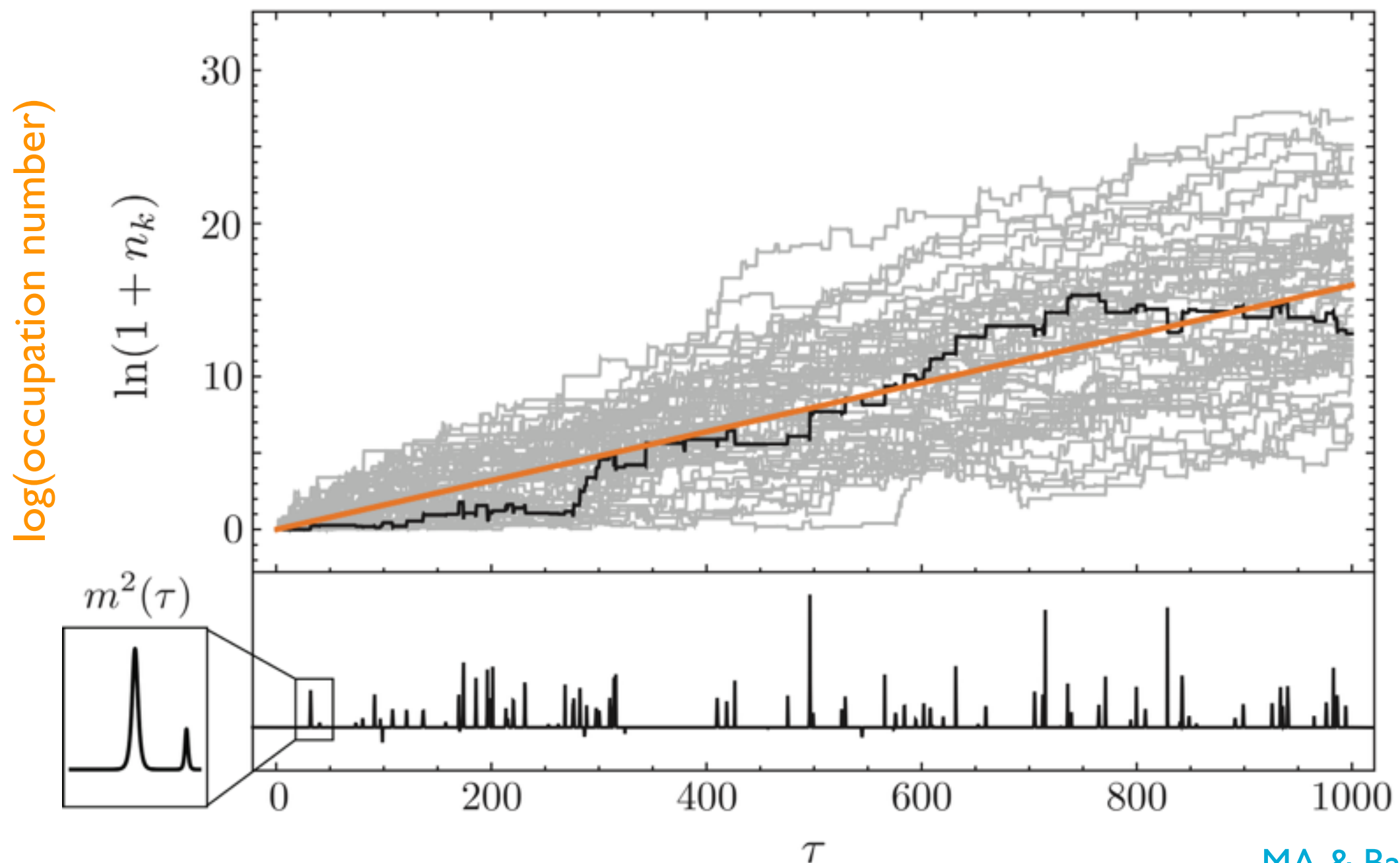
occupation number's  
“Brownian” motion





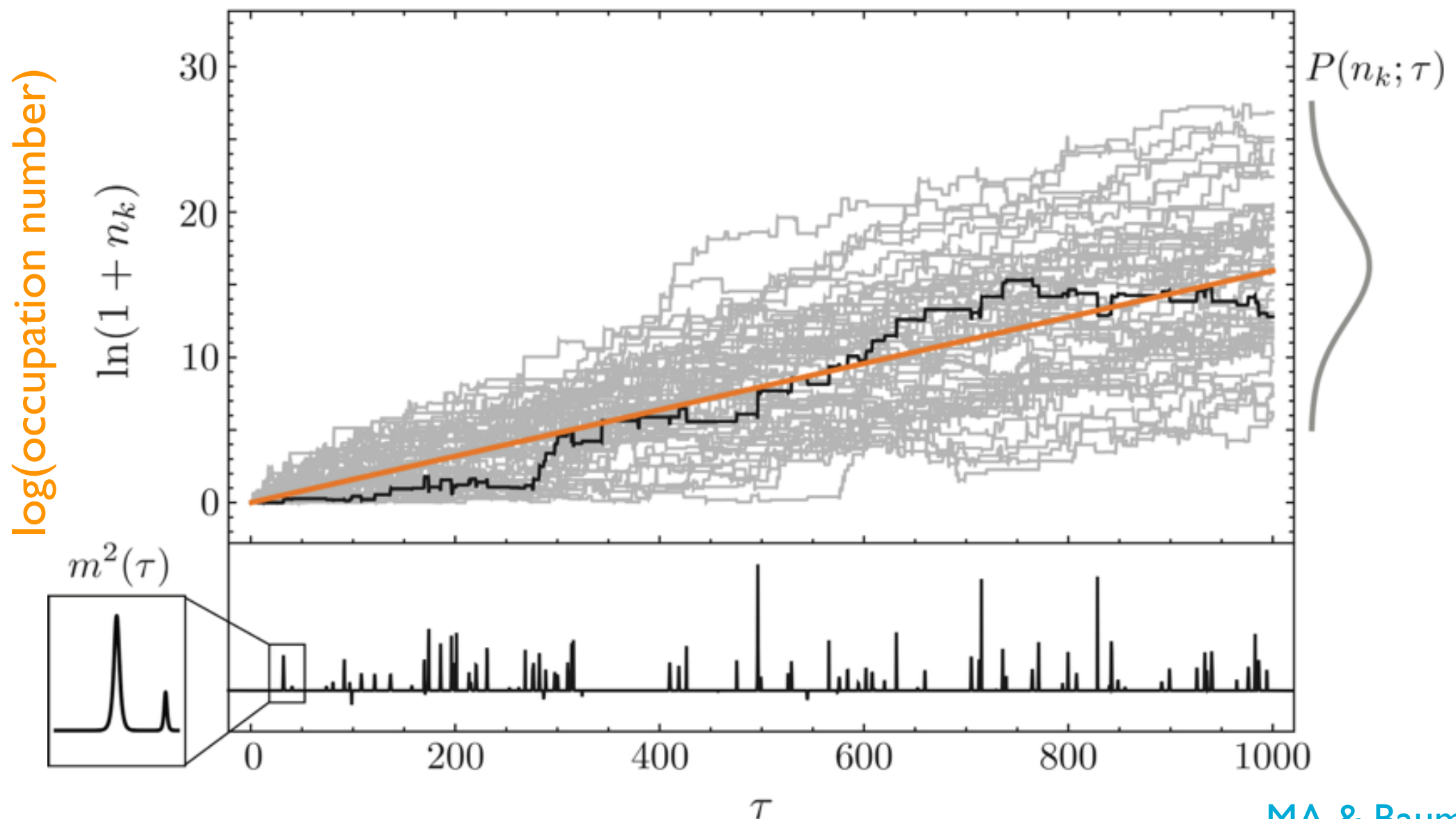
# occupation number performs a drifted random walk

$$n(k, \tau) = \frac{1}{2\omega_k} (|\dot{\chi}_k|^2 + \omega_k^2 |\chi_k|^2)$$



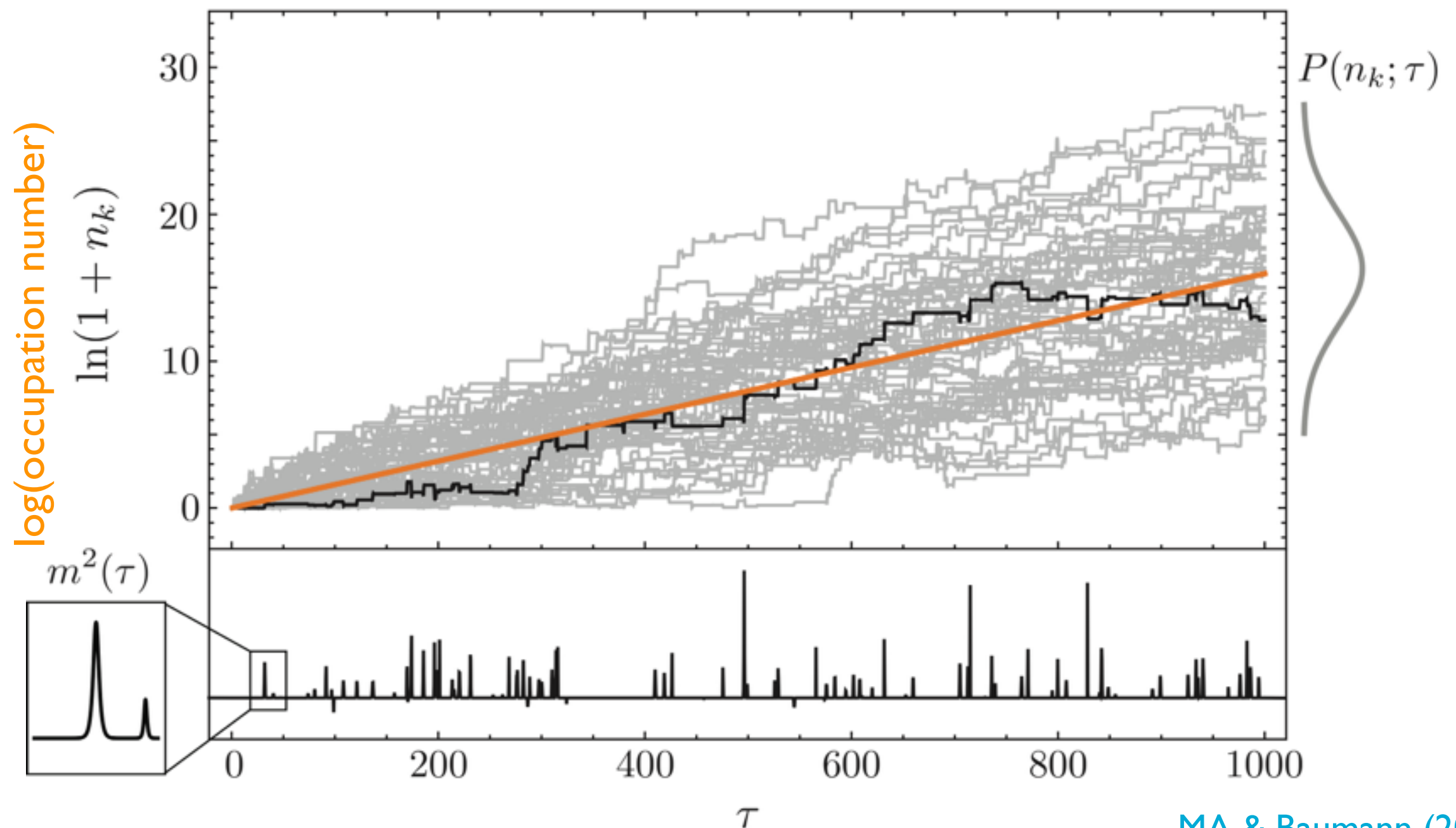
# a Fokker Planck equation

$$\frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n, \tau) = \frac{\partial}{\partial n} \left[ n(1+n) \frac{\partial}{\partial n} P(n, \tau) \right]$$

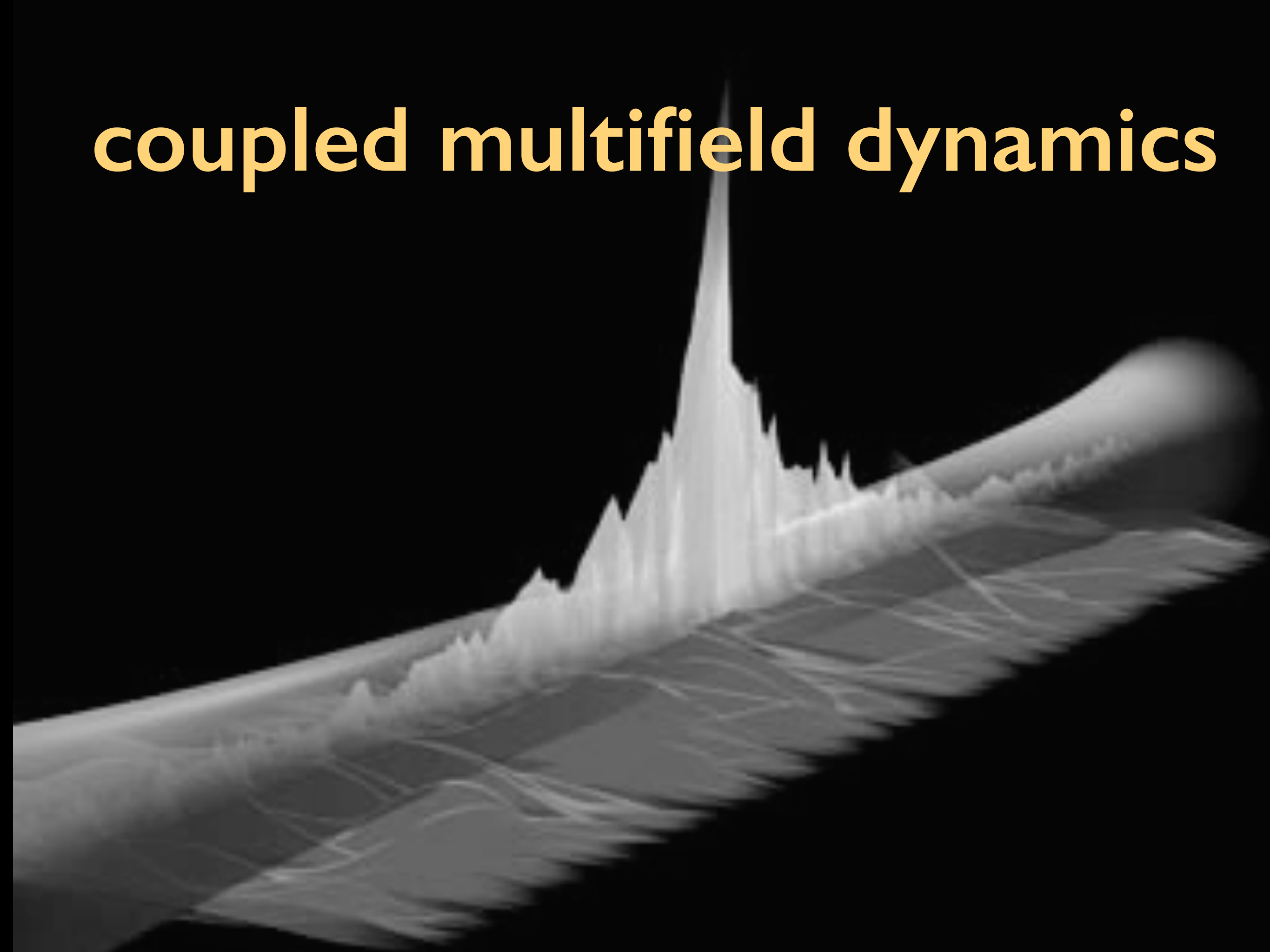


# the typical occupation number

$$n_{\text{typ}} \equiv \exp \langle \ln(1 + n) \rangle = e^{\mu_k \tau}$$



# coupled multifield dynamics





# many interacting fields (thick wires)

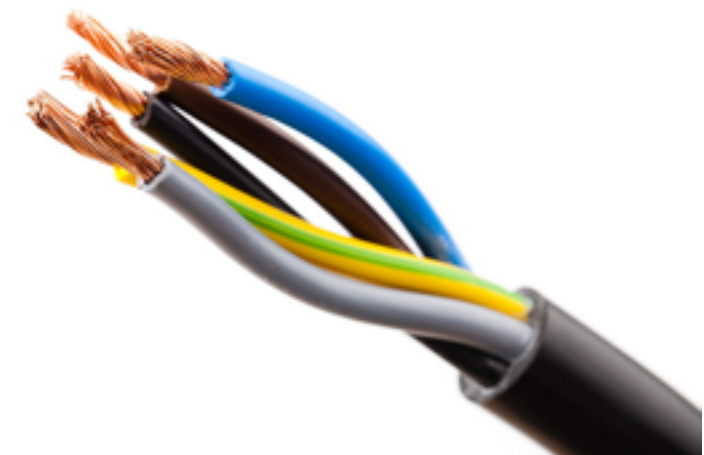
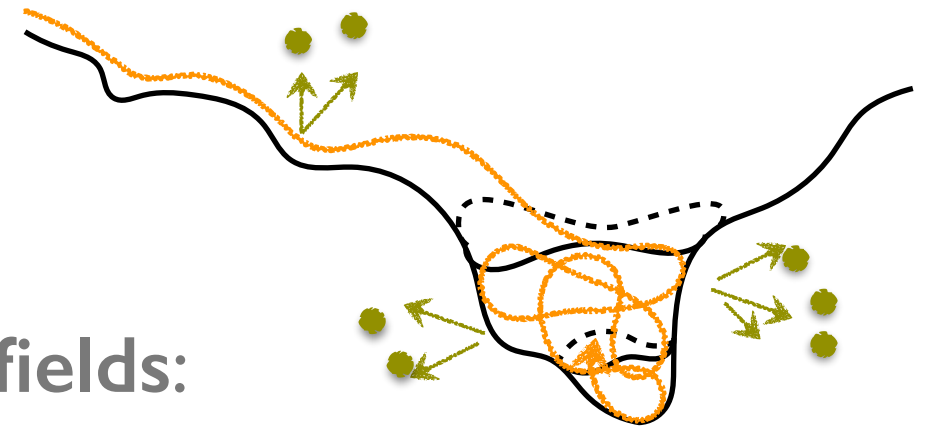
early universe: multiple interacting fields:

$$\ddot{\chi}_a + \left[ k^2 \delta_a^b + \mathcal{M}_a^b(\tau) \right] \chi_b = 0$$

$$a, b = 1, \dots, N_f$$



real wires are not one-dimensional.  
current conduction: **multiple channels**.



# multifield Fokker Planck equation

joint probability for occupation numbers satisfies the **DMPK** equation:

$$\begin{aligned} \frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n_a; \tau) = & \sum_{a=1}^{N_f} \left[ (1 + 2n_a) + \frac{1}{N_f + 1} \sum_{b \neq a} \frac{n_a + n_b + 2n_a n_b}{n_a - n_b} \right] \frac{\partial P}{\partial n_a} \\ & + \frac{2}{N_f + 1} \sum_{a=1}^{N_f} n_a (1 + n_a) \frac{\partial^2 P}{\partial n_a^2} \end{aligned}$$

Dokhorov, Mello, Pereyra & Kumar

local mean particle production rate

$$\mu_k \equiv \frac{1}{N_f} \lim_{\delta \tau \rightarrow 0} \frac{\langle n \rangle}{\delta \tau} \quad \text{where} \quad n = \sum_{a=1}^{N_f} n_a$$

# numerical tests

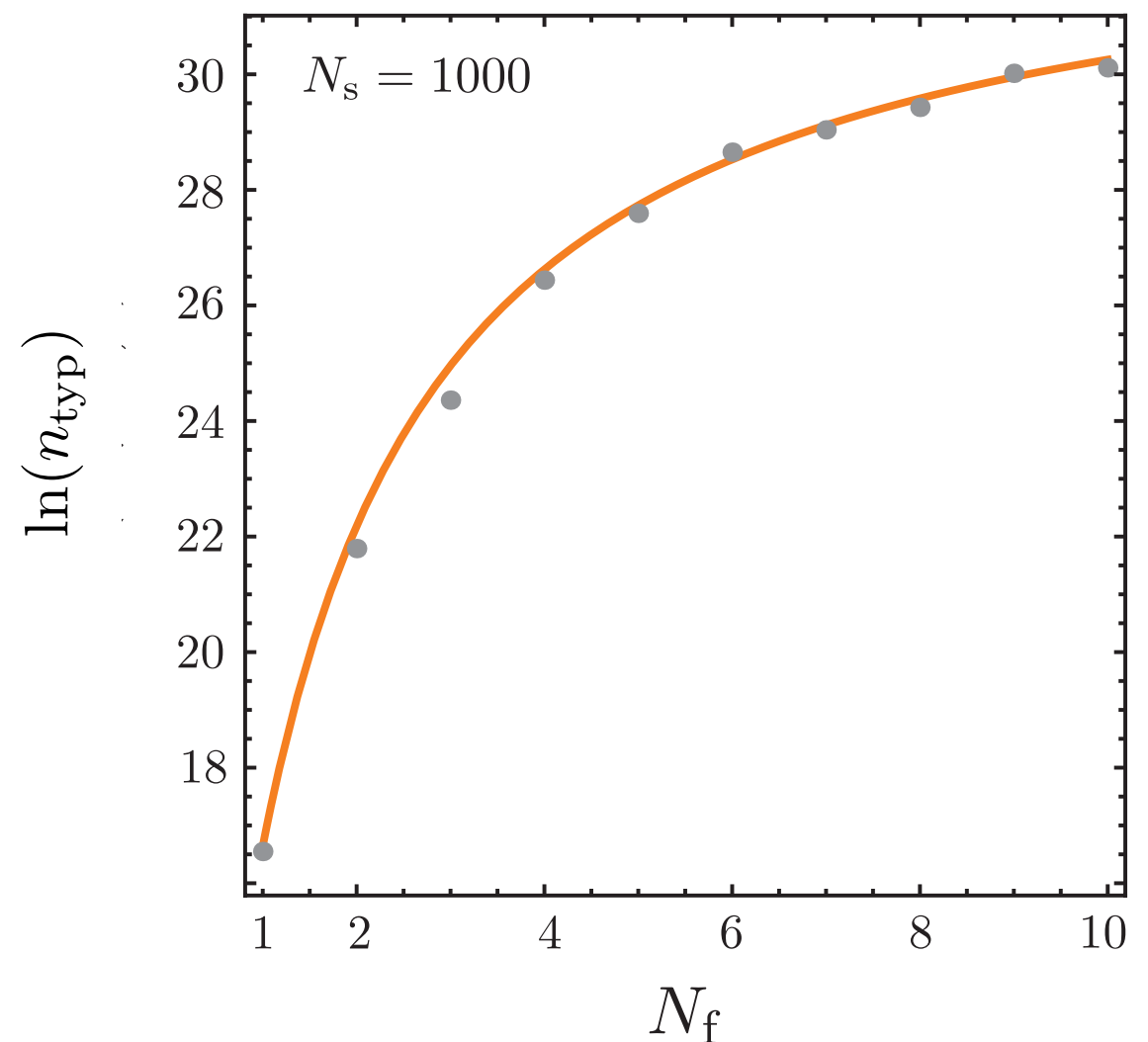
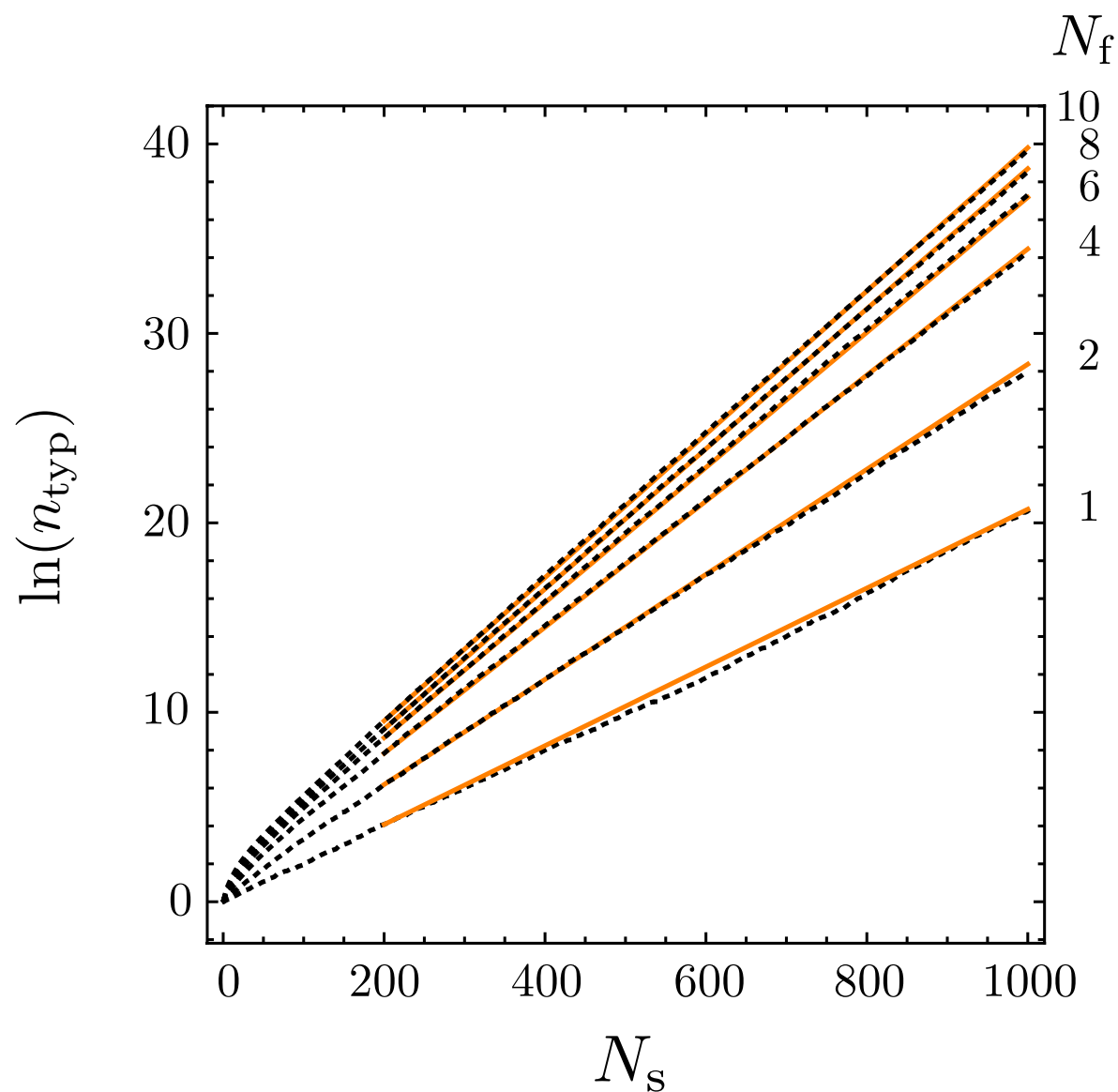
## [typical occupation numbers]

$$\ln(n_{\text{typ}}) \propto \frac{2N_f}{1 + N_f} N_s$$

where

$N_f$  = number of fields

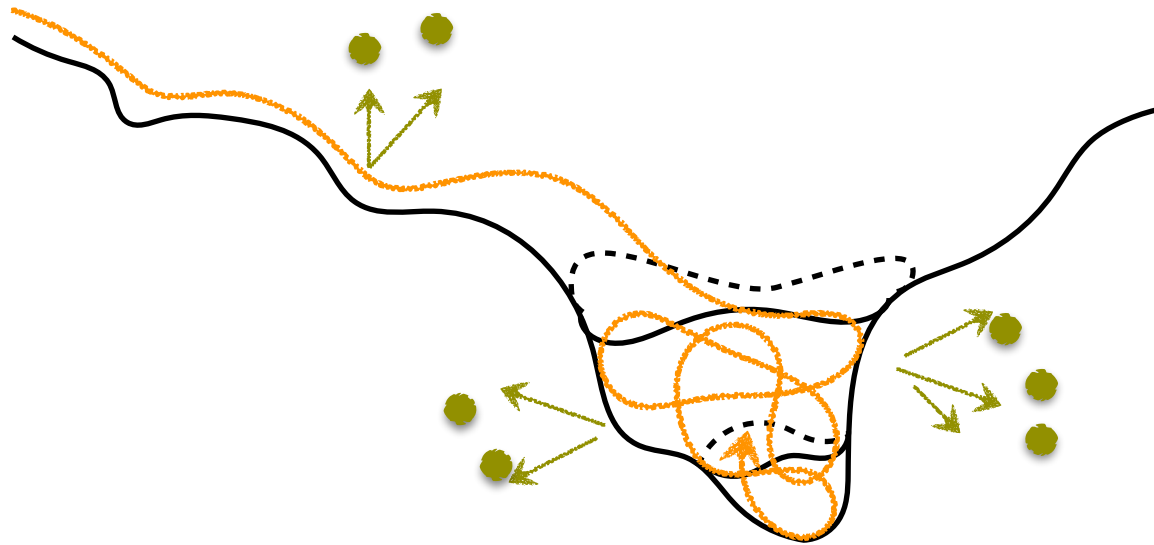
$N_s$  = number of scatterings



# simplicity/universality

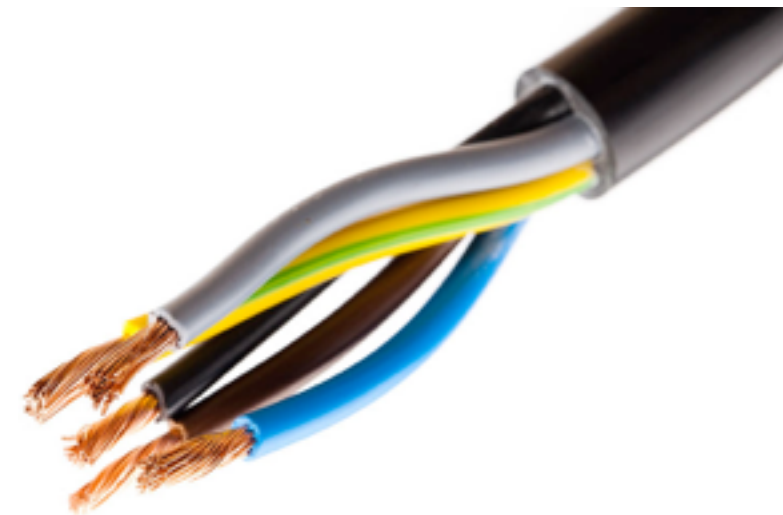
$\mu_k$  local mean particle  
production rate

$N_f$  number of fields



$l_{mf}$  mean ballistic mean  
free path

$N_f$  number of channels



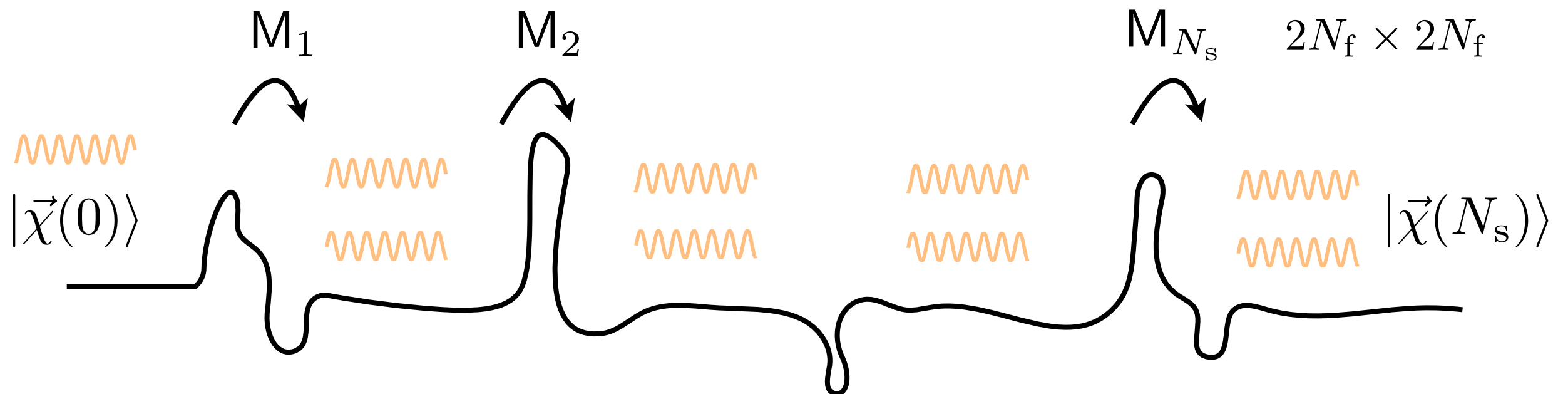
$\mu_k$  - calculate from 'local' microphysics or parametrize

$N_f$  - regimes exist where dependence vanishes



# multifield particle production as scattering

$$|\vec{\chi}(N_s)\rangle = M |\vec{\chi}(0)\rangle \quad \text{where} \quad M \equiv M_{N_s} \cdots M_2 M_1$$



total occupation number

$$n = \text{Tr}(n) = \sum_{a=1}^{N_f} n_a \quad \text{where} \quad n \sim MM^\dagger$$

↓  
particles in each “field” (eigenvalues)

# Universality from Random Matrix Theory

two large N's to make life easier:

- large number of fields:

$N_f$



- eigenvalue spectrum of  $M_j$

from RMT:

- large number of scatterings:

$N_s$



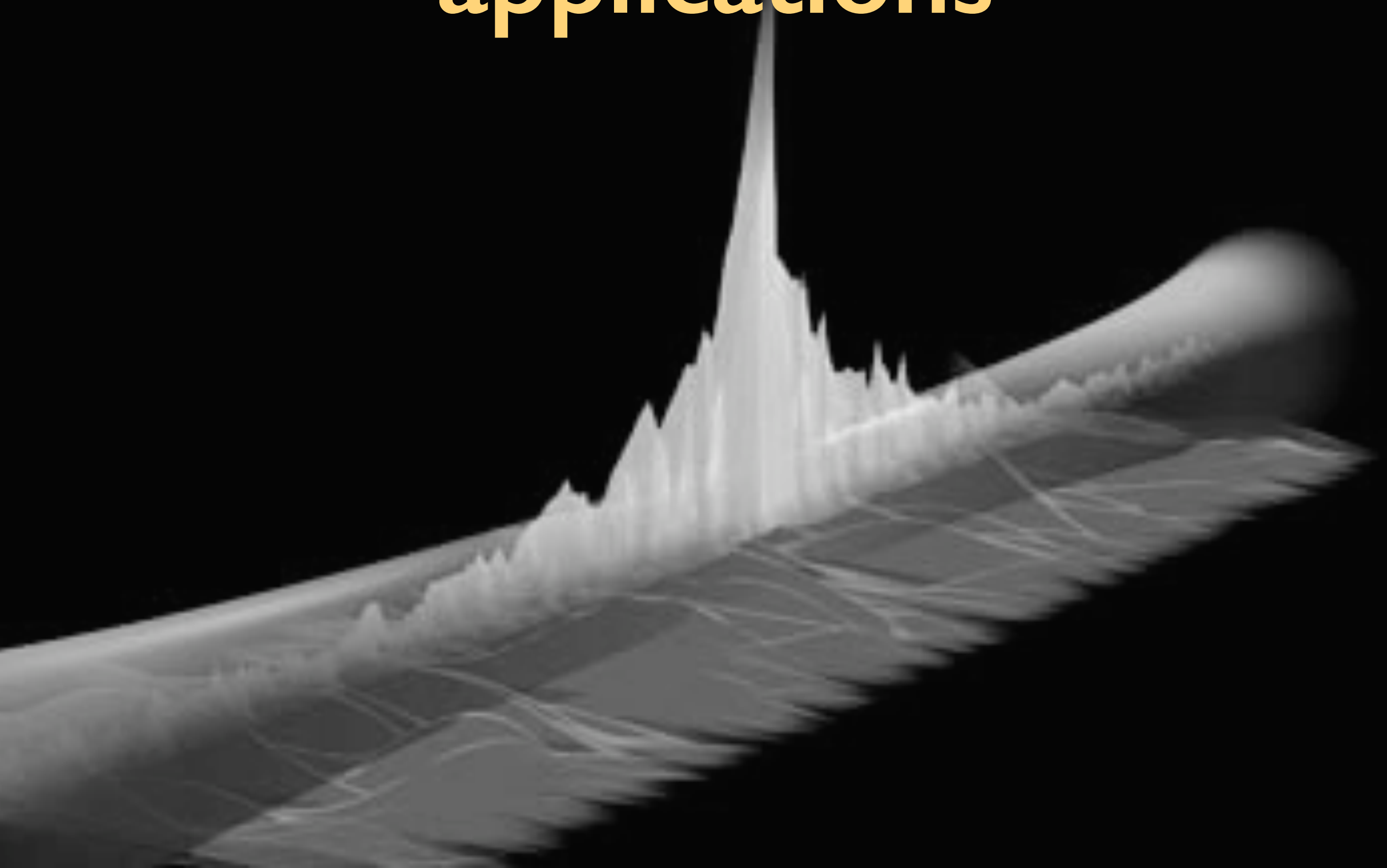
- non-random limit of  $M = \prod_{j=1}^{N_s} M_j$

Pichard and Sarma

prediction for exponential behavior in time

non-random behavior of the exponent

# applications



WORK IN  
PROGRESS

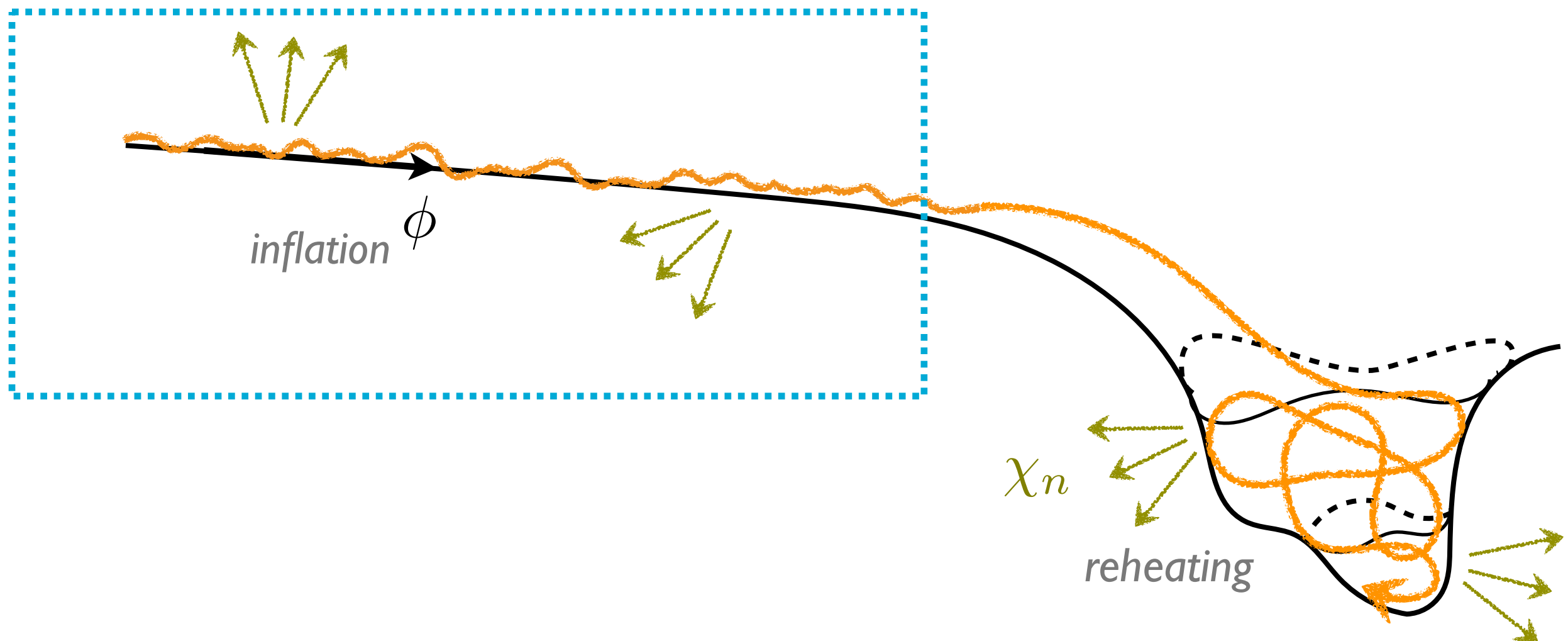
# applications: inflation

MA, Baumann, Carlsten & Green (in progress)

background dynamics  $\rightarrow$  particle production  $\leftrightarrow$  curvature fluctuations

$$\langle n_{k_1} n_{k_2} \dots \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \dots \rangle$$





WORK IN  
PROGRESS

# combine particle production & EFT with driving and dissipation

Nacir, Porto, Senatore, and Zaldarriaga  
Green, Horn, Senatore, and Silverstein

$$\mathcal{L} = \mathcal{L}_{\text{sr}} - m^2(t + \pi) \chi^2$$



Goldstone boson  $\zeta = -H\pi$

$$\left( \partial_t^2 + 3H\partial_t + \frac{k^2}{a^2} \right) \pi = \frac{dm^2}{dt} \chi^2$$

source

stochastic noise

$$(\chi^2)_S \equiv \langle \chi^2 \rangle_{\pi=0}$$

linear response

$$(\chi^2)_R \equiv \int^t dt' G_{\text{ret}}^{\langle \chi^2 \rangle}(t, t') \pi(t')$$

background dynamics



particle production



$$\langle n_{k_1} n_{k_2} \dots \rangle$$

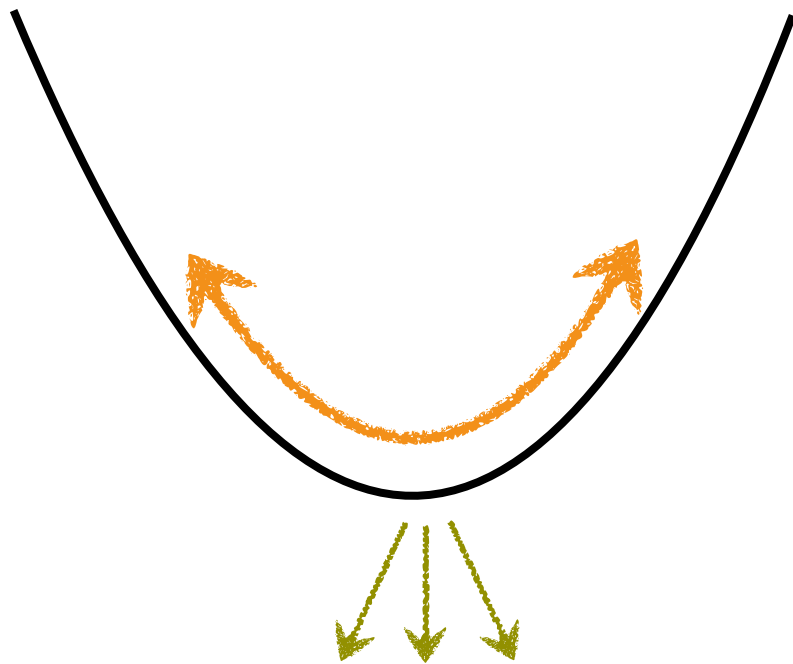
curvature fluctuations

$$\langle \zeta_{k_1} \zeta_{k_2} \dots \rangle$$

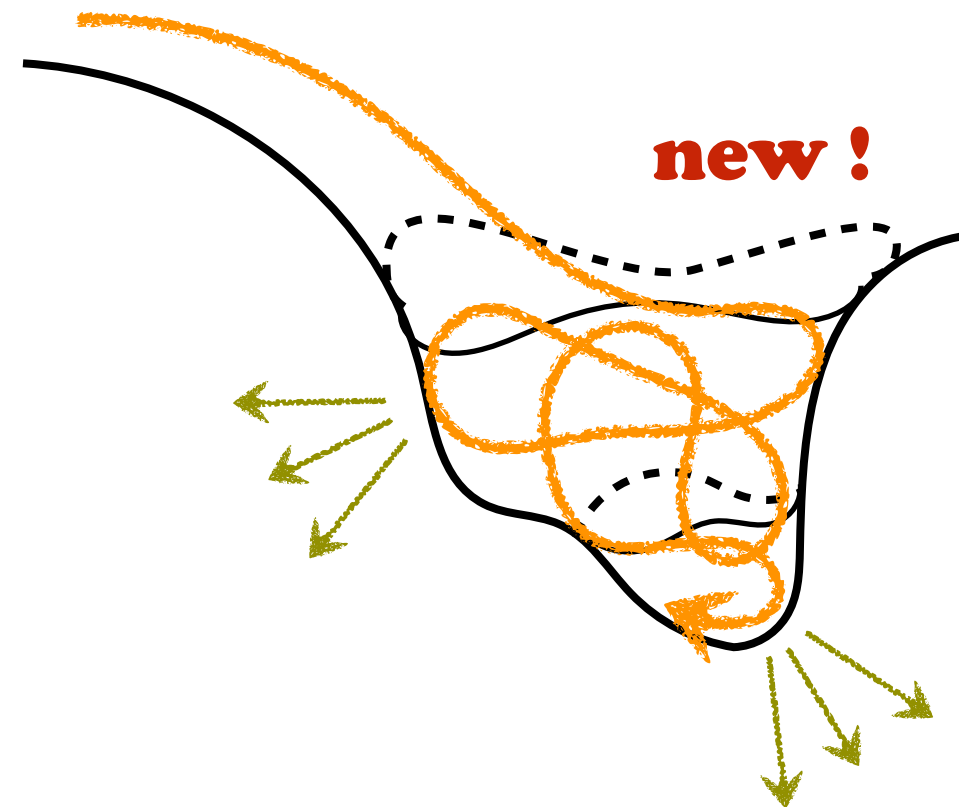
MA, Baumann, Carlsten & Green  
(in progress)

**WORK IN  
PROGRESS**

# applications : reheating



Kofman, Linde & Starobinsky (1994, 97)  
Traschen & Brandenberger (1995)  
Zanchin et. al + Bassett (1998) [noise added]



model-insensitive description of a  
complicated reheating process.

# related work:

## condensed matter + cosmology

Anderson

*Absence of diffusion in certain random lattices*  
(1957)

Mello, Pereyra Kumar

*Macroscopic approach to multichannel disordered wires*  
(1987)

C. Beenakker,

*Random matrix theory of quantum transport*  
(1997)

C. Muller and D. Delande,

*Disorder and interference: localization phenomena*  
(2010)

Kofman, Linde & Starobinsky  
(1994, 1997)

Traschen and Brandenberger  
(1997)

Bassett  
(1998)

Zanchin, Maia, Craig & Brandenberger  
(1998)

Nacir, Porto, Senatore and Zaldarriaga  
(2012)

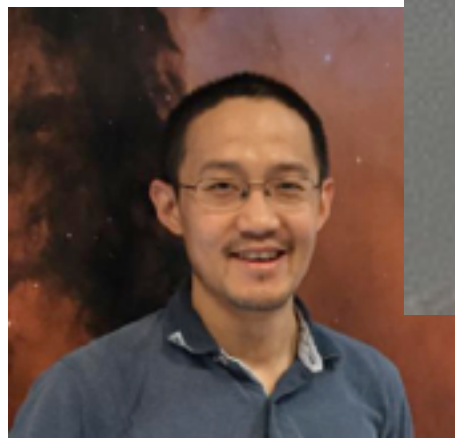
Marsh, McAllister, Pajer, Wrase  
(2013)

Green  
(2015)

Dias, Fraser & Marsh  
(2016)

+ many works on particle production during and after inflation.

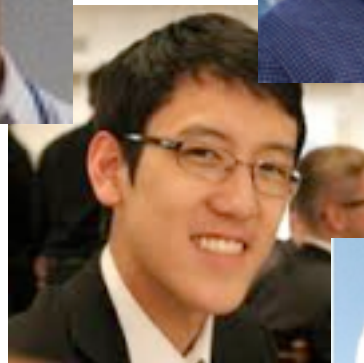
# diverse collaboration(s) for a diverse problem



H. Xie  
(condensed matter)



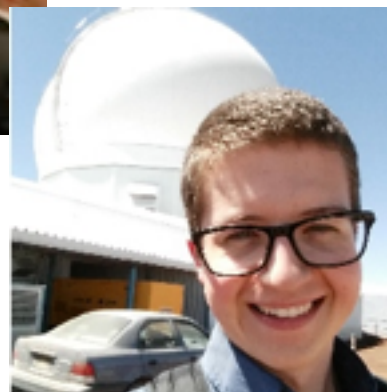
J. Shen



O. Wen




M. Garcia



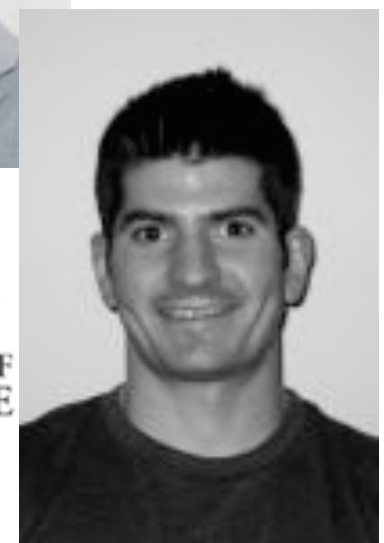
S. Carlsten



D. Baumann

 UNIVERSITY OF AMSTERDAM

 UNIVERSITY OF  
CAMBRIDGE



D. Green

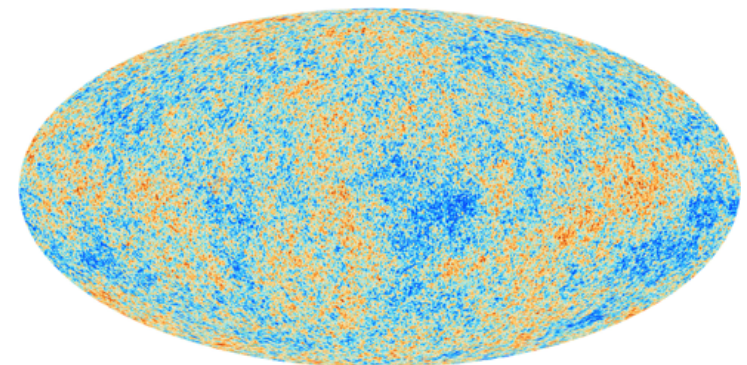
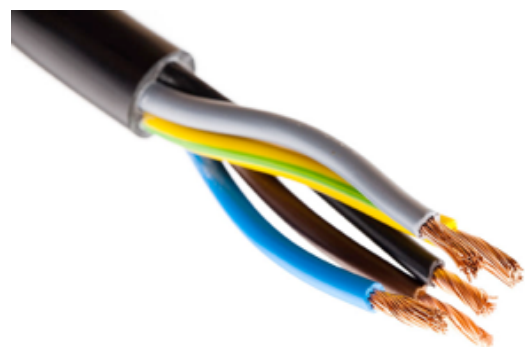
  
UNIVERSITY OF CALIFORNIA





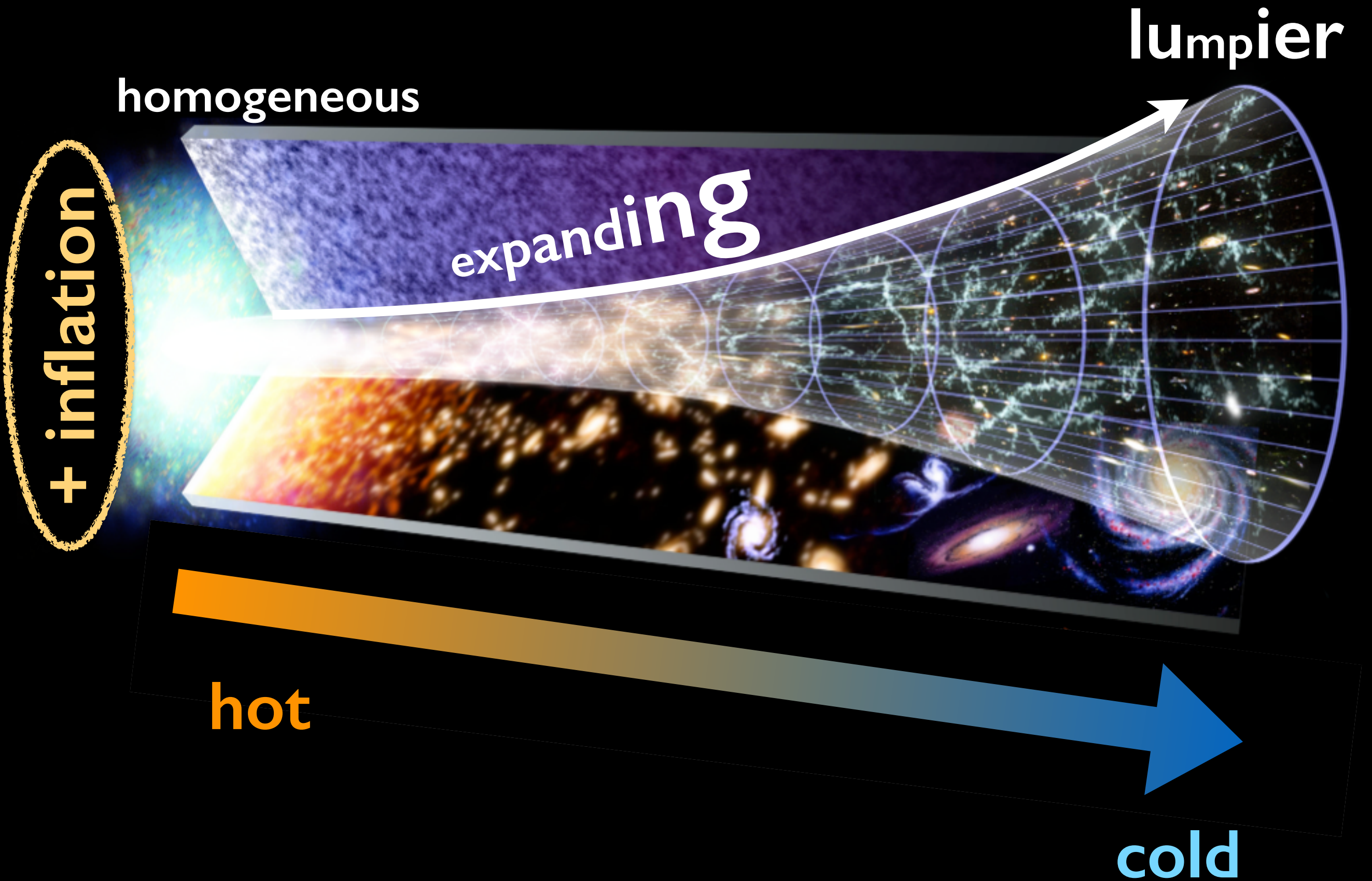
# complex enough models: summary

- statistical tool for theoretical complexity
- simplicity & hints of universality
- *observed simplicity* in spite of underlying complexity ?



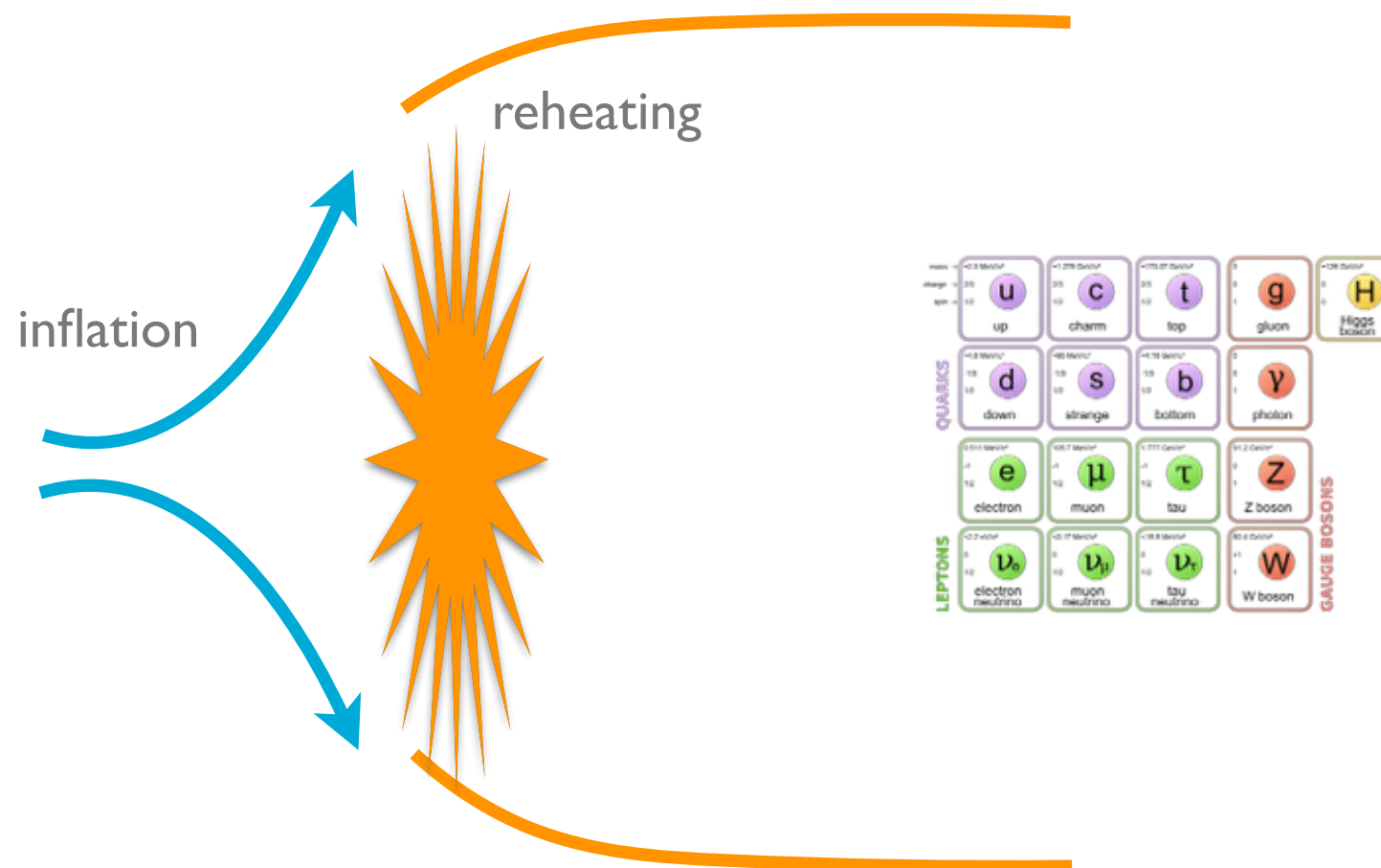
**ORIGINS ?**

# Inflationary Cosmology



# outstanding theory questions

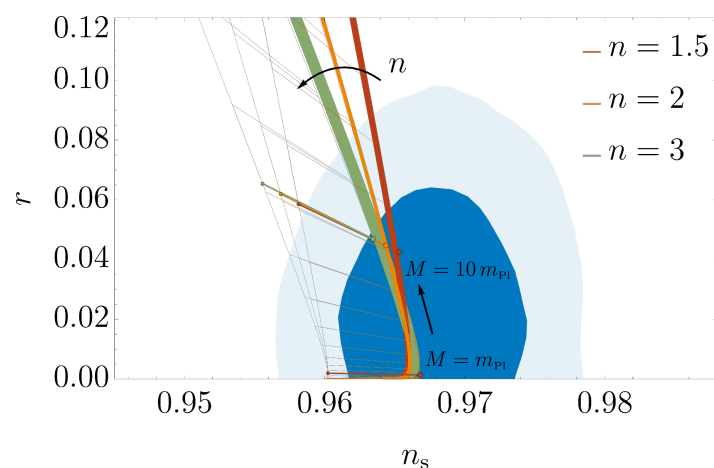
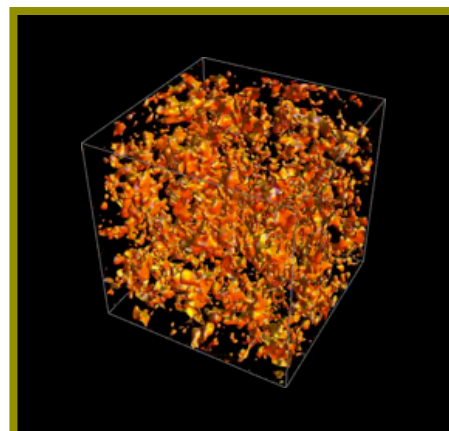
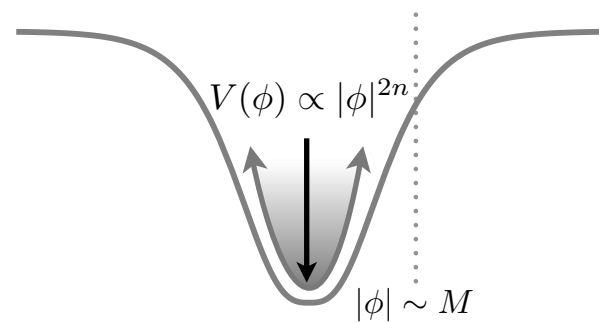
- what is the physics of **inflation** ?
- how did the universe get populated with particles after inflation ? (**reheating**)
- connecting with the standard model?





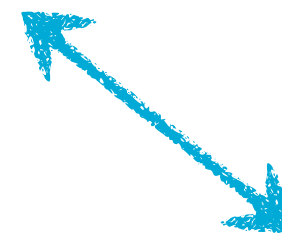
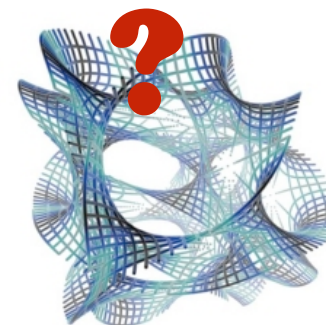
# our origins: two approaches

**SIMPLE enough**

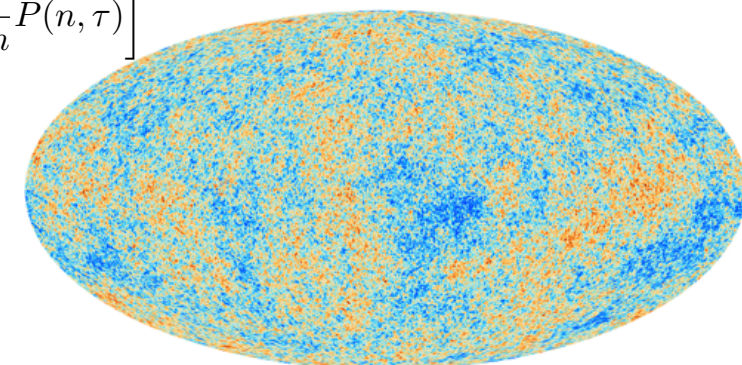


$$\Delta N_{\text{rad}} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\text{Pl}}, \\ \frac{n+1}{3} \ln \left( \frac{\kappa}{\Delta \kappa} \frac{10M}{m_{\text{Pl}}} \right) & M \gtrsim 10^{-2} m_{\text{Pl}}. \end{cases}$$

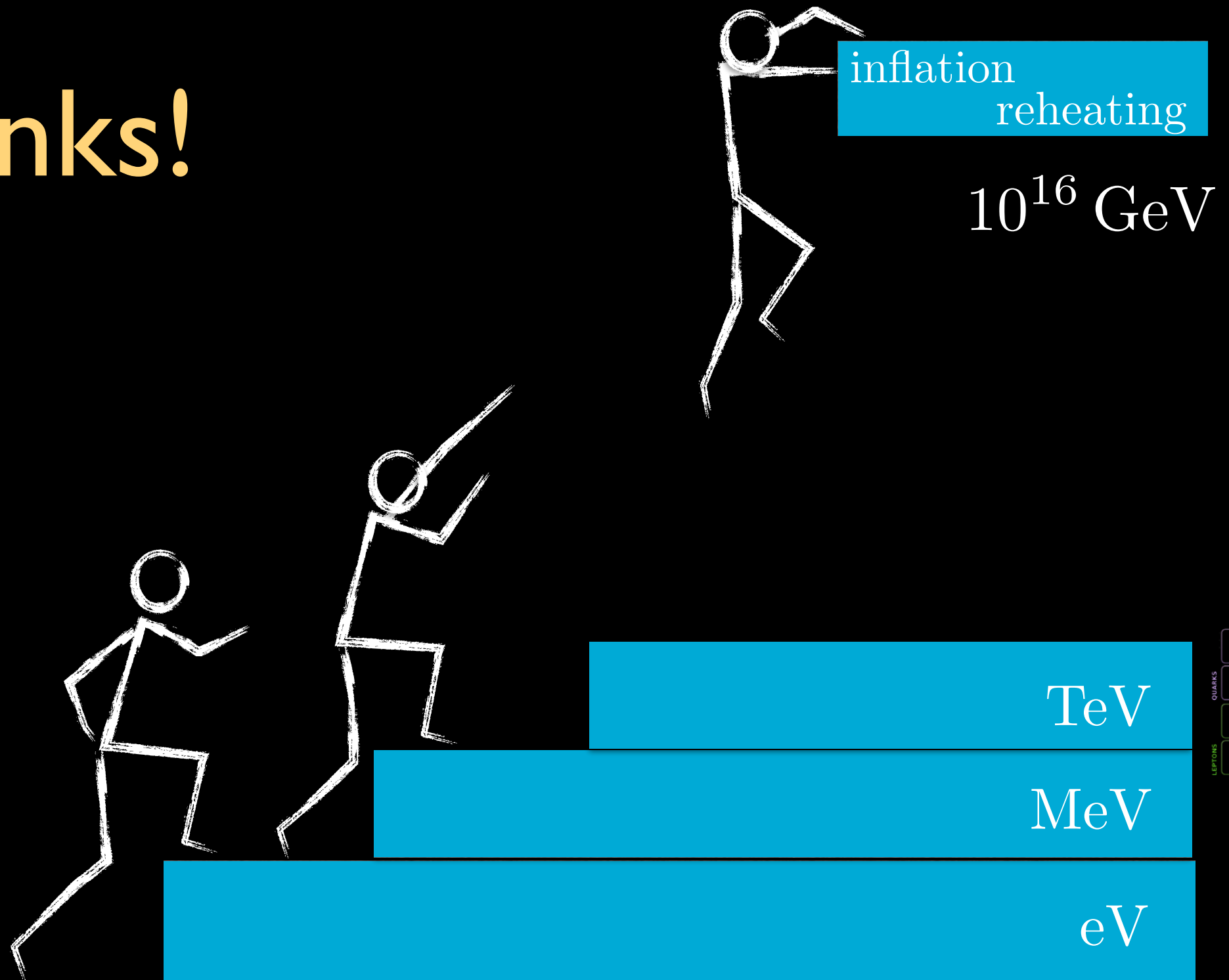
**COMPLEX enough**



$$\frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n, \tau) = \frac{\partial}{\partial n} \left[ n(1+n) \frac{\partial}{\partial n} P(n, \tau) \right]$$



# Thanks!



QUARKS	u	c	t	g	H
	d	s	b	γ	
	e	μ	τ	Z	
LEPTONS	ν <sub>e</sub>	ν <sub>μ</sub>	ν <sub>τ</sub>	W	
GAUGE BOSONS					